THE DEVELOPMENT OF RISK ANALYSIS AND GROUNDWATER MANAGEMENT TECHNIQUES FOR SOUTHERN AFRICAN AQUIFERS

Report to the
WATER RESEARCH COMMISSION

Section 1 : The Development of Groundwater Management Models
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Section 2 : Risk Analysis in Groundwater Management
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Section 3 : Management of the Grootfontein Aquifer in the Western Transvaal
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EXECUTIVE SUMMARY

1. BACKGROUND AND MOTIVATION

As a result of limited water resources in South Africa, it is necessary to utilise the available resources optimally. Groundwater forms an important component of the country's total resources with more than a 100 municipalities dependant on groundwater (e.g. De Aar, Dewetsdorp and Kuruman). It is therefore vitally important that the concerned municipalities manage their groundwater resources in a sustainable manner. Development of groundwater management models applicable to South African aquifer conditions, will therefore assist water resource planners and managers to utilise their resource in a sustainable manner.

To address the need for the development of appropriate groundwater management models for South Africa, the CSIR and the Institute for Groundwater Studies at the University of the Orange Free State, submitted a joint proposal to the Water Research Commission to develop groundwater management models for South African aquifer conditions. These models need incorporated risk assessment techniques in order to provide a measure of the uncertainties inherent in the results obtained with the simulation and management models and the geohydrological parameters used in these models.

2. STATEMENT OF OBJECTIVES

The two primary objectives of the study, as stated in the original contract with the Water Research Commission were :

- The development and implementation of user friendly groundwater management models; and
- The identification and measurement of the risks associated with the exploitation of groundwater resources in order to evaluate optimal management policies.

In order to achieve the first primary objective, the following secondary objectives were addressed :

- The development and implementation of an inverse-parameter program which would enhance the current calibration methods of groundwater flow and transport models;

- The development and application of groundwater management models addressing :

  (b) groundwater allocation;
  (c) groundwater operations; and
  (d) capacity expansion.

- The development and application of a conjunctive ground- and surface water planning model; and

- The development and application of a groundwater quality management model.

To achieve the second primary objective the behaviour of the water levels of an aquifer were studied under stochastic variation representing the uncertainty of the physical properties of the aquifer and recharge (rainfall) values. For this, the following secondary objectives were listed :

- evaluation of existing risk analysis techniques for the management of surface and surface and groundwater resources;

- identification of the geohydrological variables applicable in risk analysis assessments;

- determination of the data requirements necessary to estimate the geohydrological variables;

- the selection and development of risk analysis methodology for application in groundwater management;

- evaluation of the importance and role of geohydrological variables in risk analysis studies by using simulated data;

- application of geostatistical techniques to calculate geohydrological parameters for use in risk analysis studies;

- sensitivity analysis of these geostatistically derived estimates; and

- application of selected and developed methodologies and techniques in a case study.

The techniques developed was tested on a South African aquifer (Grootfontein aquifer in the Western Transvaal) in order to assess the performance of the models and the different risk analysis techniques.

3. DISCUSSION OF RESULTS

3.1 The development of groundwater management models

The Institute for Groundwater Studies developed a groundwater management module referred to as the AQUAMOD package. This package consist of the following components :

AQUA-NET (GRID GENERATOR)

AQUA-NET is a triangular mesh generating program to use in the numerical simulation of groundwater flow or pollution. The program generates a finite triangular mesh between a finite set of user-defined data points.

AQUA (FLOW PROGRAM)

AQUA solves the Galerkin finite element method in two dimensions for groundwater flow. Special features of AQUA include the ability to specify :

- variable pumping rates;
- time-dependant recharge values as a percentage of monthly rainfall; and
- a confined or water-table aquifer.

AQUA-INV (INVERSE PROGRAM FOR FLOW)

AQUA-INV is an automated parameter identification program which uses the flow program AQUA and the Marquardt optimisation algorithm to obtain parameter combinations such as transmissivity, storage, recharge and inflow flux at boundary.

AQUAMAS (MASS TRANSPORT PROGRAM)

This program solves the convection diffusion equation in two dimensions for mass transport problems.

MASS-INV (INVERSE MASS TRANSPORT PROGRAM)

This program is the equivalent of the AQUA-INV program, and can be used for the automated calibration of the mass transport problem.

AQUA-MAN (MATHEMATICAL OPTIMISATION)

AQUA-MAN links the distributed parameter groundwater flow simulation model, AQUA, with mathematical optimisation methods using a technique known as the response matrix approach.

SVF (RECHARGE ESTIMATION)

This program estimates the groundwater recharge of an aquifer with the aid of the Saturated Volume Fluctuation (SVF) method.

AQFSTOC (ESTIMATION OF PERCENTILES)

AQUASTOC was written to simulate the storage in an aquifer based on the recharge results as obtained with the SVF-method as well as the storage value and the abstraction from the system. Percentiles obtained from the AQFSTOC program show probability estimates of confidence for storage of the aquifer on a monthly basis.

The AQUAMOD groundwater management model was used in the following applications :

- Hydraulic management (maximise pumpage, minimise drawdown);
- Economic management (minimise pumping costs);
- Allocation of water among categories of water users.

3.2 Risk analysis in groundwater management

When assessing a proposed aquifer management plan, information on the geohydrological parameters, the recharge rate and the proposed extraction rate are required. The purpose of risk assessment is to incorporate all the uncertainties in the available information into an evaluation of an aquifer management proposal. The output would then provide the planners with statistics enabling them to evaluate the long term viability of a management plan.

In order to obtain the risk statistics, a simulation approach was followed allowing for many different but equally likely scenarios to be considered. A Monte Carlo type of system which would involve an aquifer simulation model using simulated values of storativity and transmissivity was considered the most appropriate method of risk assessment.

Two methods for simulating the geohydrological parameters were proposed. The first of these, the zonal simulator, involved dividing the aquifer into a number of homogeneous zones and using the zonal means and covariances between the zones. Simulations can be obtained for the zones. The second involved the geostatistical method of turning bands which simulates values on a fine grid. The inputs for this method are the means and variograms of the geohydrological parameters.

Risk is described in terms of risk events such as system failures and extreme values and measured as the probability of one of these events occurring. Risk events were defined in terms of the difference between the simulated and base water levels in a number of boreholes falling below a chosen tolerance level. Tolerances were defined as a proportion of effective depth. For this study the following tolerance values were used :

- Tolerance 1 = 1/6 (maximum water level - base water level)
- Tolerance 2 = ⅓ (maximum water level - base water level)
- Tolerance 3 = ½ (maximum water level - base water level)
- Tolerance 4 = ⅔ (maximum water level - base water level)

The risk evaluation was achieved by repeatedly running the aquifer simulation with different simulated values of the input parameters (storativity, transmissivity and rainfall). The risk evaluation model generates an immense number of intermediate results and one of the main objectives of this project was to summarise these into a few informative bits of information to be used by the decision maker.

The statistics were calculated by accumulating the results generated by the individual realizations and then summarising them into meaningful measures. The statistics are presented as expected values. The summary statistics included the minimum, maximum and average values of water levels relative to the base of the aquifer. The risk statistics were defined as the expected period (duration) and expected probability of a failure.

3.3 Management of the Grootfontein Aquifer

For the case study reported in Section 3 the Grootfontein aquifer was selected as it is a prime example of one of the best utilised aquifers in South Africa. It supports two towns as well as a substantial agricultural area under irrigation. The aquifer forms part of a Subterranean Water Control Area and permits for the abstraction of 11 million cubic metres per annum for irrigation and 4,5 million cubic metres per annum for the towns have been issued.

Two different methods of generating the transmissivity and storativity values required for the Monte Carlo study were compared. The first was a zonal simulation method based on the subdivision of the aquifer into zones and the second a geostatistical method generating values on a fine grid. In both cases the simulators were calibrated from variograms computed from the available data. For storativity there was no spatial continuity and a nugget effect variogram was fitted. Calibration of the zonal simulator could not be carried out via the method of inversion due to lack of convergence of the inversion program. The rainfall simulator based on an algorithm developed by Zucchini and Adamson (1984) was used to simulate rainfall values.

For the case study four tolerances were defined in terms of effective depth. Five different risk event were defined. These events were in terms of a percentage of boreholes falling below a particular tolerance level. The values were found to remain fairly constant until a pumping rate of 9.5 million cubic metres per year, or about 10% more than the previous determined "safe" abstraction rate is reached. Thereafter they decline steadily.

Risk statistics including period and probability of a failure occurring were computed. For both the expected periods and expected probabilities the results showed that the aquifer is unable to sustain an abstraction rate of more than 9.5 million cubic metres per year for significant periods. It is concluded that both the summary and risk statistics performed as predicted and are a suitable measure of evaluating a management plan.

The research identified a restriction imposed by the aquifer modelling software's inability to switch pumps on and off as water level varies, resulting in exaggerated negative results when over-exploitation takes place. The implications of this are discussed fully in the report. This issue needs to be carefully understood when using a aquifer simulation model as part of a risk assessment.

4. CONCLUSION AND RECOMMENDATIONS

The management of an aquifer by means of a groundwater model, involves the following :

- Development of a simulation model;
- Estimation of the exploitation potential of the aquifer;
- Defining of specific objectives or practical applications;
- Setting constraints on variables;
- Obtain optimised solution (optimization model).

The development and application of groundwater optimisation models for the control of groundwater hydraulics and water quality, and the inverse problem of parameter estimation, are the primary aspects of the groundwater management package, AQUAMOD, developed during the present study. The computer programs developed, can be used for :

- constructing a finite element mesh;
- automated calibration of the aquifer parameters (inverse problem);
- obtaining an estimate of natural groundwater recharge;
- risk evaluation by means of a groundwater balance;
- optimisation and groundwater flow simulations; and

- mass transport solutions.

The construction of a reliable flow model, and thus a management model for an aquifer, may be premature, unless extensive monitoring of water levels and abstraction rates were performed for a number of years.

A non-achievement of the present project was the lack of a suitable practical example on quadratic optimisation. The need for such an application may become important in the near future, as more emphasis may be placed on the energy required to operate pumps (this is because costs vary with pumping rates and pumping lifts, and lifts depend on pumping rates).

From the risk analysis methodology developed for the long term assessment of risk when using groundwater, the following conclusions can be drawn :

- There are two sources of uncertainty when considering the use of groundwater. Firstly the geological properties of the aquifer are only known up to a degree of certainty an secondly the recharge is a function of unpredictable rainfall. This study has developed methods for combining both these sources of uncertainty to provide an overall assessment of risk.

- The aquifer modelling software plays a central role in the Monte Carlo study used to make the risk assessment. This software must be capable of addressing a particular management plan if this plan is to be assessed. Thus, if one would like to simulate a process whereby a pump is switched on and off according to water level, the software must accommodate this.

- A failure event needs to be defined. This can be done in a number of ways and can be chosen to meet the specific needs of a particular study. In the case study a failure was defined as the water level at a certain number of nodes falling below a chosen tolerance. This definition was found to work well in practice.

- After thorough investigation a definition of failure in terms of effective depth was formulated. This enabled a number of risk statistics to be considered of which probability of failure and period of failure were finally selected.

- The concept of failure return time was found to be unsuitable for this type of study.

- The methodology depends on the geohydrological modelling of the aquifer, the stochastic representation of the rainfall process and the recharge rate. As with any study the accuracy of these components will determine the value of the final results. In the case of the aquifer two different geostatistical techniques were used. From the case study we concluded that the method of zonal simulation was the most suitable. In the case of rainfall a simulator based on Fourier series representation of transition probabilities as proposed by Zucchini and Adamson (1985) was used.

- It was found that although the aquifer in a sense could be equated to a surface water network, the aims of surface water management differed from the aims in this study. When this work is extended to short term risk assessment the methods used in surface water will become appropriate for groundwater management.

- The proposed methods were successfully tested for the Grootfontein aquifer and found to be practically applicable.

- The computing time involved in the developed Monte Carlo based methodology was found to be considerable, ruling out the routine generation of the risk statistics. However, in practice the long term viability will only be assessed during the planning stage. A logical progression of the current work will be toward the short term risk assessment as a routinely applied management tool. It is anticipated that for such an application computing time will be kept to practical proportions.

- The overall objectives of the project relating to risk assessment, especially in terms of longer term viability assessment, Section 2 were successfully achieved and the scene is set to extend the methodology to the application of short term risk management.

The Grootfontein Case Study, presented as part of the present project in Volume 3, is a typical and well-illustrated example of what could be achieved as a result of application of the AQUAMOD package and it is foreseen that many other case studies in South Africa will be based on the methods used in this study.

Water balance calculations have conclusively indicated that the long term recharge to the aquifer is in the order of 8,5 million cubic metres per year and that the S value is within the range of 2,15% to 2,45%. With the present water allocations amounting to about 15 million cubic metres per year it is clear that permits should be cut back to ensure that the aquifer is not depleted in the near future.

Forecasts of water level reactions using stochastic rainfall record sets and different abstraction from the system indicate that the long term potential of the aquifer is 8,5 million cubic metres per annum. It can be inferred from the simulations that it is likely that the water levels will recover due to a normal rainfall pattern. Abstraction would have to be cut back to achieve this.

A dynamic model was constructed to simulate the reaction of the aquifer by using the results obtained with the water balance study. A good fit between the actual and the simulated water levels was obtained which confirms that the parameter value obtained from the water balance study is acceptable. This model can be used to predict groundwater flow dynamics in the aquifer at any selected location of interest. It is also possible to use the calibrated flow model together with a response matrix technique to manage drawdowns, abstraction etc., at any point of interest within the aquifer.

The risk analysis methodology developed in the course of the project was tested and successfully implemented in the case study of the Grootfontein aquifer. The methodology developed adequately describes the risks involved when managing an aquifer under different management plans.

A mass transport model was coded as part of the present study, but due to the complex nature of most of our geological formations, it was decided by the Steering Committee that the work on water quality monitoring has to be scaled down and that no case study must be evaluated. Due to the importance of water quality modelling under South African conditions, this must receive attention in the near future.

The AQUAMOD package is based on a two-dimensional approach and whenever a three-dimensional model may be required for an aquifer, the techniques which were developed during this study, could be easily incorporated into a three-dimensional model.

The problem of Geohydrological Decision Analysis fell beyond the scope of the present study, but should receive attention in the near future. In this regard, the Bayesian updating and Kalman filter techniques could provide a sound basis for future research on decision analysis.

The case study reported in Section 3 concluded that the methodology proposed did provide meaningful results. The most appropriate application of the risk analysis methodologies would be for long term viability studies undertaken during the planning stages of a long term management plan. However, heavy computing time requirements make it an unsuitable method for operational management. Despite this the elements of the system could be incorporated in a system suitable for day to day management of an aquifer. This extension is recommended by the project team. It is envisaged that such a system would enable a manager to make short term predictions of the effect a management plan will have on an aquifer situation taking the current situation into account.

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"THE DEVELOPMENT OF RISK ANALYSIS AND GROUNDWATER MANAGEMENT TECHNIQUES FOR SOUTHERN AFRICAN AQUIFERS"

The Steering Committee responsible for this project, consisted of the following members :

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Mr H Maaren	Water Research Commission
Mr E Braune	Department of Water Affairs and Forestry, Geohydrology
Dr D B Bredenkamp	Department of Water Affairs and Forestry, Geohydrology
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Mr P Smit	Water Research Commission (Secretary)
Mr P W Weideman	Water Research Commission (Secretary)

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REPORT CONTENTS

Executive Summary

Acknowledgements

Section 1	:	The Development of Groundwater Management Models By G J van Tonder, H Janse van Rensburg, S Staats and V E Cogho
Section 2	:	Risk Analysis in Groundwater Management By C D Elphinstone, M I Viviers, R Meyer and A G Watson
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THE DEVELOPMENT OF RISK ANALYSIS AND GROUNDWATER MANAGEMENT TECHNIQUES FOR SOUTHERN AFRICAN AQUIFERS

SECTION 1

THE DEVELOPMENT OF GROUNDWATER MANAGEMENT MODELS

by

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FOREWORD

The overall purpose of this section of the project, as stated in the original contract with the Water Research Commission, was

The development and implementation of user friendly groundwater management models with special reference to:

(i) The development and implementation of an inverse-parameter program which will enhance the current calibration methods of groundwater flow and transport models.

(ii) The development and application of groundwater management models addressing:
  (a) groundwater allocation,
  (b) groundwater operation and
  (c) capacity expansion.

(iii) The development and application of a conjunctive ground- and surface water planning model.

(iv) The development and application of a groundwater quality management model.

The Steering Committee agreed that the principle of conjunctive use should receive attention without becoming the focal point of the research. It was also decided that the emphasis of water quality modelling should concentrate on blending solutions as opposed to the more complicated mass transport models.

Water is a resource which requires effective management throughout the world. This is particularly true of South African conditions where there is an ever-increasing demand on the limited water supply by the domestic, industrial, agricultural and rural sectors.

Many small and large towns in the more semi-arid regions of the country are either fully or partially dependent on groundwater. In addition, large farming areas under irrigation depend on the availability of groundwater as their supply of water. The future economical development in all semi-arid environments in South Africa is closely linked to the availability of groundwater, which, although often occurring in smaller quantities than surface water, nevertheless provides the bulk of the water to rural areas. This is especially true during periods of prolonged droughts when the groundwater supplies are often the only exploitable and reliable resource available for survival of man and the environment.

The proper and optimal management of our groundwater resources therefore is very crucial and involves the allocation of groundwater supplies and water quality to competing water demands and uses. The motivation for the project becomes clear when a closer look is taken at the following important questions in groundwater management for which answers are usually required:

How much water is required? When? Where?
How much water is available? When? Where?
Of what quality?
At what cost?
What are the associated risks?
How many boreholes are needed? At what positions?
At what rate must the boreholes be pumped to ensure no undesirable effects?

Although simulation models provide the resource planner with important tools for managing an aquifer, the predictive models do not identify the optimal groundwater development and operational policies. Therefore optimization models are required. The development of these models are discussed in Section 1 of the report.

In Chapter 1, an introduction is given regarding groundwater management models. Chapter 2 describes the approaches and general solutions in groundwater management and in Chapter 3 the inverse problem for determining aquifer parameters is discussed. Chapter 4 is devoted to a description of the different management models and a decision analysis framework is given. Chapter 5 gives the computer program structure of the AQUAMOD package of programs developed during this project, while a number of illustrative management examples are discussed in Chapter 6. Section 1 of this combined report is concluded with Chapter 7 in which the conclusions and recommendations are summarized. Although the AQUAMOD model package forms an integral part of Section 1 of the report, the operational manual is the subject of a separate WRC report.

CONTENTS

CHAPTER 1 1-1
INTRODUCTION 1-1

1.1 SCOPE OF INVESTIGATION 1-1
1.2 PREVIOUS WORK 1-3
1.2.1 Groundwater Hydraulic Management Models 1-3
1.2.2 Ground-Water Policy Evaluation And Allocation Models 1-7
1.3 WATER RESOURCES MANAGEMENT 1-7
1.4 SYSTEMS APPROACH IN WATER RESOURCES 1-8
1.5 OBJECTIVES, CRITERIA AND CONSTRAINTS IN WATER RESOURCES MANAGEMENT 1-9

CHAPTER 2 1-11
APPROACHES AND SOLUTIONS IN GROUNDWATER MANAGEMENT 1-11

2.1 YIELD ANALYSIS 11
2.2 ESTIMATION OF NATURAL GROUNDWATER RECHARGE 1-12
2.3 GROUNDWATER MANAGEMENT MODELS 1-13
2.4 PROBLEM FORMULATION FOR LINEAR PROGRAMMING 1-14
2.5 CONJUNCTIVE MANAGEMENT OF GROUNDWATER AND SURFACE WATER 1-14
2.6 GROUNDWATER QUALITY MANAGEMENT 1-15
2.7 TWO- AND THREE-DIMENSIONAL MODELS 1-15
2.8 RISK ANALYSIS 1-16
2.9 DATA REQUIREMENTS FOR GROUNDWATER MANAGEMENT 1-16

CHAPTER 3 1-18
INVERSE METHODS OF DETERMINING AQUIFER PARAMETERS 1-18

3.1 INTRODUCTION 1-18
3.2 INVERSE SOLUTION METHODS 1-20
3.3 DATA REQUIRED TO DEVELOP A GROUND-WATER MODEL 1-21
3.4 CALIBRATION OF THE GROUND-WATER MODEL 1-22

CHAPTER 4 1-24
GROUNDWATER MANAGEMENT MODELS AND DECISION ANALYSIS 1-24

4.1 INTRODUCTION 1-24
4.2 APPROACH AND PHILOSOPHY (from Willis and Yeh, 1987). 1-24
4.3 CATEGORIES OF GROUNDWATER MANAGEMENT MODELS 1-25
4.4 GROUNDWATER SUPPLY AND DEMAND MODELS. 1-25
4.4.1 Groundwater Operational Models 1-26
4.4.2 Groundwater Allocation Models 1-27
4.4.3 The Capacity Expansion Model 1-27
4.5 CONJUNCTIVE GROUNDWATER AND SURFACE WATER PLANNING MODEL. 1-28
4.6 PROBLEM FORMULATION 1-28
4.7 SIMULATION MODEL. 1-31
4.8 STEPS IN THE CONSTRUCTION OF AN AQUIFER MANAGEMENT MODEL. 1-32
4.9 DECISION ANALYSIS RELATING TO GROUNDWATER PROBLEMS. 1-32
4.9.1 Introduction 1-32
4.9.2 Difference between Hydrogeological Decision Analysis and Optimization. 1-33
4.9.3 Components of a Design Framework (Freeze, et al. (1990)). 1-34
4.9.4 Decision Model. 1-36
4.9.5 Summary 1-37

CHAPTER 5 1-38
COMPUTER PROGRAM STRUCTURE FOR MANAGEMENT MODELS 1-38

5.1 PROGRAM AQUA-NET (GRIT GENERATOR) 1-38
5.2 PROGRAM AQUA (FLOW PROGRAM) 1-39
5.3 PROGRAM AQUA-INV (INVERSE PROGRAM FOR FLOW) 1-39
5.4 PROGRAM AQUA-MAS 1-42
5.5 PROGRAM MASS-INV (INVERSE MASS TRANSPORT PROGRAM) 1-42
5.6 PROGRAM AQUA-MAN (MATHEMATICAL OPTIMIZATION) 1-42
5.7 PROGRAM SVF (RECHARGE ESTIMATION) 1-43
5.8 PROGRAM AQFSTOC (ESTIMATION OF PERCENTILES) 1-43

CHAPTER 6 1-44
ILLUSTRATIVE MANAGEMENT EXAMPLES 1-44

6.1 EXAMPLE 1. 1-44
6.2 EXAMPLE 2. 1-47
6.3 EXAMPLE 3. 1-49
6.4 EXAMPLE 4. 1-50
6.5 EXAMPLE 5 : ALLOCATION OF WATER AMONG ....
CATEGORIES OF WATER USERS 1-56
6.6 EXAMPLE 6 : DE AAR AQUIFER 1-60
6.6.1 Background 1-60
6.6.2 Optimization Problem. (Illustrative exercise). 1-63

CHAPTER 7 1-72
CONCLUSIONS AND RECOMMENDATIONS 1-72
7.1. CONCLUSIONS 1-72
7.2. RECOMMENDATIONS 1-73
REFERENCES End of report
AQUAMOD Manual End of Report

CHAPTER 1

INTRODUCTION

1.1 SCOPE OF INVESTIGATION

Water is a resource which requires effective management throughout the world. This is particularly true of South African conditions where there is an ever increasing demand on the limited water supply by the domestic, industrial and agricultural sectors. The approach by the Department of Water Affairs with respect to the demand, supply and quality management of water, is closely in line with international thinking as expressed in the Mar del Plata recommendations and in the Third International Hydrological Program of Unesco (DWA, 1986). Braune (1993) proposed an overall strategy for groundwater management in South Africa in terms of utilization management and groundwater protection.

From the earliest times onwards, water has been an essential element to mankind. It is essential to life, social development and economic progress. Once nomadic man settled down, around 6 000 years ago, his restricted radius of action forced him to face both the struggle against water (floods) and the struggle for water (for domestic and irrigation). In response, technological skills were developed: impressive hydraulic engineering systems already existed in ancient times and testify that hydraulic engineering belongs to man's earliest technological achievements. Examples are the antique irrigation schemes in the Egyptian Nile Valley and Mesopotamia, khanats in Iran and neighbouring countries, the Marib Dam in Arabia and the Roman aqueducts (Van der Gun, 1988).

Groundwater is a valuable resource and is especially important as a source of drinking-water, providing 75 percent of drinking water supplies in Europe, more than 50 percent in the USA and 13 percent of all water used in 1980 in South Africa (DWA, 1986). Roughly, 280 small and large towns in South Africa are either fully or partially dependent on groundwater. Approximately 5 million head of cattle, 26 million small livestock and the irrigation of more than 240 000 ha of land depend on the underground water supplies. The future economical development in all semi-arid environments in South Africa, is closely linked to the availability of groundwater, which although often occurring in smaller quantities than surface water, nevertheless provides the bulk of the water to rural areas. This is especially true during periods of prolonged droughts when the groundwater supplies are often the only exploitable and reliable resource available for survival of man, animals and the environment (Bredenkamp *et al.*, 1993). The importance of integrated quality and quantity management of our groundwater resources is widely recognized. The distinctive character of groundwater creates a level of uncertainty in management that significantly differs from, and in many ways exceeds, that encountered in surface water management. It is therefore crucial that groundwater management models are developed for South African aquifers.

Important questions for which answers are usually required are : How much water is required? When? Where? How much water is available? When? Where? Of what quality? At what costs? What are the risks associated? How many boreholes are needed? At what positions? At what rate must the boreholes be pumped to ensure no undesired effects? It quickly becomes obvious that answering even these basic questions raises the complexity of data and aquifer management.

The research in this report emanated from a project funded by the Water Research Commission entitled:

' The development of risk analysis and groundwater management techniques for Southern African aquifers ' :

This project was subdivided into two sections. The institution responsible for the specific sections are as follow:

Section 1 : The development of groundwater management models (IGS)
Section 2 : Risk analysis in groundwater management (EMATEK, CSIR)

The objectives of the study under the terms of reference were the development and implementation of management models, with special reference to:

(i) The development and implementation of an inverse-parameter program which will enhance the current calibration methods of groundwater flow and transport models.
(ii) The development and application of groundwater management models addressing:
- (a) groundwater allocation,
- (b) groundwater operation and
- (c) capacity expansion.

(iii) The development and application of a conjunctive ground- and surface water planning model.
(iv) The development and application of a groundwater quality management model.

The Steering Committee (May 1992 meeting) agreed that the principle of conjunctive use should receive attention without becoming the focal point of the research. It was also decided, that the emphasis of water quality modelling should concentrate on blending solutions as opposed to the more complicated mass transport models.

The estimation of natural groundwater recharge is regarded as one of the most important steps in the management of an aquifer. Techniques developed for this purpose during the project of Bredenkamp *et al.* (1993) on the estimation of natural groundwater recharge, were used in this study and were incorporated into the aquifer management package developed during this project.

1.2 PREVIOUS WORK

The project was initiated with a computerized literature search. This data was used to select a number of existing management models of which the source code is available. Program codes were subsequently requested from the respective authors.

Optimization methods have been used in groundwater management for over 25 years, with mixed success. Two types of management models have been developed during this time:

- o Lumped parameter models have been used to study economic and policy matters that involve ground-water resources. They do not normally consider the governing ground-water flow equations, but conceptualize aquifers with simple water mass balances. An example of such a model is discussed by Khepar and Chaturvedi (1982).

- o Distributed parameter models combine aquifer simulation with optimization methods and explicitly solve the partial differential equation that governs groundwater flow. These models have been used successfully to manage well fields, to evaluate efficient conjunctive use and to inspect the impacts of water resource policies upon the hydrogeology and economics of ground-water use.

The latter model will be discussed in some detail. According to Gorelick (1983), groundwater management models can be mainly divided into two groups:

- o Groundwater hydraulic management models.
- o Groundwater policy evaluation and allocation models.

The first category is aimed at managing groundwater stresses such as pumping and recharge. Stresses and hydraulic heads are treated directly as management variables.

The second category involves models that can be used to inspect economic interactions in water allocation, distribution, treatment and conjunctive use problems.

1.2.1 Groundwater Hydraulic Management Models

These models generally incorporate a simulation of a specific aquifer as constraints in the management model, i.e. management as well as aquifer simulation is accomplished simultaneously.

Two methods exist for achieving this:

- o The embedding method.
- o The response matrix method.

Embedding method

This method uses linear programming formulations that incorporate numerical approximations of the ground-water equations as constraints. This means that either the finite difference or finite element approximations of the governing flow equations are treated as part of the constraint set of the optimization model. This results in an extremely large constraint matrix (e.g. 1 000 nodes x 30 time steps with 30 000 decision variables). The advantages and disadvantages of the technique can be summarized as follows:

Advantage

- o Yields complete information regarding the aquifer behaviour.

Disadvantages

- o Yields a large constraint matrix, which may lead to numerical difficulties during the factorization of the banded matrices.
- o Yields redundant information, including redundant decision variables and constants.

This method has been mainly applied to steady state problems. However, the aquifer management problems which have been solved successfully using such models have been limited to a very small scale (Gorelick, 1983).

Two management models using the embedding technique were evaluated:

- o EMBED-PC by J. Jaime Gomez-Hernandex, C. Tiedeman & S.M. Gorelick (1990). This uses the finite difference model developed by Trescott *et al.* (1976) as the aquifer simulation model, while the mathematical optimization is done by MINOS.

- o A groundwater management model by W. Kinzelbach (1986). This model is based on a simple finite difference model, whereas the mathematical optimization is done by a two-phase simplex algorithm.

No further effort was spent on this technique due to the fact that these models are mainly applicable to steady state conditions, which rarely (if ever) occur in real life situations.

Response matrix method

This technique uses an external finite difference or finite element aquifer simulation model to develop unit responses. Each response describes the influence of a short event (e.g. pump, recharge) upon the hydraulic heads at the points of interest, which are generally the production and observation boreholes in the aquifer. In this case, an assemblage of unit responses is included in the management model as constraints. This yields a small system of equations, hence making large-scale problems more manageable. As in the case with the embedding method, this technique has advantages and disadvantages:

Advantages

- o Yields a small system of equations, such that additional constraints can be added, while redundancy is eliminated.
- o Efficient utilization of computer time.
- o Can handle large transient systems efficiently.
- o Easier to use with linear, mixed integer and quadratic programming techniques.

Disadvantage

- o Yields incomplete information regarding the functioning of a specific aquifer system. However, this aspect is difficult to confirm, since the results are verified and calibrated against existing data.

Van der Heijde *et al.* (1986) lists a summary of numerous ground-water management models, which are indexed at the International Ground Water Modelling Center. Eight models from this list were selected, covering:

- o ground-water operation and allocation models,
- o conjunctive use models

during a time-span of roughly 20 years. In most cases, only relevant publications were received from the respective authors of the selected models. This is perhaps an indication that although quite a number of management models were developed, only a few can be successfully applied in practice. It should, however, also be mentioned that a number of the selected models are quite old and have probably been replaced by newer updated versions, e.g. AQMAN (1987) or GWMAN (1985).

The response matrix method is generally also known as an algebraic technological function (ATF) as reported by Maddock (1972).

The ATF approach generates a unit response matrix by solving the simulation model several times, each with unit pumpage at a single pumping node. Superposition is used to determine total drawdowns. This yields a smaller optimization problem, but the method has a limitation. It is only applicable to a confined aquifer or an unconfined aquifer with relatively small drawdowns compared to the aquifer thickness. A drawdown correction method may be used with some acceptable accuracy for an unconfined aquifer with larger drawdowns. Considerable work stemmed from this approach including Maddock (1972, 1974), Maddock & Haimes (1975), Morel-Seytoux (1975), Morel-Seytoux & Daly (1975), Illangasekare & Morel-Seytoux (1982) and Willis (1984).

Another approach has been to combine the simulation model and optimization implicitly in such a way that the simulation model functions as a separate module. Gorelick *et al.* (1984) applied this method to an aquifer reclamation design to overcome the non-linearity incurred by the contaminant transport equations. However, the hydraulic response was handled by the ATF method.

In 1985, Wanakule & Mays developed a model (GWMAN) for determining optimal pumping and recharge for large-scale artesian and/or non-artesian aquifers. The model methodology was closely related to the approach used by Gorelick *et al.* (1984). The overall problem was viewed as one of discrete time optimal control, where variables describing the aquifer system were divided into system state (head) and control (pumpage). By expressing head as an implicit function of pumpage, the model constraints were conceptually eliminated, yielding a smaller reduced problem involving only the pumpage variables. Head bounds were incorporated into the objective using an augmented Lagrangian algorithm. The major contribution of their work was an analytic scheme to compute the reduced gradient needed for optimization. This requires the solution of a set of linear difference equations backwards in time, and has major speed and accuracy advantages over finite differencing. The advantage of this method is that it can overcome the non-linearity of an unconfined aquifer. The method is, however, only applicable if a finite difference simulation model is used. The mathematical optimization in this model was performed by a generalized reduced gradient routine GRG2 developed by Lasdon *et al.* (1978).

In 1987, Lefkoff & Gorelick developed an aquifer management model (AQMAN) that combines groundwater flow simulation with mathematical optimization, in order to develop and evaluate aquifer management strategies.

When AQMAN is used in conjunction with a mathematical programming code, the computer program identifies the pumping or recharge strategy that achieves a user's management objective, while maintaining groundwater hydraulic conditions within desired limits. The objective may be linear or quadratic, and may involve the minimization of pumping and recharge rates or of variable pumping costs. The problem may contain constraints on groundwater heads, gradients and velocities to simulate a complex, transient hydrologic system.

Linear superposition of solutions to the transient, two-dimensional groundwater flow equation is used by the computer program in conjunction with the response matrix method. A unit stress is applied at each decision well and transient responses at all control locations are computed using a modified version of the U.S. Geological Survey two-dimensional aquifer simulation model. The program also computes discounted cost coefficients for the objective function and accounts for transient aquifer conditions. Mathematical optimization is done by MINOS. This model has, as in the case of the older models, the limitation that non-linear constraints cannot be imposed, e.g. unconfined aquifer. However, in some cases it may be possible to linearize such systems if the drawdown is small compared to the saturated thickness of the aquifer, or by solving sequential linear problems, where the saturated thickness is given by the last iterate (Danskin & Gorelick, 1985).

A major problem of many of the models is that they either use computer based optimization routines which are unavailable and computer dependent, or they use optimization packages which are subject to stringent licence agreements, e.g. MINOS. Optimization algorithms (linear and quadratic) obtained from Kuester & Mize (1973), will be used to solve ground-water management options in this report.

Simulation and inverse parameter models used by the Institute for Ground-water Studies and the DWA&F, are based on the finite element technique. Bearing the above-mentioned in mind, the Institute for Groundwater Studies has developed a management module which is based on the AQMAN model of the USGS and will be discussed in more detail in Chapter 5.

Conjunctive use models

Two conjunctive use models were obtained from Peters and Morel-Seytoux (1980) and Haimes (1973). However, both models are based on a stream - aquifer relationship, which is not applicable within the South African context.

1.2.2 Ground-Water Policy Evaluation And Allocation Models

These models are valuable for problems where hydraulic management is not the sole concern of the water planner. They are applied to water allocation problems which include economic management considerations. According to Gorelick (1988), three types of models have been developed for groundwater policy evaluation and allocation:

- Hydraulic economic response models (e.g. AQMAN).
- Linked simulation optimization models (e.g. GWMAN).
- Hierarchical models.

It is evident from the literature, that the above models are generally combined in the hydraulic management model by means of user-definable objective functions.

1.3 WATER RESOURCES MANAGEMENT

Water resources management means intervention in matters concerning water, which may be the planning, design and operation of hydraulic works. It presupposes that an authority exists powerful enough to impose decisions upon individuals or, at least, to influence people's behavior.

Hall and Dracup (1970) grouped the many different objectives of water resources management into three fundamental classes:

- conserve and control the water resources of an area, so as to provide for protection against consequences of excesses or deficiencies in quantity or quality;
- provide or maintain water in such places and times, and according to the individual quantity and quality requirements; and
- minimizing the expenditures involved in accomplishing all of the above.

Water resources management takes care of these objectives in a co-ordinated way in order to maximize the overall net benefit that can be obtained from the water resources of the area considered.

Water resources management consists of a number of steps, carried out in succession (Van der Gun, 1988):

- assessment of the water resources system;
- assessment of water demands/requirements;
- identification of water resources management problems, objectives, constraints and uncertainties;
- development and analysis of alternative strategies and instruments of implementation;
- decision-making
- preparing for implementation (legislation, organization, planning);
- implementation; and
- monitoring (water availability, quality, demands, supply).

1.4 SYSTEMS APPROACH IN WATER RESOURCES

The systems approach to solve a real-world problem is a logically coherent and chronologically ordered sequence of steps and procedures which lead from the formulation of the problem to its final solution. The way each step is performed is subject to choice. However, a basic feature of the systems approach is the use of mathematical or logical functions which describe the structure of a system and the interactions between its components, models for predicting outcomes of activities, models for evaluating and screening proposed alternatives and, in some instances even models for selecting the alternative which is best for satisfying the desired objective.

A list of common terms in system theory comprises systems, subsystems, interrelations, inputs, outputs, system parameters, state and state variables, feedback, closed and open systems, decision variables, objectives and criteria, objective function, constraints and optimization.

Planning for water resources development involves seeking optimum designs (Stephenson and Peterson, 1991). The most efficient design of a structure is usually that which achieves the objectives in the most economic manner. In water resources planning, the cheapest design is not always the optimum, since other factors such as employment, encouragement of industries and environmental impacts must be considered.

Optimization theory has become a routine technique in water resources management. Of the optimization techniques, linear programming has been the most used technique in linear systems. The permanent process of making decisions about actions to be taken upon a system in order to achieve desired objectives with limited resources has certain premises (Bachmat, 1992):

The system must be well-defined with objectives that are definite and feasible.
There are controllable means which can affect the behaviour of the system.
There are decision-makers who select and implement these means with the help of their power and resources.
There are definite cause-effect relationships between the decisions and their outcomes.
Decision outcomes can be valued and ranked on the basis of well-defined measures of success in pursuing the objectives and their relative priorities.

1.5 OBJECTIVES, CRITERIA AND CONSTRAINTS IN WATER RESOURCES MANAGEMENT

All human activities related to water have specific objectives, being either economic, social, recreational or aesthetic. The general objective is to maximize the net benefit that may be obtained from the water resources system.

Specific objectives have to be defined for practical applications. Each of the objectives should be accompanied by a criterion or objective function, that allows the quantitative comparison of the predicted or achieved effects under alternative water resources management strategies. An optimal approach is the one that maximizes or minimizes the objective function considered.

Usually, constraints have to be taken into consideration when developing appropriate water resources management strategies. These constraints are boundary conditions to the problem to be solved and thus limit the field of feasible solutions. Some of the constraints are absolute or physical constraints (e.g. the amount of water present in the aquifer), while other may be less absolute and are indicating the limits imposed by decision-makers (e.g. economic-technical constraints)

Table 1.1 lists some of the more common water resources management objectives, together with corresponding criteria as have been suggested by Delft Hydraulics (1987).

Table 1.1: A number of specific objectives and criteria in water resources management **(After Delft Hydraulics, 1987).**

Objective	Criteria
1. Economic efficiency	Benefit/cost ratio
	Present value of net benefits
	Internal rate of return
2. Water use efficiency	Physical losses of water
3. Improve equity	Variability of net incomes (range, variance or other parameters)
	Shares of population classes in the benefits from groundwater
4. Improve employment	Number of jobs
	Unemployment percentage
5. Improve health	Water supply per capita per day
	Occurrence of water-borne diseases
	Water supply as a percentage of demand
	Coverage of sewerage and other sanitary facilities
6. Increase agricultural production	Monetary value of crops
	Production as a percentage of local demand
7. Environmental conservation	Annual rate of erosion
	Difference between observed water quality concentrations and standards
	Number of valuable species (flora and fauna)
8. Risk minimization	Probability of under accomplishment or failure in achieving engineering goals
9. Sustained development	Long-term picture of economic output

CHAPTER 2

APPROACHES AND SOLUTIONS IN GROUNDWATER MANAGEMENT

2.1 YIELD ANALYSIS

The objective of many groundwater resource studies is the determination of how much water is available for pumping, i.e. determination of the maximum possible pumping compatible with stability of the groundwater supply.

The term, safe yield, is an indication of this maximum use rate. According to Domenico (1972), Lee (1915) first defined safe yield as

> ... the limit to the quantity of water which can be withdrawn regularly and permanently without dangerous depletion of the storage reservoir.

Meinzer (1923) defined safe yield as

> ... the rate at which water can be withdrawn from an aquifer for human use without depleting the supply to the extent that withdrawal at this rate is no longer economically feasible.

Conking (1946) expanded on Meinzer's definitions and described safe yield as an annual extraction of water which does not:

(i) exceed average annual recharge,
(ii) lower the water table so that the permissible cost of pumping is exceeded and
(iii) lower the water table so as to permit intrusion of water of undesirable quality.

Todd's (1959) compact definition of safe yield as

> ... the amount of water which can be withdrawn from a groundwater basin annually without producing an undesired result

does not clarify what the term undesired result means. Thomas (1951) and Kazmann (1956) have suggested abandonment of the term because of its vagueness.

Methods of determining safe yield include (i) the Hill method, which is merely a plot of annual pumping versus average water-level change, allowing identification of the pumping draft associated with zero water-level change, (ii) the Harding method, which is a plot of retained flow (subsurface inflow minus outflow) versus average water-level change, the zero-change in water level again designating safe yield and (iii) the zero water-level change method, which is based on the premise that if the groundwater storage elevation is the same at the beginning and end of a long period of pumping, the average net draft over this period is an estimate of safe yield (Domenico, 1972).

Safe yield has no unique or constant value and is time-dependant depending on the spacing and location of wells and their influence on the dynamics of interchange between groundwater and other elements of the hydrologic cycle. In concept, the idea of safe yield encompasses a great deal more and is considerable more sophisticated than the methods proposed to ascertain its value.

In an effort to remove some of the ambiguity in the meaning of the term safe yield, the Committee on Groundwater of the American Society of Civil Engineers (1961), introduced four concepts of yield:

1. Maximum sustained yield is the maximum rate at which water can be withdrawn perennially from a particular source.
2. Permissive sustained yield is the maximum rate at which water can economically and legally be withdrawn perennially from a particular source for beneficial purposes without bringing about some undesired results.
3. Maximum mining yield is the total volume of water in storage that can be extracted and utilized.
4. Permissive mining yield is the maximum volume of water in storage than can economically and legally be extracted and used for beneficial purposes, without bringing about some undesired results.

Sustainable yield represents a groundwater abstraction rate that allows a long-term input and outputs of water to be balanced over the domain of the aquifer, thus leading to a stable state of the aquifer. Maximum sustainable yield is not equal to the aquifer's recharge: sustainable yield is the groundwater capture, which is the difference between recharge and natural discharge and consequently, maximum sustainable yield is the maximum capture attainable, which is often considerably less than the average recharge.

2.2 ESTIMATION OF NATURAL GROUNDWATER RECHARGE

Kirchner *et al.* (1991), showed that the groundwater balance method (SVF-method: saturated volume fluctuations) is the only method which yielded reliable estimates of the groundwater recharge in the Karoo aquifers of South Africa. The SVF-method also forms part of a WRC-project which is concerned with the preparation of a manual on quantitative estimation of groundwater recharge and aquifer storativity undertaken by Bredenkamp *et al.*(1993).

Judicious management of the groundwater resources of the RSA and other areas with a semi-arid climate requires an assessment of not only the recoverable resources, but also on the reliable evaluation of the recharge. Because the estimation of recharge and storativity of an aquifer is of the utmost importance to the management of an aquifer, the reader is referred to the manual by Bredenkamp *et al.* (1993). Some important findings of the report by Bredenkamp *et al.* (1993) can be summarized as follows:

- The cumulative rainfall departure method, together with the SVF-method, is the most valuable tool in the assessment of the natural groundwater recharge of an aquifer.
- The Cl-profiling method can be a valuable method to obtain a first approximation of recharge.
- The numerical modelling of an aquifer may be premature, unless extensive monitoring of water levels and abstraction rates were performed for a number of years.
- Care must be taken in obtaining the S-value from pumping tests in fractured rock aquifers. They showed that the S-value obtained from pumping tests is a function of the distance between the observation and abstraction borehole (the greater the distance, the smaller the estimated S-value).

