

# Optimal dimensioning model of water distribution systems

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## Abstract

This study is aimed at developing a pipe-sizing model for a water distribution system. The optimal solution minimises the system's total cost, which comprises the hydraulic network capital cost, plus the capitalised cost of pumping energy. The developed model, called Lenhsnet, may also be used for economical design when expanding existing hydraulic networks. The methodology developed includes an iterative dynamic calculation process as well as a hydraulic simulation model. The performance of the method is tested against 4 benchmark examples in the literature. The results obtained show the feasibility of this model, presenting it as a viable alternative for water distribution systems. The method is easily used, once it is performed under EPANET2 software interface.

**Keywords:** pipe-sizing model, economic optimisation, energy

## Notation

$C$	Hazen-Williams coefficient
$C(D_i)$	cost per unit length of the $i^{\text{th}}$ pipeline with diameter $D_i$
$C(D)$	hydraulic network cost
$C(D, Q, H)$	total distribution system cost
$C_1$	initial cost, related to initial diameter
$C_2$	cost of the diameter which is immediately larger than the diameter related to $C_1$
$D$	internal diameter of the pipe
$e$	annual rate of increase in the unit cost of energy
$Ec$	unit cost of electrical energy
$Eg$	energy cost gradient
$H$	pumping head
$H_L$	head loss
$j$	annual interest or discount rate
$L$	pipeline length
$n$	number of pipeline
$Np$	number of annual pumping hours
$Pg$	cost gradient value
$Pg^*$	optimum value of the cost gradient
$PWF$	present worth factor
$Q$	pipe flow rate
$s^*$	potential section
$t$	expected period of service for the network
$\alpha$	co-efficient of Hazen-Williams equation
$\Delta p$	pressure gain in the most unfavourable node
$\eta$	pump-motor unit efficiency

## Introduction

Water distribution systems (WDS) design optimisation have been receiving special attention from engineers and researchers of water resources and other related areas, due to the high implementation and operational costs of such systems.

Generally, the variables that determine WDS conception and expansion project optimisation are the hydraulic network pipe-line diameters and pumping head.

WDS dimensioning is mathematically undetermined, thus allowing for innumerable solutions. Throughout history, several dimensioning methods have been proposed. The first of them, namely classical ones, were restricted to choosing network diameters that provided the hydraulic balance of system. However, the scientific community has been looking to minimise WDS cost for decades.

In the late 1960s, the consolidation of micro-computing in global research centres enabled the development of techniques focusing on water network dimensioning, therefore the first optimisation systems appeared based on mathematical linear programming models (LP) (Karmeli et al., 1968), as well as nonlinear programming (NLP) (Jacoby, 1968) and dynamic programming (DP) (Liang, 1971).

More recently, models based on genetic algorithms (GA) (Dandy et al., 1996), which are known as evolutionary algorithms (EA), have come into use. Some researchers have been using methods based on the organisation and/or evolution of other living species. Eusuff and Lansey (2003) proposed shuffled frog leaping algorithms (SFLA), a meta-heuristic algorithm based on the transformation of frogs and information exchange among the population. Maier et al. (2003) and Zecchin et al. (2006) used a new technique called ant colony optimisation (ACO) to WDS optimisation, based on the analogy of the foraging behaviour of a colony of searching ants, and their ability to determine the shortest route between their nest and a food source. Suribabu and Neelakantan (2006) and Montalvo et al. (2008) applied the particle swarm optimisation (PSO) algorithm. PSO is an EA which utilises swarm intelligence to achieve the goal of optimising a specified objective function. This algorithm uses the cognition of individuals and social behaviour in the optimisation process.

Numerous algorithms have been tested on distribution systems by researchers to obtain the most reliable solutions, using the shortest possible computational time (Biscos et al., 2003). EA methods have presented good results, but they require much more computer time. Cui and Kuczera (2003) highlight the problem of long computation times of some models and

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propose that such analyses be done using super-computers or by parallel computation. This makes their application by technicians difficult.

Abebe and Solomatine (1998) implemented global optimisation algorithms; 2 algorithms, adaptive cluster covering with local search (ACCOL) and GA, yielded promising solutions enabling a choice between accuracy and required computer time. The proposed optimisation set-up can handle any type of loading condition and neither makes any restriction on the type of hydraulic components in the network nor does it need analytical cost functions for the pipes. Liong and Atiquzzaman (2004) applied a powerful optimisation algorithm, shuffled complex evolution (SCE), in order to find solutions with low processing time. SCE deals with a set of population points and searches in all directions within the feasible space based on objective function. Gomes and Bezerra (2007) and Gomes et al. (2008) proposed an iterative method with a relatively short processing time for the optimisation of the total costs for the expansion and rehabilitation of WDS.

However, in spite of considerable developments as detailed in literature, these techniques have not been accepted in practice (Savic, 2002). Dimensioning of new networks and trial-and-error analyses for extensions are frequent. Jimenez et al. (2007) states that optimisation of water networks is not a rule in engineering yet and programs with a user-friendly interface have only just begun to appear.

This study aims at presenting an optimisation model called Lenhsnet, which is designed to obtain an optimal solution of WDS and provide a friendly interface for engineers. This model, which is connected to EPANET 2.00.12 (Rossman, 2008), provides network pipeline diameters and pumping head as a response to dimensioning so as to find the total minimum cost of the system (implementation cost plus energy cost).

## Methodology

The methodology includes an iteration process, based on an initial solution, in which the distribution network is designed, according to minimum accepted diameters in the project. Such an initial solution has the minimum cost of network implementation, once it is made up by the minimum diameters. However, such a solution is usually not a feasible option, since it provides excessive head losses in network pipelines, resulting in high pumping heads.

Based on the initial solution, the calculation process develops iteratively, in a way in which each consequent solution depends on the previous one. The following solutions will be obtained by increasing, in each iteration, the diameter of one of the pipelines, in a way which will keep the additional cost as low as possible. The iterative process finishes when the configurations of network diameters comply with the restrictions imposed by the project (maximum velocity in pipes and the minimum pressure at nodes).

The algorithm method is associated with a hydraulic simulator, which will provide, at each iteration, the hydraulic balance of the system and the values of the variables of the outflow status of the network water flow (flows, velocities, head losses and pressures). EPANET2 was chosen as simulator as it is widely accepted as the world standard in hydraulic and water quality modelling of WDS. Most of the existing WDS have been modelled within EPANET2 (Biscos et al., 2003).

Once the initial solution is established, the simulation of network outflow is done in order to obtain the pressure in all nodes. Once the most unfavourable node is detected, the

optimisation process begins. In each iteration, several diameter configurations will be tested. The effective diameter change, in one iteration, will be defined according to the lowest additional network cost in relation to pressure relief given to the network. The pipeline whose change is confirmed will be the one which provides the lowest cost gradient. The cost gradient ( $Pg$ ) related to a particular pipeline is given through Eq. (1). It represents the marginal cost of the additional pressure of the most unfavourable node, brought by the change of diameter of the network pipeline by its superior adjacent.

$$Pg = \frac{(C_2 - C_1)}{\Delta p} \quad (1)$$

where:

$Pg$  is the cost gradient value (\$/m)

$C_1$  is initial cost, related to current diameter (\$)

$C_2$  is the cost of the pipe diameter which is immediately larger than the current pipe diameter (\$)

$\Delta p$  is the pressure gain in the most unfavourable node (m)

In each iteration there will be ' $i$ ' cost gradients, corresponding to ' $i$ ' configurations of diameter changes in the ' $i$ ' network pipelines; the optimum value of the cost gradient ( $Pg^*$ ) will be the lowest one among all those calculated. The pipeline corresponding to  $Pg^*$  will be called potential section ( $s^*$ ).  $s^*$  will have a new configuration, in which the diameter will be the one which is immediately higher (tested). This last configuration will be the start configuration for the following iteration. The iteration optimisation process follows the aforementioned methodology until the optimal solution objective is obtained.

Once the dimensioning solution is obtained, the next step is to check whether the velocities are within the acceptable maximum limit. If the velocity is greater than the maximum permitted, a new diameter is calculated based on the flow of the pipeline. After the diameter is defined, in case it is available, the initial solution for the model will be changed, and such part will be configured with the new diameter and the other ones with minimum diameters. Dimensioning will finish when the iterative process solution does not present any pipeline with a velocity greater than the maximum established one.

This method deals with 2 dimensioning options. In the first one, the network is supplied by a fixed piezometric level in the reservoir. In this case, the system total cost will correspond to the network implementation cost. In the second alternative, the water is directly propelled to the system or to an elevated reservoir, through pumping and the total cost of the system is calculated from the total cost of the pipeline network plus the capitalised energy cost of the pump station. In this last case, the level in the pumping head will be an extra decision variable in the optimisation process.

### Fixed piezometric level

In the first alternative, the condition to stop iterations will occur when the pressure in all of the network nodes reaches, at least, the minimum acceptable value. The total cost of the distribution system (objective function), which includes the network implementation cost, may be expressed by Eq. (2):

$$C(D) = \sum_{i=1}^n L_i \times C(D_i) \quad (2)$$

where:

$C(D)$  is the cost per unit length of the ' $i$ ' pipeline with diameter  $D_i$

$L_i$  is the length of  $i^{th}$  pipeline  
 $C(D_i)$  is the unitary price of the pipeline 'i' under diameter  $D_i$   
 $D$  is the internal diameter of the pipe  
 $n$  is the number of the pipeline

$$Eg = \frac{9,81 \times Q}{\eta} \times Ec \times Np \times PWF \quad (5)$$

### Variable pumping head

In the system dimensioning, in which the network is pressurised through a pumping station, this method takes the pumping energy cost during the project service life into account. The total cost of the distribution system (objective function) is the network implementation cost plus energy cost, Eq. (3):

$$C(D, Q, H) = \sum_{i=1}^n L_i C(D_i) + \frac{9,81 \times Q \times H}{\eta} \times Np \times Ec \times PWF \quad (3)$$

where:

$C(D, Q, H)$  is the total cost of the distribution system  
 $Q$  is the pipeline flow rate ( $m^3/s$ )  
 $H$  is the pumping head (m)  
 $\eta$  is the efficiency of the pump-motor unit  
 $Np$  is the number of the hours of pumping per annum  
 $Ec$  is the unit cost of electrical energy (unit cost/kWh)  
 $PWF$  is the present worth factor, Eq. (4)

The  $PWF$  for the project extent ( $t$  years), which does the conversion of several annual costs to a present value is provided by Eq. (4), for annual interest or discount rate  $j$  and annual rate of increase in the unit cost of energy  $e$ :

$$PWF = \left[ \frac{(1+e)^t - (1+j)^t}{(1+e) - (1+j)} \right] \times \left[ \frac{1}{(1+j)^t} \right] \quad (4)$$

The dimensioning optimisation system takes into account the energy cost through the unit called energy cost gradient ( $Eg$ ). The  $Eg$  represents the updated cost of water pressurisation per elevation meter and is provided by Eq. (5):

Similarly to the previous procedure, the iterative process is executed, following the methodology which has been previously described. At the end of each process iteration the  $Pg^*$  is compared to the calculated  $Eg$ . In case the  $Pg^*$  is lower than the  $Eg$ , the investment cost to reduce energy losses in network pipelines - and consequently to increase the pressure in the most unfavourable node - will be lower than the energy cost to increase its load in the network. The iterative process will continue increasing the diameters of the portions until the  $Pg^*$  value exceeds the  $Eg$  value. Once the latter is obtained, the pumping head is determined in a way which the minimum pressure of the system is equal to the required minimum pressure.

This methodology is synthesised in the flow chart presented in Fig. 1.