2.3 GROUNDWATER MANAGEMENT MODELS

Models that solve the governing groundwater flow or solute transport equations in conjunction with optimization techniques, such as linear and quadratic programming, are powerful aquifer management tools. Groundwater management models fall in two general categories: hydraulic and policy evaluation and water allocation. Groundwater hydraulic management models enable the determination of optimal locations and pumping rates of numerous wells under a variety of restrictions placed upon local drawdown, hydraulic gradients and water production targets. Groundwater policy evaluation and allocation models can be used to study the influence of institutional policies such as taxes and quotas upon regional groundwater use.

Different spatial patterns of wells may be chosen given a certain demand to be satisfied from a specific aquifer. Some of these patterns are more favorable than others. The problem is to find an optimal spatial distribution of abstraction wells. Depending on the details of the problem and the management objectives chosen, the analysis will focus either on optimization (if the decision process is complex) or on simulation (if the system's behavior is complex) or on a combination of both. The allocation of water to users, depends very much on the management objectives which has to be consistent with prevailing rights.

Furthermore, optimal management decisions aim to maximize the net discounted benefits from allocation of the groundwater supplies over a planning horizon, while minimizing effects and water quality problems between production boreholes. In order to achieve this, management models that are based on the so-called response matrix equations can be used. Programs which combine simulation and optimization are best suited to this task.

The key behind the response matrix method is that since the aquifer is described by a system of linear equations, the influence of each source or sink may be calculated separately and then superposed to compute the complete distribution of stresses over space and time under any pumping schedule.

The response matrix is an assemblage of coefficients, each of which relates pumping at one *location* to drawdown at another (Gorelick, 1987). Suppose, we have a pumping system consisting of n wells distributed in any manner over the aquifer. Hydrodynamic laws for a porous medium show that the drawdowns s_i occurring at every point i of the aquifer due to pumping Q_j ,(j=1,------,n) are linear functions of these yields:

$$s_i = \sum_{j=1}^{n} a_{ij} Q_j$$

The coefficient a_{ij} is the drawdown at point i, due to a unit abstraction rate at well j and is called the response matrix.

2.4 PROBLEM FORMULATION FOR LINEAR PROGRAMMING

The process of solving a linear or quadratic programming problem begins with the formulation of a management problem as a mathematical model. This problem formulation is certainly the most important (and difficult) part of management modelling. A management model consists of an objective (goal) which is to be minimized or maximized, and a series of linear constraints (restrictions) that must be obeyed. Decision variables (e.g. pumping rates) are the unknown quantities of concern that can be controlled in a managed system.

This will be illustrated with an example of problem formulation: Obtain the abstraction rate at n pumping locations so that the sum of these yields will be maximal and so that the drawdown at m control points will be equal or less than a fixed value.

Objective function: Maximize $F = \sum_{j=1}^{n} Q_j$

subject to m+n constraints:

	$a_{ij}Q_j \leq s_{max}$	(m inequalities)
and	$\mathbf{U}Q_j \leq Q_j^{max}$	(n inequalities)

where **U** being a square unit matrix of order n and Q_j^{max} = maximum capacity of well j.

For the optimization phase, any general linear programming program can be used (usually the simplex routine is used).

2.5 CONJUNCTIVE MANAGEMENT OF GROUNDWATER AND SURFACE WATER

Ground- and surface water tend to be strongly interrelated, in the sense that groundwater may feed surface water bodies and vice versa. Variations of flow, storage or quality of water in one of the subsystems may directly affect the state of the other. The availability of both surface- and groundwater in an area opens the possibility of conjunctive use of these two water sources. This approach may have distinct advantages over the isolated exploitation of the individual resources, especially, under conditions of scarcity.

2.6 GROUNDWATER QUALITY MANAGEMENT

Protection of our aquifers against pollution has become a major concern during recent years, as more and more cases of severe contamination come to light. Braune and Hodgson (1991) proposed the integration of all water resources aspects in ground water quality management. These aspects include land resources and the water quality and quantity within a catchment. According to these researchers, strategies and approaches for groundwater protection should include product controls, contaminant source controls and water resource protection. Contaminant clean-up is the least desirable control measure because of its complexity, the high costs involved and the long-term commitment required.

Three broad objectives have emerged for water quality management in South Africa (Braune and Hodgson, 1991). These are:

- The preservation of the water environment, so that water is of an acceptable quality for industrial, urban, agricultural and recreational use and for the propagation of the fish and wildlife species that could reasonably be expected in a particular environment. The protection of human health if of particular importance.
- The objective is not purely conservation, but rather the optimal management of all resources to obtain the maximum net benefit;
- In keeping with international practice, the principle that the polluter pays for the abatement of the pollution, is adopted.

A number of minimum requirement studies for (i) the monitoring of water at waste management facilities, (ii) handling of hazardous waste and (iii) waste disposal facilities are currently being undertaken by the DWA&F.

The use of a groundwater mass transport model to study the transport of solutes in groundwater is a valuable method. However, due to the complexity of our fractured aquifers, it was decided by the Steering Committee that the present study must be focused on a blending type of solution. Nevertheless, a mass transport and inverse model are included in the program structure of the present study, although no real-world case study is presented.

2.7 TWO- AND THREE-DIMENSIONAL MODELS

In general, flow through a porous medium is three-dimensional. However, since the geometry of most aquifers is such that they are thin relative to their horizontal dimensions (i.e. tens or hundreds of metres, as compared to thousands of metres), a simpler two-dimensional approach, can be introduced (Bear and Verruijt, 1987). According to this approach, it is assumed that flow throughout the aquifer is essentially horizontal, or that it may be approximated as such, neglecting vertical flow components. Actually, the two-dimensional approach does not totally neglect vertical flow components, as the balance equations do take the effect of vertical flow into account. The use of the two-dimensional approach greatly simplifies the mathematical analysis and the error introduced by this assumption is small in most cases of practical interest (Bear and Verruijt, 1987).

Verwey and Botha (1993) are of the opinion that the two-dimensional approach cannot provide a reliable framework for the efficient management of an aquifer. The reason for this statement, is not surprising if the hypothetical aquifer they used to compare two- and three-dimensional models, is considered. They used a two-layered aquifer of 40 meter thickness and compared results of the 2- and 3-D models at a distance of 11 m from the abstraction borehole. Clearly, at this distance, vertical flow components cannot be neglected and the two models would not produce the same results. However, even in this example, the results of the 2- and 3-D models were exactly the same after a period of four days of pumping. However, it is clear from their analysis that on a pumping test scale, a two-dimensional model could produce unreliable results. The same holds true for modelling pollution on a small scale in a heterogeneous aquifer.

Nonetheless, it was decided to opt for a two-dimensional approach during the present study, since there is insufficient three-dimensional data for South African aquifers. It is therefore currently impossible to utilize three dimensional models for management purposes. However, the techniques which were developed during this study, could be easily incorporated into a three-dimensional model should it be required for a specific aquifer analysis.

A well-calibrated flow model is essential for the hydraulic management of an aquifer and the automated inverse-parameter technique plays an important role in enhancing the calibration of a flow or transport model.

2.8 RISK ANALYSIS

The purpose of risk analysis is to identify and measure the risks associated with exploitation of groundwater resources, and hence develop optimal management policies. Either deterministic or stochastic methods can be used to evaluate the risk associated with an aquifer management policy. Quantities which are usually modelled stochastically in aquifer studies are the transmissivities, storativities and recharge.

Risk analysis associated with the management of an aquifer, is undertaken by EMATEK (CSIR) as part of the present study and is presented in Section 2.

2.9 DATA REQUIREMENTS FOR GROUNDWATER MANAGEMENT

It is obvious that better information leads to better decisions. It follows then that the purpose of research, modelling, analysis and data collection is to produce information to make better decisions. It is easy to acknowledge the value of information in decision-making. For example, one may consider a military commander before an attack; without good intelligence about both the friendly and opposing forces, the commander cannot make an effective decision. If the commander delays the attack too long to gather additional information, the opportunity for a victory may be lost. What the commander needs is access to the right amount of accurate information at the right time. This illustrates the need for the following aspects of decision information: amount, quality, timeliness and clarity; all of which are important in planning.

The quantity and quality of data collection is a function of cost. The behavior of both the supply and demand of water operations is stochastic, meaning that it varies in time and place. Data is therefore necessary to provide realistic estimates of the design and planning parameters. This will in turn dictate the investment decisions and the commitment of finances. In other words, the greater the financial investment the greater the need for accurate information.

The greater the environmental and political consequences or financial risk of a specific water management strategy, the more diverse the data requirements become. A variety of socio-economic and environmental information becomes crucial to the decision making process. However, even hydrologic data may amount to a large amount of information. We might begin by asking simple questions. How much water is required? When? Where? How much water is available? When? Where? Of what quality? At what costs? It quickly becomes obvious that answering even these basic questions raises the complexity of data management.

It is obvious that data is essential for all kinds of decision-making and that data reduce uncertainty. A great deal of information is currently available from the National Groundwater Data Base.

It is impossible to develop an adequate groundwater management strategy for a certain region without a good knowledge of the aquifers that are present, their properties, state and exploitation, their economic and social significance, their potential and limitations in view of the regions' development, etc. In other words, assessment of the groundwater resources of an area is a first step in the process of strategy development. The assessment should be rather detailed and includes items such as:

- Physical framework: Hydrogeologic maps showing boundaries of all aquifers and non-water bearing rocks; topographic maps; water-table maps; bedrock configuration; transmissivity and storativity maps.
- Hydrologic stresses: Type and extent of recharge areas; groundwater pumpage (distribution in time and space); precipitation.
- Model calibration: Water-level change maps and hydrographs; history of pumping rates and distribution of pumpage; rainfall and recharge.
- Prediction and optimization analysis: Economic information on water supply and demand; legal and administrative rules; environmental factors and other social considerations.

Without a reliable amount of geohydrological and other relevant data available for an aquifer, the development of a groundwater management model for the area may be premature and beyond reach at this stage.

CHAPTER 3

INVERSE METHODS OF DETERMINING AQUIFER PARAMETERS

3.1 INTRODUCTION

The problem of parameter identification in distributed parameter systems has been studied extensively during the last two decades (Yeh, 1986). The term "distributed parameter system" implies that the response of the system is governed by a partial differential equation and parameters imbedded in the equation are spatially dependent. The inverse problem of parameter identification concerns the optimal determination of the parameters (say T and S) by observing the dependent variable (water levels) collected in the spatial and time domains.

Perhaps the simplest method of parameter identification is trial-and-error. In this method, a subjective set of parameters is first related and inserted into the aquifer model, together with the known past history of excitation (pumping, etc.). Usually, the selection of values of aquifer parameters is based on geological and geohydrological information. The calculated response of the aquifer model (water levels) are then compared to the actual historical data observed in the field. The procedure is repeated for different sets of the considered parameters until the calculated and actual water levels match. The main disadvantage of the trial-and-error technique is that it does not involve any algorithm for approaching the "best" solution in a systematic way. A lot depends on the skill and experience of the modeller. Because of the non-uniqueness of the inverse solution, it is not really known if the optimal estimate has really been attained (Bear, 1978). The trial-and-error method is recognized to be labour intensive and expensive, frustrating and subjective. To obtain a "good" fit between actual and simulated water levels, may require many repeated computer runs. For example, it took three months and 110 model runs to calibrate the Grootfontein Aquifer in the western-Transvaal (Van Rensburg, 1985) with the trial-and-error method.

The above considerations led researchers to seek methods for obtaining, in a systematic and rational way, the parameters which will lead to the best fit between observed and predicted aquifer responses. In most inverse methods currently in use, an optimal set of aquifer parameters is obtained by minimizing some error criterion. Neuman (1973) classified the techniques as either direct or indirect. The direct approach treats the model parameters as dependent variables in a formal inverse boundary value problem, whereas the indirect approach is based on an output error criterion where an existing estimate of the parameters is iteratively improved, until the model response is sufficiently close to that of the measured output. In practice, spatial variables are approximated by a finite difference or finite element scheme, where the aquifer system is subdivided into several sub regions with each sub region characterized by a constant parameter (Yeh, 1986). The reduction of the number of parameters from the infinite dimension to a finite dimensional form is called parameterization.

In essence, the indirect approach is an automated version of the manual trial-and-error calibration procedure. Although more expensive than direct techniques, indirect methods lead to better solutions than the former; this is due to the fact that whether done consciously or unconsciously, least square methods based on head residuals are always capable of filtering out some of the noise in the head data, an accomplishment which the direct method is unable to match (Carrera & Neuman, 1986). However, the minimization of a criterion based solely on head residuals is usually insufficient to guarantee a stable and unique solution.

Attempts to automate the calibration process have only been marginally successful according to Carrera & Neuman (1986). The primary reason for this lacklustre performance of automated methods is failure to recognize, or treat adequately, the three major perils of parameter estimation : non-identifiability, non-uniqueness and instability.

According to Neuman & Yakowitz (1979), two major sources of difficulties must be overcome in order to solve the inverse problem. The first difficulty arises from the extremely high sensitivity of transmissivity estimates to noise in water-level data. The reason for this high sensitivity stems from the fact that the noise terms appear in the governing equations as derivatives. It is well-known that errors in derivatives of noisy data can be arbitrarily large; no matter how well the data is approximated. The second source of difficulty stems from the fact that even in the hypothetical case, where the data is precise, the solution to the inverse problem may nevertheless be non-unique. This may be caused by insufficient information about lateral flow rates or by the lack of sufficiently large hydraulic gradients in some parts of the aquifer (Neuman, 1973). Both of these difficulties manifest themselves by similar symptoms and are therefore difficult to isolate. The most common symptom is for the transmissivities to exhibit spatial oscillations whose frequency and amplitude are higher than those anticipated on physical grounds. When this happens, the solution to the inverse problem is said to be "unstable".

Neuman (1973) suggested that the problem of instability in the solution of the inverse problem can be reduced by including prior information on the transmissivities . As has been shown by Gavalas *et al.* (1979) and Shah *et al.* (1978), this is indeed the case, in the sense that it usually leads to a smaller variance for the error of estimation.

Chavent (1974) studied the uniqueness problem with regard to parameter identification in distributed parameter systems. Chavent pointed out that the uniqueness problem has a great practical importance, because in the case of non-uniqueness, the identified parameters will differ according to the initial estimates of the parameters, and there will be no reason for the estimated parameters to be close to the true parameters. In a distributed parameter system, if only point measurements are available, the inverse problem is always non-unique. The uniqueness problem is intimately related to identifiability, which addresses the question of whether it is at all possible to obtain unique solutions of the inverse problem for unknown parameters of interest in a mathematical model, from data collected in the spatial and time domains.

Most inverse models are based on the zonation approach (e.g. Emsellem & De Marsily, 1971, and Cooley, 1977). Other inverse models deal with interpolation methods. Often the transmissivity shows a high degree of spatial correlation, and the geostatistical structure has been used in estimation models based on kriging (e.g. Kitanidis & Vomvoris, 1983; Hoeksema & Kitanidis, 1984 and De Marsily *et al.*, 1984). The incorporation of a certain spatial structure, together with field observations in the model, increases the stability of the inverse solution. Neuman and Yakowitz (1979) incorporated prior information about the transmissivity into the method of zonation.

As stated earlier in parameter estimation in ground-water modelling, measured piezometer heads are compared to calculate ones on the basis of a least square criterion, and the best model parameters being determined by a minimum of the least square objective function.

3.2 INVERSE SOLUTION METHODS

For modelling purposes, the objective is to determine T (x,y) and S (x,y) from a limited number of observations of h (x,y,t) scattered in the field, so that a certain criterion is optimized. In the classical least square criterion, the objective function to be minimized is (Yeh, 1986):

$$\min_{T(x,y);S(x,y)} J = [h - h^*]^T [h - h^*]$$

where h = vector of observed heads at observation boreholes

h^* = vector of calculated heads at observation boreholes

For identification purposes, T and S can be parameterized by either a zonation or interpolation method.

In practice, usually the Gauss-Newton or modified algorithm is used for minimization, and is proven to be very effective. The rate of convergence of the Gauss-Newton method is superior when compared to classical gradient searching procedures. Constraints, such as lower and upper bounds, are usually incorporated in the algorithm.

The Marquardt algorithm (Levenberg, 1944 and Marquardt, 1963) is a very popular modification of the Gauss-Newton method, and is used in many inverse computer codes for ground-water flow (e.g. Medina *et al.*, 1990; Kauffmann & Kinzelbach, 1989 and Keidser *et al.*, 1990). The procedure is an extension of the Gauss-Newton method to allow for convergence with relatively poor starting guesses for the unknown parameters. In this method, the Gauss-Newton normal equations are modified by adding a factor λ (Kuester & Mize, 1973):

$$[A^T A + \lambda I]\Delta x = A^T (h - h^*)$$

where λ = value added to the increment

I = identity matrix

A = matrix of partial differential coefficients

Δx = increment towards the minimum

It can be shown that when λ approaches ∞ , the Marquardt method is identical to Steepest Descent. When λ equals 0, the technique reduces to Gauss-Newton. In general, a Steepest Descent procedure would be expected to converge for poor starting values, but requires a lengthy solution time. Gauss-Newton, on the other hand, will converge rapidly for good starting estimates. Thus in the Marquardt procedures, the initial value of λ is large and will decrease toward zero as the optimum is approached.

3.3 DATA REQUIRED TO DEVELOP A GROUNDWATER MODEL

The first phase of a groundwater model study consists of collecting all existing geological and hydrological data on the groundwater system in question. This will include information on surface and subsurface geology, water tables, precipitation, pumped abstractions, aquifer characteristics, aquifer boundaries and groundwater quality. If such data do not exist or are very scanty, a program of field work must first be undertaken, for no model whatsoever makes any hydrological sense if it is not based on a rational hydrogeological conception of the system (Boonstra & De Ridder, 1990). All the information is then used to develop a conceptual model of the aquifer.

Developing and testing a numerical model, requires a set of quantitative hydrogeological data that fall into two categories:

- data that defines the physical framework of the aquifer and
- data that describe its hydrological stress.

Boonstra & De Ridder (1990) listed the data required to develop a ground-water model:

Physical framework	**Hydrological stress**
1. Topography	1. Water-table elevation with time
2. Geology	2. Type and extent of recharge areas
3. Type of aquifer	3. Rate of recharge
4. Aquifer thickness and lateral extent	4. Type and extent of discharge areas

5. Aquifer boundaries
6. Lithological variations within the aquifer
7. Aquifer characteristics

5. Rate of discharge

3.4 CALIBRATION OF THE GROUND-WATER MODEL

Before a model can perform its task of predicting the future water-table behaviour, it must be calibrated, which means that a check must be made to see whether the model can correctly generate the past behaviour of the water table, as it is known from historical records. Calibration is the most difficult part of ground-water modelling and requires great skill and team-work.

Useful comments on calibration

1) Selecting a period in which the water table shows a recession, for which historical records are available. (A period of recession is necessary because it is nearly impossible to calibrate for S and recharge simultaneously.) If, however, the recharge rate is known for each month, any historical period can be selected. Obviously, the longer the period used for calibration, the better the results will be.

2) Many modellers (e.g. Kauffmann & Kinzelbach, 1989) calibrate the flow model, both in steady and non-steady state, by starting with the steady state case. In steady state, the computed heads are compared to the measured and time average areal head distribution. It is important to note that for steady state simulations at least one constant head node must be used. The steady state solution is only dependent on the areal T-distribution, discharge and recharge rates and boundary conditions (and not on the areal S-distribution). The areal T-distribution obtained from the steady state calibration is then used in the non-steady case to calibrate for the S-distribution in the aquifer.

3) It is important to supply the inverse program (AQUA-INV) with realistic upper and lower bounds on T and S for each zone. To be able to do this, a sound judgement of the parameters in question is necessary, and a possible range of errors in the T- and S-values must be provided. It may happen that some of the calculated water-table elevations cannot be matched with the historical ones, even if the values of the parameters have been varied up to their maximum error percentage. Possible reasons for this include:

   (i) Wrong zonation of the aquifer.
   (ii) Wrong boundary conditions
   (iii) Wrong recharge estimates.
   (iv) Wrong input parameters such as abstraction rates, type of aquifer, etc.

   Faced with the situation that the above-mentioned factors are correct, one has no alternative but to return to the field for additional measurements.

4) Proper model calibration depends above all on the integrity of the modeller. It is always possible to "calibrate" a model if one has a free hand in changing the input parameters and disregards the maximum error limits. But then, one is not calibrating the model, but is merely playing with it, which is a dangerous game (Boonstra & De Ridder, 1990).

5) Although not necessary, it is advisable to verify the model after it is calibrated. For verification purposes, another historical period is chosen (preferably the period succeeding the calibration period) and the flow model is run for this period. If the fitting between the simulated and actual water-level responses is still "good", the model is regarded to be suitable for future applications.

6) It is important to realize that the parameters that give a good match with the observed data, however, may not be the real parameters for the aquifer (Khan, 1980). Because of this, Labadie (1975) named the calibrated parameters surrogate parameters. It is obvious that to get a suitable model to predict the future behaviour of the aquifer, the surrogate parameters must reflect, as closely as possible, the underlying physical structure of the aquifer.

7) Theory, as well as practical experience (De Marsily, 1978), suggests that in dealing with the inverse problem, it is advisable to work in terms of log transmissivities instead of transmissivities, i.e. $Y = \log T$. If T is log normal, then it can be shown that if a subregion (zone) of the aquifer is made small enough so that the hydraulic gradient over it stays more or less uniform, the transmissivity of the zone can be represented by an effective T-value which is the geometric mean of T_i (Neuman, 1980). This implies that the effective log transmissivity of the zone is simply the arithmetic mean of Y_i.

8) To obtain realistic initial groundwater levels, it is recommended that the Bayes method of interpolation (Program TRIPOL developed at IGS, 1993) must be used to obtain these values. In the Bayes method, the topographic values are used as qualified guesses for the water levels. A number of case studies in South Africa showed that the topography mimics the water levels very closely.

CHAPTER 4

GROUNDWATER MANAGEMENT MODELS AND DECISION ANALYSIS

4.1 INTRODUCTION

The management of groundwater resources involves the allocation of groundwater supplies and water quality to competing water demands and uses. The resource allocation problem is often characterized by conflicting objectives and complex hydrologic, environmental and economic constraints. The development of mathematical simulation models in the early 1970's provided groundwater planners with quantitative techniques for analysing alternative groundwater pumping or recharge schedules and identifying the probable environmental impacts associated with subsurface waste disposal. Costs and benefits could, in principle, be developed for each planning, design or management alternative and optimal control schedules could be developed for the groundwater system.

Although simulation models provide the resource planner with important tools for managing the groundwater system, the predictive models do not identify the optimal groundwater development, design or operational policies for an aquifer system. Instead, the simulation models provide only localized information regarding the response of the groundwater system to pumping and/or artificial recharge. In contrast, groundwater optimization models can identify the optimal groundwater planning or design alternatives in the context of the system's objectives and constraints.

4.2 APPROACH AND PHILOSOPHY (from Willis and Yeh, 1987)

The planning problems associated with the management of groundwater supply are:

- the determination of the optimal pumping pattern,
- the timing and staging of well-field development and
- the design of surface storage and transport facilities to distribute the groundwater supply to the water demands.

The optimal management decisions maximize the net discounted benefits from allocating the groundwater supplies over a design or planning horizon, while minimizing interference effects between the wells in the system, land subsidence, salt water intrusion or other water quality problems.

The physical basis of these planning models are the hydrodynamic response equations of the groundwater system. These response equations, which relate the state and decision variables, are found either through the analytical or numerical solution of the groundwater system's equations.

The response equations have a dual role in groundwater planning, because hydraulic equations can be used for simulation or optimization of the groundwater system. In simulation the response equations are used to predict
1) the hydraulic or water quality response of the aquifer system to a set of pumping or recharge schedules and
2) the probable hydrologic and environmental impacts associated with groundwater or conjunctive use.
The simulation approach, because it can consider only a limited number of alternatives, generally does not identify the optimal pumping schedules in the context of all the objectives of the planning or design problem.

In contrast, however, optimization models develop optimal planning, design, and operational policies for the groundwater system. Because the response equations can be incorporated into the management models - the same equations that would normally be used for simulation - the optimal decisions define not only the optimal pumping and recharge schedules, but also predict the time and spatial variation in the hydraulic head. As a result, the response equation method combines simulation and optimization in a single management model.

4.3 CATEGORIES OF GROUNDWATER MANAGEMENT MODELS

Groundwater management models can generally be divided into three groups:

(i) Water supply and demand models.
(ii) Conjunctive use models.
(iii) Water quality models.

The models in each of the respective groups can then be further classified into two more categories, namely:

- Hydraulic management models.
- Policy evaluation and allocation models.

In this report attention will mainly be focused on the groundwater supply and demand models. These models will be discussed in terms of hydraulic management and policy evaluation and allocation.

4.4 GROUNDWATER SUPPLY AND DEMAND MODELS.

Groundwater supply and demand models can be divided into three categories:

- Groundwater operational models.

- Groundwater allocation models.
- Capacity expansion models.

Inspection of the above models in more detail reveals that they basically consist of two components, being:

(i) a physical (hydraulic) component and
(ii) an economic component

The physical (hydraulic) component is performed by hydraulic management models which:

- manage groundwater stresses (e.g. pumping and recharge) and
- treat stresses and hydraulic heads directly as management model decision variables.

The economic component is accomplished by means of policy evaluation and allocation models when:

- water allocation problems entail economic management objectives
- hydraulic management is not the sole concern.

4.4.1 Groundwater Operational Models

An important class of groundwater planning problems involves the determination of optimal operational (water extraction) schedules of a groundwater system. The planning problem is to determine how the existing well-field should be operated over the entire planning horizon to satisfy a water demand. The objective of these types of planning models is to minimize the total discounted operational costs for extraction of the resource.

The operational planning problem can be formulated as a non-linear optimization model which is constrained by:

i) the water target requirements,
ii) the well capacity restriction and
iii) the response equations of the groundwater system.

The planning model can incorporate additional restrictions or objectives to reflect different management strategies or environmental constraints (e.g. salt water intrusion in a coastal aquifer).

It is also important to consider the possible impacts of parameter and economic uncertainty on the optimal planning policies. The dual variables associated with the optimal solution of the planning models provide a partial answer to this problem. The dual variables represent the marginal change in the system objectives for a unit relaxation in the constraints of the model. In terms of the water targets of the system, the dual variables or shadow prices indicate how much the system costs are increased (decreased) by increasing (decreasing) the water demand. Similarly, if, for example, the state variables are strictly binding at certain control locations within the aquifer, the shadow prices will determine how the objective will change as these constraints are relaxed. This information is an indication of the economic and hydrologic trade-offs occurring in the basin.

The possible effects of parameter uncertainty on the optimal planning policies are a more difficult problem to assess. The parameters of the aquifer system are often random variables and, under certain conditions, the direct water demands may form a stochastic process. If it is possible to characterize the underlying distributions of these variables, then Monte Carlo optimization methods or stochastic dynamic programming can be used to determine probabilistic operating rules for the groundwater system.

4.4.2 Groundwater Allocation Models

From an economic perspective, the management of groundwater is a problem in resource allocation. The allocation, distribution or development of groundwater are controlled by the economic valuation of the water supply or water quality resource by agricultural, industrial and municipal demands within a specific catchment. The optimal allocation of the groundwater supply maximizes the net discounted benefits from developing and extracting the resource, while minimizing well interference effects and potential groundwater quality problems. The optimal decisions of the planning model define the expected water targets for the principal agricultural, municipal and industrial demand centres.

4.4.3 The Capacity Expansion Model

The Operational and Allocation Models were based on the assumption that the well sites of the groundwater system were developed prior to the operational analysis. The capital costs associated with the development of the well-field were neglected. In the capacity expansion problem, both operational and capacity investment decisions are considered over the entire planning horizon. Specifically, the problem involves:

(1) The selection over the planning horizon of the well sites that are to be developed. This is accomplished in such a way as to always satisfy the water demand.

(2) The determination in any time period of the optimal groundwater pumping pattern.

It is assumed that the discounted capital and operational costs are to be minimized. The optimal decisions will define the timing and staging of well-field development and the optimal production pattern of the aquifer system. The policies of the planning model are constrained by:

(1) The water target in each planning period.

(2) The well capacity limitation.
(3) The response equations.
(4) Possible state variable restrictions.
(5) The non-negativity of the decision variables.

There may also be a limitation on the availability of capital in any planning period. Finally, we have the constraints reflecting the time lag between well and pipeline construction and the actual operation of the wells.

The capacity expansion model is most useful for long-term groundwater planning. The optimal policies of the model will define whether or not the basin can satisfy the imposed water targets of the problem and the potential role of groundwater in regional water resources management.

4.5 CONJUNCTIVE GROUNDWATER AND SURFACE WATER PLANNING MODEL

Groundwater and surface water planning models optimally distribute, over a planning or design horizon, the water resources of a catchment to competing water demands or water uses. By controlling the total water resources of a region, conjunctive use planning can increase the efficiency, reliability and cost-effectiveness of water use, particularly in river basins with spatial or temporal imbalances in water demands and natural supplies. Conjunctive water management can reduce surface water deficiencies by using groundwater to supplement scarce surface water supplies during the drier seasons.

The conjunctive use, planning or operational problem can again be formulated as an optimization model of the water resources system. The decision or control variables of the model are the groundwater and surface water allocations in each planning period. The optimal decisions maximize the objectives of the water resource system, while satisfying the hydraulic response equations of the surface and groundwater systems, and any constraints limiting the head variations and the surface water availability.

The system objective will include the capital, operational and maintenance costs. The objective function of the planning model is to maximize the net discounted economic benefits. The groundwater and surface water allocations are constrained within any planning period by the hydraulic response equations, balance equations for pumping and recharge and possible limitations on surface water availability and head variations in each groundwater system. The groundwater recharge constraints prescribe that the recharge target for each groundwater system is satisfied for any planning period.

4.6 PROBLEM FORMULATION

According to Lefkoff and Gorelick (1987) and Kinzelbach (1986), problem formulation is certainly the most important and often the most difficult part of a management model. Any optimization problem is characterized by:

- an objective function, stating the quantity to be maximized or minimized and its functional dependence from decision variables and
- the constraints which are always linear on the decision variables under which an optimum is to be searched

Objective functions may be non-linear, depending entirely on the problem formulation Typical objective functions are listed in Table 1.2

TABLE 1.2. TYPICAL OBJECTIVE FUNCTIONS AND CONSTRAINTS IN GROUND-WATER MANAGEMENT PROBLEMS

Hydraulic objective (goal) functions	
Objective Function	**Formular expressions**
Maximize total pumpage	$\sum_{i} Q_i \rightarrow \max$
Minimize total pumpage	$\sum_{i} Q_i \rightarrow \min$
Maximum water volume	$\sum_{j} \sum_{i} Q_i^j \Delta t_j \rightarrow \max$
Maximize total recharge	$\sum_{i} R_i \rightarrow \max$
Minimize the drawdown	$\sum_{i} s_i \rightarrow \min$
Maximize the hydraulic head	$\sum_{i} h_i \rightarrow \max$
Minimize sum of squared deviations from target heads, drawdowns or pumping rates	$\sum_{i} (Q_i - Q^*_i)^2 \rightarrow \min$
Economic objective (goal) functions	
Minimize cost	$\sum_{i} c_i Q_i - \sum_{j} c'_j h_j \rightarrow \min$
Maximize revenue from pumping	
Maximize net present value (NPV) of alternative pumping schedules	
Maximize internal rate of return (IRR) of alternative pumping schedules	
Minimize energy demand	$\sum_{i} e_i Q_i + \sum_{i} e'_i h_i \rightarrow \min$
Maximum benefits	$\sum_{i} p_i Q_i - \sum_{i} p'_i h_i \rightarrow \max$

Constraints

Prescribed minimum heads $\quad h_i(Q_1, \ldots, Q_n) \geq h_i, \min$

Prescribed maximum heads $\quad h_i(Q_1, \ldots, Q_n) \leq h_i, \max$

Prescribed minimum supply $\quad \sum_{i} Q_i \geq D$

Positivity of pumping rates $\quad Q_i \geq 0$

- Water levels cannot drop below the physical bottom of an aquifer.
- Water levels may not drop below the elevations of the pump inlets, whereas pump characteristics may also limit the drawdown.
- Changes in water levels (ground-water gradients) may produce undesirable flow patterns.
- Subsidence may be caused by dewatering certain formations.
- When artificial recharge is implemented, it is undesirable to let water levels rise above a maximum depth below the ground surface.
- Considerations of energy required to lift the water to the ground surface.

The above objective functions listed in the table may be controlled by the following constraints:

Physical (Hydraulic) Constraints

- Prescribed minimum heads
- Prescribed maximum heads
- Prescribed minimum supply
- Positivity of pumping rates
- Prescribed minimum water quality

Economical Constraints

- Prescribed maximum capital budget
- Prescribed minimum return required

The above-discussed objective functions can be constrained by either physical (hydraulic) or economic constraints or a mixture of both defined in the same problem which provides a very powerful management tool.

Hydraulic management models are concerned with the best selection of borehole locations and pumping rates that achieve certain goals with aquifer yields, drawdowns, hydraulic heads and hydraulic gradients.

4.7 SIMULATION MODEL

As stated in Section 4.2, the Groundwater Management Model is a combination of both a simulation model and an optimization model.

The simulation model used in this project is based on two-dimensional groundwater flow in confined aquifers:

$$\frac{\partial}{\partial x}[T_x \frac{\partial h}{\partial x}] + \frac{\partial}{\partial y}[T_y \frac{\partial h}{\partial y}] = S\frac{\partial h}{\partial t} + W \qquad (4.1)$$

where

where	h	=	hydraulic head [L]
	T_x, T_y	=	transmissivity [L^2/T]
	S	=	storage coefficient
	t	=	time [T]
	W	=	source (recharge) or sink (pumping) per unit area [L/T]
	x,y	=	spatial co-ordinates [L]

This equation can also be used to approximate an unconfined aquifer if the drawdown in the aquifer is small compared to its thickness. For the case where the drawdown is not small compared to the aquifer thickness, the T-values are calculated according to T = KD. The T-values are thus updated after every time step during the simulation, according to the saturated thickness of the previous time step. The fundamental partial differential equations governing the flow in confined aquifers are linear, while the dependent variable h (x,y,t) varies in space and time in a continuous fashion.

According to Bear (1979), it is difficult to use partial differential equations as constraints, since it would require the solution of the equation each time one wishes to check whether a constraint is violated. It is much easier to use a system of linear equations which approximates the partial differential equation. Furthermore, in many cases, it is convenient to use the solutions of the equations in conjunction with superposition to deal with linear flow problems.

By employing the principle of superposition, one can obtain, for example, the combined drawdown at a specific location after a particular time, produced by a number of production boreholes in the aquifer (each with its own pumping schedule) by means of an influence function or an algebraic technological function. If this is considered for all the observation boreholes in the aquifer, an influence matrix is formed, which represents the operation of the aquifer for a particular time-span. This influence matrix can also be referred to as a response matrix.

Thus, the key behind the response matrix method is that because the aquifer is described by a system of linear equations, the influence of each source or sink may be calculated separately and then superposed to compute the complete distribution of stresses over space and time under any pumping schedule.

4.8 STEPS IN THE CONSTRUCTION OF AN AQUIFER MANAGEMENT MODEL

According to Lefkoff and Gorelick (1987), hydraulic aquifer management modelling is a multistaged procedure comprising the following steps:

- a site specific simulation model is developed,
- a management problem is formulated,
- the simulation model is used to generate a response matrix,
- a special data file is created that contains the response matrix and represents the management formulation,
- an optimal solution is obtained by means of a linear or quadratic optimization algorithm,
- the optimal solution is verified using the original simulation model and
- the sensitivity of the solution to uncertainties is explored.

This seven-step approach adopted, provides the necessary philosophy whereby the software for this study is discussed in detail in Chapter 5.

4.9 DECISION ANALYSIS RELATING TO GROUNDWATER PROBLEMS

4.9.1 Introduction

The process of engineering design is often described as a sequence of decisions between alternatives under conditions of uncertainty. This is a particularly apt definition for the types of engineering projects that arise in a hydrogeological context. Moreover, in engineering projects that require a knowledge of the hydrogeological environment, uncertainty as to the system properties and expected conditions is far greater than in most traditional engineering practice. Not only do we have uncertainty as to the parameter values needed for our design calculations, we even have uncertainty about the very geometry of the system we are trying to analyse. The uncertainties of lithology, stratigraphy and structure introduce a level of complexity to geotechnical and hydrogeological analysis that is completely unknown in other engineering disciplines.

Freeze *et al.* (1990) provides a framework for hydrogeological decision analysis to engineering design for projects in which the hydrogeological environment plays an important role. It is based on a risk-based philosophy of engineering design and involves the coupling of three separate models: a decision model based on a risk-cost-benefit objective function, a simulation model for groundwater flow and transport and an uncertainty model that encompasses both geological uncertainty and hydrogeological parameter uncertainty. Janse van Rensburg (1992) proposed an expansion of the Freeze *et al.* (1990) uncertainty models to include both hydrological and financial uncertainties.

4.9.2 Difference between Hydrogeological Decision Analysis and Optimization

Systems analysis can be carried out in either a decision-analysis framework or an optimization framework. Optimization involves the determination of optimal values for a set of decision variables in a system. Optimality is defined with respect to a specified objective function and is subject to a set of constraints as described in Sections 4.1 to 4.8.

Decision analysis involves the determination of the best alternative from a discrete set of specific alternatives. Preference is based on a specified objective function. Optimization is a more general approach than decision analysis in that it provides the optimal alternative from the set of all possible alternatives, whereas decision analysis provides only the best alternative from a specified set of alternatives.

There are two types of limitation that can constrain the applicability of currently available optimization models. Firstly, most methods are based on a linear programming approach and this approach requires a linear objective function, linear constraints and linear flow equations in the simulation model (if such a model is part of the optimization system). The latter part of this limitation obviates application to unconfined surficial aquifers, unless simplifying assumptions can be invoked to linearize the flow equations. Secondly, it has been traditional to apply optimization techniques in a deterministic framework, so that such methods are not suitable for risk analysis or other probabilistic approaches to making decisions.

In summary, decision analysis is less general than optimization, but it suffers no limitations with respect to linearity or stochastic application, and it is well-suited to the risk-based philosophy of engineering design.

4.9.3 Components of a Design Framework (Freeze *et al.*,1990)

The design framework as proposed by Freeze *et al.* (1990) is shown in Figure 1.1 and consists of a decision model, a hydrogeological simulation model, an engineering reliability model, a geological uncertainty model, a hydrogeological parameter uncertainty model and a field investigation program.

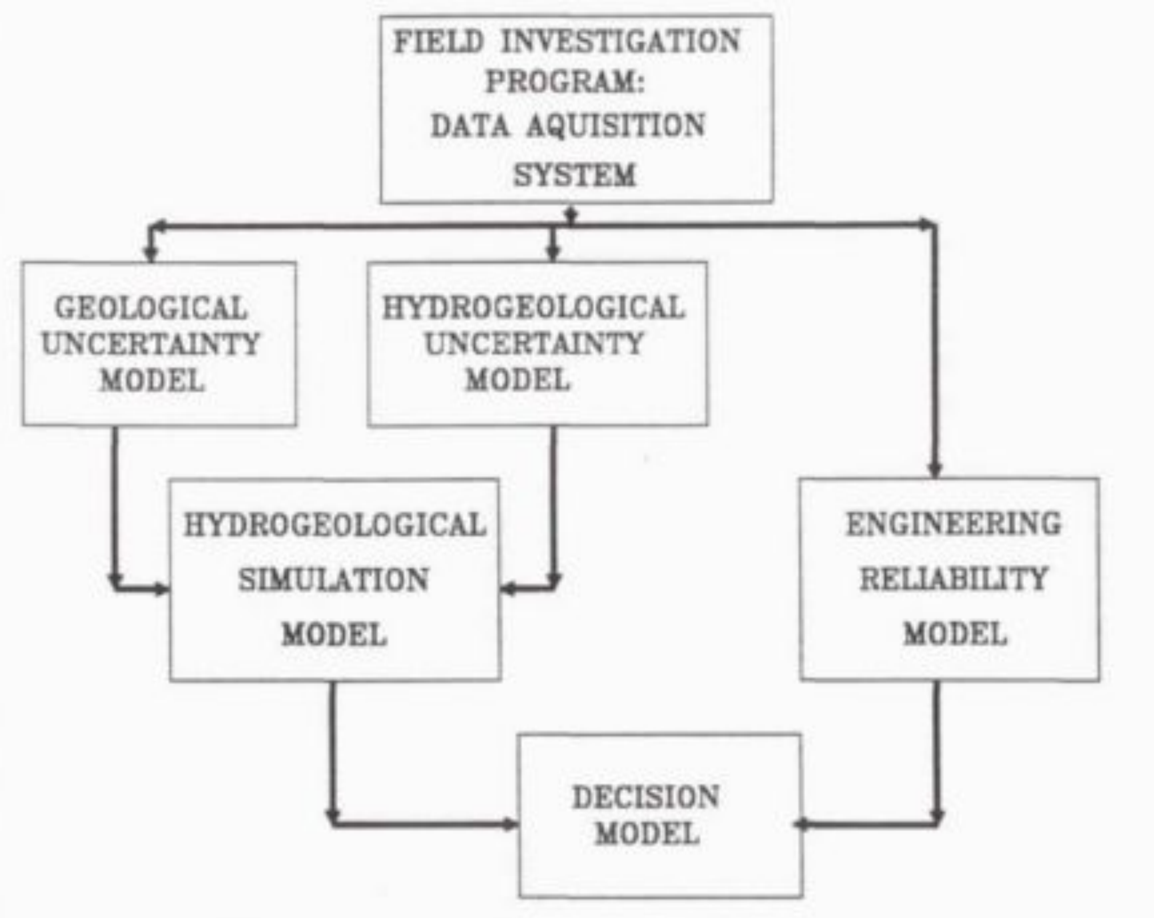


Fig. 1.1 Framework for Hydrogeological Decision Analysis (Freeze *et al*, 1990)

During his study, Janse van Rensburg (1992) concluded that a hydrological simulation model, financial simulation model, hydrological parameter uncertainty model and financial parameter uncertainty model should form part of the design framework for decision analysis. This expanded framework is shown in Figure 1.2. The decision model allows for the comparison of alternative sites and/or engineering designs. It is an economic analysis based on a risk-cost-benefit objective function.

The hydrological simulation model is used to represent the expected performance of the hydrological component of the system. The engineering reliability model is used to represent the expected performance of engineered components of the system. The hydrogeological simulation model is used to represent the expected performance of the hydrogeological component of the system. It can be an analytical solution or a numerical model of the hydrogeological system at the site: most often it will be a finite difference or finite element model of flow and transport. The financial simulation model is used to represent the expected performance of the financial component of the system. The hydrogeological simulation model, hydrological simulation model, financial simulation model and the engineering reliability model are utilized in a stochastic mode; their purpose is to predict the probability of failure which is a component of the risk term in the decision model.

The simulation must be stochastic in order to take into account the uncertainty in the hydrogeological system that always exists in heterogeneous hydrogeological environments. The uncertainty in the geological boundaries is described by the geological uncertainty model and the uncertainty in hydrogeological parameter values is described by the hydrogeological parameter uncertainty model. The uncertainty in the hydrological parameters is described by the hydrological uncertainty model and the uncertainty in financial parameter values is described by the financial parameter uncertainty model. In stochastic analysis, uncertainty in the input parameters is specified in the form of a probability density function or by the mean and the variance of such a distribution. There are three basic approaches used to propagate these uncertainties through the simulation models to estimate uncertainties of output variables: (1) first-order analysis, (2) perturbation analysis and (3) Monte-Carlo analysis (Freeze *et al.*, 1990).

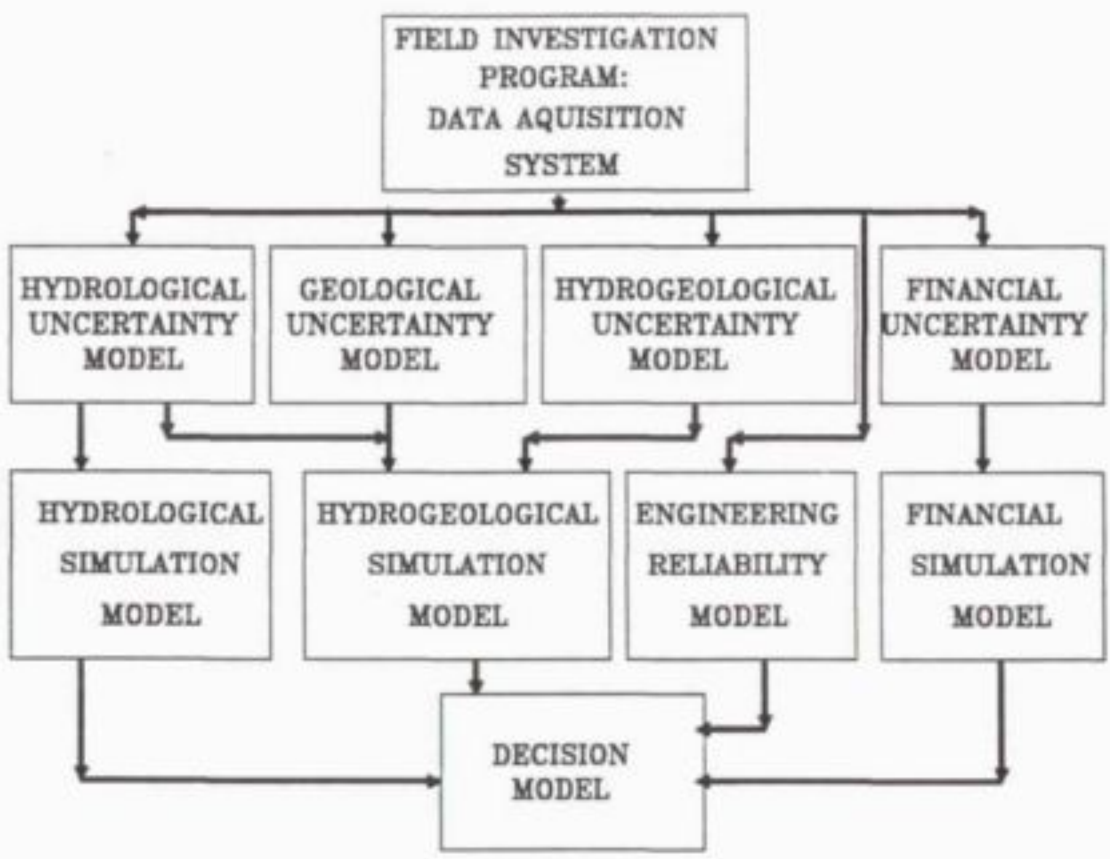


Fig.1.2 Framework for Hydrogeological Decision Analysis (Janse van Rensburg, 1992)

Monte Carlo simulation provides the most general approach to uncertainty propagation. With this approach, a large number of equally likely realizations of each parameter field are generated and the simulation models are run for each realization. The method requires that the full PDF be known for each input parameter. The PDF of the output variables is obtained from a statistical analysis of the output from the Monte Carlo runs.

The form of the uncertainty models and the values of their parameters are determined from the data generated by the field investigation program.

4.9.4 Decision Model

The economic objective of design must be to meet the technical objective in such a way so as to maximize the profit (or minimize the loss) to the owner. From this perspective, we can define an objective function as the net present value (NPV) of the expected stream of benefits, costs, and risks, taken over a planning horizon, and discounted at the market interest rate (Crouch and Wilson, 1982). If an objective function, Φ_j, is defined for each j = 1....N alternatives, then the goal is to maximize Φ_j.:

$$\Phi_j = \sum_{t=0}^{T} \frac{1}{(1+i)^t} \left[B_j(t) - C_j(t) - R_j(t) \right] \qquad (4.2)$$

where

Φ_j = objective function for alternative j [Rand]; $B_j(t)$ = benefits of alternative j in year t [Rand]; $C_j(t)$ = costs of alternative j in year t [Rand]; $R_j(t)$ = risks of alternative j in year t [Rand]; T = time horizon [years] and i = discount rate [decimal fraction].

The risks R(t), in the equation are defined as the expected costs associated with the probability of failure:

$$R(t) = P_f(t) C_f(t) \gamma(C_f) \qquad (4.3)$$

where $P_f(t)$ = probability of failure in year t [decimal fraction]; $C_f(t)$ = costs associated with failure in year t [Rand] and $\gamma(C_f)$ = normalized utility function [decimal fraction, $\gamma \geq 1$]. In this equation, the C(t) term represents the costs to the decision-maker and the B(t) term represents the benefits. The costs would be the capital costs and operational costs of each design alternative, and the benefits would be the revenues for each alternative. The probabilistic costs, $C_f(t)$, that appear in the risk term are those costs that would be incurred only in the event of failure. The utility function, $\gamma(C_f)$, allows one to take into account the possible risk-averse tendencies of some decision-makers. For risk-averse behaviour, $\gamma > 1$, and for risk-neutral behaviour, $\gamma = 1$. Small owners who do not have a large net worth are the most likely to use a risk-averse utility function. Larger companies are more likely to take a risk-neutral approach.

In Equation (4.2) , the C(t) term represents an actual cost, whereas the R(t) term represents a probabilistic cost. It is worth noting in the equations that almost all the terms are economic. The technical input to the objective function comes only in the probability term, $P_f(t)$. This identifies more clearly that the role of the simulation models in the decision-making process is to predict the probabilities of failure of the design alternatives under assessment.

Risk will be discussed in more detail in the Risk-analysis part (Section 2) of this research report.

4.9.5 Summary

Decision analysis involves an iterative process between analysis and field measurement. A subjective prior interpretation of the hydrogeological environment is first analysed with an unconditional stochastic simulation. The level of uncertainty reduction that can be achieved by field measurements is assessed with posterior conditional stochastic simulation. After each phase of field measurements, the available engineering alternatives are compared with an economic decision model. Before each phase of field measurements, the available engineering alternatives are compared with an economic decision model. Before each new phase of field measurements, a data worth assessment is carried out to determine whether further measurements are economically justified. When the point is reached where they are not, the best alternative is selected as the design decision.

CHAPTER 5

COMPUTER PROGRAM STRUCTURE FOR MANAGEMENT MODELS

The computer programs developed and used in the project are called the **AQUAMOD** package, and include the following programs:

5.1 PROGRAM AQUA-NET (GRID GENERATOR)

There is an ever-increasing demand for a well-developed triangular mesh to use in the numerical simulation of groundwater flow or pollution. The need therefore arises for a graphics interactive and user-friendly mesh generating program. Programs to generate networks do exist, but some have shortcomings, including lack of the following:

User-friendliness

Graphics representation

Dividing the region into zones with certain properties

Band-width optimization

Mouse support

The AQUA-NET program (Staats, 1993), was developed to meet these shortcomings. The program can read various data files to accommodate fixed locations, border co-ordinates and previous generated networks. Additional single or multiple data points can be inserted or deleted with a mouse. There is also a facility to generate points on the border line to ensure perfect triangulation next to the border.

AQUA-NET generates a finite triangular mesh between a finite set of user-defined data points. It uses an algorithm developed by Watson (1982). This algorithm constructs triangles by subdividing an initial triangle that encloses all the data points. Data points are introduced one at a time and triangles are formed, so that no node lies within another triangle's circumcircle. Where this happens, these triangles are deleted and new ones are formed including this new data point. On completion of this process, the triangles with connections to the initial triangle are deleted. Triangles which are situated outside the boundary are deleted simultaneously. This triangulation scheme complies with the Delaunay criteria as defined in Watson and Philip (1984) and may therefore be classified as a Delaunay triangulation.

After the triangulation, the band-width can be reduced considerably with a built in option. The algorithm for band-width reduction was taken from Burgess and Lai (1986). It builds a level structure from the connectivities of the nodes. An initial starting node with lowest connectivity (degree) is listed in the first level. All nodes connected to this node are listed in the second level. For each of the nodes in the second level, all nodes connected to this node that were not listed before, are listed in the third level. The same is done for the third level and levels to follow, until all nodes are listed in the level structure.

This level structure is then reduced, as described in Burgess and Lai (1986), to have a new structure with levels of smaller height. This new structure is then renumbered to give the same network with a smaller band-width.

Marking of zones, hereafter called zoning, is accomplished in a novel manner. An area is enclosed by a few lines and all triangles in this area are colored with the chosen zone color. After all the triangles are marked, the user enters the T- and S-value for each color. Zoning or zone refinement can also be done, using old networks. Figure 1.3 and 1.4 show example grids generated by AQUA-NET.

5.2 PROGRAM AQUA (FLOW PROGRAM)

AQUA solves the Galerkin finite element method in two dimensions for groundwater flow. The development of this program is completed and has been used by the IGS and the DWA in a number of case studies.
Special features of AQUA include the ability to specify:

(i) variable pumping rates,
(ii) time-dependent recharge values as percentage of monthly rainfall and
(iii) a confined or water-table aquifer.

The output of AQUA yields:

(i) monthly simulated water levels at each node (e.g. for contouring purposes) or water levels at specific user-defined nodes;
(ii) groundwater velocities in the center of each element (optional);
(iii) a groundwater balance.

5.3 PROGRAM AQUA-INV (INVERSE PROGRAM FOR FLOW)

AQUA-INV is an automated parameter identification program which uses the flow program AQUA and the Marquardt optimization algorithm to obtain the following choice of parameter combinations for zones in an aquifer which simultaneously produce the best fit between observed and simulated historical water-level data:

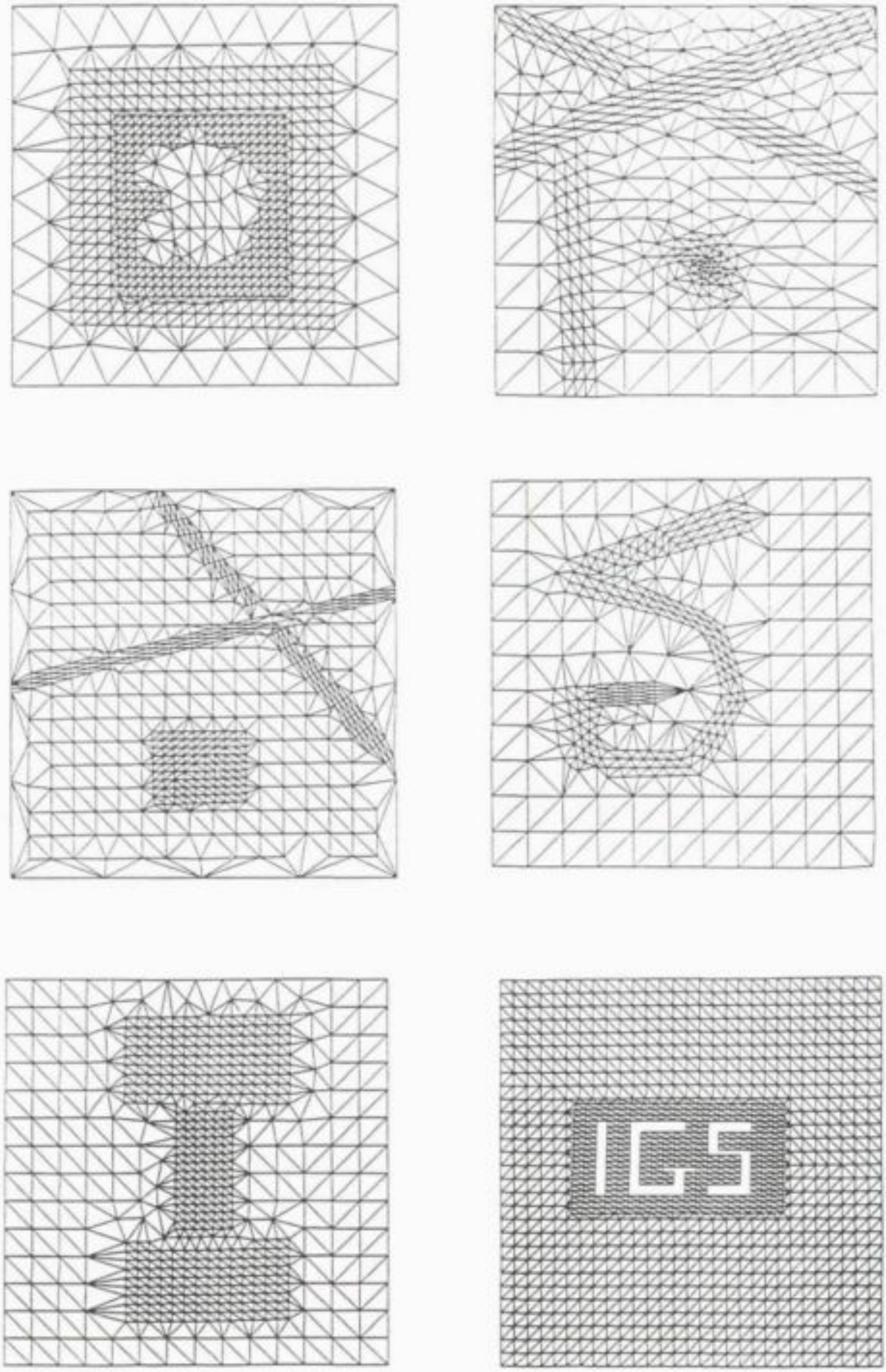


Fig. 1.3. Finite element meshes generated with program AQUA-NET.

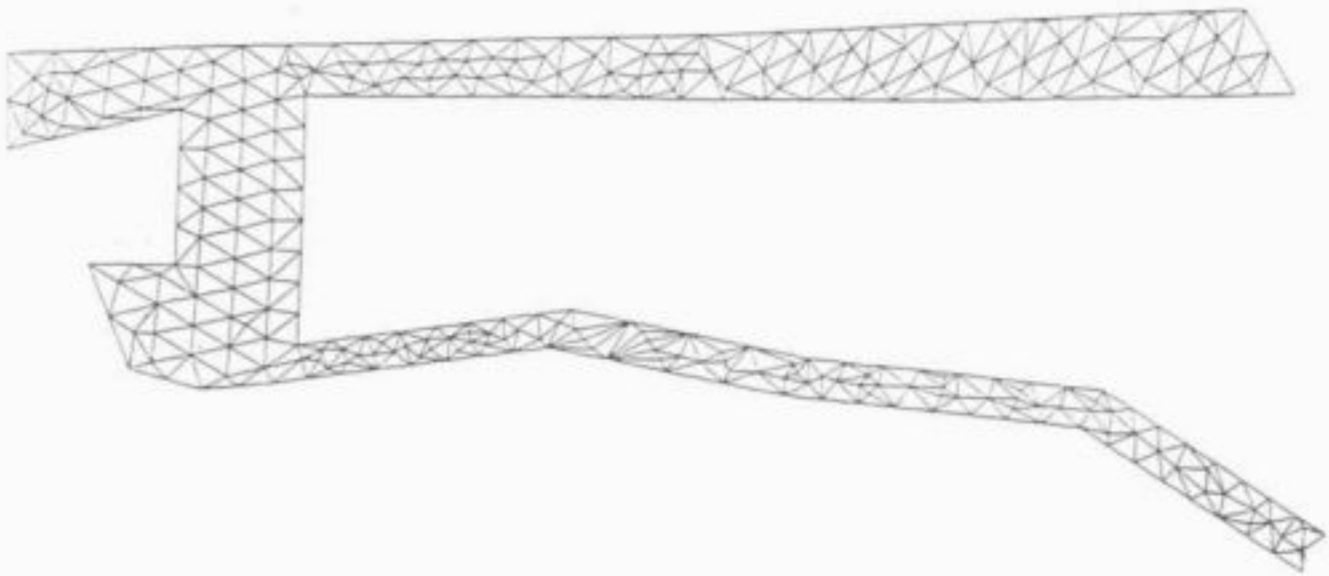


Fig. 1.4. Finite element meshes of the Greefswald- and Otjiwarongo Marble aquifer generated with the program AQUA-NET.

1. T - and S-variables

2. T

3. S

4. Recharge

5. T and recharge

6. Neumann inflow flux at boundary

7. T, S and inflow flux at boundary.

The program is based on the Ph.D. study of Christophe Kauffmann under Professor Wolgang Kinzelbach of Kassel University in Germany.

5.4 PROGRAM AQUA-MAS

This program solves the convection diffusion equation in two dimensions for mass transport problems. The program was used by Robert Scott for his M.Sc. project on the investigation into the fate of groundwater contaminants at Vansa Vanadium Mine.

5.5 PROGRAM MASS-INV (INVERSE MASS TRANSPORT PROGRAM)

This program is the equivalent of the AQUA-INV program, and can be used for the automated calibration of the mass transport problem. The parameters which could be inverse are the groundwater velocities, porosity and the longitudinal and transversal dispersivities.

5.6 PROGRAM AQUA-MAN (MATHEMATICAL OPTIMIZATION)

AQUA-MAN links the distributed parameter groundwater flow simulation model, AQUA, with mathematical optimization methods using a technique know as the response matrix approach. Linear programming formulation of the management problem is solved by a simplex routine (program **SIMPLEX**), obtained from Kinzelbach (1986). If slack and surplus values are needed, the routine by Erikson and Hall (1987) can be used, while quadratic problems are solved with the Wolfe routine obtained from Kuester and Mize (1973).

5.7 PROGRAM SVF (RECHARGE ESTIMATION)

This program estimates the groundwater recharge of an aquifer with the aid of the SVF-method and was originally programmed for the WRC-project of Kirchner *et al* (1991). Modifications to the program, as part of the recharge project of Bredenkamp *et al.* (1993, WRC-project), were made and will be reported on in that project.

5.8 PROGRAM AQFSTOC (ESTIMATION OF PERCENTILES)

AQFSTOC was written by P. van Rooyen (1992) of Bruinette, Kruger and Stoffberg, to simulate the storage in an aquifer based on the recharge results as obtained with the SVF-method as well as the S-value and the abstraction from the system. AQFSTOC requires a generated sequence of rainfall. This is done with a program called **RAINGEN** and is based on work done by Zucchini and Adamson (1991). Percentiles, as obtained from the AQFSTOC program, show probability estimates of confidence for storage of the aquifer on a monthly basis.

A complete description of the AQUAMOD package is given in Appendix A.

CHAPTER 6

ILLUSTRATIVE MANAGEMENT EXAMPLES

In this chapter, a few hypothectical management examples will be presented to demonstrate how simulation and optimization models can be used to develop a conceptual basis for groundwater management.

6.1 EXAMPLE 1.

Consider the three abstraction boreholes located at the following nodes (see figure 1.5):

Pumping hole 1 = node 24
Pumping hole 2 = node 56
Pumping hole 3 = node 88

The question is to obtain the maximum pumping rates at the three pumping boreholes such that the following constraints are valid at six observation boreholes (located at nodes 24, 7, 89, 96, 53 and 16 respectively) after 90 days of pumping:

Maximum drawdown at obs. borehole 1 < 2.0 m
Maximum drawdown at obs. borehole 2 < 2.0 m
Maximum drawdown at obs. borehole 3 < 0.4 m
Maximum drawdown at obs. borehole 4 < 2.5 m
Maximum drawdown at obs. borehole 5 < 2.5 m
Maximum drawdown at obs. borehole 6 < 0.9 m

Objective function: Maximize F = Q1 + Q2 + Q3

Solution: Prepare the following input data file

Input data: file **aqua.man (see page A-46 in Appendix A for the input parameters)**

```
        3       6       1       (number of boreholes, number of constraints and maximize code)
Ob 1 < 2.0
Ob 2 < 2.0
Ob 3 < 0.4              (constraints)
Ob 4 < 2.5
Ob 5 < 2.5
Ob 6 < 0.9
1       (coefficients of objective function)
1
1
```

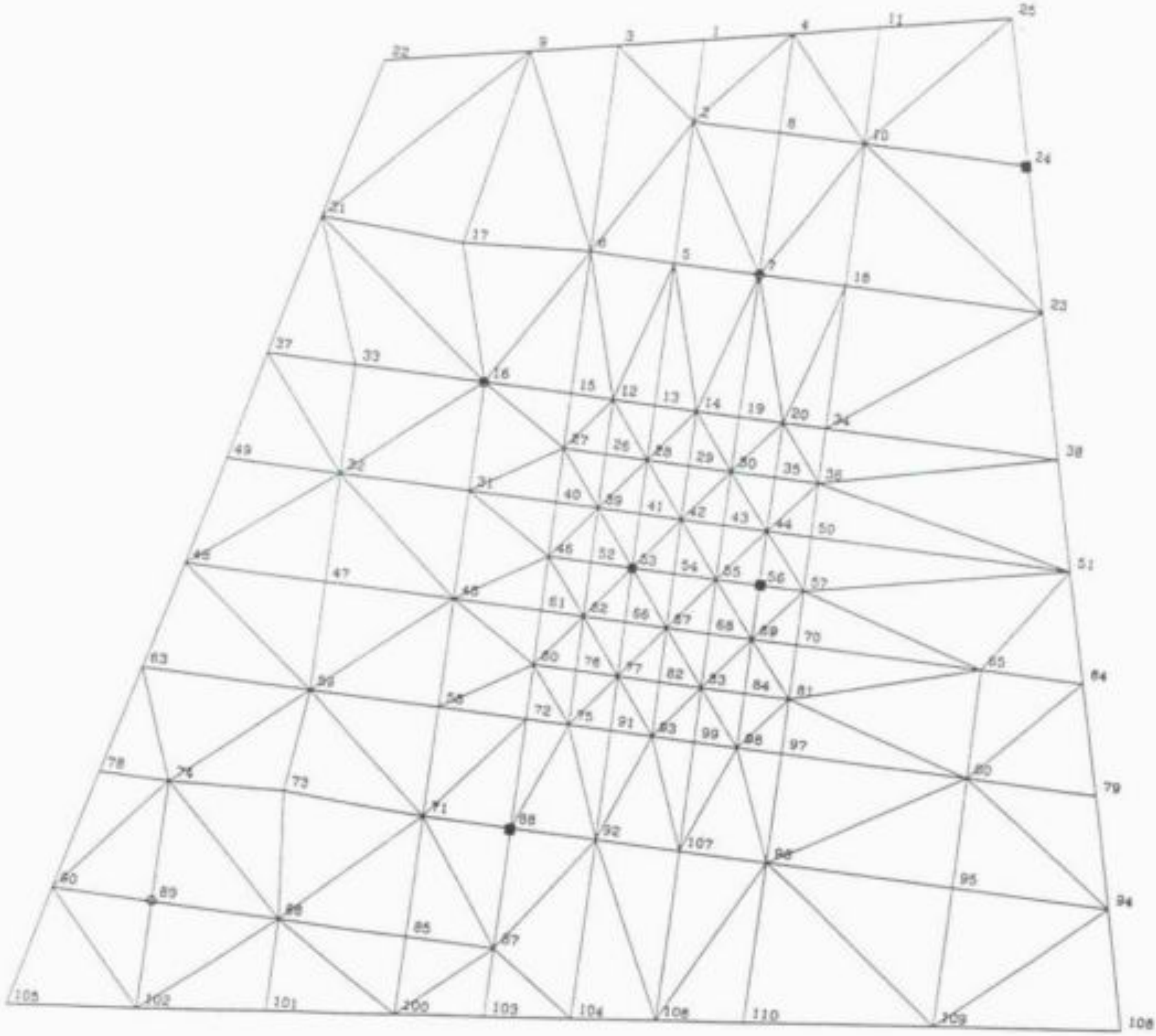


Fig. 1.5. Finite element mesh used for the management examples.

Execution of program **AQUA-MAN** yielded the following output file (file name aqua-man.out):

Output file : aqua-man.out

```
        3       6       1
Ob 1 < 2.0
Ob 2 < 2.0
Ob 3 < 0.4
Ob 4 < 2.5
Ob 5 < 2.5
Ob 6 < 0.9
1
1                          coefficients of objective function
1
0.441132E-02 0.131663E-02  0.341610E-03
0.220353E-02 0.125971E-02  0.328627E-03
0.641480E-04 0.134838E-03  0.248847E-03          Response
0.676188E-03 0.149491E-02  0.113611E-02           matrix
0.939208E-03 0.194959E-02  0.733874E-03
0.559076E-03 0.528029E-03  0.252322E-03
```

Interpretation of output file aqua-man.out:

The meaning of the data in the aqua-man.out can be put in matrix form as:

0.441132E-02	0.131663E-02	0.341610E-03	Q1		2.0
0.220353E-02	0.125971E-02	0.328627E-03			2.0
0.641480E-04	0.134838E-03	0.248847E-03	Q2	<	0.4
0.676188E-03	0.149491E-02	0.113611E-02			2.5
0.939208E-03	0.194959E-02	0.733874E-03	Q3		2.5
0.559076E-03	0.528029E-03	0.252322E-03			0.9

with objective function: maximize $F = 1*Q1 + 1*Q2 + 1*Q3$

This output file from program **AQUA-MAN** is now supplied to the simplex routine, which yields the following solution:

$Q1 = 152$; $Q2 = 699$ and $Q3 = 1189$ m^3/d and sum $Q = 2041$

6.2 EXAMPLE 2.

It is now easy to change file aqua-man.out to incorporate other constraints, e.g. suppose that the maximum yield of the three pumping boreholes is as follows:

Maximum yield of borehole 1 = 200 m^3/d
Maximum yield of borehole 2 = 600 m^3/d
Maximum yield of borehole 3 = 1000 m^3/d
Suppose further that the water demand is 1 500 m^3/d.

Task : Obtain the rate at which each of the three boreholes must be operated such that the same constraints as in Example 1 are still satisfied.

Solution

Change the input of file aqua-man.out as follows:
(The altered input is highlighted)

```
        3      10      1
Ob 1   < 2.0
Ob 2   < 2.0
Ob 3   < 0.4
Ob 4   < 2.5
Ob 5   < 2.5
Ob 6   < 0.9
Q1     < 200.0
Q2     < 600.0
Q3     < 1000.0
sumQ   = 1500.0
1
1
1
0.441132E-02      0.131663E-02      0.341610E-03
0.220353E-02      0.125971E-02      0.328627E-03
0.641480E-04      0.134838E-03      0.248847E-03
0.676188E-03      0.149491E-02      0.113611E-02
0.939208E-03      0.194959E-02      0.733874E-03
0.559076E-03      0.528029E-03      0.252322E-03
1                 0                 0
0                 1                 0
0                 0                 1
1                 1                 1
```

The simplex routine yields the solution Q1 = 200, Q2 = 600 and Q3 = 700 m^3/d.

COMMENT

To obtain the response matrix, the program **AQUA-MAN** is executed with a zero stress (abstraction rate) applied at all nodes. **The aim** is to obtain a drawdown value (difference between initial head value and head value at the end of the management period) at each observation or manageable node. This value is added to the original drawdown constraint at the manageable node. This implies that the original drawdown constraint in the aqua.man input file could have been changed to some new constraint value in the aqua-man.out file.

6.3 EXAMPLE 3.

Consider the same problem as in **6.2** above, except that the **new** objective is to minimize the total drawdown at the six observation boreholes. To obtain the required solution for the problem, file aqua-man.out must be changed as follows:
(The altered input is highlighted again)

```
3          10        -1
Ob 1  > 0.0  (in stead of a value of zero, a very large drawdown (e.g.Ob1 < 100) can be used too)
Ob 2  > 0.0
Ob 3  > 0.0
Ob 4  > 0.0
Ob 5  > 0.0
Ob 6  > 0.0
Q1     < 200.
Q2     < 600.
Q3     < 1000.
sumQ = 1500.
.008853
.006683
.003041
0.441132E-02     0.131663E-02     0.341610E-03
0.220353E-02     0.125971E-02     0.328627E-03
0.641480E-04     0.134838E-03     0.248847E-03
0.676188E-03     0.149491E-02     0.113611E-02
0.939208E-03     0.194959E-02     0.733874E-03
0.559076E-03     0.528029E-03     0.252322E-03
1                0                0
0                1                0
0                0                1
1                1                1
```

(Note: this is a minimization and not a maximization problem. The values of the objective function coefficients are obtained by summing the drawdowns in each column of the response matrix, e.g.

.441132E-02 + .220353E-02 + ... + .559076E-03 = **.008853**)

The simplex routine yields the solution as Q1 = 0, Q2 = 500 and Q3 = 1000 m^3/d. The value of the objective function is 6,38 m, which means that the mentioned abstraction rates will yield a maximum sum of drawdowns at the observation boreholes of 6,38 m.

6.4 EXAMPLE 4.

Consider the same problem as in **6.2**. Let us introduce the following information into the problem.
The pumping cost from the abstraction boreholes is as follows:

Borehole 1: R 0,30/m^3
Borehole 2: R 0,20/m^3
Borehole 3: R 0,50/m^3
The average quality (TDS) of the water pumped from each borehole was analyzed as follows:

Borehole 1: 500 mg/l
Borehole 2: 600 mg/l
Borehole 3: 450 mg/l

The user of the aquifer irrigates a very sensitive crop and he places a restriction of 550 mg/l as the maximum concentration his crops can tolerate.
He wants to find out what the pumping rate from each borehole should be to satisfy the demand constraint, the drawdown constraints, the yield constraints, the quality constraint, as well as to minimize the costs of pumping.

Defining the Decision Variables.

Let Q1 be the pumping rate from borehole 1.
Let Q2 be the pumping rate from borehole 2.
Let Q3 be the pumping rate from borehole 3.

Defining the Object Function.

Minimize F = 0,30Q1 + 0,20Q2 + 0,50Q3

Defining the Constraints.
Drawdown constraints:

$Q1*0.441132E\text{-}02 + Q2*0.131663E\text{-}02 + Q3*0.341610E\text{-}03 < 2.0$
$Q1*0.220353E\text{-}02 + Q2*0.125971E\text{-}02 + Q3*0.328628E\text{-}03 < 2.0$
$Q1*0.641480E\text{-}04 + Q2*0.134838E\text{-}03 + Q3*0.248847E\text{-}03 < 0.4$
$Q1*0.676188E\text{-}03 + Q2*0.149491E\text{-}02 + Q3*0.113611E\text{-}02 < 2.5$
$Q1*0.939208E\text{-}03 + Q2*0.194959E\text{-}02 + Q3*0.733874E\text{-}03 < 2.5$
$Q1*0.559076E\text{-}03 + Q2*0.528029E\text{-}03 + Q3*0.252322E\text{-}03 < 0.9$

Yield constraints:

$Q1 < 200$
$Q2 < 600$
$Q3 < 1000$

Demand constraint:

$Q1 + Q2 + Q3 = 1500$

Quality constraint:

$500Q1 + 600Q2 + 450Q3 < 550*1500$

Solution.

The information above is now supplied to the simplex routine (Erikson and Hall, 1989), which yields the following solution:

$Q1 = 200m^3/d$
$Q2 = 600m^3/d$
$Q3 = 700m^3/d$

The objective function's value is R 530 which is the minimum pumping cost per day.
If the demand is stepped up to 1600 m^3/d, the simplex routine yields the following solution:

$Q1 = 200m^3/d$
$Q2 = 600m^3/d$
$Q3 = 800m^3/d$
with the value of the objective function being R 580 per day.

COMPUTER MODELS FOR MANAGEMENT SCIENCE

LINEAR PROGRAMMING 07-06-1992 - 23:10:50

-=*=- INFORMATION ENTERED -=*=-

NUMBER OF VARIABLES : 3
NUMBER OF <= CONSTRAINTS : 10
NUMBER OF = CONSTRAINTS : 1
NUMBER OF >= CONSTRAINTS : 0

MIN COST = .3 X1 + .2 X2 + .5 X3

SUBJECT TO:

.004X1 +	.001X2 +	0 X3	<=	2
.002X1 +	.001X2 +	0 X3	<=	2
0 X1 +	0 X2 +	0 X3	<=	.4
.001X1 +	.001X2 +	.001X3	<=	2.5
.001X1 +	.002X2 +	.001X3	<=	2.5
.001X1 +	.001X2 +	0 X3	<=	.9
1 X1 +	0 X2 +	0 X3	<=	200
0 X1 +	1 X2 +	0 X3	<=	600
0 X1 +	0 X2 +	1 X3	<=	1000
500 X1 +	600 X2 +	450 X3	<=	825000
1 X1 +	1 X2 +	1 X3	=	1500

-=*=- RESULTS -=*=-

VARIABLE	VARIABLE VALUE	ORIGINAL COEFFICIENT	COEFFICIENT SENSITIVITY
X1	200	.3	0
X2	600	.2	0
X3	700	.5	0

CONSTRAINT NUMBER	ORIGINAL RIGHT-HAND VALUE	SLACK OR SURPLUS	SHADOW PRICE
1	2	.089	0
2	2	.573	0
3	.4	.132	0
4	2.5	.673	0
5	2.5	.629	0
6	.9	.295	0
7	200	0	.2
8	600	0	.3
9	1000	300	0

10	825000	50000	0
11	1500	0	.5

OB IVE FUNCTION VALUE: 530

-- SENSITIVITY ANALYSIS --

OBJECTIVE FUNCTION COEFFICIENTS

VARIABLE	LOWER LIMIT	ORIGINAL COEFFICIENT	UPPER LIMIT
X1	NO LIMIT	.3	.5
X2	NO LIMIT	.2	.5
X3	.3	.5	NO LIMIT

RIGHT-HAND-SIDE VALUES

CONSTRAINT NUMBER	LOWER LIMIT	ORIGINAL VALUE	UPPER LIMIT
1	1.911	2	NO LIMIT
2	1.427	2	NO LIMIT
3	.268	.4	NO LIMIT
4	1.827	2.5	NO LIMIT
5	1.871	2.5	NO LIMIT
6	.605	.9	NO LIMIT
7	0	200	221.778
8	300	600	690.902
9	700	1000	NO LIMIT
10	775000	825000	NO LIMIT
11	800	1500	1611.111

---------- E N D O F A N A L Y S I S ----------

COMPUTER MODELS FOR MANAGEMENT SCIENCE

LINEAR PROGRAMMING 07-06-1992 - 23:08:19

-*=*- INFORMATION ENTERED -*=*-

NUMBER OF VARIABLES	:	3
NUMBER OF <= CONSTRAINTS	:	10
NUMBER OF = CONSTRAINTS	:	1
NUMBER OF >= CONSTRAINTS	:	0

MIN COST = .3 X1 + .2 X2 + .5 X3

SUBJECT TO:

.004X1 +		.001X2 +		0	X3		<=	2
.002X1 +		.001X2 +		0	X3		<=	2
0	X1 +	0	X2 +	0	X3		<=	.4
.001X1 +		.001X2 +		.001X3			<=	2.5
.001X1 +		.002X2 +		.001X3			<=	2.5
.001X1 +		.001X2 +		0	X3		<=	.9
1	X1 +	0	X2 +	0	X3		<=	200
0	X1 +	1	X2 +	0	X3		<=	600
0	X1 +	0	X2 +	1	X3		<=	1000
500	X1 +	600	X2 +	450	X3		<=	880000
1	X1 +	1	X2 +	1	X3		=	1600

-*=*- RESULTS -*=*-

VARIABLE	VARIABLE VALUE	ORIGINAL COEFFICIENT	COEFFICIENT SENSITIVITY
X1	200	.3	0
X2	600	.2	0
X3	800	.5	0

CONSTRAINT NUMBER	ORIGINAL RIGHT-HAND VALUE	SLACK OR SURPLUS	SHADOW PRICE
1	2	.054	0
2	2	.541	0
3	.4	.107	0
4	2.5	.559	0
5	2.5	.555	0
6	.9	.27	0
7	200	0	.2
8	600	0	.3
9	1000	200	0

10	880000	60000	0
11	1600	0	.5

OB IVE FUNCTION VALUE: 580

-- SENSITIVITY ANALYSIS --

OBJECTIVE FUNCTION COEFFICIENTS

VARIABLE	LOWER LIMIT	ORIGINAL COEFFICIENT	UPPER LIMIT
X1	NO LIMIT	.3	.5
X2	NO LIMIT	.2	.5
X3	.3	.5	NO LIMIT

RIGHT-HAND-SIDE VALUES

CONSTRAINT NUMBER	LOWER LIMIT	ORIGINAL VALUE	UPPER LIMIT
1	1.946	2	NO LIMIT
2	1.459	2	NO LIMIT
3	.293	.4	NO LIMIT
4	1.941	2.5	NO LIMIT
5	1.945	2.5	NO LIMIT
6	.63	.9	NO LIMIT
7	0	200	213.384
8	400	600	655.865
9	800	1000	NO LIMIT
10	820000	880000	NO LIMIT
11	800	1600	1733.333

---------- E N D O F A N A L Y S I S ----------

6.5 EXAMPLE 5 : ALLOCATION OF WATER AMONG CATEGORIES OF WATER USERS

Problem 6.5.a: (adapted from Van der Gun, 1988)

- An aquifer is to be exploited according to a sustained yield policy. The maximum sustained yield is 10 millions m^3 per year.
- Water is used for domestic water supply and for irrigation of agricultural land. Domestic use has an absolute priority.
- There are 50 000 inhabitants, and the per capita domestic water demand is 70 l/day.
- There are 5 different crops under consideration , with the following characteristics:

Crop No.	Water demand	Net return	Labour input
	($x10^6$ m^3/ha)	(R/ha)	(persons/ha)
1	.011	300	.2
2	.012	400	.5
3	.009	200	.3
4	.015	600	.2
5	.014	700	.4

- There are 1000 ha of irrigable land in the area.
- Water should be allocated in such a way that the net economic return is maximized, but a few additional restrictions are made:

  (a) there should be employment in irrigated agriculture for at least 250 persons;

  (b) each crop should occupy at least 100 ha;

  (c) crop 5 should not occupy more than 500 ha.

SOLUTION

- Some 1.278 million m^3 should be allocated for domestic supply, therefore 8.722 millions annually remain available for irrigation.

Solution in terms of hectares:

Let X_i be the amount of ha annually to be allocated to crop i.
Maximize:
Objective function: $F = 300X_1 + 400X_2 + 200X_3 + 600X_4 + 700X_5$
subject to:

$X_1 > 100$
$X_2 > 100$
$X_3 > 100$
$X_4 > 100$

$X_5 > 100$
$X_5 < 500$

$1X_1$	+	$1X_2$	+	$1X_3$	+	$1X_4$	+	$1X_5$	<	1000
$.011X_1$	+	$.012X_2$	+	$.009X_3$	+	$.015X_4$	+	$.014X_5$	<	8.722
$.2X_1$	+	$.5X_2$	+	$.3X_3$	+	$.2X_4$	+	$.4X_5$	>	250

The data should be supplied to the simplex routine as follows:

	5	9	1	
1	> 100			
2	> 100			
3	> 100			
4	> 100			
5	> 100			
5	< 500			
SUMH<	1000			
SUMQ<	8.722			
SUMA>	250			
300				
400				
200				
600				
700				
1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1
0	0	0	0	1
1	1	1	1	1
.011	.012	.009	.015	.014
.2	.5	.3	.2	.4

The simplex routine yields the results:

$X_1 = 100$ ha
$X_2 = 196$ ha
$X_3 = 100$ ha
$X_4 = 100$ ha
$X_5 = 205$ ha
with object value = R 331 900
or in terms of amount of water

Crop 1 = 100 ha * ,011 = 1,1 million m^3 per year
Crop 2 = 2,35
Crop 3 = 0,9
Crop 4 = 1,5
Crop 5 = 2,87

Solution in terms of amount of water:

The problem can be solved directly in terms of the amount of water allocated to each crop as follows:

Object function:

$F = 27272X_1 + 33333X_2 + 22222X_3 + 40000X_4 + 50000X_5$
(where e.g. the value 27272 is calculated from 300/0,011 and X_i = amount of water allocated to crop i)
subject to

$X_1 > 1.1$
$X_2 > 1.2$
$X_3 > .9$
$X_4 > 1.5$
$X_5 > 1.4$
$X_5 < 7$
$1X_1 + 1X_2 + 1X_3 + 1X_4 + 1X_5 < 8.722$
$90.9X_1 + 83.3X_2 + 111.1X_3 + 66.7X_4 + 71.4X_5 < 1000$
$18.2X_1 + 41.6X_2 + 33.3X_3 + 13.3X_4 + 28.6X_5 > 250$

(where e.g. 90.9 is calculated from 1/.011 and 18.2 from .2/.011)

The simplex routine yields the result:

X_1 = 1,1 million m^3 per year
X_2 = 2,36
X_3 = 0,9
X_4 = 1,5
X_5 = 2,86
with maximum value of F = R 331 900
which is the same result as previously obtained.

Problem 6.5.b:

The previous example indicated that the employment constraint is really constraining the solution - the extent of available land (1000 ha) is more than what is used (701 ha). If the same problem as in 5.a is performed without this employment constraint, the result is:

X_1 = 100 ha
X_2 = 100 ha
X_3 = 100 ha
X_4 = 100 ha
X_5 = 287 ha with objective F = R 351 100
If, furthermore, the constraints on the number of hectares to be occupied by each of the crops are dropped, the result is:

$X_1 = 0$ ha
$X_2 = 0$ ha
$X_3 = 0$ ha
$X_4 = 0$ ha
$X_5 = 623$ ha with objective F = R 436 100
which will produce the highest net return

EXAMPLE 6.6 DE AAR AQUIFER

6.6.1 Background

De Aar lies approximately 100 km south of the Orange River in the north-western part of what is called the Great Karoo, at an elevation of 1240 m. De Aar has been dependent on groundwater supplies from the pioneer farming days in the previous century up to the present time (Vegter, 1992)

The geology comprises Beaufort Group mudstones, siltstones and sandstone east and southeast of De Aar and Ecca Group shale and subordinate siltstone west and norhtwest of De Aar. The beds have been intruded by undulating and furcating dolerite sheets on two horizons and by dolerite and kimberlite dykes. Superficial deposits consist of calcrete forming extensive terraces in certain parts, of colluvium around the base of hills and of alluvium along river couses (Vegter, 1992).

Numerous groundwater investigations have been performed at De Aar, e.g. De Villiers 1945; Dziembowski, 1971; De Bruin en Vegter, 1972; Carter, 1972; Kirchner *et al.*, 1991; Woodford, 1989,1990 and Vegter, 1992).

De Aar is currently (1992) supplied by the following groundwater schemes (Vegter, 1992):
Zewe Fountain (Burgerville) ; Subarea A
Southwestern (Zwartekopjes, Vaalbank, Renosterpoort) ; Subarea F
Southeastern (Riet Fountain 6 - Wagt en Bittje 5) ; Subarea B
Caroluspoort ; Subarea C

In his report " An evaluation of groundwater exploitation and its potential for urban use, De Aar, " (Vegter, 1992), Vegter identified several areas around the town of De Aar from where groundwater can be exploited. Each of these subareas are subdivided into smaller areas. For each area the following information is supplied:

- the current potential from the existing boreholes
- the potential if the additional drilled boreholes are commissioned
- the potential by further development of each area.

A summary of these data is shown in Table 1.3.

These subareas as well as other potential groundwater resources are depicted in Figure 1.6.

Table 1.3. Groundwater units near De Aar.

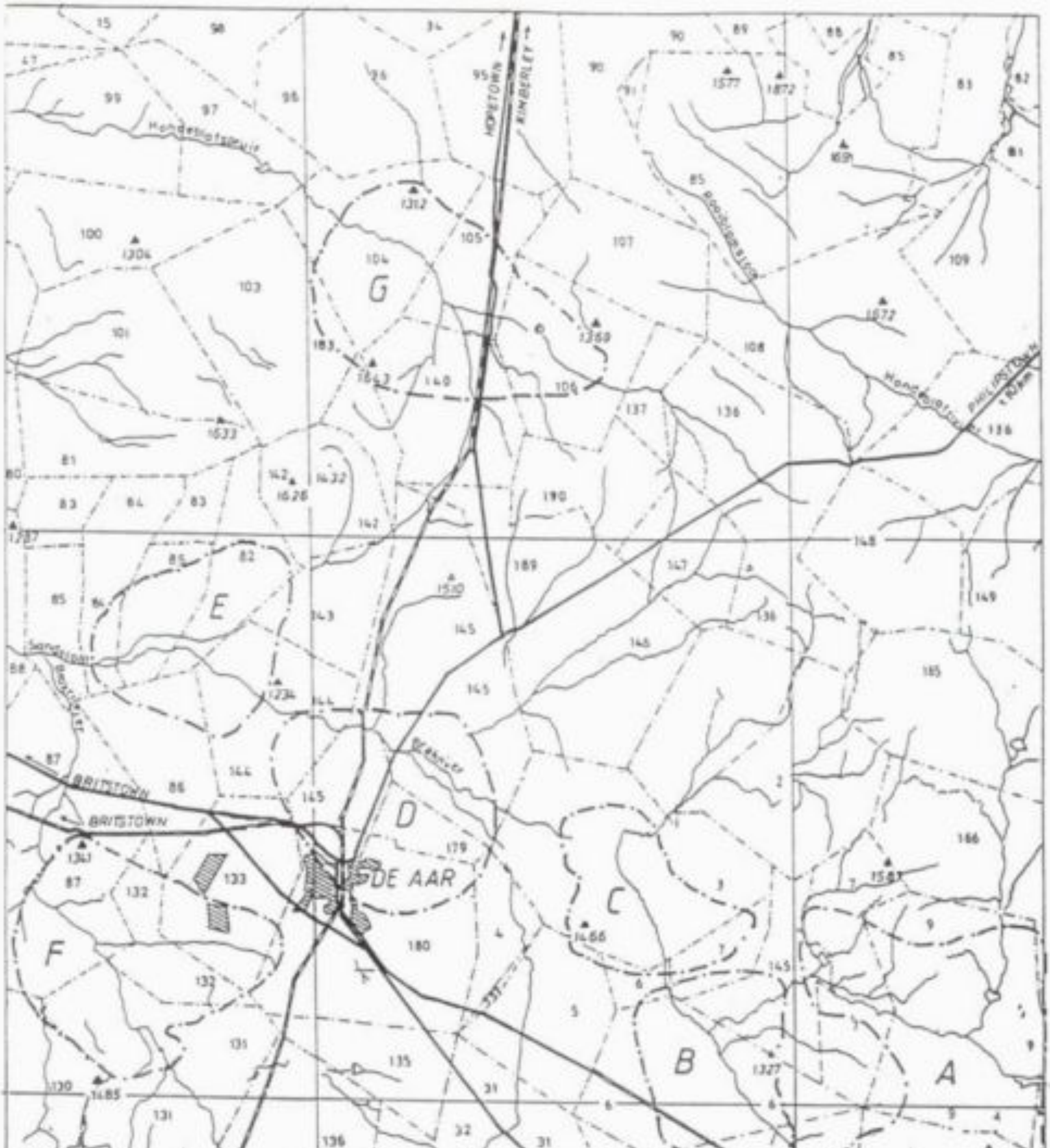
GROUNDWATER UNIT	YIELD-EXISTING SCHEME		YIELD-NEW BOREHOLES		ADDITIONAL POTENTIAL	
	LOW	HIGH	LOW	HIGH	LOW	HIGH
SUBAREA A(BURGERVILLE/SEWEFONTEIN)	0,56	0,73	0,78	0,82	0,42	0,78
SUBAREA B (SOUTHEAST)	0,41	0,52	0,65	0,90	0,45	0,52
SUBAREA C (CAROLUSPOORT)	0,38	0,41	0,38	0,41	0,29	0,47
SUBAREA D (CENTRAL)	-	-	0,06	0,06	0,42	0,42
SUBAREA E (NORTH)	-	-	0,67	0,79	0,39	0,74
SUBAREA F (SOUTH WEST)	0,72	0,88	0,79	1,01	0,53	0,78
SUBAREA G (FAR NORTH)	-	-	0,26	0,41	-	-
TOTAL	2,07	2,54	3,59	4,40	2,50	3,83

The TDS-concentrations for each of these subareas are shown in Table 1.4.