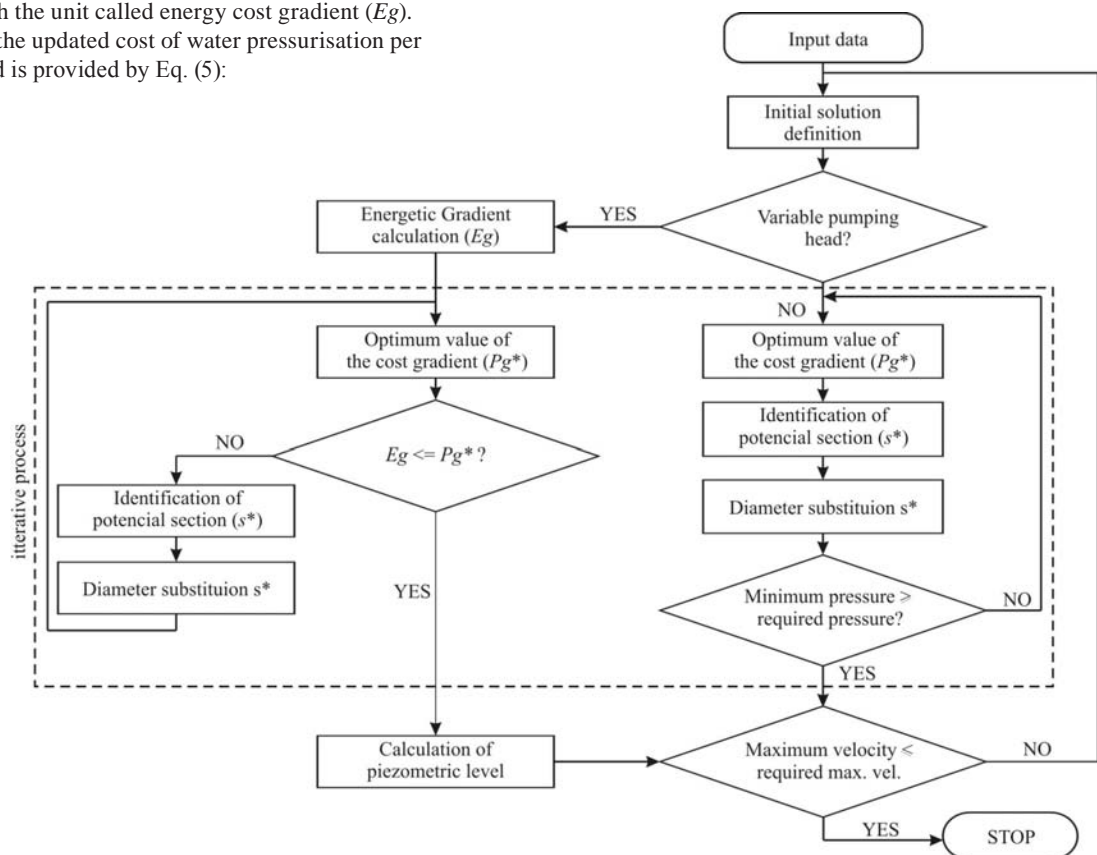
### Examples of application

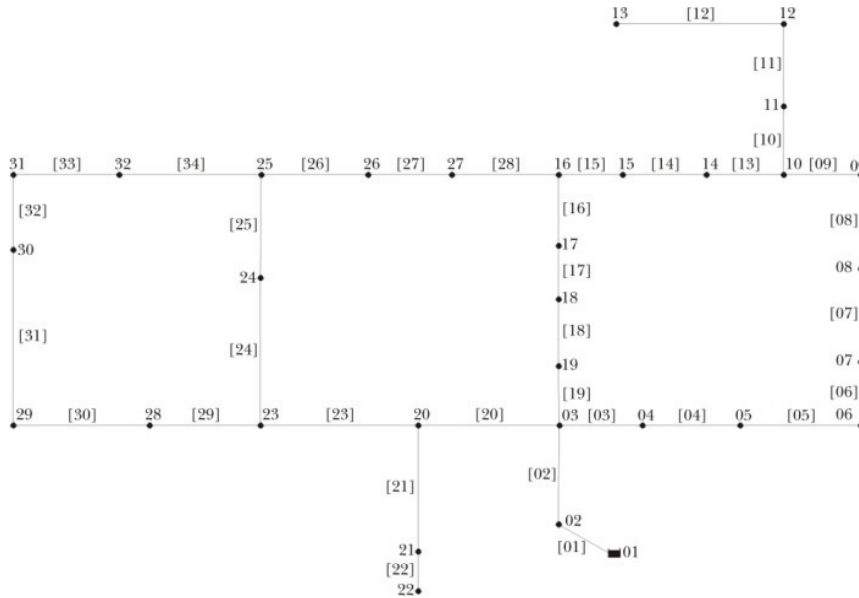
This method was applied in the optimised dimensioning of the Hanoi Network problem (Fujiwara and Khang, 1990), New York Tunnels Problem (Schaake and Lai, 1969), Bessa Network (Gomes and Formiga, 2001) and the R-9 Network (Leal, 1995).

#### Example 1: The Hanoi Network problem

The water distribution system in Hanoi (Vietnam) comprises 3 rings, 34 sections, 31 nodes and 1 fixed-level reservoir (Fig. 2, next page). This network was originally investigated by Fujiwara and Khang (1990), and was later used by several

Figure 1  
Flow chart of the methodology





**Figure 2**  
Network layout for  
the Hanoi problem

**TABLE 1**  
Network data for the Hanoi problem

Pipeline data		Node data	
Node number	Demand (m <sup>3</sup> /h)	Pipeline	Length (m)
01	-	[01]	100
02	890	[02]	1 350
03	850	[03]	900
04	130	[04]	1 150
05	725	[05]	1 450
06	1 005	[06]	450
07	1 350	[07]	850
08	550	[08]	850
09	525	[09]	800
10	525	[10]	950
11	500	[11]	1 200
12	560	[12]	3 500
13	940	[13]	800
14	615	[14]	500
15	280	[15]	550
16	310	[16]	2 730
17	865	[17]	1 750
18	1 345	[18]	800
19	60	[19]	400
20	1 275	[20]	2 200
21	930	[21]	1 500
22	485	[22]	500
23	1 045	[23]	2 650
24	820	[24]	1 230
25	170	[25]	1 300
26	900	[26]	850
27	370	[27]	300
28	290	[28]	750
29	360	[29]	1 500
30	360	[30]	2 000
31	105	[31]	1 600
32	805	[32]	150
		[33]	860
		[34]	950

authors (Cunha and Sousa, 1999; Eusuff and Lansey, 2003; Liong and Atiquzzaman, 2004; Suribabu and Neelakantan, 2006; Zecchin et al., 2006; Van Dijk, 2008). The system data are presented in Table 1 and the cost data for pipes in Table 2. The piezometric level is 100 m and the minimum network node pressure is 30 m.

To design the system by Lenhsnet 2 contour systems were considered, concerning the maximum velocity acceptable in network pipelines. In the first, only the diameters which were available through the original reference were considered (Fujiwara and Khang, 1990), which provided high velocity, which were higher than the values which are acceptable in practice. In the second situation, new commercial diameters were added to the original series (1 231.2 to 1 435.4 mm), which increased the possibility search space from 6<sup>34</sup> to 8<sup>34</sup>, and which reduced the maximum velocity to acceptable limits in practice ( $v < 3.5$  m/s). The unitary costs values of new diameters added in Table 2 were determined through the linear tendency curve of the series of original diameters.

**TABLE 2**  
Cost data for pipes

Diameter	Unit cost (\$/m)
304.8	45.73
406.4	70.40
508.0	98.38
609.6	129.30
762.0	180.80
1 016.0	278.30
1 231.2 *	375.27
1 435.4 *	477.76

\* Internal diameters of ductile iron pipes

The equation used for load loss was Hazen-Williams formula, Eq. (6), using  $\alpha$  value equal to 10.67 (default value EPANET2) and the Hazen-Williams coefficient ( $C$ ) equal to 130.

$$H_L = \frac{\alpha L Q^{1.852}}{C^{1.852} D^{4.871}} \quad (6)$$

The execution of this example through the proposed model brought the results of dimensioning of pipeline diameters and

Pipeline number	Eusuff and Lansey (2003)*	Liong and Atiquz-zaman (2004)	Suribabu and Neelakantan (2006)	Van Dijk et al. (2008)	Lenhsnet	
					$v_{max} \geq 3,5$	$v_{max} \leq 3,5$
[01]	1 016.0	1 016.0	1 016.0	1 016.0	1 016.0	1 435.4
[02]	1 016.0	1 016.0	1 016.0	1 016.0	1 016.0	1 435.4
[03]	1 016.0	1 016.0	1 016.0	1 016.0	1 016.0	1 016.0
[04]	1 016.0	1 016.0	1 016.0	1 016.0	1 016.0	1 016.0
[05]	1 016.0	1 016.0	1 016.0	1 016.0	1 016.0	1 016.0
[06]	1 016.0	1 016.0	1 016.0	1 016.0	1 016.0	1 016.0
[07]	1 016.0	1 016.0	1 016.0	1 016.0	1 016.0	762.0
[08]	1 016.0	762.0	1 016.0	1 016.0	1 016.0	762.0
[09]	1 016.0	762.0	1 016.0	1 016.0	1 016.0	762.0
[10]	762.0	762.0	762.0	762.0	1 016.0	609.6
[11]	609.6	762.0	609.6	609.6	1 016.0	609.6
[12]	609.6	609.6	609.6	609.6	609.6	508.0
[13]	508.0	406.4	508.0	609.6	609.6	508.0
[14]	406.4	304.8	406.4	304.8	609.6	508.0
[15]	304.8	304.8	304.8	304.8	609.6	406.4
[16]	304.8	609.6	304.8	304.8	304.8	304.8
[17]	406.4	762.0	406.4	406.4	406.4	406.4
[18]	508.0	762.0	609.6	609.6	508.0	508.0
[19]	508.0	762.0	609.6	609.6	508.0	508.0
[20]	1 016.0	1 016.0	1 016.0	1 016.0	1 016.0	1 016.0
[21]	508.0	508.0	508.0	508.0	508.0	406.4
[22]	304.8	304.8	304.8	304.8	304.8	304.8
[23]	1 016.0	762.0	1 016.0	1 016.0	1 016.0	762.0
[24]	762.0	762.0	762.0	762.0	762.0	609.6
[25]	762.0	609.6	762.0	762.0	609.6	508.0
[26]	508.0	304.8	508.0	508.0	406.4	406.4
[27]	304.8	508.0	304.8	304.8	508.0	406.4
[28]	304.8	609.6	304.8	304.8	609.6	406.4
[29]	406.4	406.4	406.4	406.4	406.4	406.4
[30]	406.4	406.4	304.8	304.8	304.8	406.4
[31]	304.8	304.8	304.8	304.8	304.8	304.8
[32]	304.8	406.4	406.4	508.0	406.4	304.8
[33]	406.4	508.0	406.4	406.4	406.4	304.8
[34]	508.0	609.6	609.6	609.6	609.6	508.0
Cost (\$M)	6.07*	6.22	6.10	6.11	6.42	5.50

\*An  $\alpha$  value was used, Eq. (6), different from EPANET2.

of optimised cost, which are presented in Table 3. The last 2 columns of this table contain the diameters obtained for the dimensioning done with and without maximum velocities restriction of network sections. The table also presents the results of the dimensioning done through other models mentioned in the literature.

Table 4 (next page) shows the pressures and costs obtained with Lenhsnet and other solutions mentioned in the literature.