Table 1.4. (Haasbroek, 1991).

SUBAREA		QUALITY RANGE
REGION ON MAP	NAME	mg/l
A	BURGERVILLE	600-950
B	SOUTHEAST	1000-1500
C	CAROLUSPOORT	1100-1400
D	CENTRAL	600-1200
E	NORTH	1200-1600
F	SOUTH WEST	1000-1350
G	FAR NORTH	700-1500

Water with a TDS of 1250 mg/l is acceptable to the municipality of De Aar.
The anticipated water demand for De Aar is depicted in Table 1.5.



GROUNDWATER RESOURCES
Subareas around DE AAR

SCALE :
0 1 2 3 4 5 10km

Fig. 1.6

Table 1.5. (Stewart, Sviridov and Oliver, 1990).

YEAR	DEMAND($\times 10^6 m^3$)
1995	2,90
1997	3,00
1998	3,04
2002	3,28
2005	3,50
2010	3,89

The unit cost of water per cubic metre from the Far Northern area is estimated as R1,32 (Haasbroek, 1991). The distance from this area to De Aar is approximately 30 km. If the assumption holds that there exists a direct relationship between the distance from De Aar and the cost per cubic metre of water, costs per cubic metre of water can be calculated for the other areas. (No data is available for the other areas). The calculated cost per cubic metre for each area is shown in Table 1.6.

Table 1.6. (Assuming a direct relationship between costs and distance).

SUBAREA	DISTANCE (km)	COST (R/m^3)
A	31	1,36
B	23	1,01
C	15	0,66
D	10	0,44
E	15	0,66
F	13	0,57
G	30	1,32

6.6.2 Optimization Problem. (Illustrative exercise).

The main goal of the exercise is to supply water of acceptable quantity as well as quality at minimum costs to De Aar. **It is important to note the deficiencies in the data sets and the results of the exercise must be viewed as an illustrative example only**.

Defining Decision Variables.

Let XA = The quantity of water pumped from area A per annum
XB = The quantity of water pumped from area B per annum
XC = The quantity of water pumped from area C per annum
XD = The quantity of water pumped from area D per annum
XE = The quantity of water pumped from area E per annum
XF = The quantity of water pumped from area F per annum
XG = The quantity of water pumped from area G per annum

Defining the Goal Function.

The goal is to supply water at minimum cost to De Aar. It is therefor a problem of cost minimization. From the information in Table 1.6. the goal function can be written as:

Minimize F = 1,36XA + 1,01XB + 0,66XC + 0,44XD + 0,66XE + 0,57XF + 1,32XG restricted by the following constraints:

Defining the Constraints

Yield constraints: (From Table 1.3)

XA < 0,82
XB < 0,90
XC < 0,41
XD < 0,06
XE < 0,79
XF < 1,01
XG < 0,41

Quality constraints: (The average quality from Table 1.4)

775XA + 1250XB + 1250XC + 900XD + 1400XE + 1175XF + 1100XG < 1250 x 2,9

where 1250 represents the quality limit of 1250 mg/l and 2,9 represents the demand from the system for 1995.

Demand constraint: (From Table 1.5.)

XA + XB + XC + XD + XE + XF + XG > 2,9

where 2,9 represents the demand from the system for 1995.

Solution of Problem.

The solution for the problem was computed by using the simplex optimization routine (Erikson and Hall, 1989).
The results of the optimization exercise are shown in Table 1.7.

Discussion of results.

The following results are shown in Table 1.7.

- The quantity that can be abstracted from each region per annum to minimize the costs.
- The minimum costs (total and unit) to supply the water.

A further exercise was carried out to investigate what the minimum quality would be if costs are not taken into account. For this exercise the goal function was changed from minimizing costs to minimizing quality. This results are shown in Table 1.8. From this table it can be seen that a much better quality can be obtained if costs are neglected from the solution. On average however the water costs about 20 cents per cubic metre more if costs are not minimized. Purifying costs however are 30 cents per cubic metre. It can thus be concluded that a "better" solution can be obtained by minimizing quality instead of minimizing costs if the purifying costs are taken into account.

Table 1.7

MINIMIZE COSTS

DEMAND = 2.9 MILLION CUBIC METRES PER ANNUM

DEC VARIABLE	OPTM VALUE	QUALITY	QUALTOT	COST FROM	TOTAL COST
XA	0.046	775	35.65	1.36	62560.00
XB	0.584	1250	730.00	1.01	589840.00
XC	0.410	1250	512.50	0.66	270600.00
XD	0.060	900	54.00	0.44	26400.00
XE	0.790	1400	1106.00	0.66	521400.00
XF	1.010	1175	1186.75	0.57	575700.00
XG	0.000	1100	0.00	1.32	0.00
TOTAL	2.900		3624.90		2046500.00
		QUAL(mg/l)=	1249.97	R/m^3=	0.71

DEMAND = 3.0 MILLION CUBIC METRES PER ANNUM

DEC VARIABLE	OPTM VALUE	QUALITY	QUALTOT	COST FROM	TOTAL COST
XA	0.046	775	35.65	1.36	62560.00
XB	0.684	1250	855.00	1.01	690840.00
XC	0.410	1250	512.50	0.66	270600.00
XD	0.060	900	54.00	0.44	26400.00
XE	0.790	1400	1106.00	0.66	521400.00
XF	1.010	1175	1186.75	0.57	575700.00
XG	0.000	1100	0.00	1.32	0.00
TOTAL	3.000		3749.90		2147500.00
		QUAL(mg/l)=	1249.97	R/m^3=	0.72

DEMAND = 3.04 MILLION CUBIC METRES PER ANNUM

DEC VARIABLE	OPTM VALUE	QUALITY	QUALTOT	COST FROM	TOTAL COST
XA	0.046	775	35.65	1.36	62560.00
XB	0.724	1250	905.00	1.01	731240.00
XC	0.410	1250	512.50	0.66	270600.00
XD	0.060	900	54.00	0.44	26400.00
XE	0.790	1400	1106.00	0.66	521400.00
XF	1.010	1175	1186.75	0.57	575700.00
XG	0.000	1100	0.00	1.32	0.00
TOTAL	3.040		3799.90		2187900.00
		QUAL(mg/l)=	1249.97	R/m^3=	0.72

DEMAND = 3.28 MILLION CUBIC METRES PER ANNUM

DEC VARIABLE	OPTM VALUE	QUALITY	QUALTOT	COST FROM	TOTAL COST
XA	0.016	775	12.40	1.36	21760.00
XB	0.900	1250	1125.00	1.01	909000.00
XC	0.410	1250	512.50	0.66	270600.00
XD	0.060	900	54.00	0.44	26400.00
XE	0.790	1400	1106.00	0.66	521400.00
XF	1.010	1175	1186.75	0.57	575700.00
XG	0.094	1100	103.40	1.32	124080.00
TOTAL	3.280		4100.05		2448940.00
		QUAL(mg/l)=	1250.02	R/m^3=	0.75

DEMAND = 3.50 MILLION CUBIC METRES PER ANNUM

DEC VARIABLE	OPTM VALUE	QUALITY	QUALTOT	COST FROM	TOTAL COST
XA	0.000	775	0.00	1.36	0.00
XB	0.900	1250	1125.00	1.01	909000.00
XC	0.410	1250	512.50	0.66	270600.00
XD	0.060	900	54.00	0.44	26400.00
XE	0.790	1400	1106.00	0.66	521400.00
XF	1.010	1175	1186.75	0.57	575700.00
XG	0.330	1100	363.00	1.32	435600.00
TOTAL	3.500		4347.25		2738700.00
		QUAL(mg/l)=	1242.07	R/m^3=	0.78

DEMAND = 3.89 MILLION CUBIC METRES PER ANNUM

DEC VARIABLE	OPTM VALUE	QUALITY	QUALTOT	COST FROM	TOTAL COST
XA	0.310	775	240.25	1.36	421600.00
XB	0.900	1250	1125.00	1.01	909000.00
XC	0.410	1250	512.50	0.66	270600.00
XD	0.060	900	54.00	0.44	26400.00
XE	0.790	1400	1106.00	0.66	521400.00
XF	1.010	1175	1186.75	0.57	575700.00
XG	0.410	1100	451.00	1.32	541200.00
TOTAL	3.890		4675.50		3265900.00
		QUAL(mg/l)=	1201.93	R/m^3=	0.84

Table 1.8

MINIMIZE QUALITY

DEMAND = 2.9 MILLION CUBIC METRES PER ANNUM

DEC VARIABLE	OPTM VALUE	QUALITY	QUALTOT	COST FROM	TOTAL COST
XA	0.820	775	635.50	1.36	1115200.00
XB	0.190	1250	237.50	1.01	191900.00
XC	0.410	1250	512.50	0.66	270600.00
XD	0.060	900	54.00	0.44	26400.00
XE	0.000	1400	0.00	0.66	0.00
XF	1.010	1175	1186.75	0.57	575700.00
XG	0.410	1100	451.00	1.32	541200.00
TOTAL	2.900		3077.25		2721000.00
		QUAL(mg/l) =	1061.12	R/m^3=	0.94

DEMAND = 3.0 MILLION CUBIC METRES PER ANNUM

DEC VARIABLE	OPTM VALUE	QUALITY	QUALTOT	COST FROM	TOTAL COST
XA	0.820	775	635.50	1.36	1115200.00
XB	0.290	1250	362.50	1.01	292900.00
XC	0.410	1250	512.50	0.66	270600.00
XD	0.060	900	54.00	0.44	26400.00
XE	0.000	1400	0.00	0.66	0.00
XF	1.010	1175	1186.75	0.57	575700.00
XG	0.410	1100	451.00	1.32	541200.00
TOTAL	3.000		3202.25		2822000.00
		QUAL(mg/l) =	1067.42	R/m^3=	0.94

DEMAND = 3.04 MILLION CUBIC METRES PER ANNUM

DEC VARIABLE	OPTM VALUE	QUALITY	QUALTOT	COST FROM	TOTAL COST
XA	0.820	775	635.50	1.36	1115200.00
XB	0.330	1250	412.50	1.01	333300.00
XC	0.410	1250 -	512.50	0.66	270600.00
XD	0.060	900	54.00	0.44	26400.00
XE	0.000	1400	0.00	0.66	0.00
XF	1.010	1175	1186.75	0.57	575700.00
XG	0.410	1100	451.00	1.32	541200.00
TOTAL	3.040		3252.25		2862400.00
		QUAL(mg/l) =	1069.82	R/m^3=	0.94

DEMAND = 3.28 MILLION CUBIC METRES PER ANNUM

DEC VARIABLE	OPTM VALUE	QUALITY	QUALTOT	COST FROM	TOTAL COST
XA	0.820	775	635.50	1.36	1115200.00
XB	0.570	1250	712.50	1.01	575700.00
XC	0.410	1250	512.50	0.66	270600.00
XD	0.060	900	54.00	0.44	26400.00
XE	0.000	1400	0.00	0.66	0.00
XF	1.010	1175	1186.75	0.57	575700.00
XG	0.410	1100	451.00	1.32	541200.00
TOTAL	3.280		3552.25		3104800.00
		QUAL(mg/l) =	1083.00	R/m^3=	0.95

DEMAND = 3.50 MILLION CUBIC METRES PER ANNUM

DEC VARIABLE	OPTM VALUE	QUALITY	QUALTOT	COST FROM	TOTAL COST
XA	0.820	775	635.50	1.36	1115200.00
XB	0.790	1250	987.50	1.01	797900.00
XC	0.410	1250	512.50	0.66	270600.00
XD	0.060	900	54.00	0.44	26400.00
XE	0.000	1400	0.00	0.66	0.00
XF	1.010	1175	1186.75	0.57	575700.00
XG	0.410	1100	451.00	1.32	541200.00
TOTAL	3.500		3827.25		3327000.00
		QUAL(mg/l) =	1093.50	R/m^3=	0.95

DEMAND = 3.89 MILLION CUBIC METRES PER ANNUM

DEC VARIABLE	OPTM VALUE	QUALITY	QUALTOT	COST FROM	TOTAL COST
XA	0.820	775	635.50	1.36	1115200.00
XB	0.900	1250	1125.00	1.01	909000.00
XC	0.410	1250	512.50	0.66	270600.00
XD	0.060	900	54.00	0.44	26400.00
XE	0.280	1400	392.00	0.66	184800.00
XF	1.010	1175	1186.75	0.57	575700.00
XG	0.410	1100	451.00	1.32	541200.00
TOTAL	3.890		4356.75		3622900.00
		QUAL(mg/l) =	1119.99	R/m^3=	0.93

LINEAR PROGRAMMING 07-06-1992 - 23:51:36

-=*=- INFORMATION ENTERED -=*=-

```
NUMBER OF VARIABLES          :  7
NUMBER OF <= CONSTRAINTS     :  8
NUMBER OF  = CONSTRAINTS     :  0
NUMBER OF >= CONSTRAINTS     :  1

MIN MICOS =     1.36 XA +   1.01 XB +    .66 XC +    .44 XD +    .66 XE
          +      .57 XF +   1.32 XG

SUBJECT TO:

     1     XA +    0     XB +    0     XC +    0     XD +    0     XE
+    0     XF +    0     XG                                  <=       .82

     0     XA +    1     XB +    0     XC +    0     XD +    0     XE
+    0     XF +    0     XG                                  <=       .9

     0     XA +    0     XB +    1     XC +    0     XD +    0     XE
+    0     XF +    0     XG                                  <=       .41

     0     XA +    0     XB +    0     XC +    1     XD +    0     XE
+    0     XF +    0     XG                                  <=       .06

     0     XA +    0     XB +    0     XC +    0     XD +    1     XE
+    0     XF +    0     XG                                  <=       .79

     0     XA +    0     XB +    0     XC +    0     XD +    0     XE
+    1     XF +    0     XG                                  <=      1.01

     0     XA +    0     XB +    0     XC +    0     XD +    0     XE
+    0     XF +    1     XG                                  <=       .41

   775     XA + 1250     XB + 1250     XC +  900     XD + 1400     XE
+ 1175     XF + 1100     XG                                  <=    3625

     1     XA +    1     XB +    1     XC +    1     XD +    1     XE
+    1     XF +    1     XG                                  >=      2.9
```

-=*=- RESULTS -=*=-

VARIABLE	VARIABLE VALUE	ORIGINAL COEFFICIENT	COEFFICIENT SENSITIVITY
XA	.046	1.36	0
XB	.584	1.01	0
XC	.41	.66	0
XD	.06	.44	0
XE	.79	.66	0
XF	1.01	.57	0
XG	0	1.32	.199

CONSTRAINT NUMBER	ORIGINAL RIGHT-HAND VALUE	SLACK OR SURPLUS	SHADOW PRICE

1	.82	.774	0
2	.9	.316	0
3	.41	0	.35
4	.06	0	.828
5	.79	0	.239
6	1.01	0	.495
7	.41	.41	0
8	3625	0	.001
9	2.9	0	1.931

OBJECTIVE FUNCTION VALUE: 2.046

-- SENSITIVITY ANALYSIS --

OBJECTIVE FUNCTION COEFFICIENTS

VARIABLE	LOWER LIMIT	ORIGINAL COEFFICIENT	UPPER LIMIT
XA	1.01	1.36	1.992
XB	.828	1.01	1.302
XC	NO LIMIT	.66	1.01
XD	NO LIMIT	.44	1.268
XE	NO LIMIT	.66	.899
XF	NO LIMIT	.57	1.065
XG	1.121	1.32	NO LIMIT

RIGHT-HAND-SIDE VALUES

CONSTRAINT NUMBER	LOWER LIMIT	ORIGINAL VALUE	UPPER LIMIT
1	.046	.82	NO LIMIT
2	.584	.9	NO LIMIT
3	.094	.41	.994
4	0	.06	.122
5	.645	.79	1.234
6	.635	1.01	1.3
7	0	.41	NO LIMIT
8	3347.5	3625	3646.75
9	2.883	2.9	3.194

---------- E N D O F A N A L Y S I S ----------

C O M P U T E R   M O D E L S   F O R   M A N A G E M E N T   S C I E N C E

LINEAR PROGRAMMING   07-06-1992 - 23:36:12

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NUMBER OF VARIABLES              :  7
NUMBER OF <= CONSTRAINTS         :  7
NUMBER OF  = CONSTRAINTS         :  0
NUMBER OF >= CONSTRAINTS         :  1

MIN QLTY =     775  XA + 1250  XB + 1250  XC +  900  XD + 1400  XE
            + 1175  XF + 1100  XG

SUBJECT TO:

     1  XA +  0  XB +  0  XC +  0  XD +  0  XE
 +   0  XF +  0  XG                           <=     .82

     0  XA +  1  XB +  0  XC +  0  XD +  0  XE
 +   0  XF +  0  XG                           <=     .9

     0  XA +  0  XB +  1  XC +  0  XD +  0  XE
 +   0  XF +  0  XG                           <=     .41

     0  XA +  0  XB +  0  XC +  1  XD +  0  XE
 +   0  XF +  0  XG                           <=     .06

     0  XA +  0  XB +  0  XC +  0  XD +  1  XE
 +   0  XF +  0  XG                           <=     .79

     0  XA +  0  XB +  0  XC +  0  XD +  0  XE
 +   1  XF +  0  XG                           <=    1.01

     0  XA +  0  XB +  0  XC +  0  XD +  0  XE
 +   0  XF +  1  XG                           <=     .41

     1  XA +  1  XB +  1  XC +  1  XD +  1  XE
 +   1  XF +  1  XG                           >=    2.9
```

-=*=- RESULTS -=*=-

VARIABLE	VARIABLE VALUE	ORIGINAL COEFFICIENT	COEFFICIENT SENSITIVITY
XA	.82	775	0
XB	.19	1250	0
XC	.41	1250	0
XD	.06	900	0
XE	0	1400	150
XF	1.01	1175	0
XG	.41	1100	0

CONSTRAINT NUMBER	ORIGINAL RIGHT-HAND VALUE	SLACK OR SURPLUS	SHADOW PRICE
1	.82	0	475
2	.9	.71	0

3	.41	0	0
4	.06	0	350
5	.79	.79	0
6	1.01	0	75
7	.41	0	150
8	2.9	0	1250

OBJECTIVE FUNCTION VALUE: 3077.25

-- SENSITIVITY ANALYSIS --

OBJECTIVE FUNCTION COEFFICIENTS

VARIABLE	LOWER LIMIT	ORIGINAL COEFFICIENT	UPPER LIMIT
XA	NO LIMIT	775	1250
XB	1250	1250	1400
XC	NO LIMIT	1250	1250
XD	NO LIMIT	900	1250
XE	1250	1400	NO LIMIT
XF	NO LIMIT	1175	1250
XG	NO LIMIT	1100	1250

RIGHT-HAND-SIDE VALUES

CONSTRAINT NUMBER	LOWER LIMIT	ORIGINAL VALUE	UPPER LIMIT
1	.11	.82	1.01
2	.19	.9	NO LIMIT
3	0	.41	.6
4	0	.06	.25
5	0	.79	NO LIMIT
6	.3	1.01	1.2
7	0	.41	.6
8	2.71	2.9	3.61

---------- E N D O F A N A L Y S I S ----------

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1. CONCLUSIONS

Practical resource development and management decisions are subject to normal project constraints and data limitations under a variety of hydrological conditions. All these constraints and limitations pose a problem to the geohydrologist, from whom it is usually required to develop a management strategy for an aquifer. Water is a resource which requires effective management throughout the world. This is particularly true of South African conditions where there is an ever-increasing demand on the limited water supply by the domestic, industrial and agricultural sectors.

The management of an aquifer by means of a groundwater model, involves the following:

- Development of a simulation model.
- Estimation of the exploitation potential of the aquifer.
- Defining of specific objectives for practical applications.
- Setting constraints on variables.
- Obtain optimised solution (optimization model).

The development and application of groundwater optimization models for the control of groundwater hydraulics and water quality, and the inverse problem of parameter estimation, are the primary emphases of the groundwater management package, AQUAMOD, developed during the present study. The computer programs developed, can be used for:

- constructing a finite element mesh,
- automated calibration of the aquifer parameters (inverse problem),
- obtaining an estimate of natural groundwater recharge,
- risk evaluation by means of a groundwater balance,
- optimization and groundwater flow simulations and
- mass transport solutions.

The construction of a reliable flow model, and thus a management model for an aquifer, may be premature, unless extensive monitoring of water levels and abstraction rates were performed for a number of years.

The Grootfontein Case Study, presented as part of the present project in Section 3, is a typical and well-illustrated example of what could be achieved as a result of application of the AQUAMOD package and it is foreseen that many other case studies in South Africa will be based on the methods used in this case study.

Although the De Aar Case Study (presented in this section) was only an illustrative example (because of data deficiencies), the ideas illustrated should provide a sound basis for practical applications.

7.2. RECOMMENDATIONS

A mass transport model was coded as part of the present study, but due to the complex nature of most of our geological formations, it was decided by the Steering Committee that the work on water quality modelling has to be scaled down and that no case study should be evaluated. Due to the importance of water quality modelling under South African conditions, this must receive attention in the near future.

A non-achievement of the present project, was the lack of application of a suitable practical example on quadratic optimization. The need for such an application could become important in the near future as more emphasis may be placed on the energy required to operate pumps (this is because costs vary with pumping rates and pumping lifts, and lifts depend on pumping rates).

The AQUAMOD package is based on a two-dimensional approach and whenever a three-dimensional model may be required for an aquifer, the techniques which were developed during this study, could be easily incorporated into a three-dimensional model.

The problem of Geohydrological Decision Analysis fell beyond the scope of the present study, but should receive attention in the near future. In this regard, the Bayesian updating and Kalman filter techniques could provide a sound basis for future research on decision analysis.

It is recommended that the use of the AQUAMOD package for groundwater management problems is to be promoted.

THE DEVELOPMENT OF RISK ANALYSIS AND GROUNDWATER MANAGEMENT TECHNIQUES FOR SOUTHERN AFRICAN AQUIFERS

SECTION 2

RISK ANALYSIS IN GROUNDWATER MANAGEMENT

by

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FOREWORD

When assessing a proposed aquifer management plan, information on the geohydrological parameters, the recharge rate and the proposed extraction rate are required. With the exception of the last of these, the determination of these parameters needs be inferred from empirical studies or historical records. The utility of any evaluation of a particular management plan will depend on the accuracy of the available information.

The purpose of risk assessment is to incorporate all the uncertainties of the available information into an evaluation of an aquifer management proposal. The output would then provide the planners with statistics enabling them to evaluate the long term viability of a management plan.

From a literature study it was concluded that the most appropriate method of risk assessment would be a Monte Carlo type of system which would involve an aquifer simulation model using simulated values of storativity and transmissivity conditioned on all available information as input, and rainfall simulations honouring the available rainfall records.

In this study a finite element aquifer model (AQUAMOD) developed by IGS was used as the aquifer simulation model after some minor modifications.

Two methods for simulating the geohydrological parameters were proposed. The first of these, the zonal simulator, involved dividing the aquifer into a number of homogeneous zones and using the zonal means and covariances between the zones, simulations can be obtained for the zones. The second involved the geostatistical method of turning bands which simulates values on a fine grid. The input for this method are the means and variograms of the geohydrological parameters. A variogram is a measure of spatial continuity within the aquifer.

The rainfall was generated using software based on an algorithm proposed by Zucchini (1 984). The validity of the rainfall simulations was verified and found to be satisfactory.

The evaluation of risk relates to specific events which can be defined to meet the requirements of a particular study. After considerable investigation risk events were defined in terms of the difference between the simulated and base water levels in a number of boreholes falling below a chosen tolerance level. Tolerances were defined as a proportion of effective depth.

The risk evaluation is achieved by repeatedly running the aquifer simulation with different simulated values of the input parameters (storativity, transmissivity and rainfall). The risk evaluation model generates an immense amount of intermediate results and one of the main objectives of this project was to summarise these into a few informative bits of information to be used by the decision maker. The study of the literature did not suggest risk evaluation statistics and these had to be developed as part of this project. Statistics fell into two categories namely summary and risk statistics.

The statistics were calculated by accumulating the results generated by the individual realizations and then summarising them into meaningful measures. The statistics are presented as expected values. The summary statistics included the minimum, maximum and average values of water levels relative to the base of the aquifer.

The risk statistics were defined as the expected period (duration) and expected probability of a failure. It should be emphasised that although these were found to be suitable for the needs of this project, other statistics could be used if these better meet the objectives of a particular investigation.

The case study reported in Section 3 concluded that the methodology proposed did provide meaningful results.

The most appropriate application would be for long term viability studies undertaken during the planning stages of a long term management plan. Heavy computing time requirements make it an unsuitable method for operational management. Despite this the elements of the system could be incorporated in a system suitable for day to day management of an aquifer. This extension is recommended by the project team. It is envisaged that such a system would enable a manager to make short term predictions of the effect a management plan will have on an aquifer situation taking the current situation into account.

CONTENTS

1. INTRODUCTION AND OBJECTIVES 2-1
   1. Introduction 2-1
   1.2 Objectives 2-1
   1.3 Structure of Section 2 of the report 2-2

2. LITERATURE SURVEY 2-3

3. TERMINOLOGY USED IN RISK STATISTICS FOR SURFACE AND GROUNDWATER SYSTEMS 2-3

4. STOCHASTIC VERSUS DETERMINISTIC APPROACH TO RISK ASSESSMENT 2-5
   4.1 Deterministic methods 2-5
   4.2 Stochastic methods 2-6
   4.3 Approach followed 2-6

5. ELEMENTS OF RISK ANALYSIS 2-7

6. THE TWO APPROACHES TO STOCHASTIC AQUIFER MODELLING . 2-8
   6.1 Inversion (Nonlinear regression) 2-9
   6.2 Geostatistical approach 2-9

7. THE RISK EVALUATION SYSTEM - SOFTWARE ISSUES 2-10
   7.1 Zonal simulation programme 2-11
   7.2 Geostatistical simulation programme 2-12
   7.4 Rainfall simulator 2-13

8. STATISTICS 2-14
   8.2 Risk events 2-17
   8.3 Risk quantification 2-17
      8.3.1 Period 2-18
      8.3.2 Probability 2-18
      8.3.3 Return time 2-19

9. CONCLUSIONS AND RECOMMENDATIONS 2-20
   9.1 Conclusions 2-20
   9.2 Recommendations 2-21

1. INTRODUCTION AND OBJECTIVES

1. Introduction

Surface water resources are managed in many different ways. The Department of Water Affairs and Forestry (DWA&F) has over the years in cooperation with consultants successfully developed management techniques and strategies for South African surface water sources on a catchment basis. Lately, the risks involved in water management decisions have been included in the overall management strategies and are currently being applied with a large degree of success. Similar techniques do not exist for the management of groundwater resources. The DWA&F identified that management techniques for groundwater resources are not as far advanced as those for surface water, and this has led to the formulation of a joint research project between the CSIR and the Institute for Groundwater Studies (IGS) at the University of the Orange Free State.

The combined project can be divided into two main components, namely

i) a risk analysis component; and

ii) an aquifer management component.

Research on the risk analysis components resided mainly at the CSIR, whereas the aquifer management section was addressed by the IGS. Cooperation and liaison between the two groups was achieved throughout the entire project and led to the application and testing of the techniques developed on the Grootfontein aquifer at Mmabatho. The case study is dealt with in Section 3 of this report.

1.2 Objectives

The overall aim of the research project is to identify and measure the risks associated with the exploitation of groundwater resources in order to evaluate optimal management policies.

In particular the behaviour of the water levels of an aquifer are studied under stochastic variation representing the uncertainty of the physical properties of the aquifer and recharge (rainfall) values. The objectives as listed in the research proposal were as follows:

- evaluation of existing risk analysis techniques for the management of surface and surface and groundwater resources;
- identification of the geohydrological variables applicable in risk analysis assessments;
- determining the data requirements necessary to estimate the geohydrological variables;
- selection and development of risk analysis methodology for application in groundwater management;
- evaluation of the importance and role of geohydrological variables in risk analysis studies by using simulated data;
- application of geostatistical techniques to calculate geohydrological parameters for use in risk analysis studies;

- sensitivity analysis of these geostatistically derived estimates; and
- application of selected and-developed methodologies and techniques in collaboration with the IGS to a case study.

1.3 Structure of Section 2 of the report

Section 2 of the report includes an overview of the software tools developed and simulated results which conform to real data. It also discusses the practical issues with regard to the implementation of the methodologies that were developed during the course of the research project.

Briefly, the approach followed uses Monte Carlo simulation for evaluating the statistics which summarize the occurrence of failures in time and space where failure was defined as some event relating to the status of the aquifer. As part of this approach, monthly rainfall and the physical properties of the aquifer were simulated to account for the uncertainty relating to both these properties. The two aquifer properties, transmissivity and storativity, were simulated by two different methods, namely a zonal simulator and geostatistical simulator. A rainfall simulator developed by Zucchini and Adamson (1984) was used to simulate rainfall.

By using the Monte Carlo method, expected values for statistics describing the uncertainties, or risk or the long term utilization of an aquifer according to a specified management plan were computed.

The risks evaluated relate to the long term performance of the aquifer and at this stage cannot be used to predict failures in the short term which was outside the defined scope of this project.

During the course of the project a number of alternative methods for representing risk were proposed. Some of these were abandoned after thorough investigation when they were either found to be unsuitable with respect to the project objectives or unpractical to implement. These intermediate results are mentioned in Section 2 of the report but are not detailed. The objective of Section 2 is to summarize the project and present the final methodology in detail.

An overview of the literature study is given in Chapter 2 and in Chapter 3 the terminology used in risk statistics for surface and groundwater systems are described.

The different approaches which could be followed are listed in Chapter 4, as well as a discussion on the selection of the Monte Carlo method.

In Chapter 5 the stochastic elements which play a role in risk assessment are identified and discussed.

The two approaches to stochastic modelling and the different modules of the risk evaluation system are discussed in Chapters 6 and 7.

In Chapter 8 the statistics used to summarize the performance of the aquifer are described and illustrated.

Chapter 9 contains the conclusions and recommendations. References for Section 2 of the report are contained in a comprenesive list of references following Section 3 of the report.

The methodology and techniques developed in the course of the project, were applied to the Grootfontein aquifer in the Western Transvaal. The results of this case study are described in Section 3 of this report.

2. LITERATURE SURVEY

The initial step in analysing the feasibility of risk management techniques for groundwater resources was an appraisal of existing work in the field. Available literature reveals three sources of possible approaches to risk analysis and risk management for aquifers:

i) risk analysis and management of surface water resources (including reservoir operation);

ii) management of the conjunctive use of surface and groundwater resources; and

iii) risk analysis and management of groundwater resources.

In accordance with its brief, CSIR has concentrated on groundwater resource management while taking note of the methodologies of surface water management and conjunctive use management. A reference list of relevant papers is given in Appendix A. Section 3 of this report concentrates on a specific example (the Grootfontein aquifer in the Western Transvaal) of groundwater management (van Rensburg, *et al.*, 1994).

Of all the references surveyed, Freeze et al. (1990), presents a consistent and complete discussion of techniques for risk analysis in aquifer management. This paper, describing stochastic simulation, conditioning, inverse simulation, Bayesian updating and aquifer management criteria, served as the seminal reference for this research.

The development of the appropriate risk statistics for the groundwater application was essentially a product of this research as little was available in the literature regarding this aspect.

3. TERMINOLOGY USED IN RISK STATISTICS FOR SURFACE AND GROUNDWATER SYSTEMS

Concepts developed jointly by the firm BKS and the Department of Water Affairs and Forestry for the analysis of surface water systems, and in particular the Vaal River system, have led to the successful planning and management of these water sources (DWA, 1986). By adjusting existing methods and developing modern techniques of optimization and risk analysis, and then implementing these, the practical operation and decision mailing on a large scale in the surface water resources field, has advanced rapidly. These concepts formed a basis for the current project and as such there are some parallels that can be drawn between the risk analysis techniques developed for the two water sources.

The approach followed in the surface water field was to develop a long term as well as a short term yield analysis. In the current project under discussion, only the long term risk analysis and management was addressed. The short term and more practical managing procedures in terms of risk, will hopefully be addressed in a follow-up project.

For the surface water systems, many stochastic sequences are fed into the system under the same conditions as the historic sequence to ascertain how the system could possibly perform in future. The purpose of this is to determine the reliability of the system by recording the values attained by certain key variables that are produced by each of the individual independent stochastic analyses. In the surface water terminology these are firm yield, average yield and base yield corresponding to the various levels of target draft. In the stochastic analysis values for these variables are calculated for each stochastic sequence. These are then compared to the values of the historic sequence. The reliability of the sequence is then inferred by determining how frequently in the stochastic trials a particular value of a key variable is exceeded. For both the surface and the groundwater systems, the number of stochastic sequences that need to be generated to give acceptable precision have to be determined experimentally. The main reason for optimising the number of sequences, is to cut computing time (and cost) without sacrificing accuracy.

A groundwater system is considerably more complex to analyse than a surface water system because of the many unknown variables that have to be estimated and cannot be measured accurately. As a result, a series of related variables used in the surface water environment, sometimes has to be represented by a single variable in the groundwater system. For example, in the analysis of surface water systems, the "target draft" is made up of components of the yield namely, "firm, base, average and total yield". These are used to describe a historic inflow sequence, whereas the groundwater equivalent for this family of terms, is abstraction or yield produced or sustainable by the aquifer per unit time. A similar separation of the total yield cannot be made in the groundwater field.

However, in practice it is possible to reduce the yield to less than the target draft when the reservoir level is low, and increase it when the reservoir is spilling. The equivalent in the groundwater case is over and under abstraction which can be achieved by operating more or fewer pumps.

The concept of firm yield, is defined as the maximum base yield that can be abstracted from the river/reservoir system for a given inflow sequence. In groundwater terminology this is equivalent to transmissivity. The difference is however that firm yield can be defined accurately to be a fixed value for a reservoir, but a great deal of uncertainty surrounds the value of transmissivity or variation thereof over the entire aquifer. The same argument can be advanced for a number of geohydrological variables, i.e. porosity, storativity, safe yield, specific capacity, and others.

Risk of failure of a water resource system can be defined in many different ways. For the surface water system it was defined as "the inability of the system to supply base yield associated with a specific target draft" (DWA, 1986). In this study a failure was regarded to occur if:

(i) the water level at one or more of the nodes drops below the minimum tolerance;

(ii) the water level at all the nodes drops below the maximum tolerance level; and

(iii) the average water levels at the nodes drops below the average tolerance.

The approach followed was to compare the nodal minimum or maximum water levels to the maximum and minimum tolerance and record a minimum and maximum failure. In addition the average nodal water level was compared to the average tolerance and an average failure recorded if this was less than the average tolerance.

It is common practice to make use of the recurrence interval concept to quantify risk of failure of a river/reservoir system. Recurrence interval is the expression used to describe the mean time of occurrence between "failures" of a system. These are typically referred to as 1 in 20, 1 in 50 years, etc. The recurrence, or return time as it is referred to in this section of the report, gives an indication of the time that elapsed before the next failure is about to occur once a failure has occurred. The application of this concept to groundwater risk analysis studies was studied intensively and evaluated to be of no value.

The surface water risk analysis relied heavily on the reliability of the values of for instance the base yield, generated by a large number of separate, independent, stochastic sequences of the stream flow records. The reliability of generated values for firm yield and supply, both over the long and the short term, were also used extensively.

The groundwater equivalent used is the reliability (or as it is termed in this section of the report the "validation") of the rainfall, transmissivity and storativity values produced by the multiple (99) stochastic simulations.

In this study the methodology developed was specifically directed at the long term analysis of risks involved in the management of groundwater resources. Now that the methodology has been successfully developed, management of the groundwater resources in the short term taking into account the risk inherent in the plans, can be addressed. The short term approach is the subject of a future research project.

4. STOCHASTIC VERSUS DETERMINISTIC APPROACH TO RISK ASSESSMENT

Two contrasting approaches to risk evaluation were evident in the papers reviewed namely, a deterministic or stochastic approach. The advantages and disadvantages of these two approaches are discussed below.

4.1 Deterministic methods

Deterministic methods use worst-case input parameters to evaluate the risk associated with an aquifer management policy. For example, Orlovski et al. (1984) used a few selected inflow sequences representative of floods and droughts.

A deterministic analysis is possible, however, only if the hydrogeological parameters are known with certainty, or if most-likely representative values are available (Freeze et al., 1990).

None of the reviewed papers makes use of a completely deterministic method, because the inputs to an aquifer system are intrinsically variable and unobservable.

4.2 Stochastic methods

In stochastic analysis, uncertainty in the input parameters is specified in the form of a probability density function. There are three basic approaches used to propagate these uncertainties: (1) first order analysis, (2) perturbation analysis, and (3) Monte-Carlo analysis.

First-order analysis is a simple and direct means of propagating uncertainty. It can be employed with either analytical or numerical solutions of the governing equations, but it is limited to linear or nearly linear systems, for which the coefficient of variation of model parameters is much less than one. The approach is unsuitable for the estimation of failure probabilities unless some assumptions can be made about the form of the probability distribution function of the output variable (Freeze et al. 1990).

With the perturbation approach both the output variable and the input parameters are defined in terms of a mean plus a perturbation about the mean. The relationship between the input and output uncertainties can be developed using two general techniques, namely Fourier-Stieljes integrals, inverse Fourier transform and finite-element, finite difference methods (Freeze et al. 1990).

Monte Carlo simulation provides the most general approach to uncertainty propagation. With this approach, a large number of equally likely realizations of each parameter field are generated, and the hydrogeological simulation model is run for each realization. The method is widely used because of its generality and simplicity. (Freeze et al. 1990).

Quantities which are usually modeled stochastically in aquifer studies are the transmissivity, storativity, hydraulic conductivity and the recharge (rainfall). Of these, most of the classical work has concentrated on rainfall using time series models (Naff et al., 1983; Grosman et al., 1985). Modelling the variability in the physical parameters of the aquifer is, however, equally important for evaluating risk (Hodgson, 1978; Davis, 1982; Nguyen et al., 1985). The parameter uncertainty model provides the methodology for calculating spatial statistics of an aquifer (Freeze et al., 1990). Reliability measures of the system performance are developed on the basis of predicted hydrologic uncertainties (Datta et al., 1984).