By using only the diameters originally used by Fujiwara and Khang (1990), the solution presented by Cunha and Sousa (1999) achieved the lowest cost (\$6.06 m.), see Table 4. However, pressures originated from these references were lower than 30 m in some network nodes, for  $\alpha$  equal to 10.67 (default value EPANET2). The dimensioning solution by Lenhsnet provided an optimal system cost of \$6.42 m.

Considering the contour condition, in which a maximum velocity in pipeline is admitted; dimensioning by

Lenhsnet provided an optimised cost of \$5.50 m. The processing time for this example was less than 8 s, through the use of a 1.60 GHz Intel® Core™ Duo processor with 2 GB of RAM.

#### **Example 2: The New York tunnels problem**

The 2<sup>nd</sup> example is the New York tunnels problem (see Fig. 3), which is gravity-fed from a single reservoir and comprises 20 nodes connected via 21 pipes. The reservoir is at an elevation of 91.44 m and all the nodes are at zero elevation. The objective of the New York tunnels problem was to determine the most economically effective design for addition to the existing system of tunnels that constituted the primary water distribution system of the city of New York. Pipe diameters are considered as design variables. There are 15 available discrete diameters and one extra possible decision which is 'do nothing' option.

**TABLE 4**  
**Results of the pressures of optimised dimensioning of Hanoi Network**

Node	Abebe and Solomatine (1998)		Cunha and Sousa (1999)*	Eusuff and Lansey (2003)*	Liong and Atiquzzaman (2004)	Zecchin et al. (2006)	Lenhsnet	
	GA	ACCOL					$v_{max} \geq 3.5$	$v_{max} \leq 3.5$
2	97.14	97.14	97.14	97.14	97.14	97.14	97.14	99.47
3	61.67	61.67	61.63	61.67	61.67	61.67	61.67	92.88
4	58.59	57.68	56.82	56.88	57.54	57.08	56.28	87.66
5	54.82	52.75	50.86	50.94	52.43	51.38	49.59	81.19
6	39.45	47.65	44.57	44.68	47.13	45.40	42.45	74.29
7	38.65	42.97	43.10	43.21	45.92	44.01	40.73	72.65
8	37.87	41.68	41.33	41.45	44.55	42.36	38.60	64.42
9	35.65	40.70	39.91	40.04	40.27	41.06	36.85	57.73
10	34.28	32.46	38.86	39.00	37.24	40.11	35.53	52.68
11	32.72	32.08	37.30	37.44	35.68	38.55	35.14	48.05
12	31.56	30.92	33.87	34.01	34.52	35.12	34.86	44.63
13	30.13	30.56	29.66	29.80	30.32	30.91	30.65	34.40
14	36.36	30.55	34.94	35.13	34.08	37.21	32.45	46.32
15	37.17	30.69	32.88	33.14	34.08	32.89	31.58	44.69
16	37.63	30.74	29.79	30.23	36.13	32.16	31.01	41.79
17	48.11	46.16	29.95	30.32	48.64	41.36	31.02	50.54
18	58.62	54.41	43.81	43.97	54.00	48.55	44.20	71.61
19	60.64	60.58	55.49	55.57	59.07	54.33	55.66	85.58
20	53.87	49.23	50.43	50.44	53.62	50.61	51.76	83.22
21	44.48	47.92	41.07	41.09	44.27	41.26	42.41	55.50
22	44.05	47.86	35.90	35.93	39.11	36.10	37.24	50.34
23	39.83	41.96	44.24	44.21	38.79	44.53	46.62	63.22
24	30.51	40.18	38.50	38.90	36.37	39.39	42.36	53.55
25	30.50	38.95	34.79	35.55	33.16	36.18	34.97	41.29
26	32.14	36.01	30.87	31.53	33.44	32.55	30.62	38.05
27	32.62	35.93	29.59	30.11	34.38	31.61	30.66	38.35
28	33.52	36.47	38.60	35.50	32.64	35.90	40.57	50.63
29	31.46	36.45	29.64	30.75	30.05	31.23	30.31	42.28
30	30.44	36.54	29.90	29.73	30.10	30.29	30.39	36.55
31	30.39	36.64	30.18	30.19	30.35	30.77	30.64	36.61
32	30.17	36.76	32.64	31.44	31.09	32.04	32.89	38.20
Max. vel. (m/s)	6.83	6.83	6.83	6.83	6.83	6.83	6.83	3.42
Cost (\$M)	7.01	7.84	6.06*	6.07*	6.22	6.13	6.42	5.50

\*An  $\alpha$  value was used, Eq. (6), different from EPANET2.

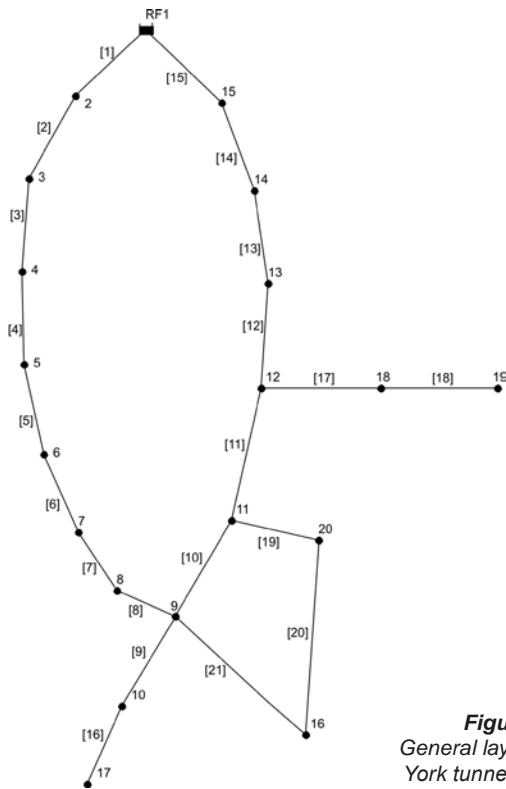
**TABLE 5**  
**Optimised solution for expansion of the New York tunnels problem – diameters and flows**

Pipe Number	Diameter (mm)	Flow (ℓ/s)
3	914.4	273.19
16	3 358.2	1 353.39
17	2 743.2	4 933.51
18	2 133.6	2 347.16
19	2 438.4	4 281.73
21	1 828.8	2 051.09

All 21 pipelines are considered suitable for duplication. A full enumeration of all possibilities would require:  $16^{21} = 1.9342 \times 10^{25}$  evaluations. The system has been used as a benchmark network since 1969 (Schaake and Lai, 1969) to compare

various optimisation procedures, particularly by Dandy et al. (1996), Savic and Walters (1997), Maier et al. (2003), Matías (2003), Zechin et al. (2006), Gomes et al. (2008) and Montalvo et al. (2008).

The results thus obtained show that the proposed solution meets with all the hydraulic conditions required for the operation of the system. The present study identifies 6 pipes to be doubled in size (see Table 5). A direct comparison of the optimum solution obtained with the developed procedure and that obtained by other researchers is shown in Table 6. The current optimal solution for this is \$38.64 m. and no pressure deficit although this can vary slightly depending on the modelling software and parameters used. The cost obtained with the proposed method was \$41.24 m. The processing time for this example was less than 3 s, through the use of a 1.60 GHz Intel® Core™ Duo Processor with 2 GB of RAM.



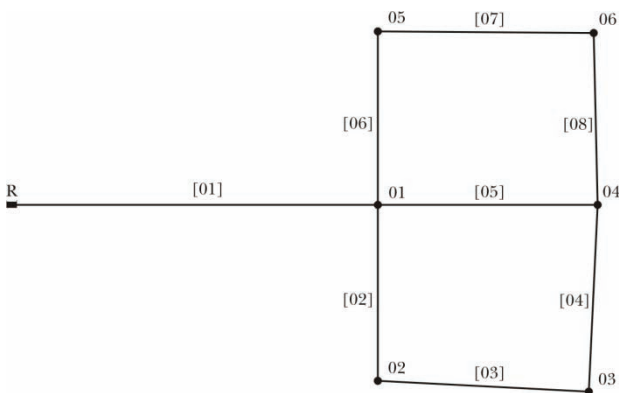
**Figure 3**  
General layout of New York tunnels problem

**Example 3: The Bessa Network**

This project consists of projecting the Bessa Network, which was originally presented by Gomes and Formiga (2001). This example was adopted because it used electric energy cost, in addition to pipeline investment costs.

It is aimed at dimensioning, as economically as possible, the network sections and the height of the elevated reservoir, considering the pipeline investment costs and capitalised energy cost of the pump, which pumps water to the reservoir. The pumping head is 30 m. Table 7 contains network data. The minimum pressure required at each node is 25 m and the maximum acceptable velocity is 3.0 m/s. Figure 4 shows the numbers of nodes and network sections.

The equation used for energy loss was Hazen-Williams formula, Eq. (6), which assumes  $\alpha$  value as 10.67 (default value EPANET2). Table 8 provides the available diameters, materials, Hazen-Williams coefficients C and unitary prices for pipe investment.



**Figure 4**  
Network layout for the Bessa Network

Researcher	Algorithm	$\alpha$ value used Eq. (6)	Cost (\$m.)
Dandy et al. (1996)	genetic algorithms	10.68	38.80
Savic and Walters (1997)	genetic algorithms	10.51	37.13
Savic and Walters (1997)	genetic algorithms	10.90	40.42
Eusuff and Lansey (2003)	shuffled frog-leaping algorithm	-	38.80
Maier et al. (2003)	ant colony optimisation	10.67	38.64
Matias (2003)	genetic algorithms	-	38.64
Zechin et al. (2006)	ant colony optimisation	10.67	38.64
Gomes et al. (2008)	heuristic method	10.67	37.23*
Montalvo et al. (2008)	particle swarm optimisation	-	38.64
Van Dijk et al. (2008)	genetic algorithms	10.67	38.65
Present study	heuristic method	10.67	41.24

\* obtained feasible solutions but utilised split pipe solutions.

Node data			Pipeline data	
Node number	Demand (ℓ/s)	Elevation (m)	Pipeline number	Length (m)
1	0.00	6.0	[01]	2 540
2	47.78	5.5	[02]	1 230
3	80.32	5.5	[03]	1 430
4	208.60	6.0	[04]	1 300
5	43.44	4.5	[05]	1 490
6	40.29	4.0	[06]	1 210
			[07]	1 460
			[08]	1 190

The costs and pumping conditions are: number of pumping hours per annum,  $N_p = 7\ 300$ , efficiency of the motor-pump unit,  $\eta = 0.75$ , expected period of service for the network,  $t = 20$  years, the average pumping discharge ( $Q$ ), equal to 420.43 ℓ/s, power tariff,  $E_c = R\$ 0.20/\text{kWh}$ , annual rate of increase in the tariff of electrical power,  $e = 6\%$ , and annual discount rate,  $j = 12\%$ .

D (mm)	Pipeline material	HW coefficient 'C'	Unit cost (R\$/m)*
108.4	PVC	145	47.09
156.4	PVC	145	63.80
204.2	PVC	145	87.62
252.0	PVC	145	118.59
299.8	PVC	145	152.24
366.2	ductile iron	130	317.86
416.4	ductile iron	130	375.00
466.6	ductile iron	130	436.23
518.0	ductile iron	130	515.60
619.6	ductile iron	130	640.30

\*R\$ - Brazilian Real (R\$ 2.00 = USD 1.00 and R\$ 0.24 = ZAR 1.00).