4.3 Approach followed

In this study which included a practical case study it was concluded that Monte Carlo simulation of the uncertainties would provide the desired flexibility. Furthermore the methods are well understood with high potential for future application. For this project extensive computing resources were available but for large scale practical application, ways of reducing the computing would need to be found.

5. ELEMENTS OF RISK ANALYSIS

In a model representation of a managed system, such as an aquifer, variables are of two kinds:

i) those corresponding to occurrences with natural (unpredictable) variation such as rainfall, and
ii) those corresponding to fixed (possibly unknown) characteristics such as the physical dimensions and properties of an aquifer.

In modelling the system both types of variables can be stochastically simulated; in the first case so as to account for the unpredictability of the system and in the second, so as to reflect the uncertainty of knowledge regarding physical parameters.

Risk is described in terms of risk events such as system failures and extreme values and measured as the probability of one of these events occurring.

Three quantities in aquifer modelling suitable for defining risk events are:

i) the hydraulic head,
ii) the pump-out rate, and
iii) the recharge rate (rainfall).

In a managed system such as an aquifer the different stochastic inputs are combined via a complex model of interactions. The stochastic elements are combined with finite element aquifer models and flow equations resulting in an analytically intractable model and the only recourse is to computer simulation. The Monte Carlo simulation generates a large number of realizations of each of the input processes which are then used as inputs to the model. The resulting outputs are then analysed using appropriate risk statistics most of which were developed as part of this project.

Variability, thus being the source of the risk, must be properly modeled in order that the output stochastic process represent a valid abstraction of reality.

The components of the risk assessment model are displayed schematically in Figure 1 . Inputs and outputs are centered around a finite element software model. An important aspect of the structure is the stochasticity of the recharge function (rainfall), and of the physical properties of the aquifer (storativity and transmissivity). As a result of these stochastic inputs, the output of the model (water levels at each node in the finite element grid), is also stochastic. The risk analysis procedure which was developed concerned ways of quantifying 'undesirable situations' arising from these two sources of randomness, as observed via the output of the model.

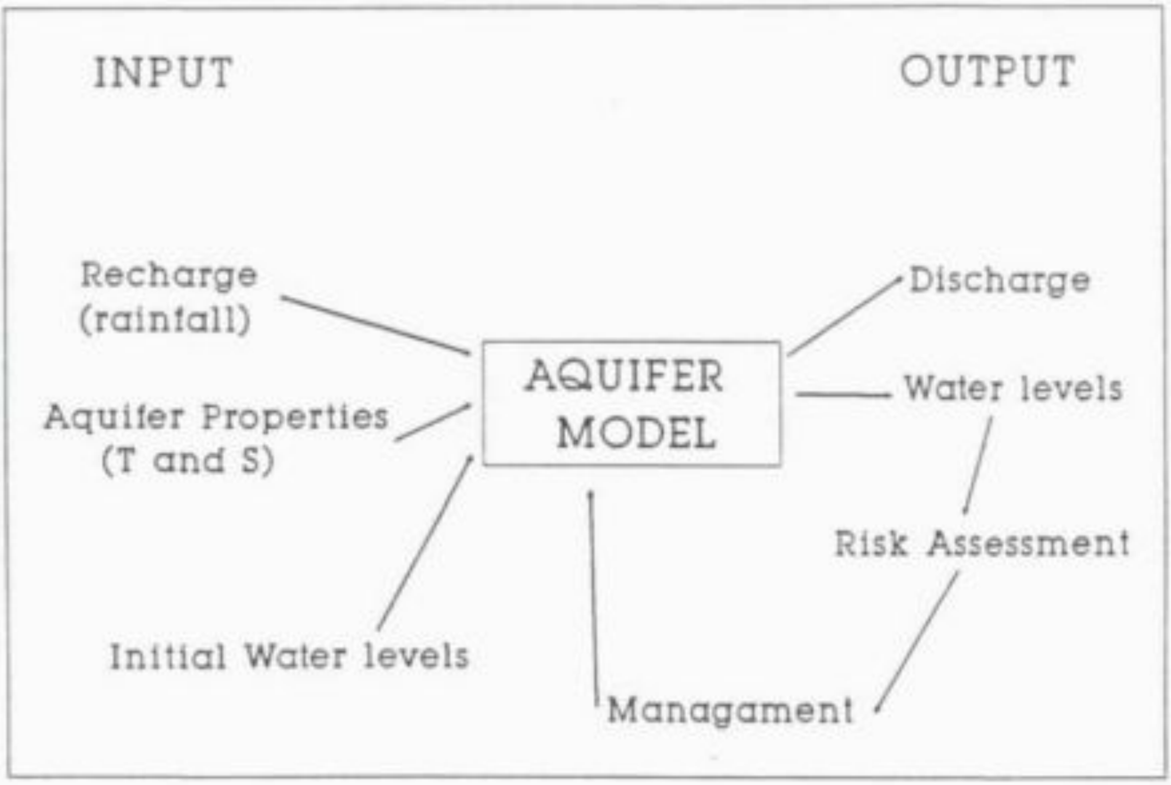


Figure 1: Schematic representation of the different components of an aquifer risk assessment model

6. THE TWO APPROACHES TO STOCHASTIC AQUIFER MODELLING

At the core of the risk assessment model is the numeric model representing the physical aquifer. In practice the actual geological and geohydrological conditions within the aquifer are never known. The important parameters are boundaries, depth, transmissivity, hydraulic conductivity and storativity. The last three of these are important when determining the operating characteristics. At best information on transmissivity and storativity is available at a limited number of positions (boreholes) in the aquifer and at worst we need to depend on intelligent deduction. The first step in the modelling is to use available data to estimate (through interpolation) the parameter values at all nodes within the aquifer. The second step is to introduce the uncertainty to this mean realization by simulating new realizations which are acceptable given the data but differ from the mean representation.

Two approaches to the stochastic modelling are considered; the first method, inversion, divides the aquifer into zones and estimates a representative (average) value for each zone while the second estimates the parameters at each point within the aquifer.

6.1 Inversion (Nonlinear regression)

A description of this method can be found in Seber and Wild (1989). For completeness a brief description of the manner in which the regression is set up and the output applied in the risk analysis follows below.

Although inversion potentially offers a good way of obtaining a covariance matrix to calibrate zonal simulations, the available software did not have adequate convergence properties. It was beyond the scope of this project to develop inversion routines.

The aquifer is divided into zones. A mathematical model of the aquifer which is a function of the required zonal parameters can be used to predict the data at the observation boreholes. Estimates of the average storativities and transmissivities for each of these zones are obtained by minimizing the difference between observed and predicted (by the model of the aquifer) transmissivity and storativity at each borehole site in the aquifer. The method corresponds to that of nonlinear regression. The inversion process has the property that estimated covariances are obtained between the zones. These covariances reflect the uncertainty inherent in the estimates on transmissivity and storativity. Multi-Gaussian simulation using the means and covariance matrices from the inversion process can then be used to generate large numbers of equally likely realizations, any of which could be the actual, but unknown, true values for the aquifer. These realizations would serve as stochastic inputs to the risk analysis. The number of realizations for each simulation will depend on the number of zones allocated to this region when the finite element grid was set up. Every element in a specific zone will be set to the value that was simulated for the whole zone.

6.2 Geostatistical approach

The geostatistical methods used in this project are standard applications and are well described in literature (Journel A.G. and Huijbregts C.H.J., 1978). The discussions of the techniques used is provided to aid the reader to gain insight into the methods as they apply to this application. It is recommended that the geostatistical methods should be applied with the assistance of someone familiar with the techniques.

Semi-variograms are used to measure spatial continuity of geological phenomena. They are directly related to the autocovariance function of a process, and quantify the variability of measurements as a function of their separation distance.

The semi-variogram is estimated by computing the square differences between all pairs of observations. Observations must be correct, plentiful, sampled over the whole region and spatially correlated, so as to give a good estimate of the semi-variogram.

The underlying random field from which the observations were drawn is described statistically by its mean value and its variogram. Given these two quantities and a set of observations taken at specific locations on a realization of the random field, it is possible to determine the marginal conditional random field at unobserved locations. The values at these locations can either be predicted or simulated. In simulation mode, software generates artificial realizations of the geology, which are statistically indistinguishable from the original. Under the assumption of Gaussianity, the technique of Kriging provides the marginal conditional mean and marginal conditional variance at an unobserved location. Using these two quantities as the parameters of a normal distribution, the unobserved random variable may be simulated and added to the data set. Proceeding sequentially in this manner, an entire set of realizations can be generated if the explosion of the data set size can be controlled. A simulation technique, which achieves this size control, is the turning bands method, based on the relationship between the autocovariance function of a random process and its convolution product with a moving average weighting function.

Both sequential multi-Gaussian and turning bands simulators yield Gaussian realizations, whereas lognormal realizations are most likely sought for aquifers. A simple approach to this problem is to generate Gaussian simulations which honour the statistics of the log-transformed observations, and then to exponentiate the results. Regrettably, there is no guarantee that this process will preserve the original covariance structure, but neither is there any existing theory to improve upon it. A technique which provides some security (and which we employed) is comparison of the simulated variograms with the original variograms.

Geo-spatial simulation is generally performed on regular, rectangular grids, groundwater modelling software, however, often uses irregular grids whose densities and structures vary over the region of interest, according to the complexity of the model. Existing geo-spatial simulation software was used to generate fine-grain grids of transmissivities and storativities. Each finite element node will then be allocated the value of the simulated grid node closest to it. In this way, the observed covariance structure will be honoured as far as possible.

7. THE RISK EVALUATION SYSTEM - SOFTWARE ISSUES

In order to obtain the risk statistics, a simulation approach was followed allowing for many different but equally likely scenarios to be considered.

Figure 2 illustrates the simulation system. For each realization of the system a transmissivity and storativity simulation over the aquifer is generated which is used to calibrate the aquifer modelling program AQUAMOD. A fixed discharge rate at a number of predefined boreholes is taken to represent the management model being evaluated. A sequence of 99 years of rainfall (a rainfall cycle is taken to be from July to June) is generated and AQUAMOD returns a sequence of predicted water levels at all nodes of the aquifer for every 30 days for the entire period of 99 years. From these water levels the statistics required for the risk statistics are accumulated. The system is re-run until enough realizations are available to compute the risk statistics.

The software is built up of a number of different modules each performing a different function as described in Section I of this report. These modules were combined into a single executable program. Although this approach speeded up the simulations considerably, it is still a very computationally intensive process. The execution time of the AQUAMOD run is proportional to the size of the finite element grid.

To ensure representative statistics and stable aquifer conditions for a 99 year period a 110 year time span is used in the simulation with the first 11 years allowed for the system to achieve stability. The results of these first eleven years are discarded. It is important to note that the 110 year period is not to predict the state of the aquifer in 110 years time but only to obtain realistic estimates of return times and periods.

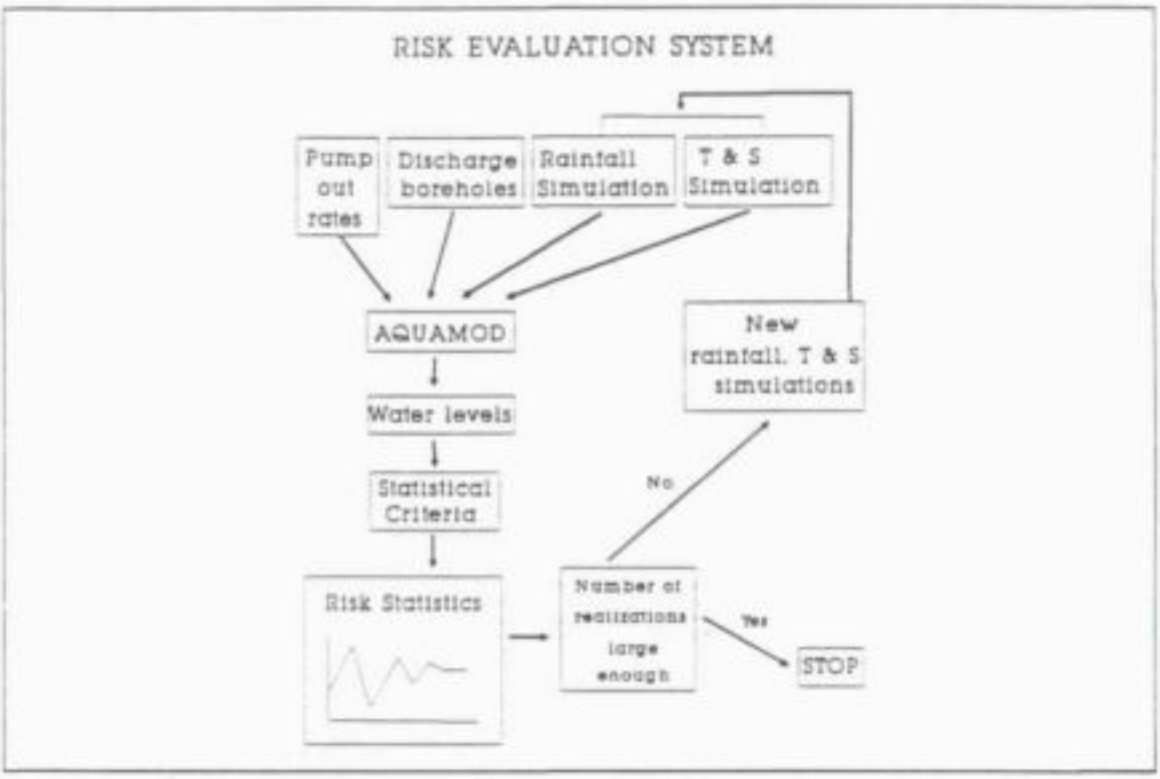


Figure 2: The simulation approach followed to obtain the risk statistics for different scenarios.

All the source code was encoded in Fortran. The program was compiled on a 486 PC with coprocessor and on an UNIX operating system. The simulations were executed on the UNIX operating system (60 MHz, 68 DHRYSTONE) to achieve the best execution speed. It takes about 23 hours CPU time to compute 99 realizations using the zonal simulation program. The geostatistical simulation program is computationally more expensive because transmissivity and storativity values are simulated on a very fine grid. Values are then allocated to all the elements defined in the finite element mesh. For this, execution time will be somewhat slower.

7.1 Zonal simulation programme

Covariance matrices and mean values for both transmissivity and storativity for each zone are used as input to generate zonal simulations using a multi-guassian simulation program. An algorithm was used based on the LU-decomposition methodology. This algorithm is efficient for reasonable number of zones (less than twenty).

Figure 3 illustrates the zonal simulation program.

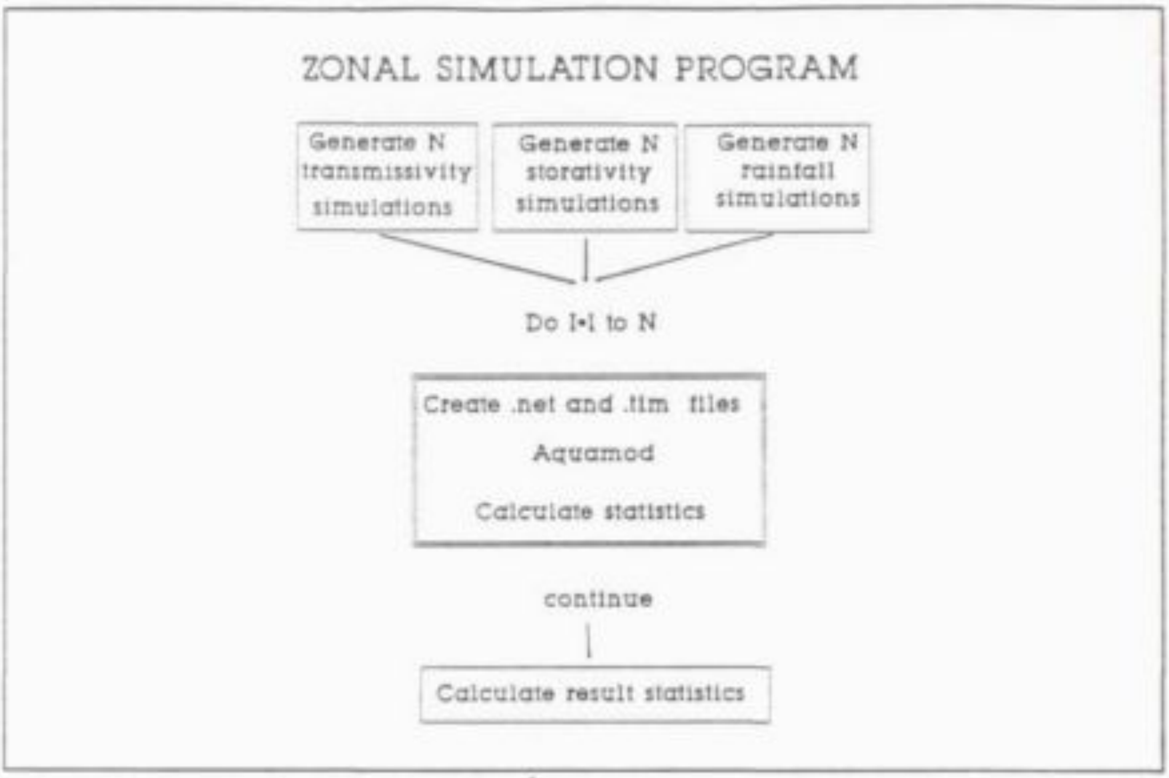


Fig 3: The zonal simulation program.

7.2 Geostatistical simulation programme

A geo-spatial simulation is a computer-generated model of a geological structure. To simulate such a structure, the shape of the body and its spatial characteristics (the variogram and distribution function) are required. This information is derived from physical samples and from expert opinion.

A properly formulated model honours the distribution, and the spatial correlation function of the sample values. There is an infinite set of simulated realities which will have these properties. To reduce the size of that infinite set, and to reflect observed reality more closely, it is further required that the model assumes the observed values at the sampling points. Such a simulation is said to be conditioned on the observations.

The risk evaluation system uses the Turning Bands method proposed by Journel and Huijbregts (1978) for geostatistical simulations. This method avoids an explosion of computer time and memory, handles large grids and is extremely fast. This method uses as input the variogram model and its parameters. The LU~decomposition method used for zonal simulations would not be suitable because of the large number of nodes.

7.3 Aquifer simulation program (AQUAMOD)

In this study the program AQUAMOD, developed by the IGS, University of the Free State, Bloemfontein was used. It is a finite element groundwater modelling program. Transmissivity and storativity can vary over the aquifer at an element-by-element resolution level. Output of the program takes the form of water levels both for selected nodes and for the entire aquifer. It became evident during the study that AQUAMOD was not entirely suitable in its present form for a Monte Carlo exercise. This was mainly due to the absence of fixed limit values for the recharge and discharge water levels which would keep the simulations realistic.

Without these limits the water levels would show an unrestricted positive growth rate if the recharge is larger than the discharge. However, in practice an aquifer can only accommodate a fixed maximum capacity of water. After consultation with IGS, it was decided, that each node on the grid must be supplied with a maximum water level. If arty water level exceeds the maximum it is reset to the limit for that node.

Similarly, if the discharge is larger than the recharge, the water levels tend to decrease infinitely. This implies an unrealistic situation corresponding to an infinite supply of water. Consider running the aquifer model for a water table aquifer with a known base water level. Any element will be regarded as dry if the difference between the calculated water level and the base water level becomes zero or negative. A pump at this element will be switched off. Thus the aquifer level will drop until all the pumps are switched off and with the current version of AQUAMOD will remain off and then a constant water level will be maintained over the whole aquifer grid.

The objective of the current exercise is to test different management models. The point of repeatedly simulating a fixed management strategy for 110 years is not to predict how it will perform over 110 years, which is not of any practical interest, but rather to obtain probabilities of extreme events occurring in the event of the particular management plan being implemented in practice. Switching off pumps changes this management strategy and invalidates the estimation of probabilities of extreme events. A more dynamic model where pumps will be switched off if the water level drops below the base level and on again if the water level rises above the base water level will be more suitable. A failure can then be defined in terms of the switching on/off events or the proportion of required water delivered per time period. With the present management model the risk management system will give good results if the aquifer is in a more stable state. If the aquifer drops below the base level and stays there, the statistics will be worse than is expected. The reason for this is that it will take longer to rise above this base level where recharge once again exceeds discharge. It must be decided in any future study whether it is important to have information about these exaggerated cases or whether it is only necessary to determine the maximum discharge the aquifer can sustain.

7.4 Rainfall simulator

Rainfall records were obtained from the Computing Centre for Water Research (CCWR) for rainfall station Slurry. This station is situated on the border of the Grootfontein aquifer and was chosen because of the length of rainfall -records that are available.

It was a complicated and tedious process to fit a SARIMA model to the time series before being able to evaluate simulated rainfall sequences against real data. Statistics for example, percentiles, means, variances, medians, the number of years with total rainfall less than 200 mm, 250 mm, 300 mm, 350 mm were calculated for the simulated data and compared with those of the real data. Rainfall records from 1916 to 1990 were used. Station Slurry had four years out of a total of 75 years with a total rainfall between 300 mm and 350 mm. A rainfall record of less than 250 mm was recorded in 1991 at this station. All the statistics were calculated on rainfall years, a cycle spanning from July to June.

After some further investigation is was decided to code the rainfall simulation algorithm which was developed by Zucchini and Adamson in a report to the Water Research Commission (Zucchini, 1984). Since their investigation was directed on the occurrence and severity of droughts in South Africa it was very applicable to our risk investigation. Parameters for almost all the rainfall stations in South Africa have been fitted by Zucchini and Adamson and is available as an appendix to the above mentioned report. If a new aquifer is modeled, no additional time will be required for fitting model rainfall parameters.

The values simulated from this rainfall simulator are daily values which were converted to monthly rainfall values for use in AQUAMOD. Although the values were simulated as point values, it was assumed that the simulated rainfall values occurred over the whole aquifer.

When the simulated rainfall values were evaluated against the real data it was noticed that this rainfall simulator, like the SARIMA simulator implemented by ourselves, constantly produced a higher average rainfall calculated over 100 years than the average of the real data calculated over 75 years. This statistic was calculated over 100 rainfall simulations each for a cycle of 100 years. In order to confirm these results a rainfall simulator distributed by Department of Botany of the University of Witwatersrand which was based on the Zucchini method was used and the results compared. These results did not show the high mean property. The discrepancy between the generated and actual mean is believed to be due to the random number generator selected to generate random numbers during the both simulation processes. To make the rainfall simulations realistic and to have enough 'dry' years in the simulations it was decided to add a correction factor to the simulations. All the statistics were calculated on a rainfall year, i.e. a cycle spanning from July to June. It must be emphasized that this correction factor only changes the overall mean but does not influence the rainfall cycle which was found to match the data adequately. This experience indicates the importance of validating the particular rainfall simulator chosen. Two example simulated rainfall series using Zucchini's rainfall simulator are shown in Figures 4(a) and 4(b).

8. STATISTICS

The simulation system described above produces vast quantities of information. To be of use this information needs to be summarized by meaningful statistics.

In this chapter several statistics are defined, discussed and illustrated. Firstly, overall statistics to summarise the performance of an aquifer system are discussed followed by the definition of risk events. Finally, methods of quantifying the risk are presented.

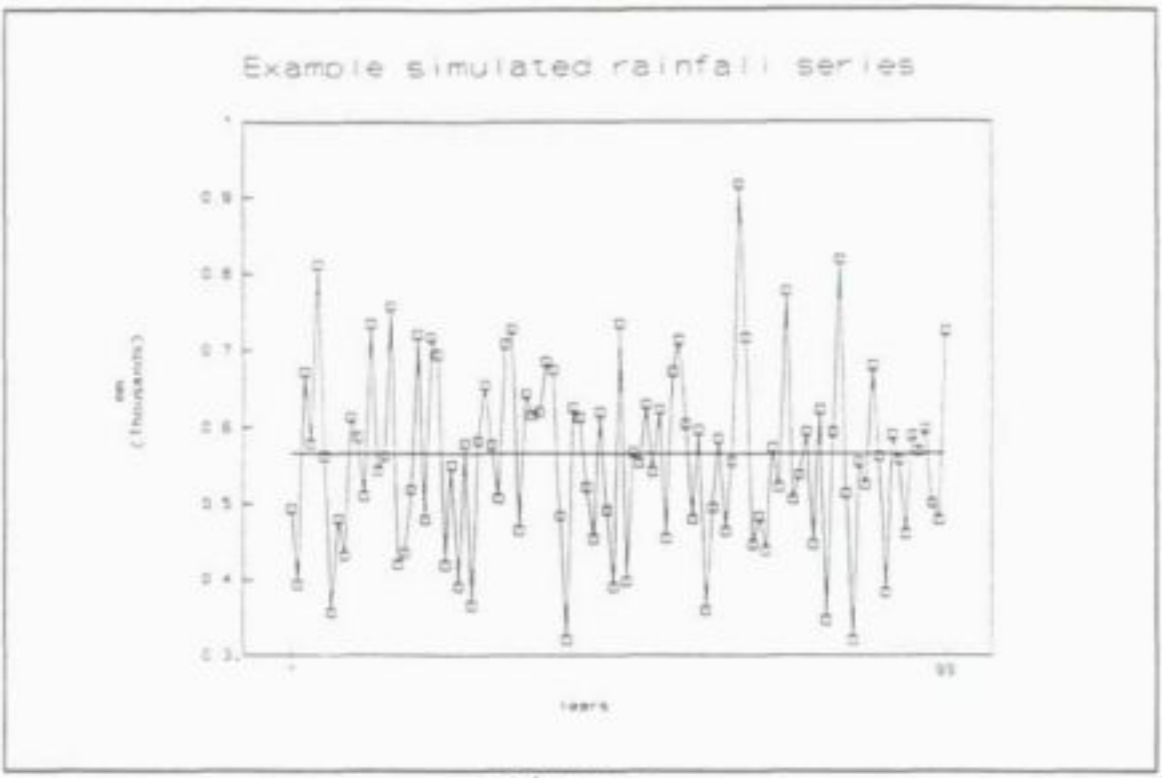
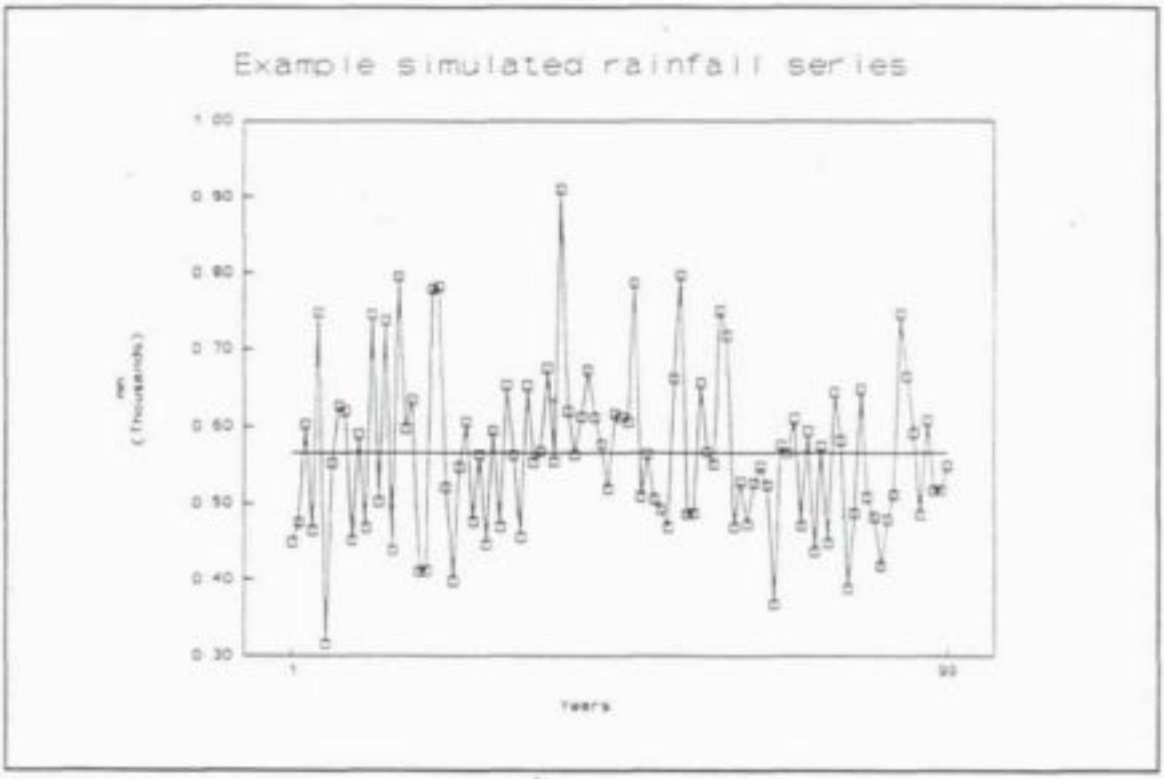


Figure 4



Figures 4(a) and 4(b): Two examples of simulated rainfall series using Zucchini's rainfall simulator.

Initially the study concentrated on water levels for determining the state of the aquifer at given times. After extensive experimentation it was realized that a distorted picture could sometimes be obtained since low water levels were not necessarily a result of the discharge but could be due to the physical topology of the aquifer. After discussion with IGS it was decided that all the statistics must be calculated from the difference between the simulated water levels and the base water level. Therefore for each node a maximum and a base water level was specified beforehand. The maximum level was used for overflow.

8.1 Summary statistics

The statistics calculated at the end of each realization of the risk evaluation system are complicated by the fact that they are calculated over space and time. Output from the aquifer modelling program takes the form of nodal water levels for each time step of 30 days. Water levels at selected user defined nodes (boreholes) also forms part of the output. A large amount of information is available after each realization. For example, if the model is run for 99 years, this means 12 * 99 = 1 188 water levels for one node: if an aquifer model has a total of 1 030 nodes, this means 1 118 * 1 030 = 1 223 640 water levels. If the number of realizations equal 100, it means that at the end the output results in 122 364 000 water levels. It is a great challenge to represent all these values in a few statistics to make it useful to planners of water resource exploitation. In comparison, when a surface water model, for example a storage dam, is run, it give only one water level. This simplifies the representation of statistics tremendously.

In order to evaluate a management model, NTOTAL realizations of the risk evaluation system will be computed with different generations of transmissivity, storativity and rainfall simulations. Each realization represents a timespan of NYEARS (rainfall years). After completion of all the realizations of the system the user will have NTOTAL values of each of the statistics defined below, stored in output files. Expected values are calculated at the end of all realizations and stored in a resulting statistics output file.

For each realization and after each timestep (30 days) a nodal **minimum**, **maximum** and **average** of the difference between the simulated water level and base water level over all nodes is calculated. Thus if the system is run for a timespan of 99 years, this represents 1 188 minimum difference water levels, 1 188 maximum difference water levels and 1 188 average difference water levels. From these the following summary statistics are computed:

Maximum {nodal minimum difference} : MaxMin
Minimum {nodal minimum difference} : MinMin
Average {nodal minimum difference} : AvgMin

Maximum {nodal maximum difference}: MaxMax
Minimum {nodal maximum difference} : MinMax
Average {nodal maximum difference} : AvgMax

Maximum {nodal average difference}: MaxAvg
Minimum {nodal average difference}: MinAvg
Average {nodal average difference} : AvgAvg

The MinMin value represents the minimum difference or lowest difference between the simulated water levels and base water levels that was generated during this realization. The MaxMax value represents the maximum or highest difference between the simulated water levels and base water levels and the AvgAvg represents the overall average difference of this realization. If the risk evaluation system is run for 99 realization, 99 values for each of these summary statistics will be available for further analysis.

The **expected** values are obtained by averaging these statistics over all realizations.

8.2 Risk events

Risk is defined in terms of a particular event occurring. In this case the risk event will be defined in terms of the effective depth of a borehole. Effective depth is defined as the difference between the maximum water level (determined during flood years) and the pump depth (base water level). A tolerance is defined as a proportion of the effective depth. A risk event occurs if the difference between the simulated water level and base water level falls below a tolerance level. For this exercise the following tolerance values were used:

- Tolerance 1 = 1/6 (maximum water level - base water level)
- Tolerance 2 = 1/3 (maximum water level - base water level)
- Tolerance 3 = 1/2 (maximum water level - base water level)
- Tolerance 4 = 2/3 (maximum water level - base water level)

Failure events were only tested at pumping boreholes, but the user can decide at which nodes this must be tested. Each borehole has a different maximum and base water level resulting in each borehole having a different tolerance value. A failure is recorded as having occurred if the difference of one of the boreholes fails below one of the tolerance values.

The concepts described above are displayed schematically in Figure 5. In this example four tolerances are shown, each reflecting a different degree of severity. By quantifying the risk for each of these tolerances additional information relating to the severity of failures can be deduced. Driscoll (1986) shows that is uneconomical to operate a well with a drawdown greater than 67 percent of the maximum. Tolerance 2 will give results in this order.

8.3 Risk quantification

In this paragraph methods of quantifying the performance of an aquifer system will be discussed. Clearly this quantification will depend on the assumed values for the aquifer model (which are actually unknown), the rainfall (which is unpredictable) and pump rates. The modelling of this variability is dealt with by computing expected values of the statistics using the Monte Carlo simulation method discussed earlier. These expected values relate to the long term ability of an aquifer to be pumped at a certain rate. This is not a method of forecasting specific failures or the behaviour in the presence of a reactive pumping program.

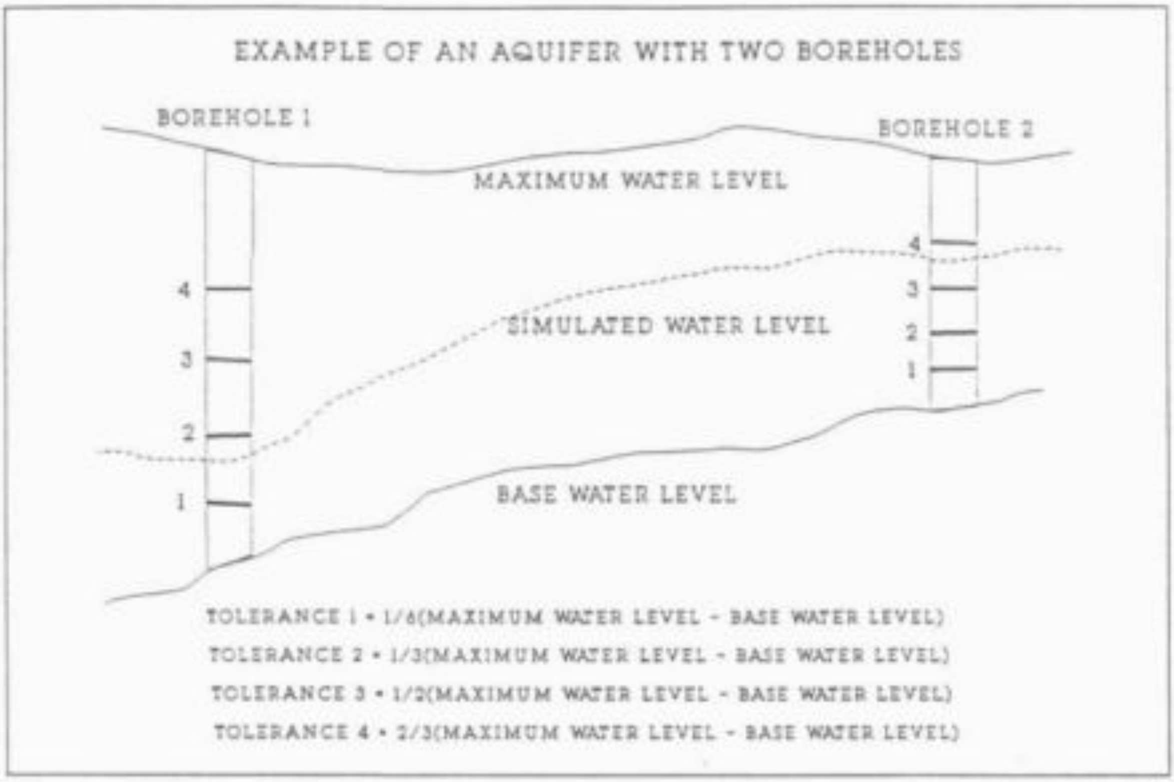


Figure 5: Schematic illustration of the tolerance values used in defining the risk of a particular event occurring.

A number of different options are available and one aim of the project was to assess these. The options was the probability of failures, return times and periods of failures.

8.3.1 Period

The period is the average length of time a failure is expected to continue for consecutive time steps. A short period indicates that although some nodes are below the tolerance they quickly return to a safe condition. A long period can indicate that the aquifer is being over exploited depending on the tolerance value used. An average period is obtained by averaging the period over a number of realizations. In Figure 6 the period is 3.67.