Based on the supplied data, the energy gradient value ( $E_g$ ) is R\$89 323.97 (R\$ - Brazilian Real (R\$ 2.00 = USD 1.00 and R\$ 0.24 = ZAR 1.00)). The iterative calculation is processed by increasing the sections diameters and reducing the piezometric level, until the solution which provides the lowest system cost is obtained. The iterative process is ended after the 37<sup>th</sup> iteration, when the optimal cost gradient ( $Pg^*$ ) reaches R\$108 879.58, which is higher than the energy gradient. The dimensioning variables values are obtained in the last iteration.

Once the stop condition is established, this method calculates the final pumping head, which was 15.79 m. When the pumping head is multiplied by the  $E_g$ , the current cost of electrical power of the system is determined at R\$1 410 488.40. Pipeline network capital cost is R\$3 260 811.50. Consequently, the water distribution system total cost is R\$4 671 299.90.

Figure 5 shows cost evolution during the iterative process, providing several feasible alternatives. In case the managing agency of the system decides that the initial network cost should be lower than R\$2.7 m., the solution to be chosen should be that indicated by the 30<sup>th</sup> iteration, in which pipeline cost is calculated to be R\$2.6 m. The dimensioning results are presented in Tables 9 and 10.

When the results obtained by Lenhsnet are compared with those obtained by Gomes and Formiga (2001), by using the nonlinear programming, it is concluded that the proposed model obtained a more favourable result in economic terms, once it reduces the total network cost value by 14.64%.

#### Example 4: The R-9 Network

The 4<sup>th</sup> example (see Fig. 6) is the R-9 Network, a medium-size municipal network in Joao Pessoa city - Brazil, implemented by an urban water company in 1982, which consists of 53 nodes connected via 72 pipes and a reservoir. The input data for this

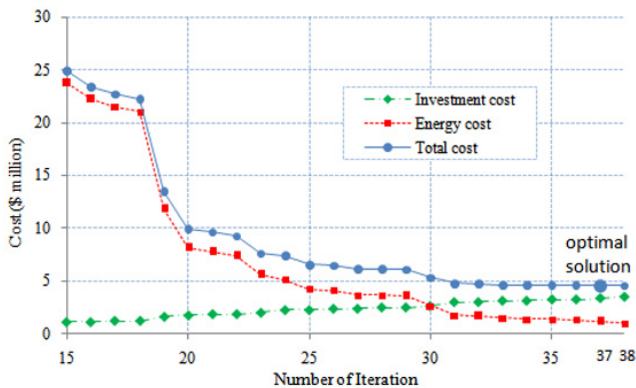
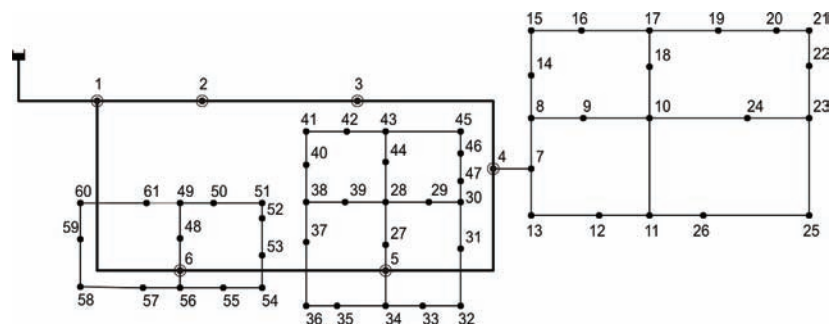


Figure 5  
Project costs based on the pumping head

Figure 6  
Layout for the R-9 Network



Pipeline	Diameter (mm)	Node number	Pressure (m)
[01]	600	1	32.97
[02]	300	2	27.17
[03]	300	3	25.02
[04]	200	4	29.16
[05]	500	5	27.63
[06]	250	6	25.00
[07]	200		
[08]	150		

problem are given in FORMIGA (2005) and are presented in Tables 10 and 11 (opposite page). This network has been used previously by some researchers (LEAL, 1995; LOPES, 2003; FORMIGA, 2005). The network data, in EPANET2 solver format, are available from <http://www.lenhs.ct.ufpb.br/html/benchmarks.html>. Table 12 provides the available diameters, Darcy-Weisbach coefficients and unitary prices for pipe investment.

Diameter (mm)	Darcy-Weisbach coefficient	Unit cost (Cr\$/m)*
100	0.01	1 629
150	0.01	4 054
200	0.01	5 769
250	0.01	7 718
300	0.10	9 237
350	0.10	11 012
400	0.10	12 397
450	0.10	15 501
500	0.10	17 686
600	0.10	23 132

\* Current Brazilian currency in 1982 (Cr\$ 90.60 = USD 1.00)

The optimum solution of \$199.39 m. cost units is obtained if the pipes as listed in Table 13 are used resulting in a minimum pressure in the system at node 6 being 17.10 m. When the results obtained by Lenhsnet are compared with those obtained by Formiga (2005) (\$202.80 m.), by using non-dominated sorting genetic algorithm - NSGA-II, it is concluded that the proposed model reduces the total cost value by 1.7%.



Node	Demand (ℓ/s)	Elevation (m)	Minimum pressure
1	2.51	5.0	25.00
2	44.07	5.0	25.00
3	41.24	4.0	25.00
4	1.04	4.5	25.00
5	0.86	4.5	25.00
6	1.32	4.5	25.00
7	1.35	4.5	15.00
8	8.59	5.0	15.00
9	6.40	4.5	15.00
10	6.07	5.0	15.00
11	4.95	3.5	15.00
12	8.38	3.5	15.00
13	11.70	3.5	15.00
14	5.63	5.0	15.00
15	5.57	6.0	15.00
16	6.30	6.0	15.00
17	3.26	6.0	15.00
18	3.60	6.0	15.00
19	4.83	6.0	15.00
20	4.50	6.0	15.00
21	2.80	5.0	15.00
22	5.46	3.0	15.00
23	62.45	3.5	15.00
24	8.19	6.0	15.00
25	58.87	3.5	15.00
26	3.26	3.5	15.00
27	4.36	4.3	15.00
28	4.25	4.0	15.00
29	4.56	2.5	15.00
30	8.32	2.5	15.00
31	4.94	3.5	15.00
32	4.09	4.5	15.00
33	3.68	5.0	15.00
34	4.04	5.0	15.00
35	3.22	6.0	15.00
36	2.53	4.5	15.00
37	2.31	4.5	15.00
38	2.50	4.0	15.00
39	2.89	4.0	15.00
40	2.48	4.0	15.00
41	4.61	4.0	15.00
42	3.47	4.0	15.00
43	3.61	4.0	15.00
44	5.17	4.0	15.00
45	6.48	4.0	15.00
46	4.91	4.5	15.00
47	6.50	4.0	15.00
48	4.97	4.5	15.00
49	2.97	3.0	15.00
50	1.80	5.0	15.00
51	2.96	4.0	15.00
52	4.66	3.0	15.00
53	4.54	4.5	15.00
54	8.80	4.5	15.00
55	4.26	4.5	15.00
56	2.98	5.0	15.00
57	3.91	5.0	15.00
58	3.70	4.7	15.00
59	1.86	5.0	15.00
60	3.12	5.0	15.00
61	3.52	4.5	15.00

Pipeline	Node 1	Node 2	Length (m)
[1]	Reservoir	1	2 540
[2]	1	2	350
[3]	2	3	1 140
[4]	3	4	1 430
[5]	4	5	1 020
[6]	6	5	1 430
[7]	1	6	1 710
[8]	4	7	220
[9]	7	8	190
[10]	8	9	295
[11]	9	10	390
[12]	11	10	370
[13]	12	11	190
[14]	13	12	310
[15]	7	13	205
[16]	8	14	305
[17]	14	15	295
[18]	15	16	300
[19]	16	17	290
[20]	18	17	180
[21]	10	18	315
[22]	17	19	300
[23]	20	19	295
[24]	21	20	215
[25]	22	21	140
[26]	23	22	220
[27]	23	24	220
[28]	10	24	285
[29]	25	23	300
[30]	26	25	315
[31]	11	26	170
[32]	5	27	110
[33]	27	28	280
[34]	28	29	225
[35]	29	30	200
[36]	30	31	190
[37]	32	31	285
[38]	33	32	210
[39]	34	33	240
[40]	5	34	250
[41]	34	35	340
[42]	35	36	270
[43]	36	37	240
[44]	37	38	160
[45]	39	38	260
[46]	28	39	250
[47]	38	40	330
[48]	40	41	230
[49]	42	41	385
[50]	43	42	160
[51]	44	43	330
[52]	28	44	210
[53]	43	45	150
[54]	45	46	255
[55]	47	46	260
[56]	30	47	230
[57]	6	48	115
[58]	48	49	180
[59]	49	50	140
[60]	50	51	215
[61]	51	52	175
[62]	52	53	180
[63]	53	54	260
[64]	54	55	205
[65]	55	56	255
[66]	56	6	260
[67]	57	56	275
[68]	58	57	315
[69]	59	58	200
[70]	59	60	175
[71]	60	61	300
[72]	61	49	250

**TABLE 13**  
**Solution for design of the R-19 Network – diameters and flows**

Pipe-line	Diameter (mm)	Flow (ℓ/s)	Pipe-line	Diameter (mm)	Flow (ℓ/s)
[1]	600	456.20	[37]	100	1.77
[2]	600	394.87	[38]	100	5.86
[3]	600	350.80	[39]	100	9.54
[4]	500	309.56	[40]	150	21.68
[5]	350	86.35	[41]	100	8.10
[6]	100	3.44	[42]	100	4.88
[7]	250	58.82	[43]	100	2.35
[8]	400	222.17	[44]	100	0.04
[9]	200	39.67	[45]	100	6.35
[10]	100	10.73	[46]	100	9.24
[11]	100	4.33	[47]	100	3.89
[12]	100	9.28	[48]	100	1.40
[13]	400	161.08	[49]	100	3.20
[14]	400	169.45	[50]	100	6.67
[15]	400	181.15	[51]	150	18.02
[16]	150	20.34	[52]	150	23.20
[17]	150	14.71	[53]	100	7.74
[18]	100	9.14	[54]	100	1.26
[19]	100	2.84	[55]	100	3.64
[20]	100	1.57	[56]	150	10.15
[21]	100	5.17	[57]	150	27.01
[22]	100	1.15	[58]	150	22.04
[23]	100	3.68	[59]	100	10.85
[24]	150	8.18	[60]	100	9.05
[25]	150	10.98	[61]	100	6.09
[26]	150	16.44	[62]	100	1.43
[27]	100	5.82	[63]	100	3.11
[28]	100	2.37	[64]	100	11.91
[29]	300	84.71	[65]	150	16.17
[30]	350	143.58	[66]	150	27.05
[31]	350	146.85	[67]	100	7.90
[32]	250	67.24	[68]	100	3.99
[33]	250	62.88	[69]	100	0.29
[34]	150	26.19	[70]	100	1.58
[35]	150	21.63	[71]	100	4.70
[36]	100	3.17	[72]	100	8.22

## Conclusions

The method presented in this study, based on a dynamic programming process, enables engineers to follow several dimensioning alternatives through the developed program. In this study, the optimisation algorithm was applied to 4 water distribution systems, whose results were compared to the ones obtained through other models, which used different optimisation techniques (NLP, ACO, GA, SFLA, PSO, etc.). This study showed that Lenhsnet presented total optimised costs, which are similar to the results obtained in the literature.

For various reasons, it is important to point out that it is not reasonable to expect that a WDS project be solved in a totally automated way. Optimisation should be seen as a decision support tool. It is within such a paradigm that Lenhsnet is presented as an attractive and practical alternative

for WDS design. The necessary computer processing time is very low, and the interface program with the user is easy, once the method runs in the EPANET2 program.

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