8.3.2 Probability

The probability of a failure is calculated by dividing the number of failures by the number of time steps during which a failure could have occurred. In other words it is the proportion of months for which a failure occurred. A value close to one would indicate that the aquifer is being over exploited while a value close to zero will indicate that it is operating well within the specified conditions. The expected probability is obtained by averaging over the number of realizations. In Figure 6 these failures are denoted by ★ and the probability of failure is 0.367.

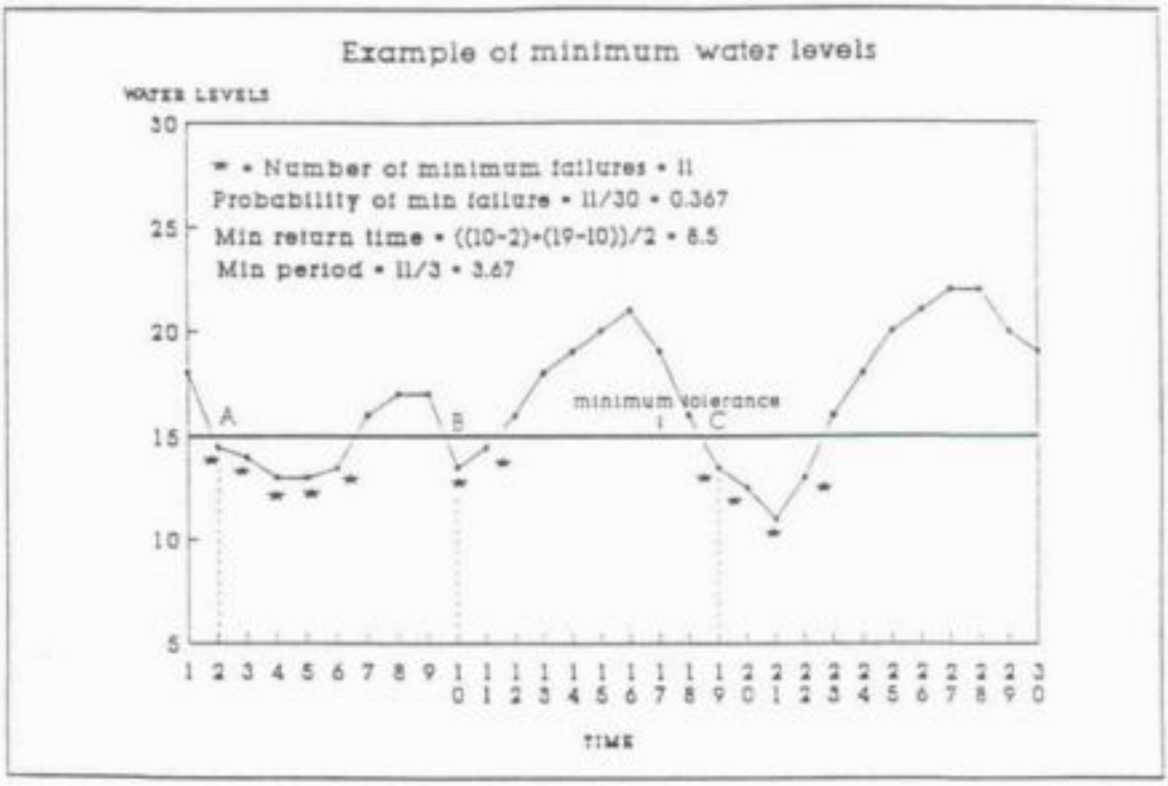


Figure 6: Example showing simulated minimum water levels indicating those instances where the water level dropped below the specified minimum tolerance level.

8.3.3 Return time

A return time is the time elapsed between a first failure, point A in Figure 7, until the second failure (point B on the graph). The time from point B to point C also represents a return time. To obtain an average return time for a realization, all the return times are added and divided by the number of times this occurs, in this case twice. The return time gives an indication of the time elapsed before the next failure will occur once a failure has occurred.

It must be emphasized that this return time differs from the return time of a tornado or flood. A drought may last an arbitrary length of time. Thus a failure may be short lived or continue for an extended time. Return time does not differentiate between a short or a long failure.

After an intensive study it was concluded that the return time is not a useful statistic for aquifer management.

9. CONCLUSIONS AND RECOMMENDATIONS

9.1 Conclusions

During this project a methodology has been developed for the long term assessment of risk when using groundwater. Below are listed some of the conclusions reached.

- There are two sources of uncertainty when considering the use of groundwater. Firstly the geological properties of the aquifer are only known up to a degree of certainty and secondly the recharge is a function of unpredictable rainfall. This study has developed methods for combining both these sources of uncertainty to provide an overall assessment of risk.

- The aquifer modelling software plays a central role in the Monte Carlo study used to make the risk assessment. This software must be capable of following a particular management plan if this plan in to be assessed. Thus, if one would like to simulate a process whereby a pump is switched on and off according to water level, the software must accommodate this. In the application to the Grootfontein aquifer this was not the case.

- A failure event needs to be defined. This can be done in a number of ways and can be chosen to meet the specific needs of a particular study. In the case study a failure was defined as the water level at a certain number of nodes failing below a chosen tolerance. This definition was found to work well in practice.

- After thorough investigation a definition of failure in terms of effective depth was formulated. This enabled a number of risk statistics to be considered of which probability of failure and period of failure were finally selected.

- The concept of failure return time was found to be totally unsuitable for these types of study.

- The methodology depends on the geohydrological modelling of the aquifer, the stochastic representation of the rainfall process and the recharge rate. As with any study the accuracy of these components will determine the value of the final results. In the case of the aquifer two different geostatistical techniques were used. From the case study we concluded that the method of zonal simulation was the most suitable. In the case of rainfall a simulator based on Fourier series representation of transition probabilities as proposed by Zucchini and Adamson (1984).

- It was found that although the aquifer in a sense could be equated to a surface water network, the aims of surface water management differed from the aims in this study. When this work is extended to short term risk assessment the methods used in surface water will become appropriate for groundwater management.

- The proposed methods were successfully tested for the Grootfontein aquifer and found to be practically applicable.

- The computing time involved in the developed Monte Carlo based methodology was found to be considerable ruling out the routine generation of the risk statistics. However, in practice the long term viability will only be assessed during the planning stage. A logical progression of the current work will be toward the short term risk assessment as a routinely applied management tool. It is anticipated that for such an application computing time will be kept to practical proportions.

- The overall objectives of the project were successfully achieved and the scene is set to extend the methodology to the application of short term risk management.

9.2 Recommendations

This study has played an important role in assembling the tools for the long term evaluation of a aquifer management plan. Such a management plan would be in terms of fixed pump rates at various boreholes. The results relate to the viability of the aquifer to deliver water at the required rate and the risk of failure to achieve this. A feature of these studies is that they require major computing facilities probably outside the resources of a aquifer manager. Furthermore, once a viable scheme is in place, short term decisions would need to be taken to ensure safe and optimal use of the groundwater.

It is thus clear that the logical extension would be into the area of short term risk assessment allowing evaluation of dynamic management schemes. Such short term risk assessment would be aimed at one or two time periods (monthly, quarterly or annual) and would enable a aquifer manager to assess a management plan in the light of the current aquifer situation and the unpredictability in the expected recharge for the period in question. As in the current study, aquifer uncertainties would also be taken into account.

The development to short term risk assessment will bring the methods much closer to those currently in use for surface water management and the potential will exist for concurrent assessment.

Although this extension will differ fundamentally from the current study in its objectives, it will make use of the same techniques which were developed and will use the results extensively.

THE DEVELOPMENT OF RISK ANALYSIS AND GROUNDWATER MANAGEMENT TECHNIQUES FOR SOUTHERN AFRICAN AQUIFERS

SECTION 3

MANAGEMENT OF THE GROOTFONTEIN AQUIFER IN THE WESTERN TRANSVAAL

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FORWORD

The Grootfontein Aquifer in the Western Transvaal is a typical example of one of the best utilized aquifers in South Africa. Not only are both the towns of Mafikeng and Mmabatho dependent on this aquifer for their water supply but substantial areas are also irrigated from it. The aquifer forms part of a Subterranean Water Control Area. Control is therefore exercised on the quantities of water that are abstracted from the aquifer. Permits are issued to farmers for irrigation which amounts to 11 million cubic metres per annum compared to 4,5 million cubic metres per annum which the towns of Mmabatho and Mafikeng are entitled to abstract. This in effect implies an accedence of the long term average recharge to the aquifer which is also the sustainable yield.

The prolonged drought that started in the early 1980's together with large scale abstraction caused a large drop in the water level of the compartment. It was soon evident that over abstraction of the aquifer occurs, which required the safe yield of the aquifer to be derived in order to make an equitable appropriation regarding future allocations and proper management of the groundwater source.

The objectives of this Section of the report was to apply the risk analysis techniques developed during this project to an aquifer which is being used extensively and for which sufficient information is available to develop and calibrate a simulation model. The modelling program AQUAMOD developed at the IGS was used for this purpose.

A water balance model using the program SVF was constructed which yielded answers on the recharge from rainfall and the effective porosity of the aquifer. The long term recharge to the aquifer was calculated as 8,5 million cubic metres per annum with effective porosity values ranging from 2,15 % to 2,45 % according to different calculation methods. The annual recharge to rainfall relationship is given by the formula

$$\text{GROUNDWATER RECHARGE} = 0{,}10 * (\text{RAINFALL} - 67 \text{ mm}).$$

By using stochastic generated rainfall records the reaction of the aquifer was forecasted by imposing different abstraction scenarios (program AQFSTOC). The result of these scenarios confirmed that the safe yield of the aquifer is at maximum about 8,5 million cubic metres per annum.

A dynamic groundwater flow model was constructed (programs AQUA-NET and AQUA) for the compartment by using as input parameters the obtained S-value (2,25%) and the above recharge/rainfall relationship. Using an inverse modelling technique (program AQUA-INV) the kd-values for the aquifer were obtained and a good fit between the actual and simulated water levels were obtained.

The calibrated flow model coupled with a response matrix technique (program AQUA-MAN) was used to determine the optimal extraction pattern from the aquifer using a simplex algorithm (program MANAGE) subject to specified demand and drawdown constraints.

Risk evaluations for different risk events were used. The risk statistics produced very similar maximum allowable abstraction figures to those generated by the water balance model.

CONTENTS

1. INTRODUCTION 3-1

2. LOCALITY OF THE STUDY AREA 3-2

3. PREVIOUS INVESTIGATIONS 3-2

3.1 Geological 3-2
3.2 Geophysical 3-2
3.3 Geohydrological 3-2

4. GEOLOGY OF THE STUDY AREA 3-3

4.1 Basement Rock 3-3
4.2 Ventersdorp Supergroup 3-3
4.3 Transvaal Supergroup 3-3
4.4 Vertical Diabase Intrusions 3-4

5. RAINFALL 3-4

6. ABSTRACTION 3-5

7. GEOHYDROLOGY 3-5

7.1 Groundwater levels in the Grootfontein Compartment 3-5
7.2 Water Balance Equation 3-6
7.2.1 Water Balance Equation applied to the Grootfontein Compartment 3-8
7.2.1.1 The Saturated-Volume-Fluctuation (SVF) 3-8
7.2.1.2 Determining the S value (Indirect approach) 3-8
7.2.1.3 Recharge and its relationship to rainfall using the S-value obtained through the indirect approach 3-9
7.2.1.4 The recharge/rainfall relationship obtained using the "equal volume method" 3-9
7.2.1.5 Calculation of the S-value using the recharge/rainfall relationship that was obtained with the "equal volume method" 3-10
7.2.1.6 Application of the modified Hill-method on the Grootfontein Compartment 3-10
7.2.1.7 Summary of results obtained with the water balance method 3-11

8. FORECASTING OF AQUIFER VOLUME (WATER LEVELS) USING THE

MODEL 3-11

9. DYNAMIC MODEL OF THE GROOTFONTEIN COMPARTMENT 3-13

9.1 Construction of the finite element network 3-13
9.2 Boundary conditions 3-13
9.3 Initial head values 3-13
9.4 Abstraction 3-14
9.5 Recharge 3-14
9.6 Porosity 3-14
9.7 Transmissivity values 3-14
9.8 Calibration of the model 3-15
9.9 Forecasting of aquifer response using the dynamic flow model 3-16

10. ILLUSTRATIVE EXAMPLE OF OPTIMAL EXTRACTION PATTERN AND ALLOCATION OF GROUNDWATER AMONG CATEGORIES OF WATER USERS 3-17

11. RESULTS OF THE RISK ANALYSIS TECHNIQUES APPLIED TO THE GROOTFONTEIN AQUIFER CASE STUDY 3-53

12. CASE STUDY CALIBRATION AND VALIDATION 3-53

12.1 CALIBRATION 3-53
12.2 VALIDATION 3-55

13. EVALUATION OF RECHARGE RATE 3-56

14. COMBINATION OF RISK EVENTS 3-58

15. ZONAL SIMULATIONS 3-58

15.1 SUMMARY STATISTICS FOR ZONAL SIMULATIONS 3-58
15.2 RISK STATISTICS FOR ZONAL SIMULATIONS 3-61

16. GEOSTATISTICAL SIMULATIONS 3-67

16.1 SUMMARY STATISTICS FOR GEOSTATISTICAL SIMULATIONS 3-67
16.2 RISK STATISTICS FOR GEOSTATISTICAL SIMULATIONS 3-70

17. CASE STUDY SENSITIVITY 3-76

18. CONCLUSIONS AND RECOMMENDATIONS 3-77

18.1 MANAGEMENT 3-77
18.2 RISK EVALUATION 3-78

1. INTRODUCTION

The Grootfontein Compartment, a dolomitic aquifer in the Western Transvaal, is a typical example of one of the best utilized aquifers in South Africa. Not only are both the towns of Mafikeng and Mmabatho in the Republic of Bophutatswana dependent on this aquifer for their water supply, but substantial areas are also irrigated from it. The aquifer forms part of an area known as the Bo Molopo Subterranean Water Control Area. Control is therefore exercised on the quantities of water that are abstracted from the aquifer. Permits are issued to farmers for irrigation according to a policy that takes into account the size of the irrigated land, which receives an allocation of 7500 m^3/ha/year. This in effect implies an excess of the long-term average recharge to the aquifer which is also the sustainable yield. A previous investigation (Van Rensburg, 1987) indicated that the permit allocation in the Grootfontein Compartment for irrigation amounts to 11 million cubic metres per annum compared to 4,5 million cubic metres per annum which the towns of Mmabatho/Mafikeng are entitled to abstract.

The prolonged drought that started in the early 80's, together with large-scale abstraction, caused a large drop in the water level of the compartment. It was soon evident that over-abstraction of the aquifer occurs, which required the sustained yield of the aquifer to be determined in order to make an equitable appropriation regarding future allocations and proper management of the groundwater source.

For this purpose, a water balance model was constructed. The water balance calculations yielded answers on the recharge and the storativity of the aquifer. The long-term recharge to the aquifer was calculated as 8,5 million cubic metres per annum with storativity values ranging from 2,15% to 2,45%, according to different calculation methods. The annual recharge to rainfall relationship is given by the formula GR = 0,10*(RF - 67 mm).

By using stochastic generated rainfall records, the reaction of the aquifer was forecasted by imposing different abstraction scenarios. The result of these scenarios confirmed that the sustained yield of the aquifer is at maximum about 8,5 million cubic metres per annum.

A dynamic groundwater flow model was constructed for the compartment by using as input parameters the obtained S-value (2,25%) and the recharge/rainfall relationship [GR = 0,10*(RF - 67 mm)]. Using an inverse modelling technique, the kD-values for the aquifer were obtained and a good fit between the actual and simulated water levels were obtained. Different scenarios comprising recharge/no-recharge at different abstraction schedules were performed to evaluate the amount of drawdown at the Grootfontein Spring.

An illustrative example of an optimal extraction pattern and allocation of groundwater among categories of users in the Grootfontein Compartment, together with a Monte

Carlo analysis for risk evaluation concludes this volume.

2. LOCALITY OF THE STUDY AREA

The Grootfontein Compartment is situated in the far Western Transvaal, about 20 kilometres south-east from Mafikeng and 25 kilometres north-west of Lichtenburg, and covers an area of about 169 km². A locality map of the study area is shown in Figure 1.

3. PREVIOUS INVESTIGATIONS

3.1 Geological

Von Backström *et al* (1952) and Davies and Prévost (1978) mapped and described the geology of the study area.

3.2 Geophysical

Several geophysical surveys were carried out in the study area. Ground magnetic and electromagnetic surveys were done by Hauger (1973) and Day (1976) carried out magnetic studies over the diabase dykes. A gravimetric survey was carried out by Palmer (1978).

3.3 Geohydrological

Geohydrological investigations in the area were executed by Porszasz (1966), Bredenkamp and Schutte (1970), Bredenkamp *et al* (1974), Vipond (1979) and Bredenkamp (1984).

Aquifer modelling studies by Hely-Hutchinson (1972), Boshoff (1980), Cogho (1982), Bredenkamp and Van Rensburg (1983), Van Rensburg (1985) and Van Rensburg (1987) added to the extensive coverage of the system.

4. GEOLOGY OF THE STUDY AREA

The study area can geologically be subdivided in:

- Basement rock;
- Ventersdorp Supergroup;
- Transvaal Supergroup;
- Vertical diabase intrusions.

The lithostratigraphical sequence of the study area is shown in Figure 2.

4.1 Basement Rock

The basement rock consists of a light grey to pinkish granite-gneiss. Although the basement rock does not outcrop over the study area, the existence thereof was confirmed by gravimetric surveys.

4.2 Ventersdorp Supergroup

The granite-gneiss basement rock is overlain by the Ventersdorp Supergroup which consists of quartzite, shale and andesitic lava.

4.3 Transvaal Supergroup

The Transvaal Supergroup consists of the Chuniespoort Group of which the Malmani Subgroup forms a part. The Malmani Subgroup can be divided into the following formations:

(a) Black Reef Formation

The rocks of this formation overlie the Ventersdorp Supergroup. It consists of feldspathic quartzite, shale and conglomerate; its thickness is in the order of 50 m (Visser, 1970).

(b) Oaktree Formation

This formation outcrops in the western portion of the study area and is characterized by dark chert poor dolomite which is not a good aquifer.

(c) Monte Christo Formation

This is the most important water-bearing formation in the area and consists of a chert-rich lighter colour dolomite.

(d) Lyttelton Formation

This formation outcrops in the north-eastern portion of the compartment. It is poor in groundwater because of the dolomite which is chert-poor and less karstified.

4.4 Vertical Diabase Intrusions

The groundwater system is bounded by diabase dykes of unknown age. In the north, the boundary is formed by the Grootfontein/Trekdrift dykes. southern and western boundaries are formed by the Blaauwbank and Mooimeisiesfontein dykes respectively (Figure 3).

5. RAINFALL

The study area falls within the summer rainfall region and rain occurs mainly in the form of thunderstorms. The yearly average rainfall is 560 mm as measured at the Slurry Rainfall Station. Seasonal fluctuations as well as persisting dry and wet years can cause much variability in monthly and yearly rainfall figures. A representation of the yearly rainfall is shown in Figure 4.

6. ABSTRACTION

Abstraction takes place from the Grootfontein Spring (which stopped flowing during 1981) to supply the towns of Mafikeng/Mmabatho and irrigation water to farmers from boreholes. The positions of the production boreholes are shown in Figure 5. A representation of the quantities that have been abstracted for the period 1979/80 to 1990/91 is shown in Figure 6. Quantities abstracted for Mafikeng/Mmabatho have been measured accurately over the entire record period by means of flow meters. Quantities abstracted for irrigation were calculated by means of the electricity consumption of the pumps (Van Rensburg, 1987). Unfortunately, these figures extended only to April 1987. Abstraction for the year 1987/88 was calculated by making use of hour meter readings on the pumps. These meters became defective over the years and the abstraction for irrigation (1987/88) was assumed constant for the following three years (1988/89 to 1990/91), but they were later on proved to be different.

7. GEOHYDROLOGY

7.1 Groundwater levels in the Grootfontein Compartment

The Department of Water Affairs monitored groundwater levels,since 1974 on a monthly basis, in the Grootfontein Compartment. The positions of the observation boreholes are shown in Figure 7. These boreholes are evenly distributed, except in the north-eastern portion of the compartment. Contours of the measured water levels are shown in Figures 8(a) to 8(g) for the month of October for the period 1979, 1981, 1983, 1985, 1987, 1989 and 1991.

From these contour maps, it is evident that:

(i) Groundwater flow takes place from the south-east and east to the west and north-west.
(ii) A steepening in the groundwater gradient extends over the farms Kliplaagte, Blaauwbank and Grootfontein. The reason for this phenomenon can be partly explained by the corresponding steepening in the gradient of the base-rock elevation, shown in Figure 9 (Van Rensburg, 1985). This steepening in the water-level gradient seems to extend further to the east as dewatering progressed (Figures 8(a) to 8(g)). The lateral inflow from the east is therefore expected to be minimal.
(iii) Another important characteristic of the compartment is the possible variation in hydraulic conductivity. Areas with a low groundwater gradient are indicative of high hydraulic conductivity, whilst areas with a steeper

groundwater gradient indicate a low hydraulic conductivity. According to this, the areas between the 1470 m and the 1462 m contour lines and between the 1452 m and 1444 m contour lines are more permeable than the area where the steep gradients exist (Figure 8(a)).

Hydrographs of the individual boreholes (October 1979 to January 1992) are shown in Figures 10(a) to 10(t). Groundwater levels in the compartment show large variations as abstraction and recharge take place. A comparison between the water levels for November 1980 and May 1981 shows a general rise in the water level. This can be attributed to above average rainfall over this period. From June 1981 till about the beginning of 1987, large drawdowns in the water levels occurred. This coincided with high abstraction rates and below average rainfall over the 1982/83 to 1986/87 period. The above average rainfall for the hydrological year of 1988/89, together with a lower abstraction from the compartment, caused a rise in the water level from October 1988 to May 1989. Average rainfall and fairly constant abstraction resulted in a water level that maintained its level for the period June 1989 till the end of the record in February 1992, although drawdowns in the eastern portion of the compartment did not recover as is the case in the western portion of the compartment.

7.2 Water Balance Equation

The classical geohydrological balance equation for a groundwater reservoir with an impermeable base (Kirchner and Van Tonder, 1989) for a time increment of $\Delta t = t_2 - t_1$ is given by the equation:

$$I - O + GR - Q = \Delta W \qquad (1)$$

where

$I = (I_1 + I2)/2$ = mean lateral inflow (m^3/d)
$O = (O_1 + O_2)/2$ = mean lateral ouflow (m^3/d)
GR = effective groundwater recharge (m^3/d)
Q = discharge out of (or into) reservoir (m^3/d)
ΔW = change in groundwater volume (m^3) = $S * \Delta V$
ΔV = change in saturated volume aquifer material ($V_2 - V_1$)
S = specific yield or effective porosity

Equation (1) can be rewritten as

$$GR + I - O - \Delta V * S = Q \qquad (2)$$

This equation is the general groundwater balance equation for an unconfined aquifer and can be applied for a number of specific conditions (Bredenkamp *et al*, 1989):

(a) If the inflow and outflow terms are equal, the change in groundwater storage is zero.

This provides the necessary conditions to derive at sustained yield estimates and to predict recharge from precipitation:

$$GR = O - I + Q \qquad (3)$$

(b) By incorporating the "no recharge" recession ($GR = 0$), when the change in saturated volume presents a maximum over a given time period t, Equation (2) reduces to:

$$I - O - \Delta V{*}S = Q \qquad (4)$$

from which S can be calculated as

$$S = \frac{I-O-Q}{\Delta V} \qquad (5)$$

(c) If the aquifer is bounded by impervious dykes or by groundwater divides, I and O in (2) are zero. For this case

$$GR - \Delta V{*}S = Q \qquad (6)$$

from which the groundwater recharge can be calculated if S is known. If the groundwater recharge is assumed as zero for the maximum change in saturated volume, a mean S-value for the aquifer can be calculated.

PC computer program SVF can be used to calculate the saturated-volume-fluctuation (SVF) for an aquifer on a month to month basis. This allows:

1) A mean S-value to be obtained, corresponding to the change in saturated volume being a maximum and implying groundwater recharge to be zero (Eq. 6). As it is difficult to determine whether the requirement of zero recharge has been met, an indirect approach was adopted by Van Rensburg *et al* (1987) to obtain the aquifer storativity. The storativity of the aquifer was calculated on an annual basis, by incorporating the recharge as part of the effective storativity. The following relationship applies:

$$S_a = S_o + S_r \qquad (7)$$

where S_a = the effective S-value if the recharge is incorporated as part of the storativity value.

S_o = the intrinsic effective porosity of the aquifer.
S_r = the equivalent porosity representing only the recharge.

Storativity values (S_a) for each year can hence be calculated and plotted against the annual rainfall. The resulting relationship between S and the annual rainfall allows the true storativity (S_o) to be inferred. S_o must correspond to the value for zero annual recharge. The storativity corresponding to the low annual rainfall values represent a maximum storativity for the aquifer.

(2) By substitution of this S-value into the water balance equation, recharge and its relationship to rainfall can be obtained.

(3) The aquifer's saturated-volume-fluctuation (SVF) is subsequently simulated using the established rainfall-recharge relationship and varying S to obtain the best match between the simulated and the observed saturated volumes. Along the same lines the storativity can be assumed and the best rainfall-recharge relationship can be obtained. (Bredenkamp *et al*, 1989).

7.2.1 Water Balance Equation applied to the Grootfontein Compartment

7.2.1.1 The Saturated-Volume-Fluctuation (SVF)

The monthly saturated-volume-fluctuations for the Grootfontein Compartment were determined from the water-level measurements at the monitoring boreholes for the period October 1979 to January 1992, as depicted in Figure 11. This shows the decline of the saturated volume for the period May 1981 to May 1988, due to abstraction that exceeded recharge as well as two major recharge events during 1981 and 1989.

7.2.1.2 Determining the S value (Indirect approach)

The inferred apparent storativity S_a (inflow = outflow) using the SVF-program for 12 month periods shifted forward one month at a time, using Equation 5, is shown plotted against rainfall in Figure 12. This indicates according to the best fit relationship, the intrinsic effective porosity, S_e, of the aquifer to be 0,0215. The 95% confidence limits show that the lower and upper bounds for storativity are 1,5% and 3,0% respectively.

7.2.1.3 Recharge and its relationship to rainfall using the S-value obtained through the indirect approach

The S-value obtained through the indirect approach is independent of the recharge to the aquifer and can be used to calculate the recharge to the aquifer which would yield the recharge-rainfall relationship if such a relationship exists. Three S-values were used namely the lower bound S-value (1,5%), the upper bound S-value (3,0%) and the S-value of the regression line (2,15%). The recharge to rainfall relationships obtained with the different three S-values are shown in Figures 13(a) to 13(c). Using the "best estimate" of S, namely 2,15%, the recharge to rainfall relationship is given as:

$$GR = 0{,}10(RF - 38mm) \qquad (8)$$

The correlation coefficient between recharge and rainfall increases as the S-value increases. However, the correlation coefficient does not increase substantially for S-values higher than 3,0%. The highest increase in correlation coefficient occurred for S-values between 2% and 3%, which is regarded as the range of the most probable value of S.

7.2.1.4 The recharge/rainfall relationship obtained using the "equal volume method"

The rationale behind this method is briefly as follows. If the inflow and outflow can be assumed constant or zero, the recharge to the aquifer is equal to the abstraction for all periods where an equal volume status of the aquifer occurs. The periods where equal volumes of saturated aquifer thicknesses occurred were calculated by the SVF-program and the abstraction over such periods represents recharge. The recharge to rainfall relationship thus obtained is shown in Figure 14 and yields the following relationship:

$$GR = 0{,}10(RF - 67mm) \qquad (9)$$

Equation 9 shows a remarkably close relationship to the rainfall-recharge relationship that had been obtained by assuming an S-value of 2,15% (Eq. 8 in Sect. 7.2.1.3).

Using the recharge to rainfall relationship (Eq.9) the average recharge to the compartment can be obtained by substituting the long-term rainfall (560 mm/a) into the equation. For the Grootfontein Compartment this yields a recharge figure of 49,3 mm per annum which, if spread over an area of 169 km^2, yields a recharge of 8,4 million cubic metres per annum.

7.2.1.5 Calculation of the S-value using the recharge/rainfall relationship that was obtained with the "equal volume method"

An optimizing technique was used to calculate the "best" S-value (using the recharge-rainfall relationship obtained in 7.2.1.4.) by comparing the actual SVF with the calculated SVF. The S-value for the best fit between the actual and simulated SVF is 2,25%. The close fit between the actual and simulated SVF is shown in Figure 15; the S-value is very close to the S-value that was inferred from the indirect approach (2,15%).

7.2.1.6 Application of the modified Hill-method on the Grootfontein Compartment

From the water balance equation, it is clear that if inflow, outflow and recharge are zero, the S-value of a system can be obtained by plotting the abstraction against the change in saturated volume that is effected from that abstraction. The S-value corresponds to the slope of the line fitted through the plotted points, according to the method proposed by Bredenkamp *et al* (1989). In the case of the Grootfontein Compartment, inflow and outflow appear to be minimal or could be assumed to cancel out. The scatter of values around the fitted line (Fig. 16) between the plotted points can therefore be attributed to varying recharge. According to this plot, the average long-term recharge to the aquifer is equal to 8,5 million cubic metres (where $dV = 0$). This is in excellent agreement with the recharge quantity that was obtained through the equal volume method. If the recharge-rainfall relationship that was derived from the equal volume method is used in the water balance equation the net effect of recharge and abstraction (GR - Q) can be obtained. Hence, if (GR - Q) is plotted against dV, as shown in Figure 17, the S-value can be obtained as the slope of the line fitted through the points. A value of $S = 0,0245$ is derived in this way. The deviation from the straight line in the upper portion of the data corresponds to the period 1988 to 1991. As was mentioned in Section 6 the abstraction rates for this period were assumed constant but according to the deviation the abstraction had been overestimated. By inferring the abstraction by means of the electricity consumption, the adjusted abstraction can be validated.

7.2.1.7 Summary of results obtained with the water balance method

For greater clarification, the findings can be summarized as follows:

An S-value for the compartment was calculated as 2,15%, according to the indirect approach. The upper and lower bounds for the S-value (at a 95% confidence limit) were calculated as 3,0% and 1,5% respectively. An S-value of 2,15% was used in the water balance equation to calculate the recharge. This calculated recharge showed a linear relationship with the rainfall exceeding a threshold value [GR = 0,10(RF - 38mm)].

Another recharge-rainfall relationship was obtained by making use of the equal volume method [GR = 0,10(RF - 67mm)]. This relationship yields a long-term recharge of 8,4 million cubic metres per annum to the aquifer. The method is independent of the systems' S-value and the obtained relationship is in close correspondence to the relationship that was found, using the S-value obtained by the indirect approach. This recharge/rainfall relationship was used to reconstruct the saturated volume fluctuation and optimizing for the S-value. The S-value that yielded the smallest mean square error (MSE) was found to be 0,0225 - again in close correspondence to the S-value calculated by using the indirect approach (2,15%).

The adapted Hill method showed the long-term recharge to the aquifer to be 8,5 million cubic metres per annum and the S-value to be 0,0245.

8. FORECASTING OF AQUIFER VOLUME (WATER LEVELS) USING THE MODEL

The groundwater balance model can be usefully applied as an ongoing management tool in preventing overabstraction of the groundwater resource in the area and in prolonging the viability of irrigation on a reduced scale.

The long-term rainfall record of Slurry (1915/16 - 1991/92) (Figure 18) was used to predict the aquifer volumes for different abstraction scenarios. This historical rainfall sequence was used to generate 41 stochastic rainfall records (Pegram, 1986) based on the statistics of the historical rainfall record. A program, AQFSTOC, was written by P. van Rooyen (1992) of Bruinette, Kruger and Stoffberg to simulate the storage in the aquifer based on the recharge/rainfall relationship, the S-value of the system and the abstraction from the system. The results show "Box and Whisker" percentiles of monthly storage estimates for the given period of 77 years. The given percentiles show probability estimates of confidence for storage of the aquifer e.g. the 1% percentile line indicates a 99% confidence of the storage being lower than x MCM and the 75% percentile line indicates a 25% confidence of the storage of the aquifer being lower than xz MCM.

In the first scenario, the abstraction of water was set at 8,5 million cubic metres per annum which is considered to be the long-term assured yield of the aquifer. By making use of the 41 stochastically generated records from the rainfall record measured at Slurry, the reaction of the aquifer was forecasted. Figure 19 shows the simulated volumes against time. According to this simulation the storage in the aquifer will stay more or less the same over the entire period of the rainfall record, if the median value (the 50% percentile line on the graph) is considered to be the most likely outcome. This means that the specified abstraction (8,5 million cubic metres per annum) would not exceed recharge to the aquifer in the long run, which confirms that it is a reliable value for the long-term recharge to the aquifer. Even the "most optimistic" situation (the 1% percentile line on the graph) indicates a very low probability that the storage will exceed 135 MCM which indicates a full storage. On the other hand, if the "worst case" (the 99% percentile line on the graph) in the rainfall sequence occurs, the aquifer will also not be depleted below a storage of about 40 million cubic metres.

Abstraction rates of 9 and 10 million cubic metres were specified and from Figures 20 and 21, it is clear that the aquifer volume will be depleted in the course of time. An abstraction rate of 7 million cubic metres was also specified and a gradual rise in the aquifer volume can be observed (Figure 22). In the best case scenario (highest recharge) the Grootfontein Spring will start flowing after about 180 months. For the "median case" (50% percentile line), the spring will start to flow after about 450 months.

The aquifer response is very sensitive to abstraction and it can be inferred that any abstraction exceeding the mean annual recharge will result in a corresponding depletion of the volume of water stored in the aquifer. The declined water levels will not recover unless the abstraction is reduced for consecutive periods.

9. DYNAMIC MODEL OF THE GROOTFONTEIN COMPARTMENT

A dynamic flow model for the Grootfontein Compartment was constructed to simulate the groundwater flow in the aquifer. The main objective of this exercise was to verify the reliability of storativity and recharge values that were obtained from the water balance calculations. The AQUA-INV and AQUA programs were used respectively to calibrate and simulate groundwater flow in the compartment. The finite element model is based on the Galerkin solution of the two-dimensional partial differential equation that describes flow subject to specific boundary and initial head conditions. The results obtained through the water balance study, namely the S-value and recharge were used as input in the flow model. With these input values a good correlation between the actual and simulated water levels was obtained by altering the kD-values in the aquifer.

9.1 Construction of the finite element network

Compilation of the finite element model necessitates the construction of triangular elements over the compartment, incorporating groundwater levels, geology, borehole positions and boundaries or dyke positions. The triangular finite element network is shown in Figure 23. In the construction of the network, it is essential that the positions of abstraction/recharge boreholes coincide with node positions. The network was constructed to reflect the geometry of the compartment accurately.

9.2 Boundary conditions

In order to construct an accurate model, the position of the boundary as well as the behaviour of the water level at that boundary must be known. The Grootfontein Compartment is bounded by diabase dykes as discussed in Section 5.4. According to investigations carried out by Van Rensburg (1985), it is unlikely that lateral flow occurs across the dykes. It was therefore assumed that the dykes are impervious.

9.3 Initial head values

The water levels for January 1983 were used as initial head values for the groundwater model. It was decided to calibrate the groundwater model for the period January 1983 to December 1986, because of the large drop in water levels that had been caused by overabstraction and low rainfall (recharge).

9.4 Abstraction

The abstraction figures used in the model were obtained from the Grootfontein State Water Scheme at the Grootfontein Spring as well as from Report No. GH 3549 (Van Rensburg, 1987).

9.5 Recharge

The recharge specified in the groundwater model was obtained from the water balance study. For this case, the recharge specified was obtained from the relationship:

$$GR = 0{,}10(RF - 67mm)$$

where GR = groundwater recharge
and RF = rainfall.

9.6 Porosity

A porosity value of 0,0225 was specified in the groundwater model. This value was obtained from water balance calculations.

9.7 Transmissivity values

Transmissivity values were specified in the groundwater model, according to the spatial variation in the groundwater levels, i.e. low T-values where the groundwater level gradient is steep and higher T-values where the groundwater level gradient is flatter. The program, TCAL (Van Tonder, 1989), was used for this purpose.

9.8 Calibration of the model

An inverse modelling technique optimizing the parameter values automatically, instead of using a trial and error modelling technique, was used to calibrate the model for the Grootfontein Compartment. During the past few years, inverse flow models have been appearing in the modelling field. The inverse modelling approach is also known as the parameter estimation or identification method. In contrast to the conventional modelling approaches, they use the dependent variables as input to estimate the independent model parameters. In this case the dependent variable, namely the configuration of the groundwater level is used to estimate the independent variables, namely the k- and S-values.

The procedure of parameter identification runs as follows:

The modeller assigns a provisional set of initial k- and S-values. The model solves the groundwater flow equation on the basis of these parameters and compares the water-level response with the actual drawdowns. A mean square error (MSE) is calculated between the actual drawdowns and the calculated drawdowns and the parameters of k and S are altered until a smallest value of the MSE is obtained. The flow equation is solved between a chosen minimum and maximum value for k and S. This range of values must represent a realistic range for the limits for the respective parameters. The goal function was minimized by making use of the Marquardt algorithm.

In the case of the Grootfontein Compartment, the model was calibrated by optimizing for only the T-values (Figure 24) as the S-value and recharge to the aquifer were assumed correct as obtained by the water balance study. The comparison between the actual water levels and the calculated water levels with the optimum calculated T-values is illustrated in Figures 25(a) to 25(n). This shows a close fit between the calculated and measured water levels at some observation points and less good fits for others. This is the result of assuming that the aquifer is homogeneous. Once better knowledge of the geological settings is obtained, the aquifer could be divided in zones of different S-values resulting in a better fit between the actual and measured water levels. For the present exercise, however, the k- and S-values, as well as the recharge obtained, are considered representative of the aquifer as a whole and the "fine tuning" of the parameters would serve no purpose before additional knowledge of the aquifer is obtained.

9.9 Forecasting of aquifer response using the dynamic flow model

The reaction of the model can be forecasted by using different recharge/abstraction schedules. In the case of the Grootfontein Compartment, it is clear that the permit allocations to farmers by far exceed the recharge to the aquifer. Reductions in permit allocations are therefore inevitable. It is not the intention of this report to prescribe how the reductions should be made, but it was decided to test the response of the aquifer at a total abstraction rate of 8,5 million cubic metres per annum (6,5 million/a from the Grootfontein Spring and 2 million/a from irrigation).

Two scenarios were performed:

1) In the first scenario, 6,5 million cubic metres per annum were abstracted from the Grootfontein Spring and 2 million cubic metres per annum from the irrigation boreholes. This scenario was performed with average recharge, half of average recharge and with no-recharge.

2). In this scenario, 4,5 million cubic metres per annum was abstracted at the Grootfontein Spring, 2 million cubic metres per annum from the farm Blaauwbank and 2 million cubic metres from the other irrigation boreholes.

This scenario was also performed with average recharge, half of average recharge and with no-recharge. The objective by this scenario was to investigate what effect the shift in abstraction of 2 million cubic metres will have on water levels close to the spring, if abstracted from Blaauwbank.

Figure 26 shows the results of the two scenarios at borehole GN41 (close to the Grootfontein Spring).

With "no-recharge", the water level in the vicinity of the spring will reach the base of the aquifer after about 1300 days (43 months), if all the supply to Mmabatho (6,5 million cubic metres per annum) is abstracted from the Grootfontein Spring. The shutdown of the pumps when the water level reaches the base of the aquifer and the recovery in the water level thereafter can be clearly seen on the graph. With 2 million cubic metres per annum of abstraction being shifted from the Grootfontein Spring to Blaauwbank this level will be reached about one year later if no-recharge is specified. The effect of the reduction in the kD-values, as dewatering prolongs, is clearly depicted in the graph.

If only half the average recharge is specified, a similar drop in water levels can be observed, although the drop is not as drastic as when the simulation is performed with no-recharge. When average recharge is specified, a small drop in the water level can be observed, should all of the 6,5 million cubic metres be abstracted from the spring area. A slight rise in the water level can be observed, if 2 million cubic metres of that abstraction are shifted to Blaauwbank.

It appears from the modelling exercise, that it seems possible to abstract 6,5 million cubic metres per annum from the Grootfontein Spring, if average recharge is assumed.

10. ILLUSTRATIVE EXAMPLE OF OPTIMAL EXTRACTION PATTERN AND ALLOCATION OF GROUNDWATER AMONG CATEGORIES OF WATER USERS

To use the Grootfontein Compartment as an example of a groundwater allocation model, consider the problem of determining the optimal groundwater extraction pattern in the compartment. We assume that a central authority (e.g. the Farmers' Association) has the authority to allocate the groundwater, so as to maximize the net return from agricultural production. The authority is also responsible for determining the optimal cropping pattern for the compartment, i.e. the amount of hectares devoted to the various crops in the area.

The compartment must be exploited according to a sustained yield policy which, in this particular case, is equal to $8,5 \times 10^6 m^3/a$. Water is also used for domestic/industrial supply (Mafikeng/Mmabatho) which is equal to $4,5 \times 10^6 m^3/a$ and must be abstracted from the Grootfontein Spring. Total drawdown must not exceed 3 m at any location within the aquifer, so as to minimize the risk of land subsidence problems.

There are five different crops under consideration with the following characteristics:

CROP	WATER DEMAND ($x10^6 m^3/ha/a$)	NET RETURN (R/ha)	LABOUR INPUT (persons/ha)
1.Maize	0,011	300	0,2
2. Wheat	0,012	400	0,5
3.Sunflower	0,009	200	0,3
4.Vegetables	0,015	600	0,2
5.Potatoes	0,014	700	0,4

There is 600 ha of irrigable land in the area and there must be at least employment in the irrigated agriculture for 250 persons. A further constraint is that each crop should occupy at least 50 ha.

The problem consists of two parts:

1) Determine the optimal extraction pattern among the production boreholes, so that drawdown does not exceed 3 m at any location over the compartment.
2) Determine the amount of water to be allocated to each crop, so as to maximize the net return and satisfy the set constraints.

Task 1. Determining the Optimal Extraction Pattern.

Solution

The optimal extraction pattern must be obtained, satisfying the following constraints:
Total abstraction from the aquifer must equal $8,5 \times 10^6 m^3/a$.
Abstraction from the Grootfontein Spring must equal $4,5 \times 10^6 m^3/a$.
Maximum permitted drawdown in the aquifer is 3 m.

The first step in the solution to the problem was to calculate the response matrix constants by means of the AQUA-MAN program. The problem was formulated as shown in the MANAGE.DAT file (Appendix 1). This file was supplied to the simplex routine (SIMPLEX.EXE) which yields the following solution:

The optimal pumping rate must be:
12500 m^3/d at the Grootfontein Eye
902 m^3/d at Borehole BB7
1567m^3/d at Borehole VL20
8012 m^3/d at Borehole VF115
628 m^3/d at Borehole VG2

The value of the objective function is 23611 m^3/d which is equal to $8,5 \times 10^6 m^3/a$.

Task 2. Determine the optimal water allocation devoted to each crop so as to maximize the net return.

Solution

The available volume for irrigation is $4,0 \times 10^6 m^3/a$.

Defining the decision variables.

Let XM the amount of hectares devoted to maize
Let XC the amount of hectares devoted to wheat
Let XS the amount of hectares devoted to sunflower

Let XV the amount of hectares devoted to vegetables
Let XP the amount of hectares devoted to potatoes.

Defining the objective function.

Maximize net return: $F = 300XM + 400XC + 200XS + 600XV + 700XP$

Defining the constraints:

Area (hectare) constraints:

$XM + XC + XS + XV + XP < 600$
$XM > 50$
$XC > 50$
$XS > 50$
$XV > 50$
$XP > 50$

Water availability constraint:

$.011XM + .012XC + .009XS + .015XV + .014XP < 4.0$

Minimum labour input constraint:

$.2XM + .5XC + .3XS + .2XV + .4XP > 250$

Solution:

The above formulated problem was supplied to the simplex routine of Erikson and Hall (1989). The output is shown in Appendix 2.

The simplex routine yielded the following results:

The following amount of hectares must be devoted to each crop:

To maize: 50 ha
wheat: 68.182ha
sunflower: 50 ha
vegetables: 50 ha
potatoes: 102.273 ha

Total: 320.455 ha

The objective function's value is R 153863.625

The amount of water devoted to each crop is as follows:

To maize : 50 ha x .011 = .55 million cubic metres

wheat : 68.182 x .012 = .818 million cubic metres
sunflower:50 x .009 = .45 million cubic metres
vegetables:50 x .015 = .75 million cubic metres
potatoes:102.273 x .014 = 1.4423 million cubic metres

Total: 4 million cubic metres

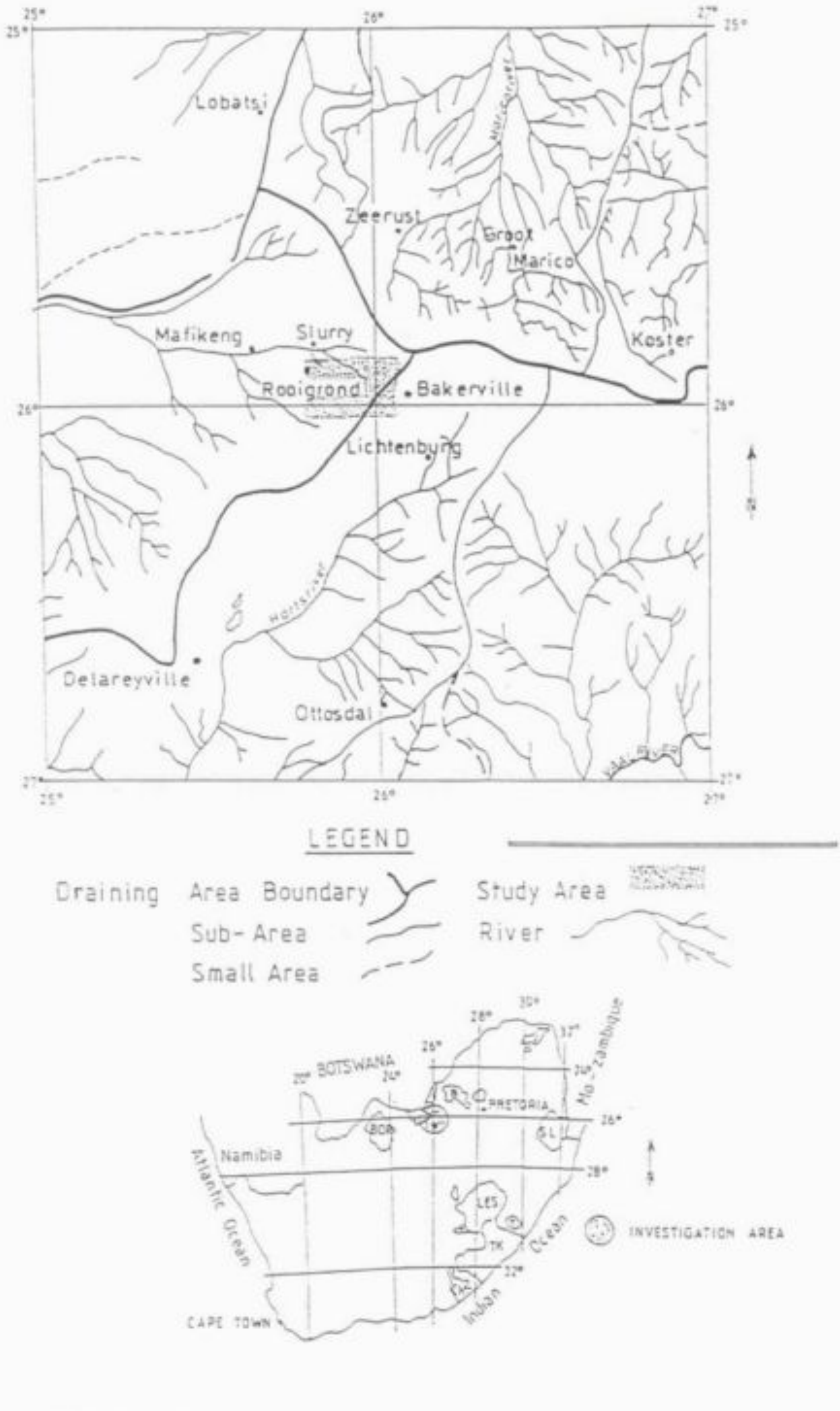

Lobatsi
Zeerust
Groot
Marico
Mafikeng
Slurry
Rooigrond
Bakerville
Koster
Lichtenburg
Delareyville
Ottosdal
LEGEND
Draining Area Boundary
Study Area
Sub-Area
River
Small Area
BOTSWANA
PRETORIA
Namibia
Atlantic Ocean
Indian Ocean
Mozambique
LES
TK
CAPE TOWN
INVESTIGATION AREA

Figure 1

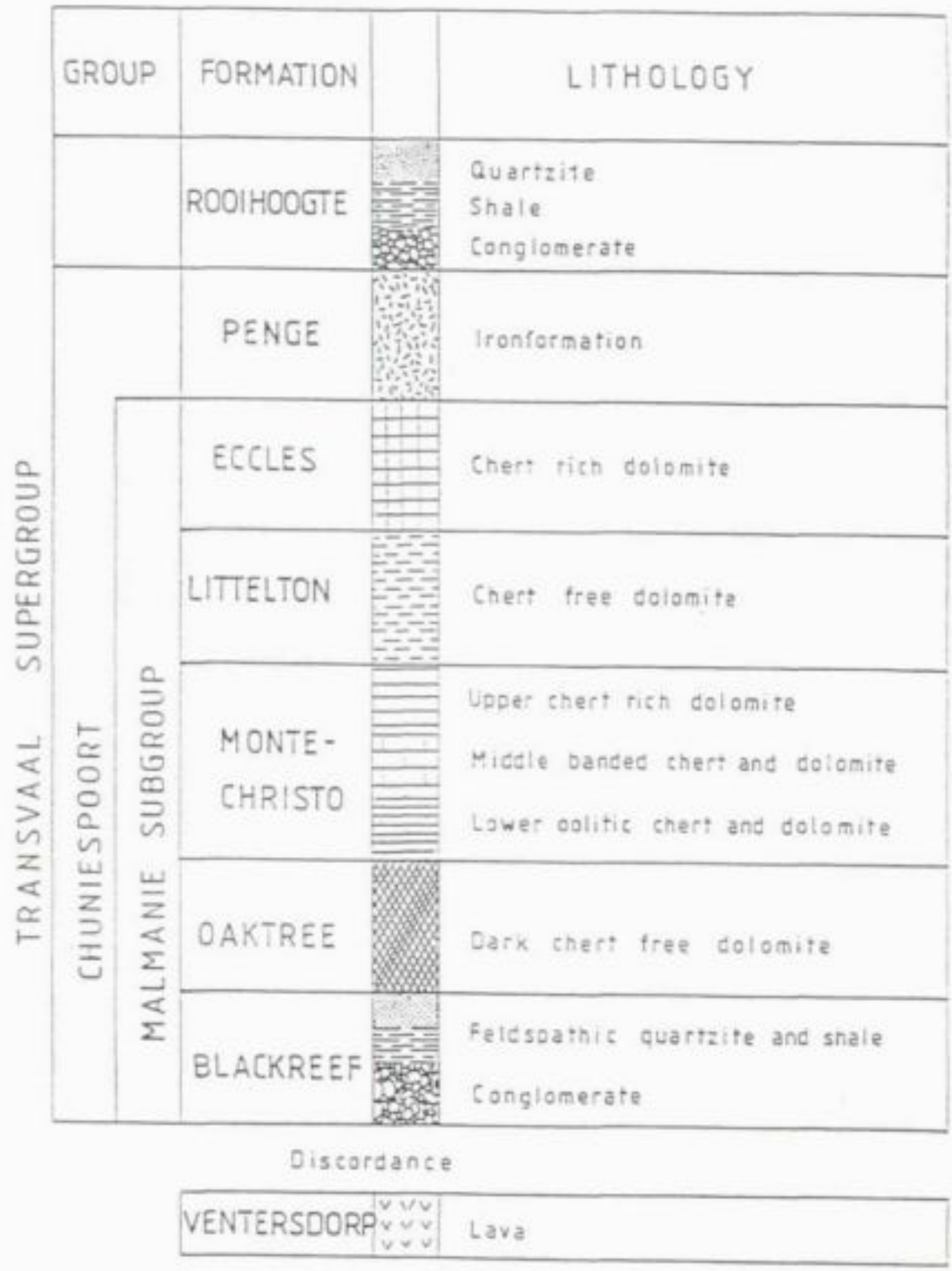

GROUP
FORMATION
LITHOLOGY
ROOIHOOGTE
Quartzite
Shale
Conglomerate
PENGE
Ironformation
TRANSVAAL SUPERGROUP
CHUNIESPOORT
MALMANIE SUBGROUP
ECCLES
Chert rich dolomite
LITTELTON
Chert free dolomite
MONTE-CHRISTO
Upper chert rich dolomite
Middle banded chert and dolomite
Lower oolitic chert and dolomite
OAKTREE
Dark chert free dolomite
BLACKREEF
Feldspathic quartzite and shale
Conglomerate
Discordance
VENTERSDORP
Lava

Figure 2

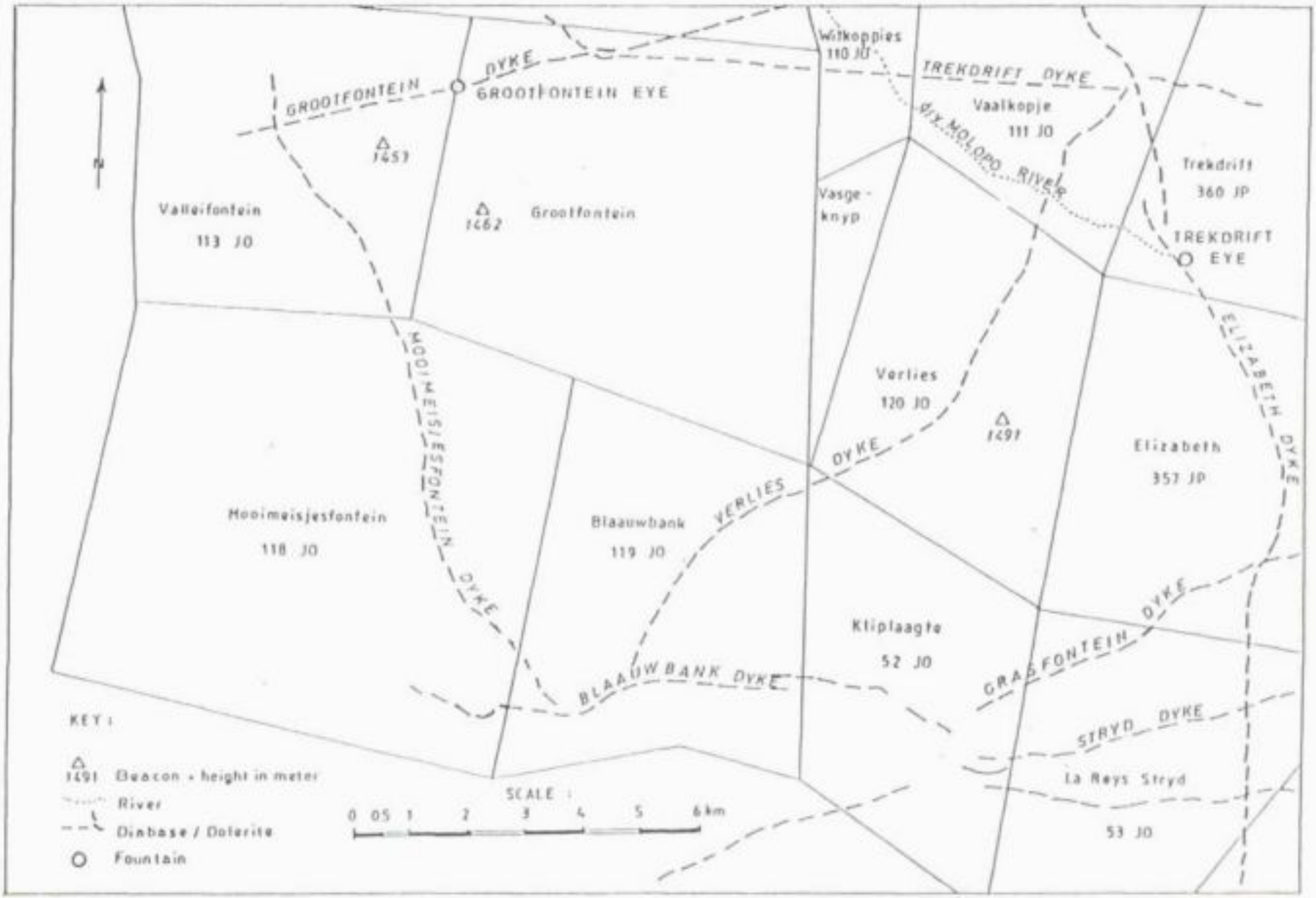

N
Witkoppies
110 JO
TREKDRIFT DYKE
Vaalkopje
111 JO
dry MOLOPO RIVER
Trekdrift
360 JP
TREKDRIFT
EYE
GROOTFONTEIN
DYKE
GROOTFONTEIN EYE
1457
Valleifontein
113 JO
1462
Grootfontein
Vasge-
knyp
Verlies
120 JO
1491
ELIZABETH DYKE
Elizabeth
357 JP
VERLIES DYKE
MOOIMEISJESFONTEIN DYKE
Mooimeisjesfontein
118 JO
Blaauwbank
119 JO
Klipplaagte
52 JO
GRASFONTEIN DYKE
BLAAUWBANK DYKE
STRYD DYKE
La Reys Stryd
53 JO
KEY:
1491 Beacon + height in meter
River
Diabase / Dolerite
Fountain
SCALE:
0 0.5 1 2 3 4 5 6km

Figure 3

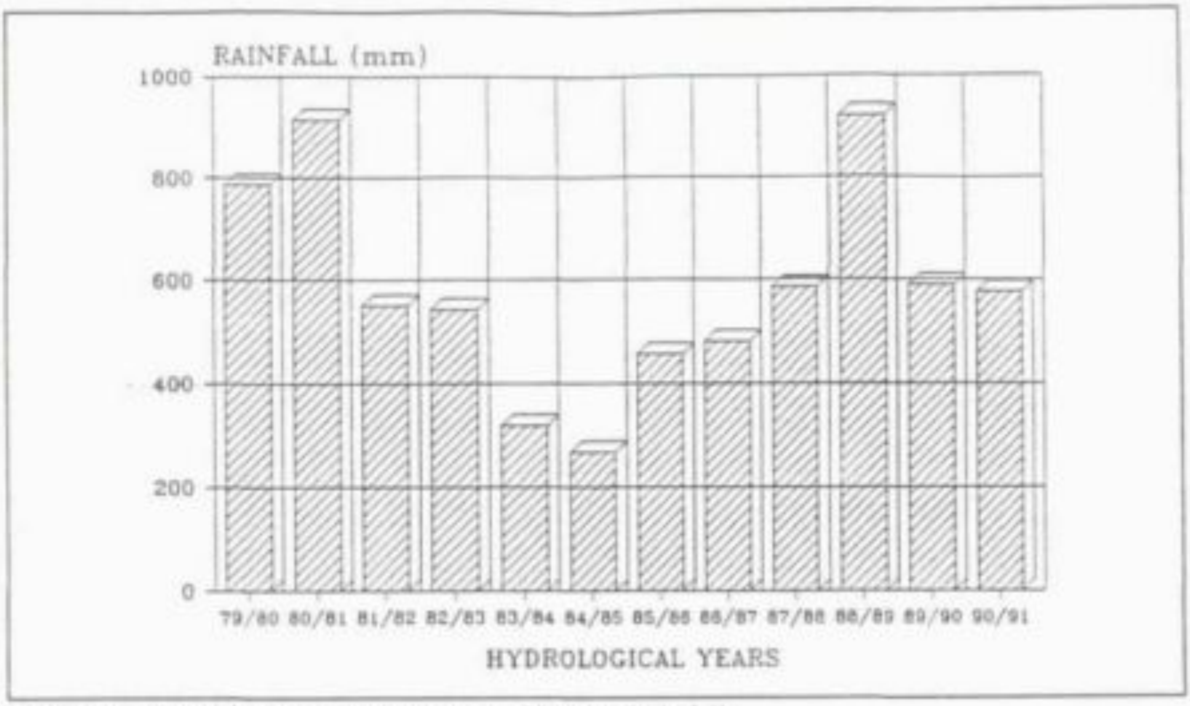


Figure 4 Grootfontein Compartment Yearly Rainfall

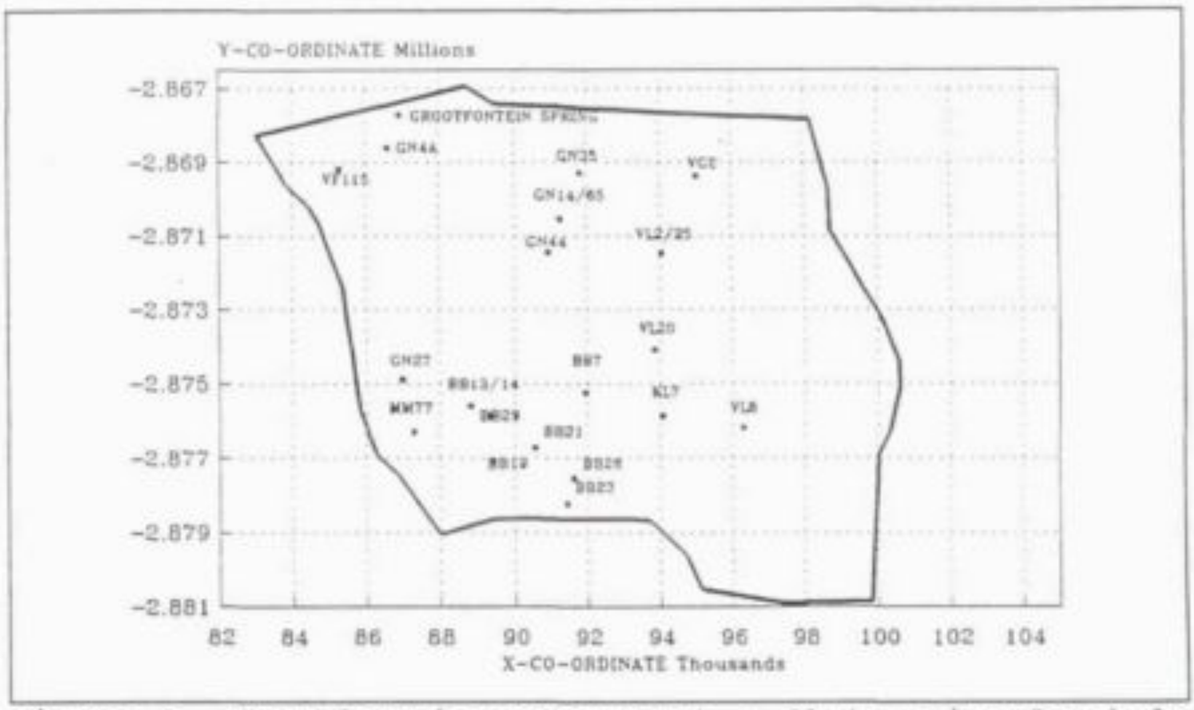


Figure 5 Grootfontein Compartment - Abstraction Borehole Positions

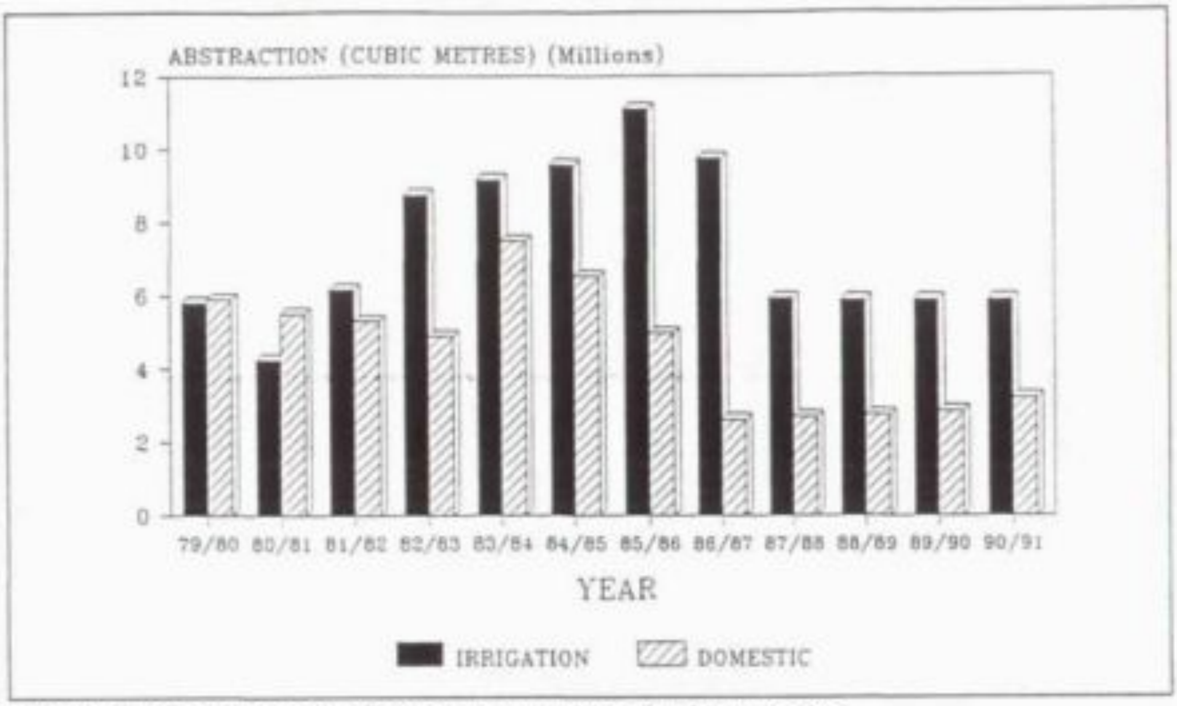


Figure 6 Grootfontein Compartment Yearly Abstraction

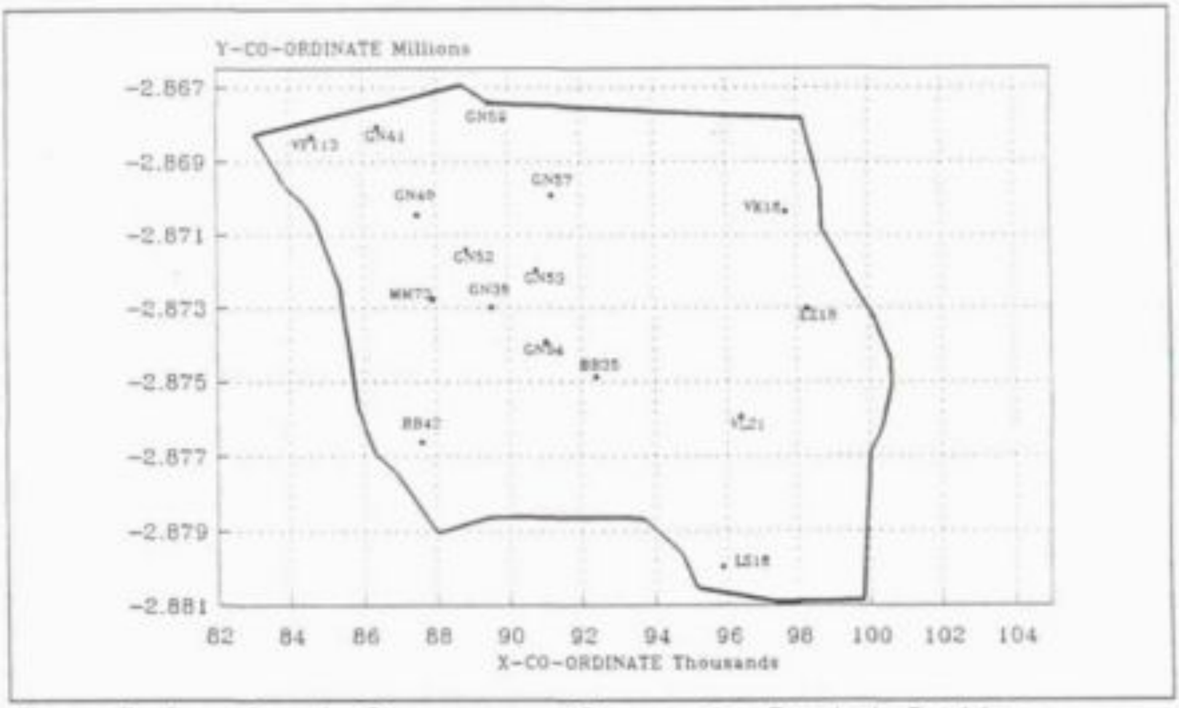


Figure 7 Grootfontein Compartment Observation Borehole Positions

Figure 8(a)

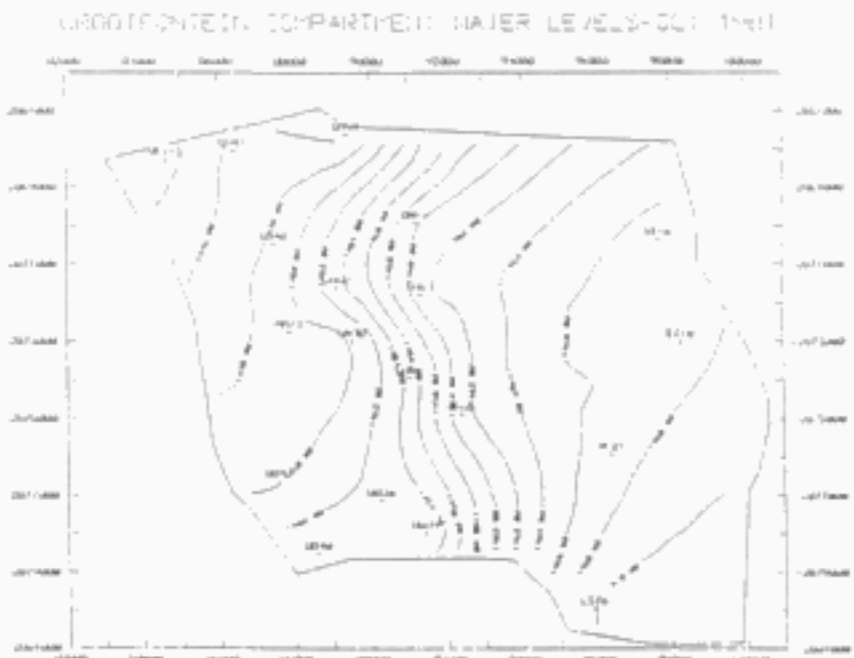


Figure 8(b)

Figure 8 (c)

Figure 8(d)

GROOTFONTEIN COMPARTMENT WATER LEVELS-OCT 1987

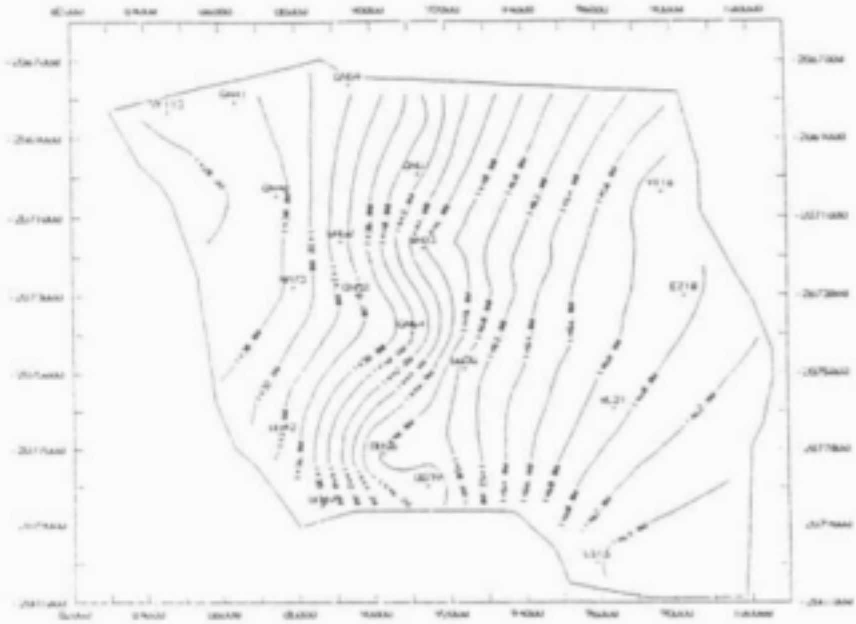


Figure8(e)

GROOTFONTEIN COMPARTMENT WATER LEVELS OCT-1989

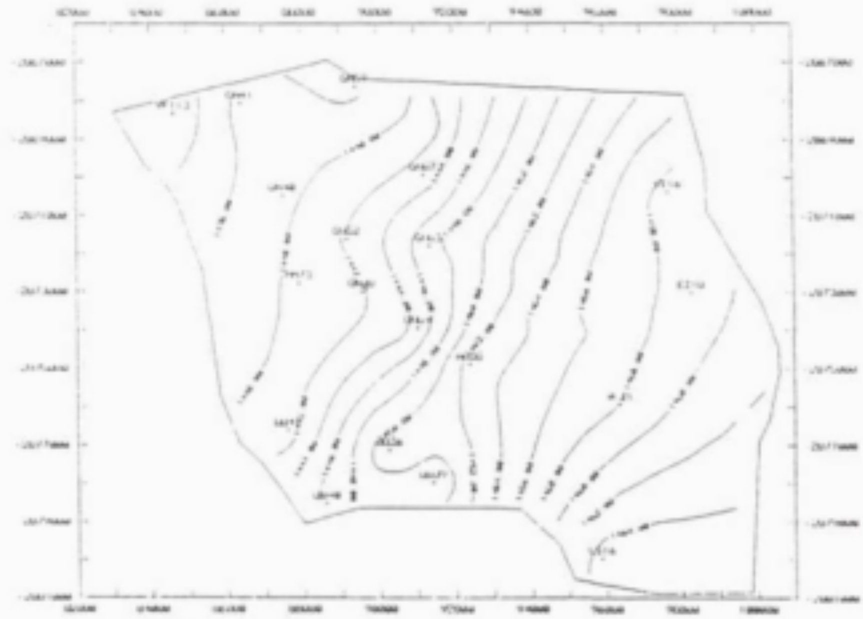


Figure 8(f)

GROOTFONTEIN COMPARTMENT WATER LEVELS-OCT 1991

Figure 8(g)

GROOTFONTEIN COMPARTMENT BASE ROCK ELEVATION

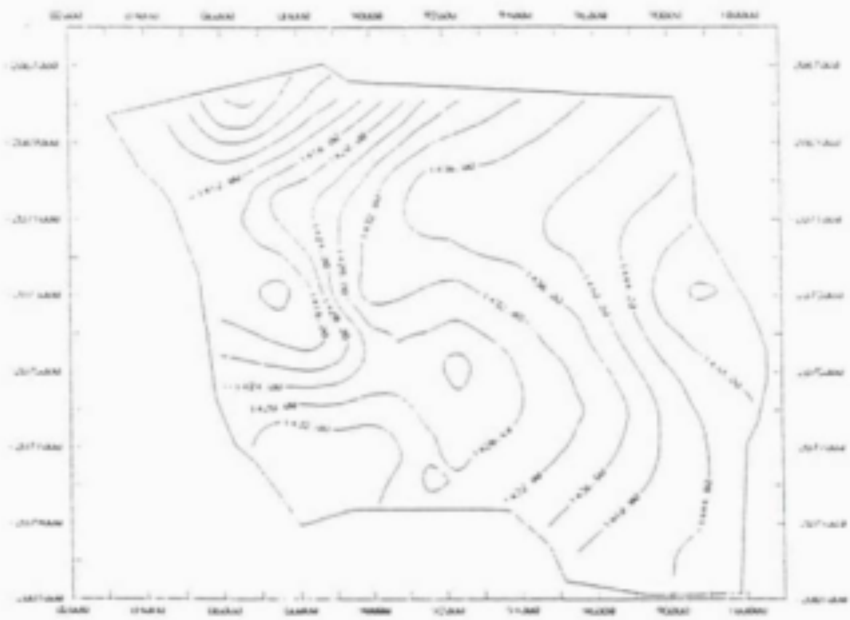


Figure 9

GN41
WATER LEVEL FLUCTUATION

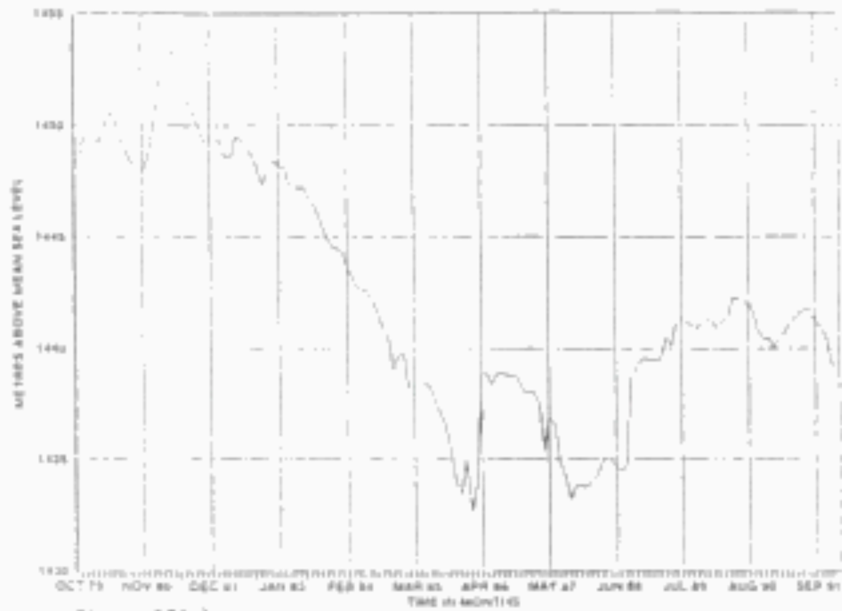


Figure 10(a)

EZ18
WATER LEVEL FLUCTUATION

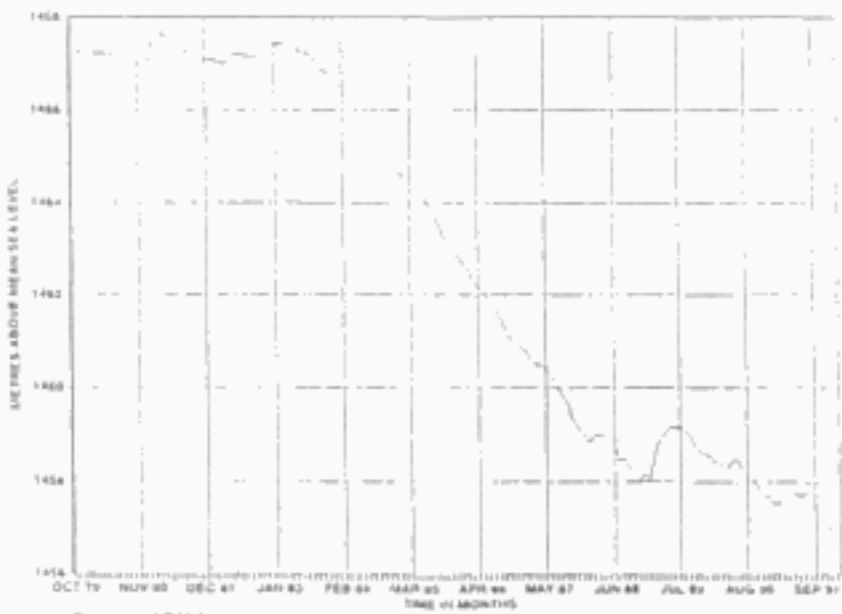


Figure 10(b)

MM73
WATER LEVEL FLUCTUATION

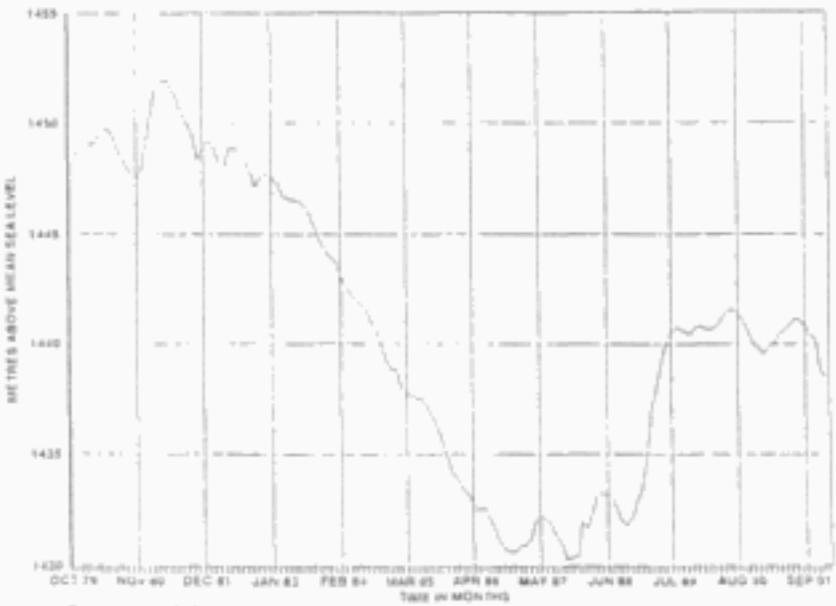


Figure 10(c)

GN53
WATER LEVEL FLUCTUATION

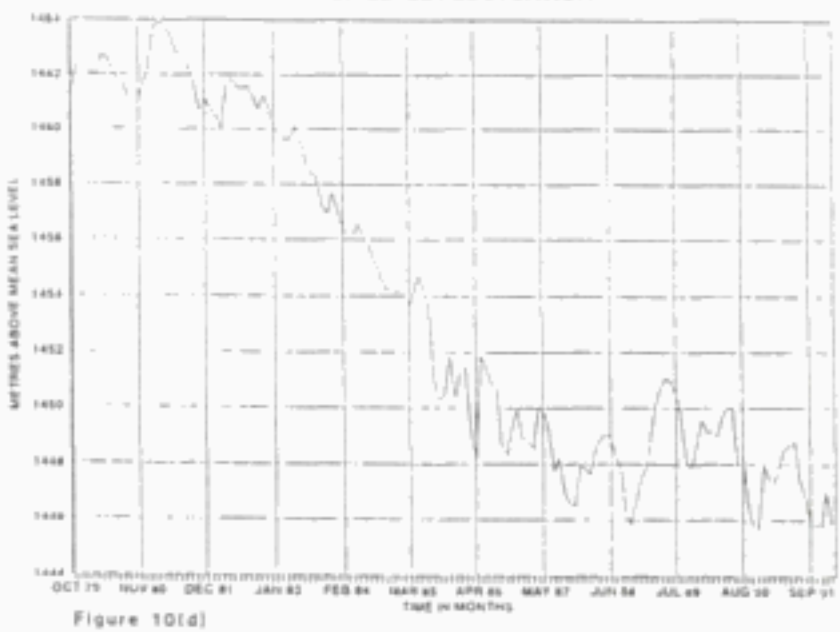


Figure 10(d)

GN54
WATER LEVEL FLUCTUATION

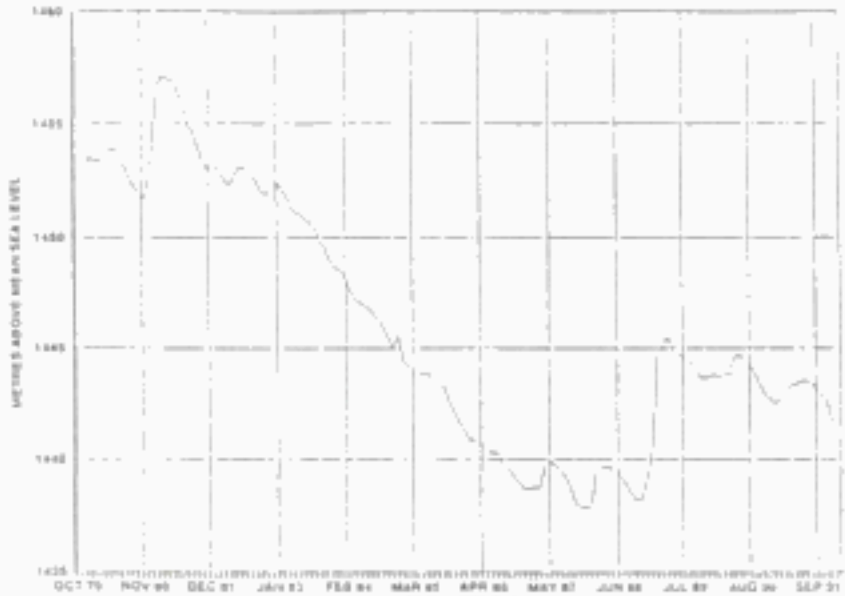


Figure 10(e)

BB35
WATER LEVEL FLUCTUATION

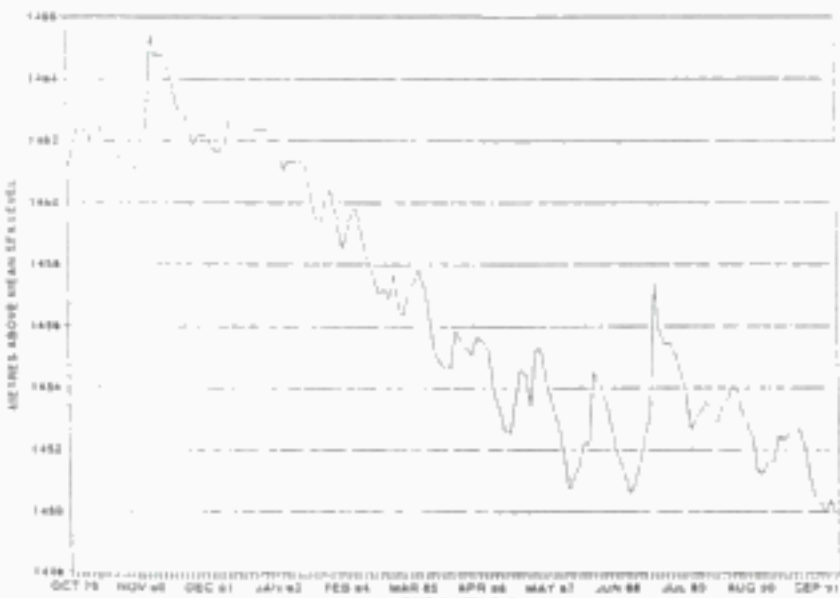


Figure 10(f)

VK16
WATER LEVEL FLUCTUATION

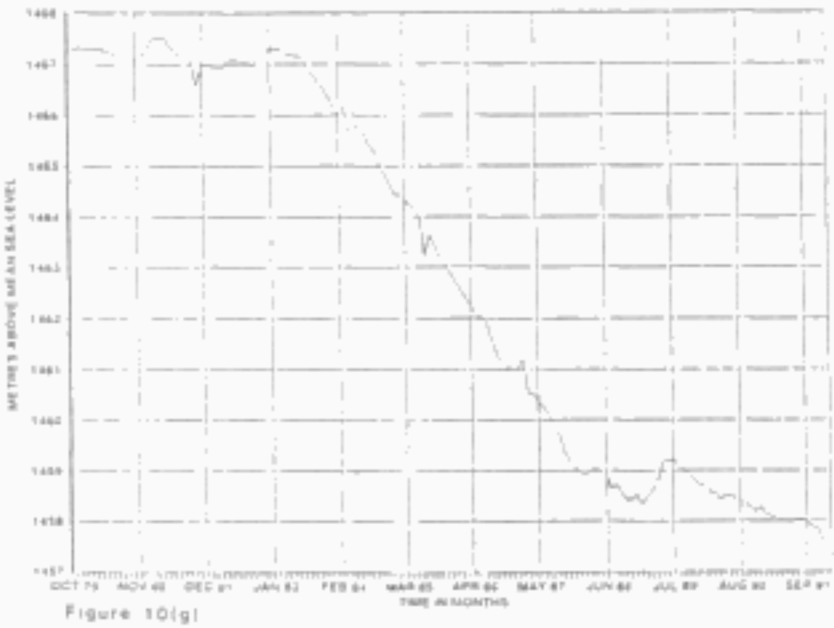


Figure 10(g)

VL21
WATER LEVEL FLUCTUATION

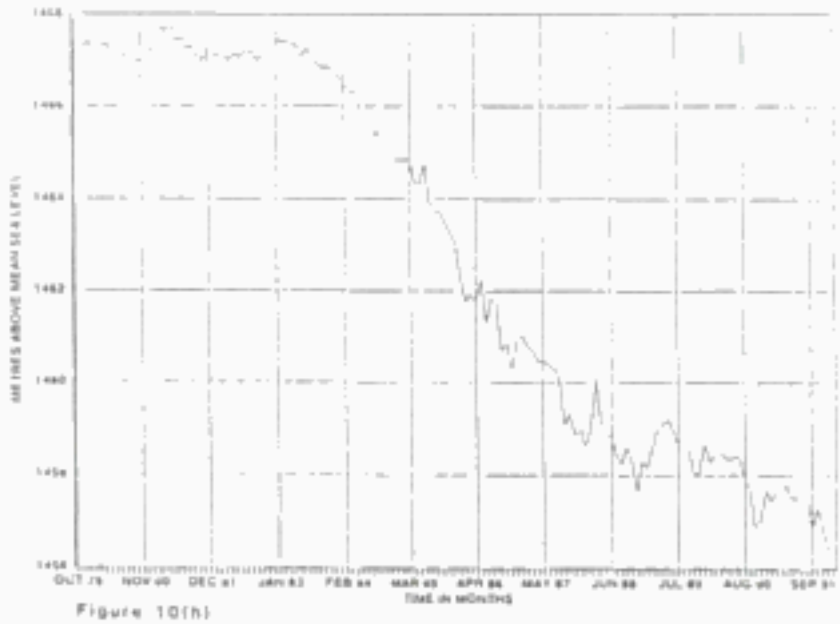


Figure 10(h)

LS16
WATER LEVEL FLUCTUATION

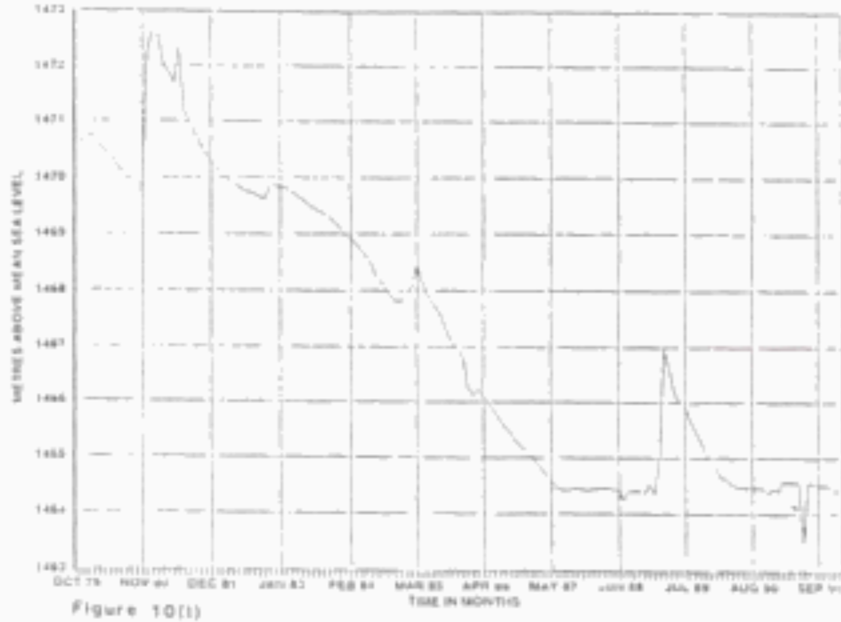


Figure 10(i)

VK19
WATER LEVEL FLUCTUATION

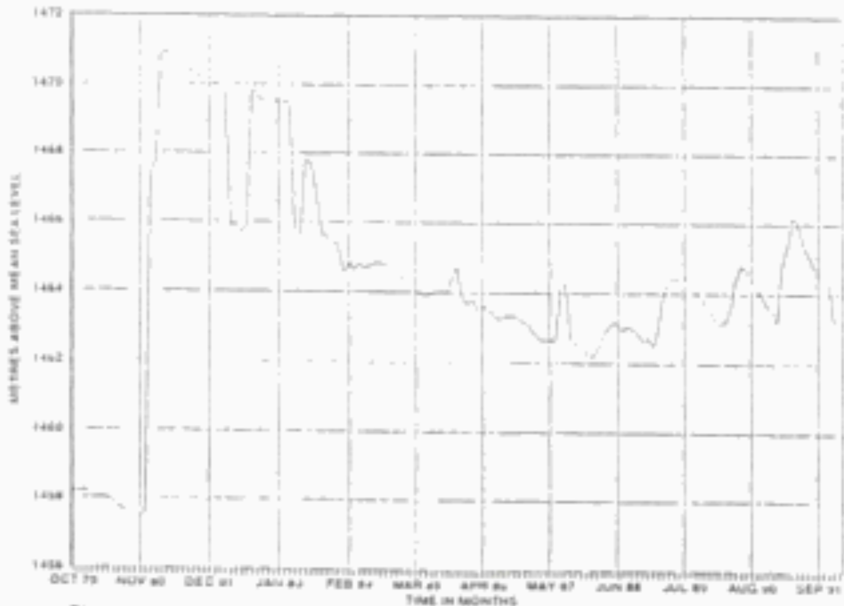


Figure 10(j)

BB36
WATER LEVEL FLUCTUATION

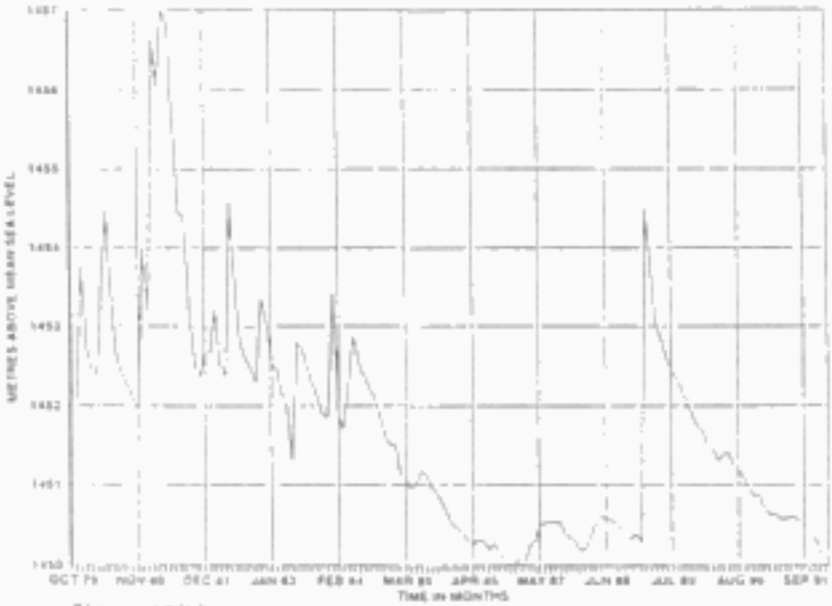


Figure 10(k)

GN59
WATER LEVEL FLUCTUATION

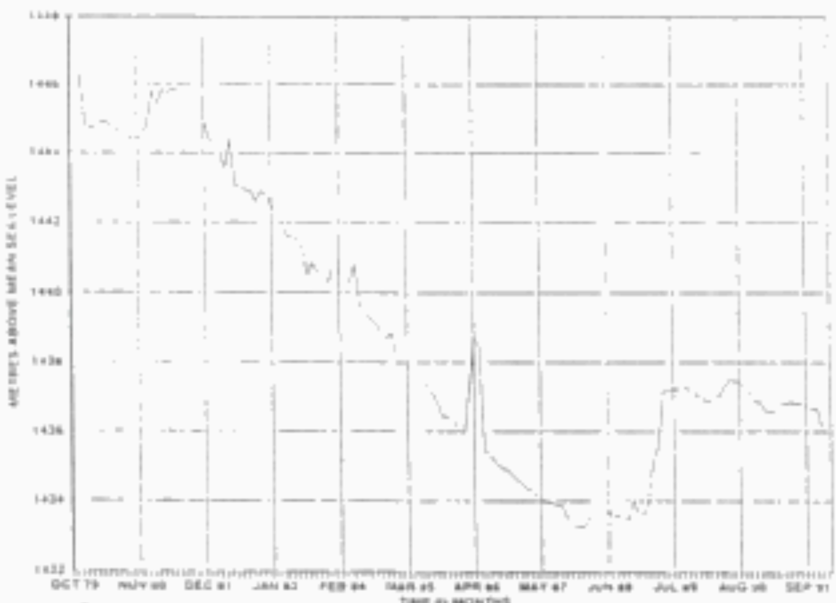


Figure 10(l)

GN38
WATER LEVEL FLUCTUATION

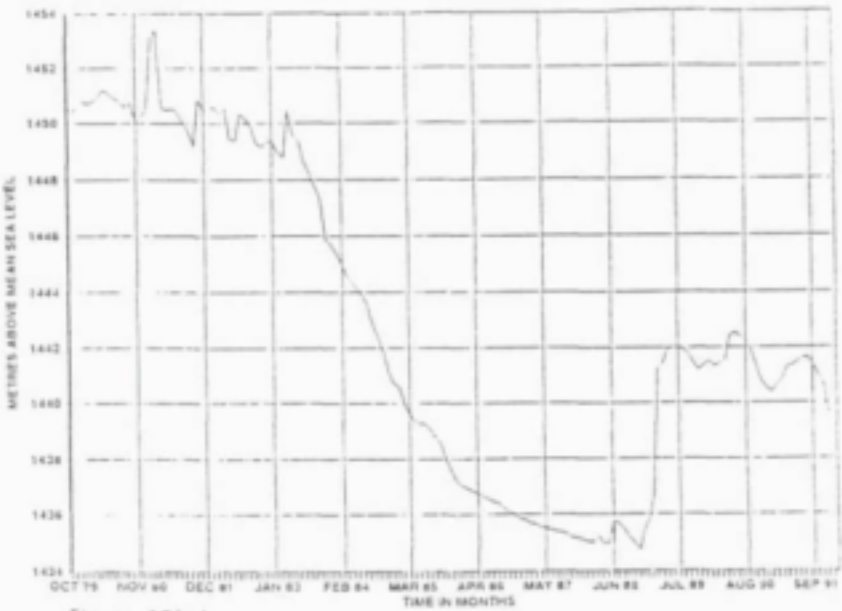


Figure 10(m)

GN57
WATER LEVEL FLUCTUATION

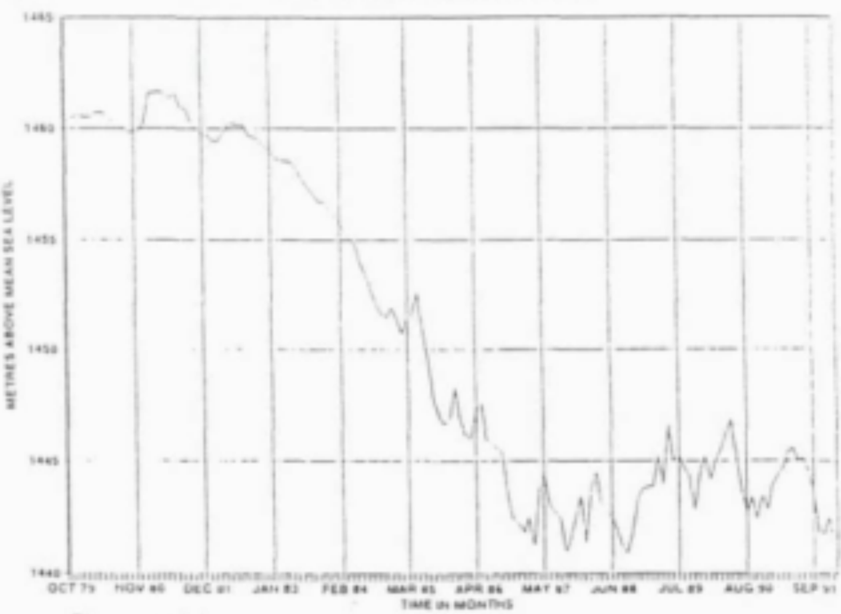


Figure 10(n)

BB35
WATER LEVEL FLUCTUATION

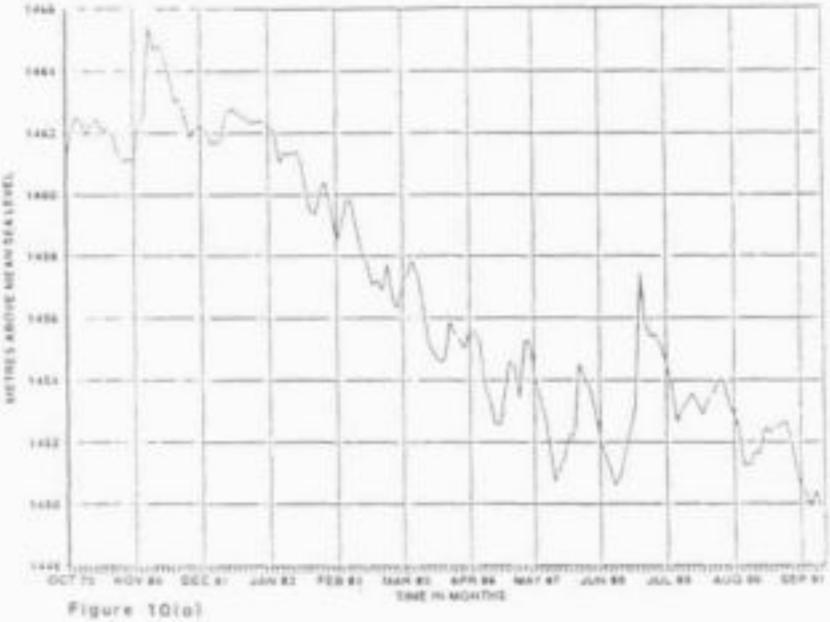


Figure 10(o)

BB39
WATER LEVEL FLUCTUATION

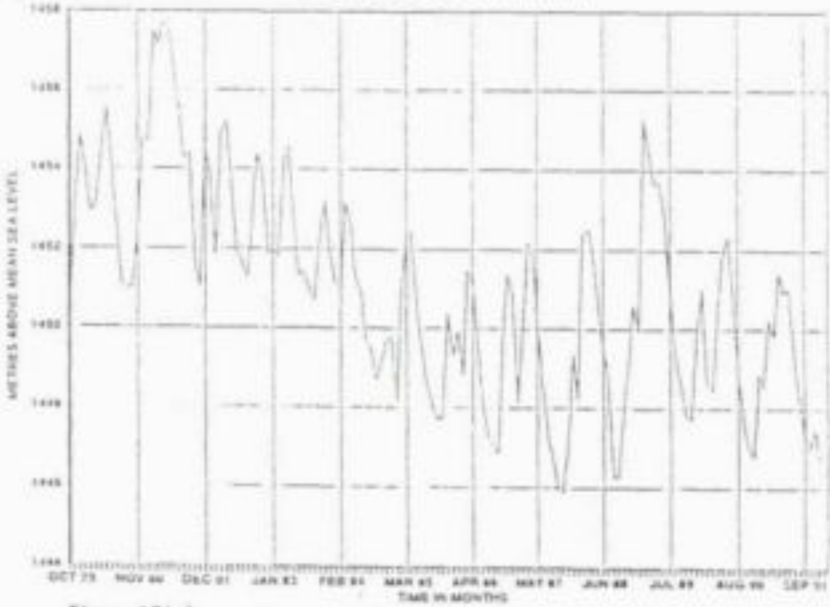


Figure 10(p)

VF113
WATER LEVEL FLUCTUATION

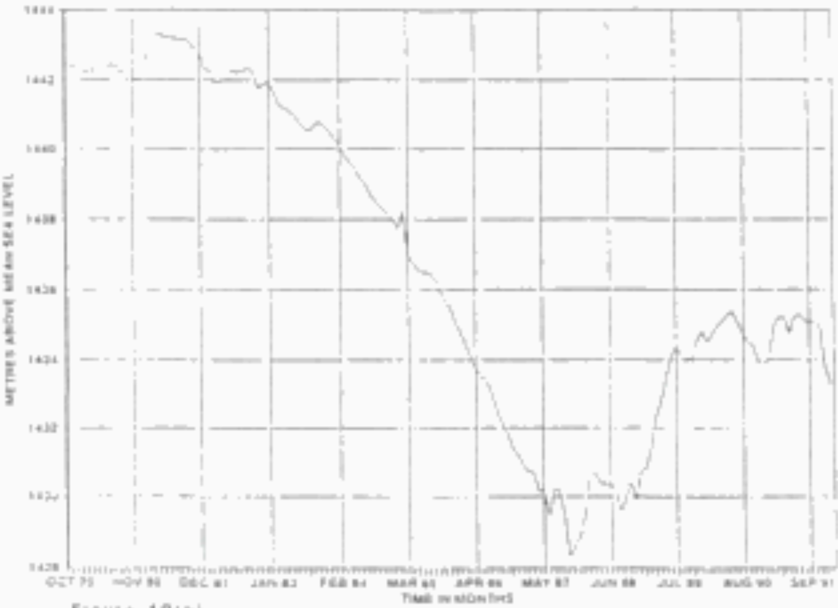


Figure 10(q)

GN52
WATER LEVEL FLUCTUATION

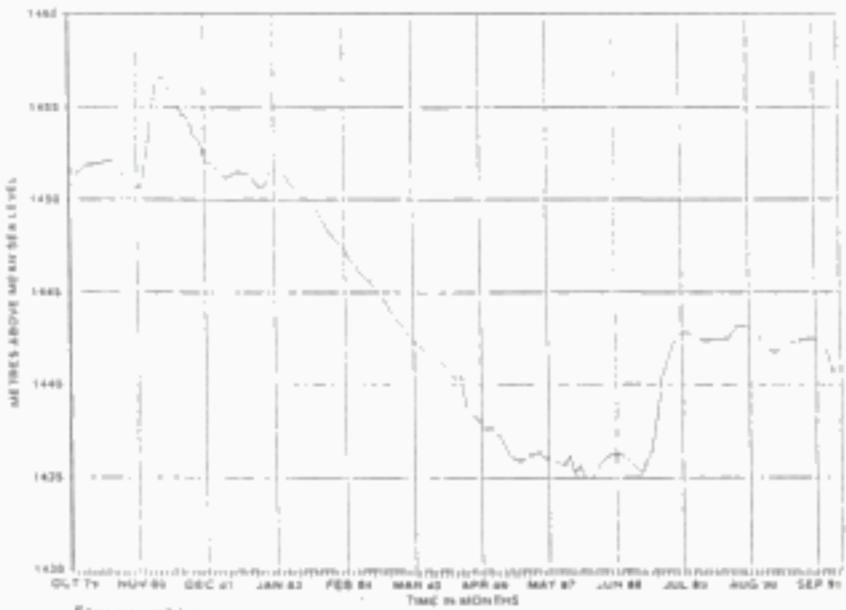


Figure 10(r)

BB40
WATER LEVEL FLUCTUATION

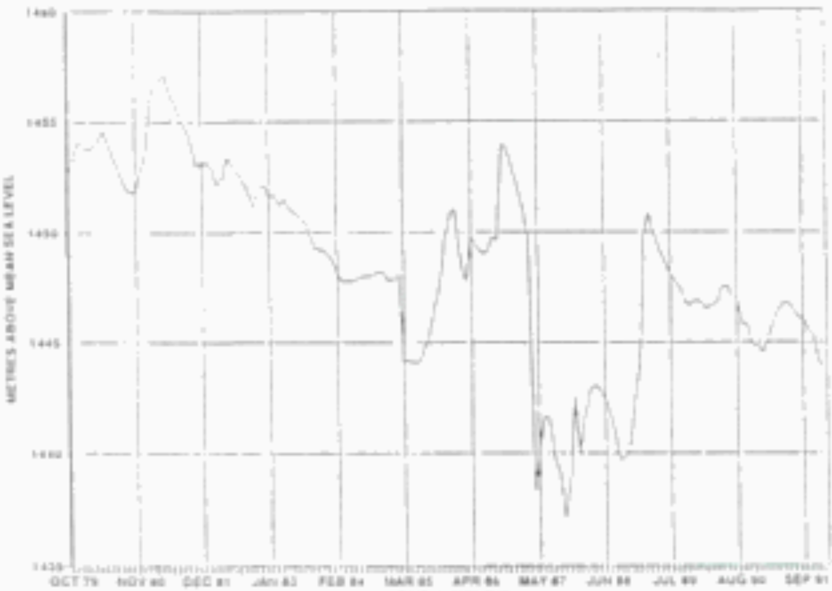


Figure 10(s)

GN40
WATER LEVEL FLUCTUATION

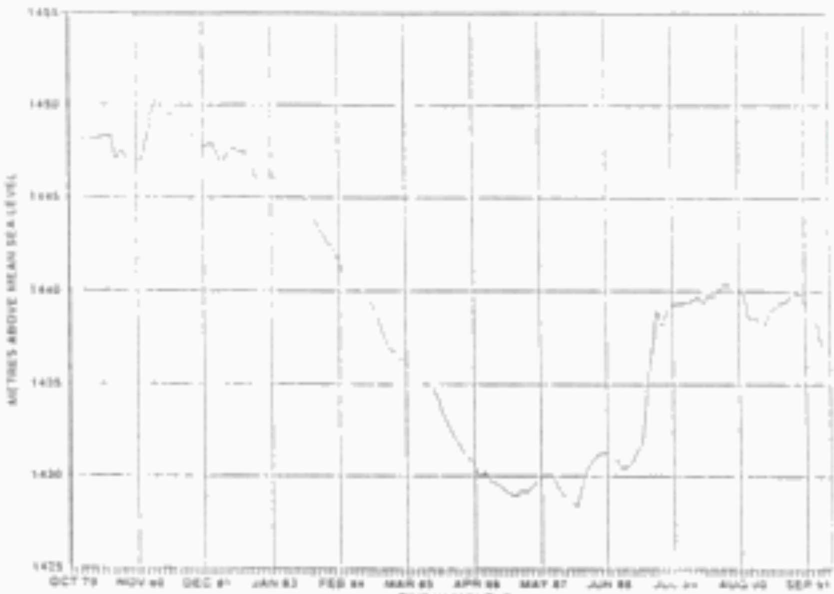


Figure 10(t)

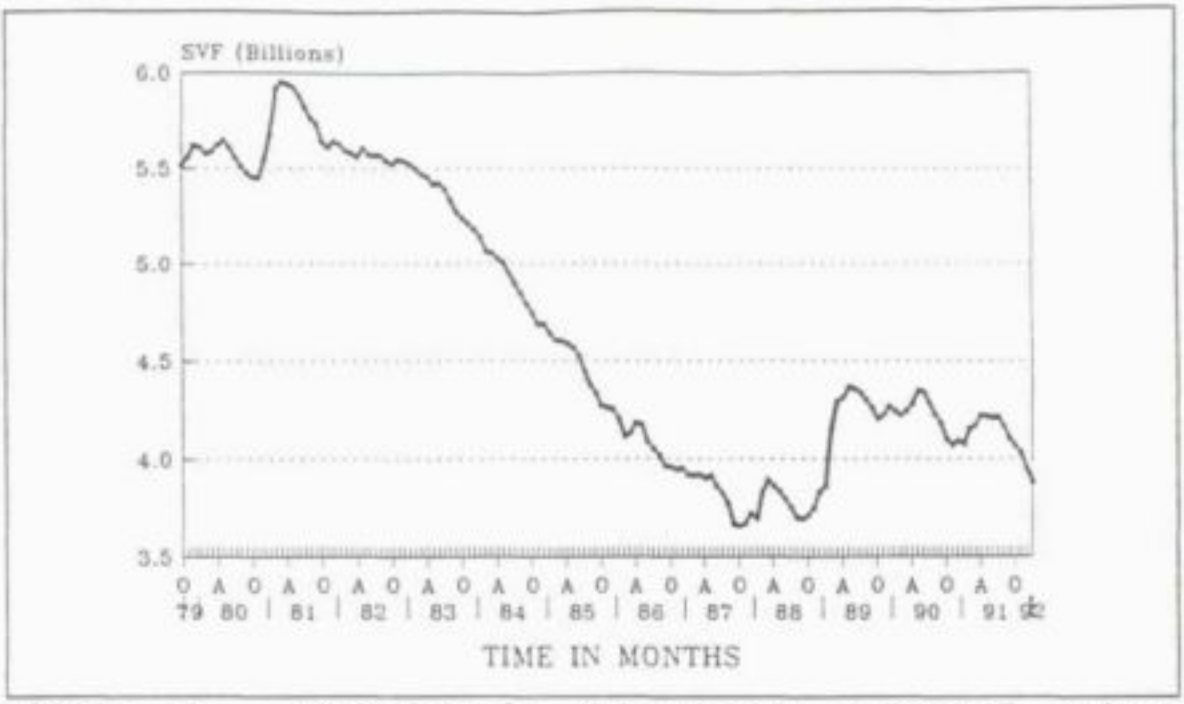


Figure 11 Grootfontein Compartment Saturated Volume Fluctuation

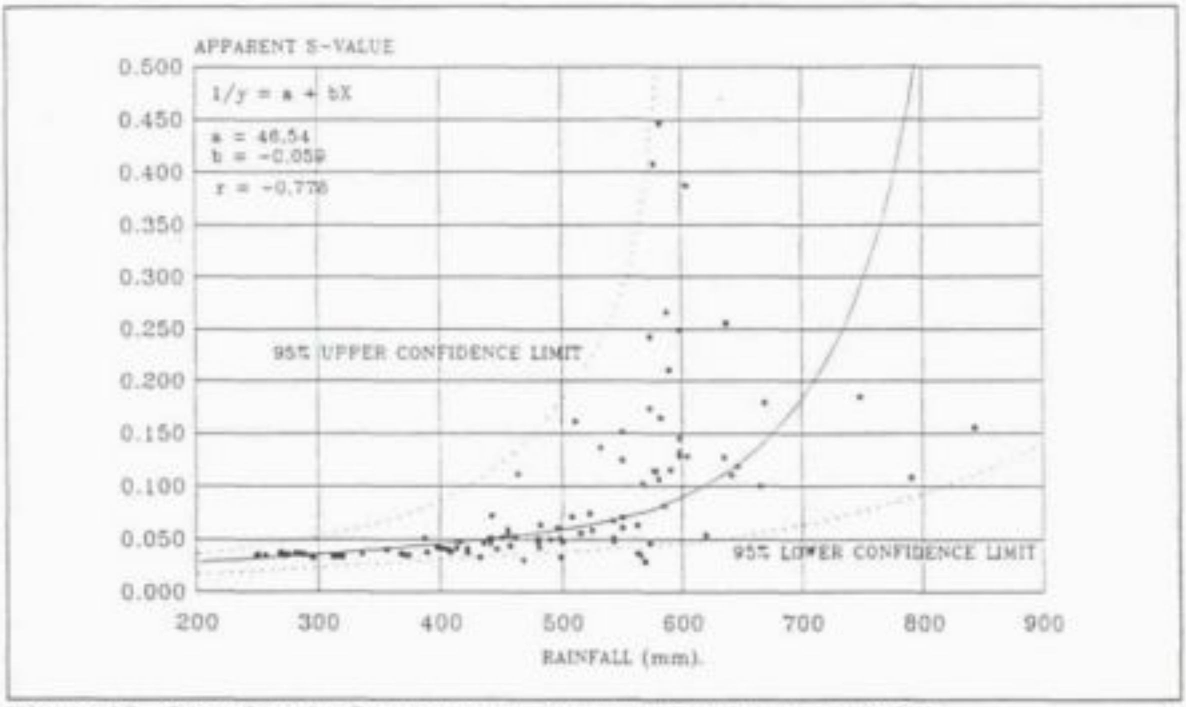


Figure 12 Grootfontein Compartment Apparent S-value vs. Rainfall

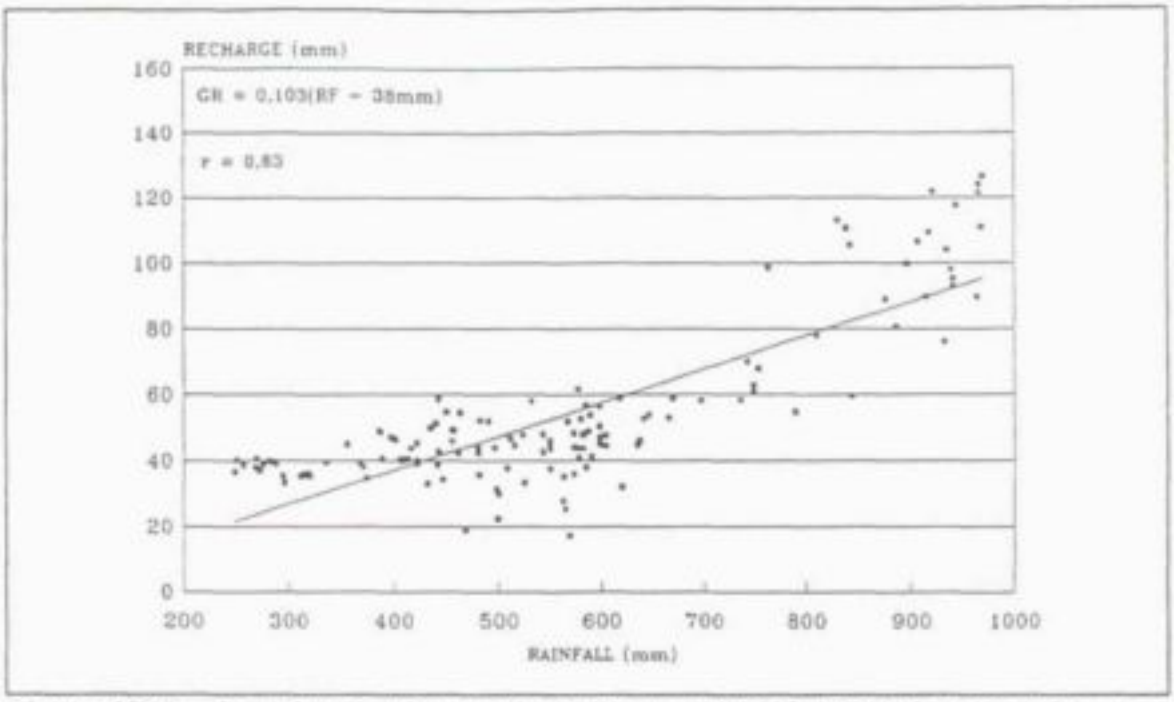


Figure 13(a) Grootfontein Compartment Recharge/Rainfall Relationship using most likely S-value of 0,0215

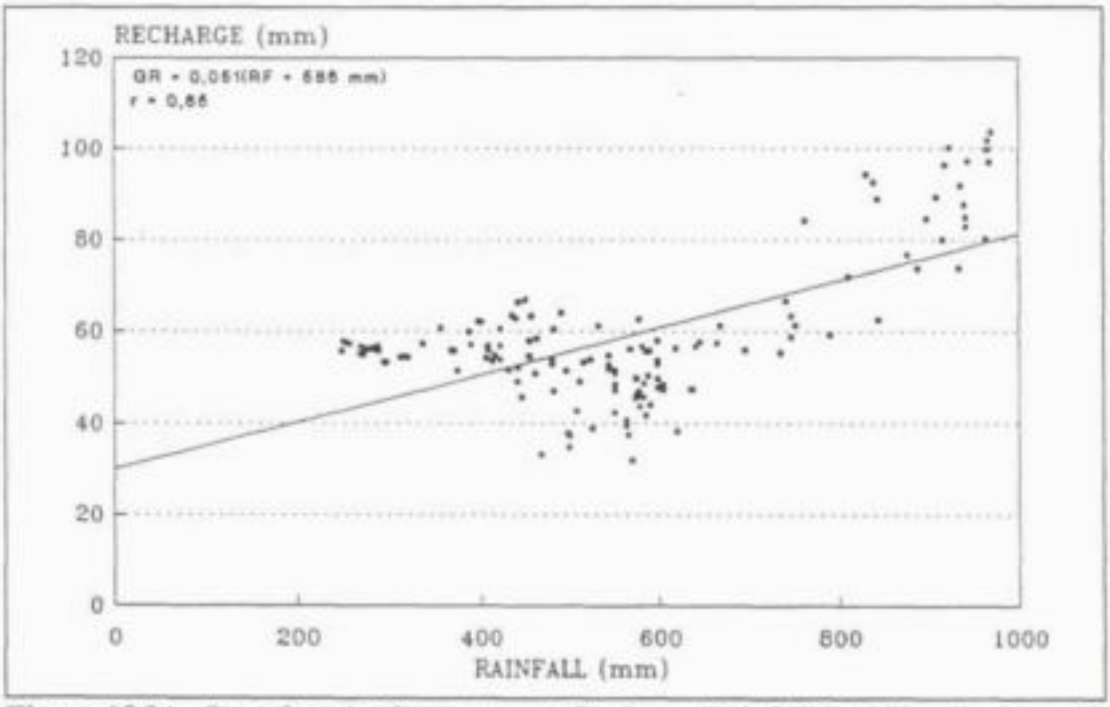


Figure 13(b) Grootfontein Compartment Recharge/Rainfall Relationship using S-value of 0,015

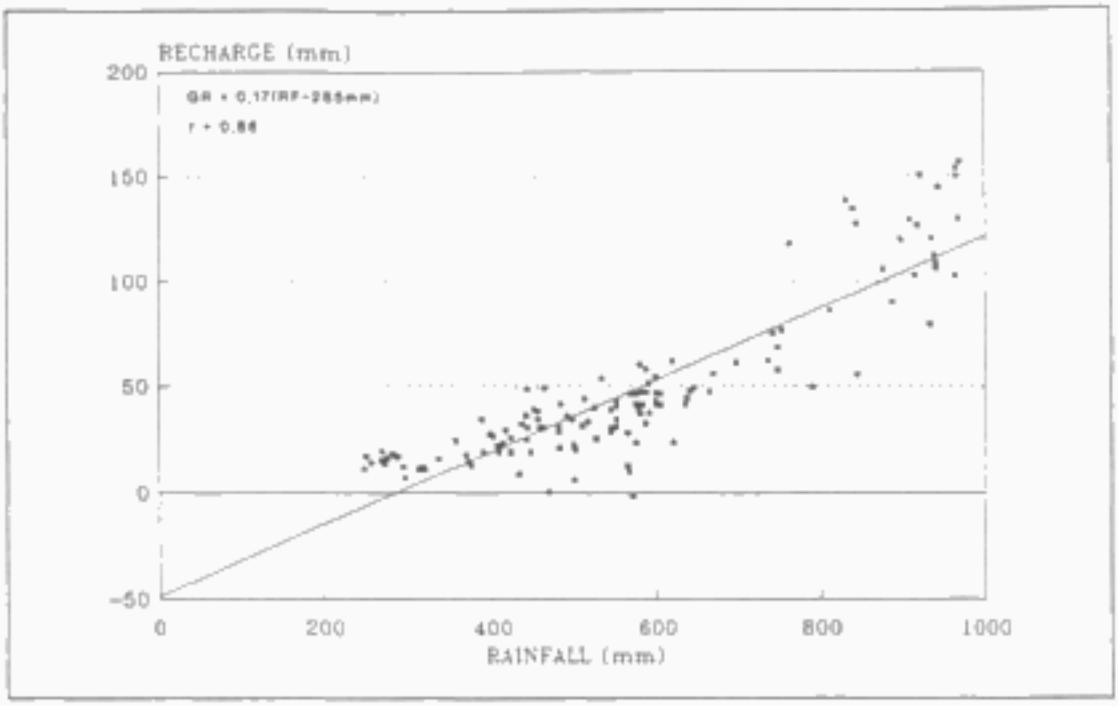


Figure 13(c) Grootfontein Compartment Recharge/Rainfall Relationship using most likely S-value of 0,03

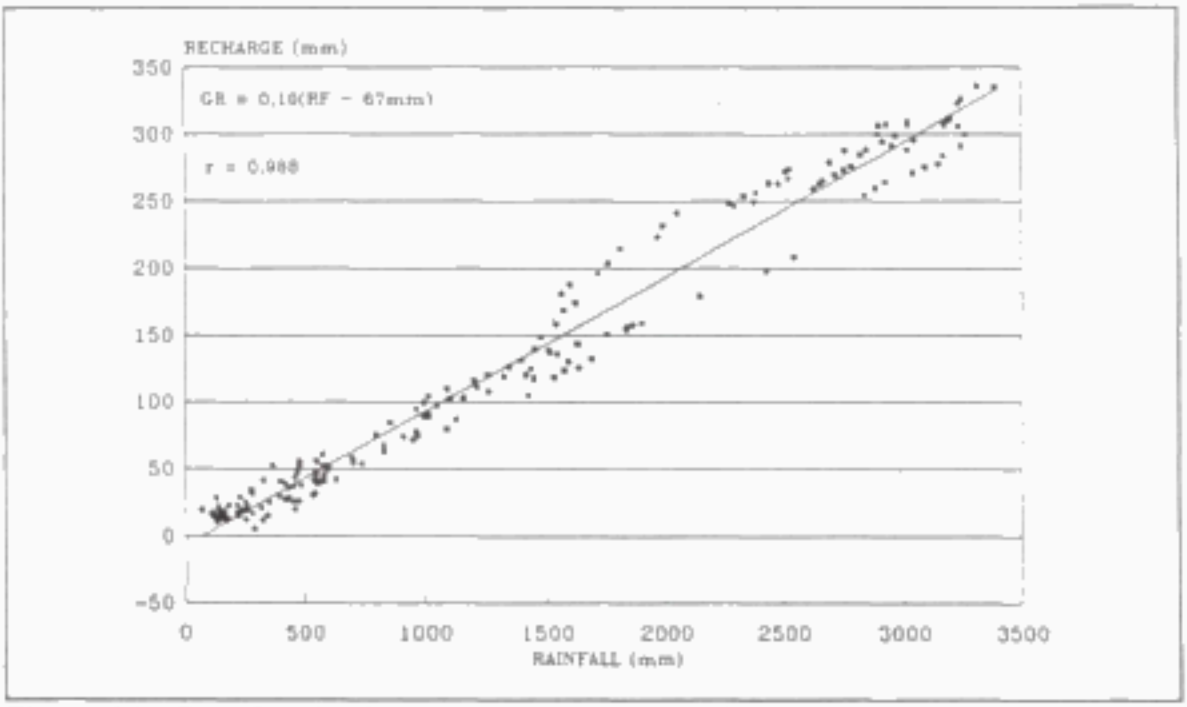


Figure 14 Grootfontein Compartment Recharge/Rainfall Relationship obtained using Equal Volume Method

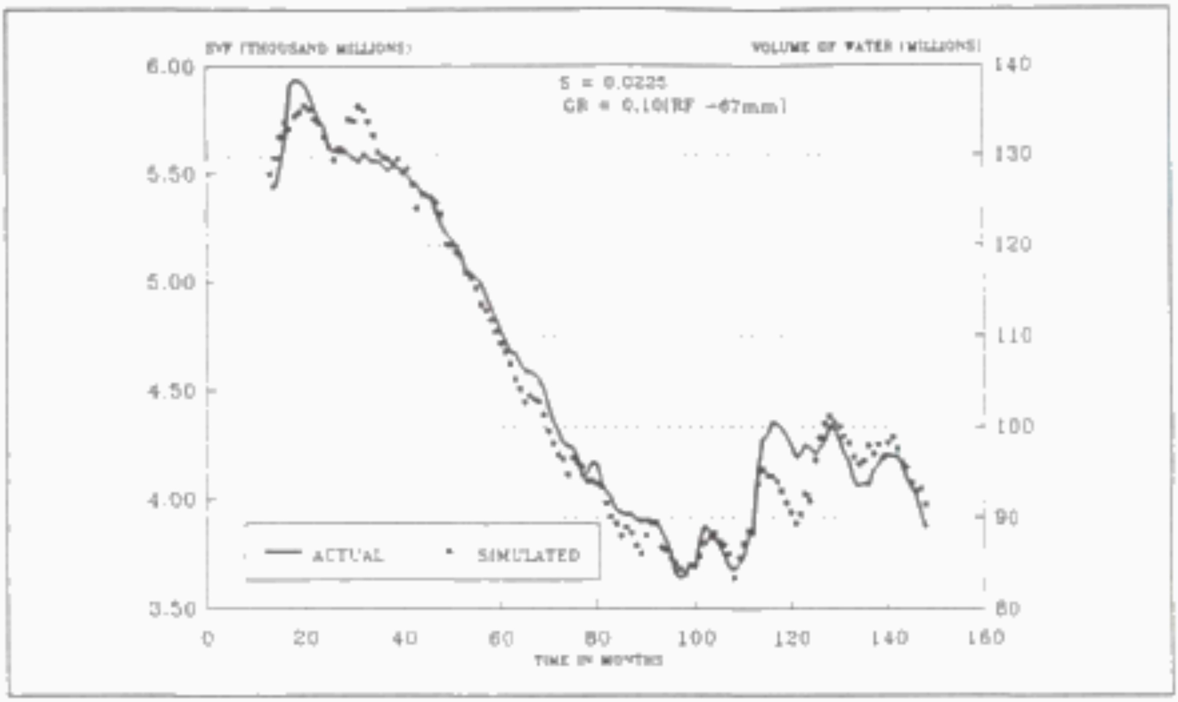


Figure 15 Grootfontein Compartment Actual vs. Simulated SVF

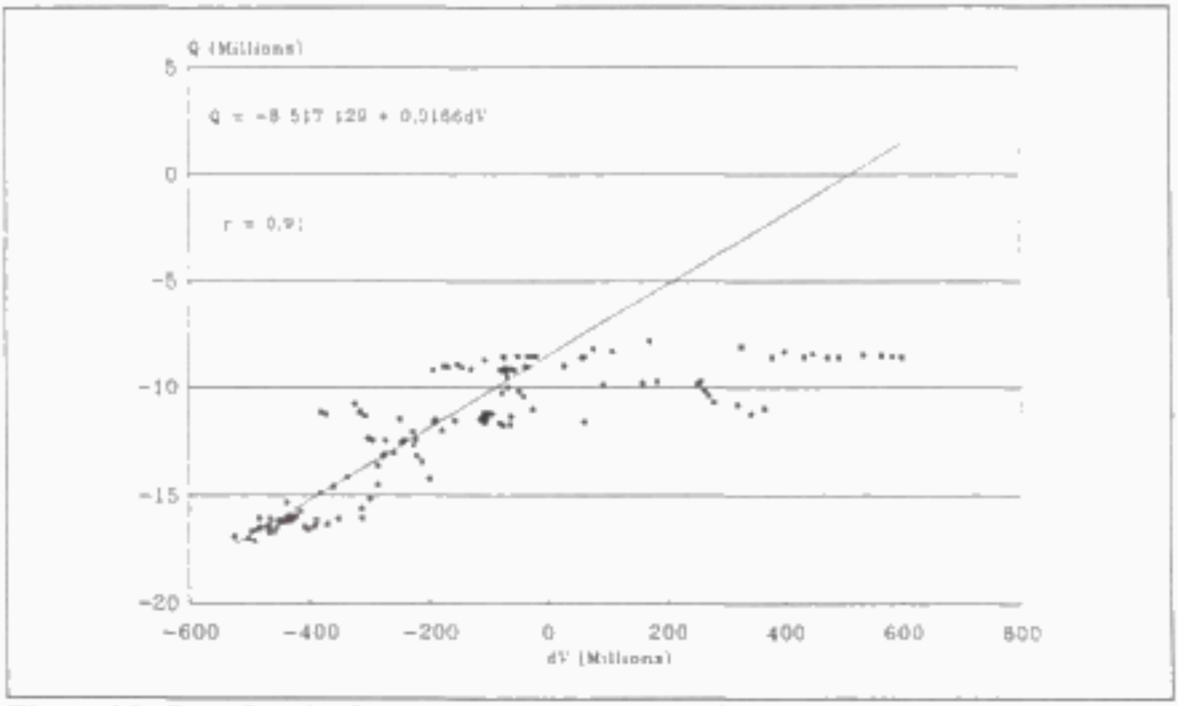


Figure 16 Grootfontein Compartment Abstraction (Q) vs. Change in SVF

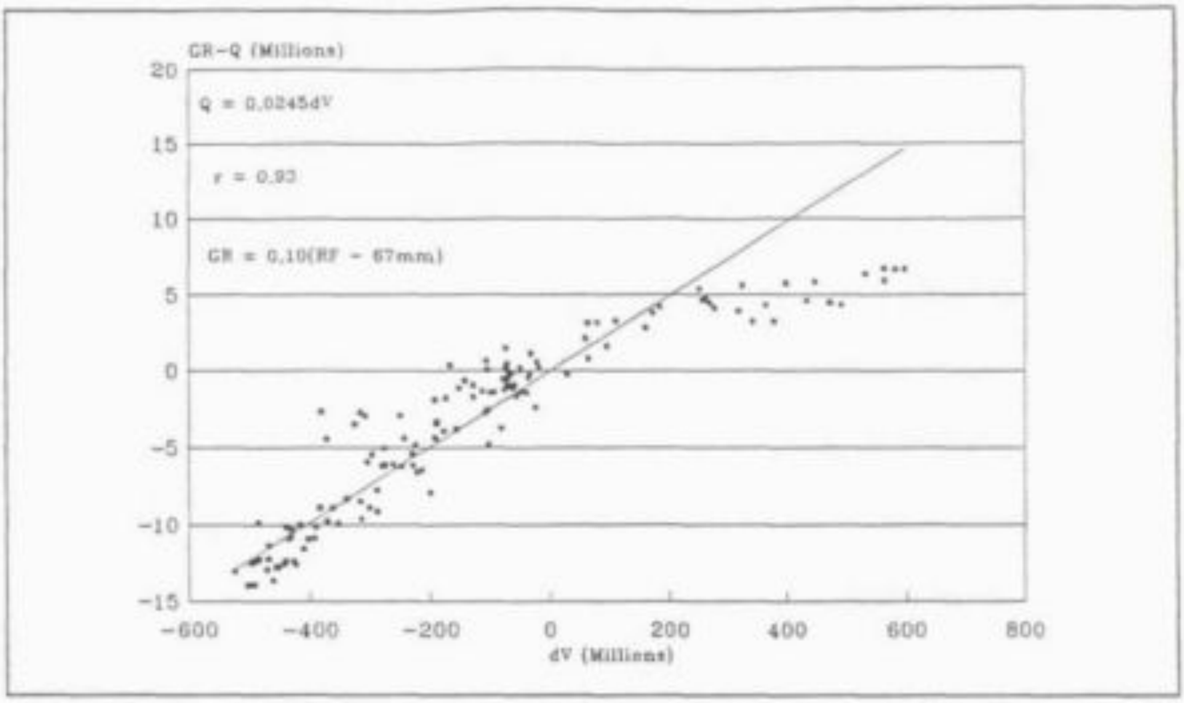


Figure 17 Grootfontein Compartment Recharge minus Abstraction (GR-Q) vs. Change in SVF

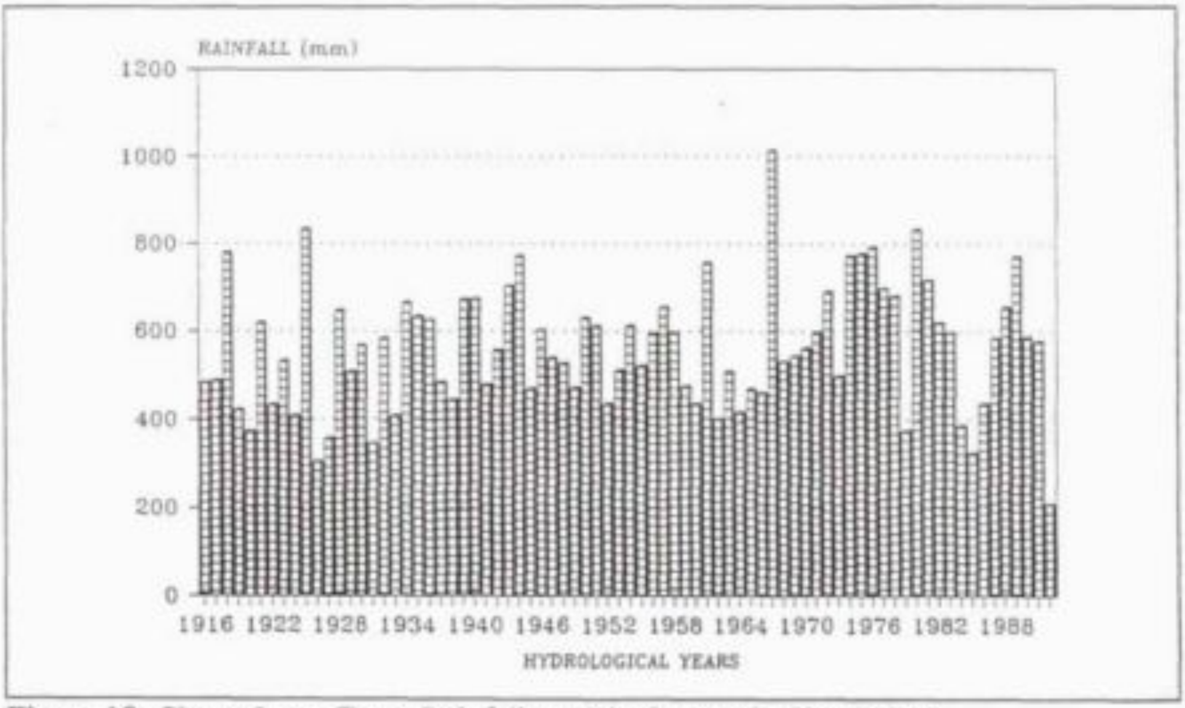
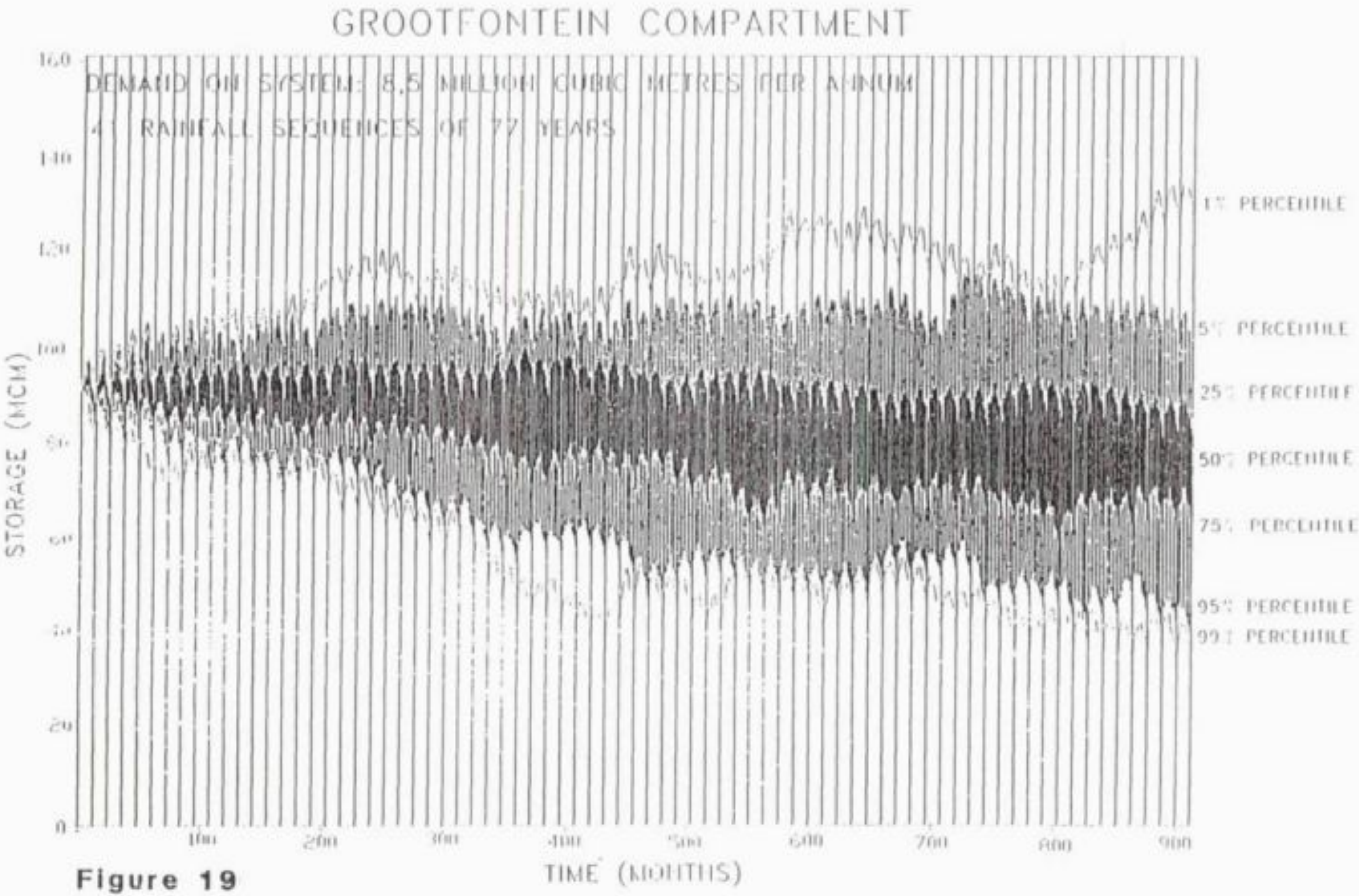


Figure 18 Slurry Long Term Rainfall used in Stochastic Simulations



GROOTFONTEIN COMPARTMENT
DEMAND ON SYSTEM: 8.5 MILLION CUBIC METRES PER ANNUM
41 RAINFALL SEQUENCES OF 77 YEARS
STORAGE (MCM)
TIME (MONTHS)
1% PERCENTILE
5% PERCENTILE
25% PERCENTILE
50% PERCENTILE
75% PERCENTILE
95% PERCENTILE
99% PERCENTILE

Figure 19

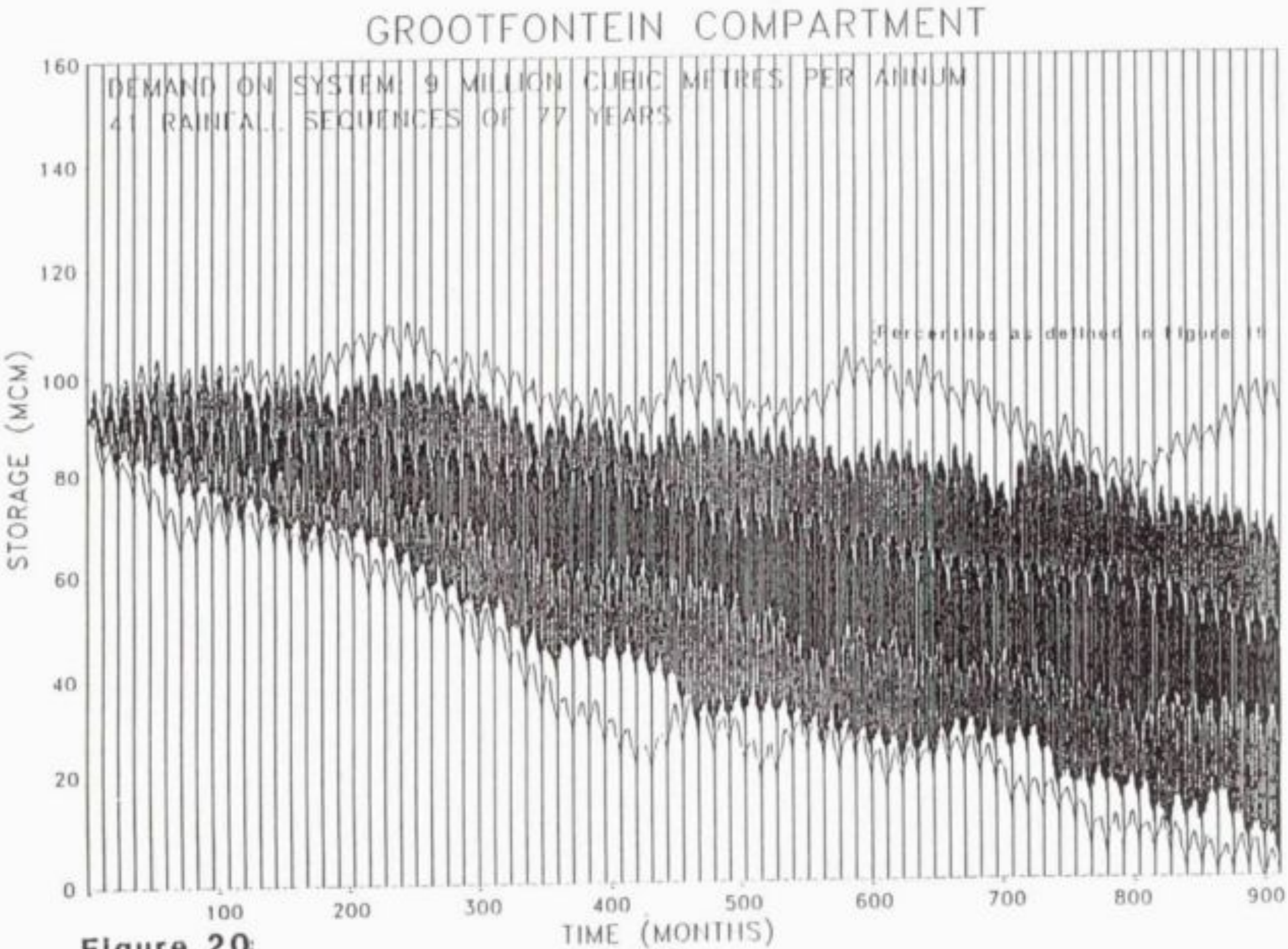

GROOTFONTEIN COMPARTMENT
DEMAND ON SYSTEM: 9 MILLION CUBIC METRES PER ANNUM
41 RAINFALL SEQUENCES OF 77 YEARS
STORAGE (MCM)
0
20
40
60
80
100
120
140
160
100
200
300
400
500
600
700
800
900
TIME (MONTHS)

Figure 20

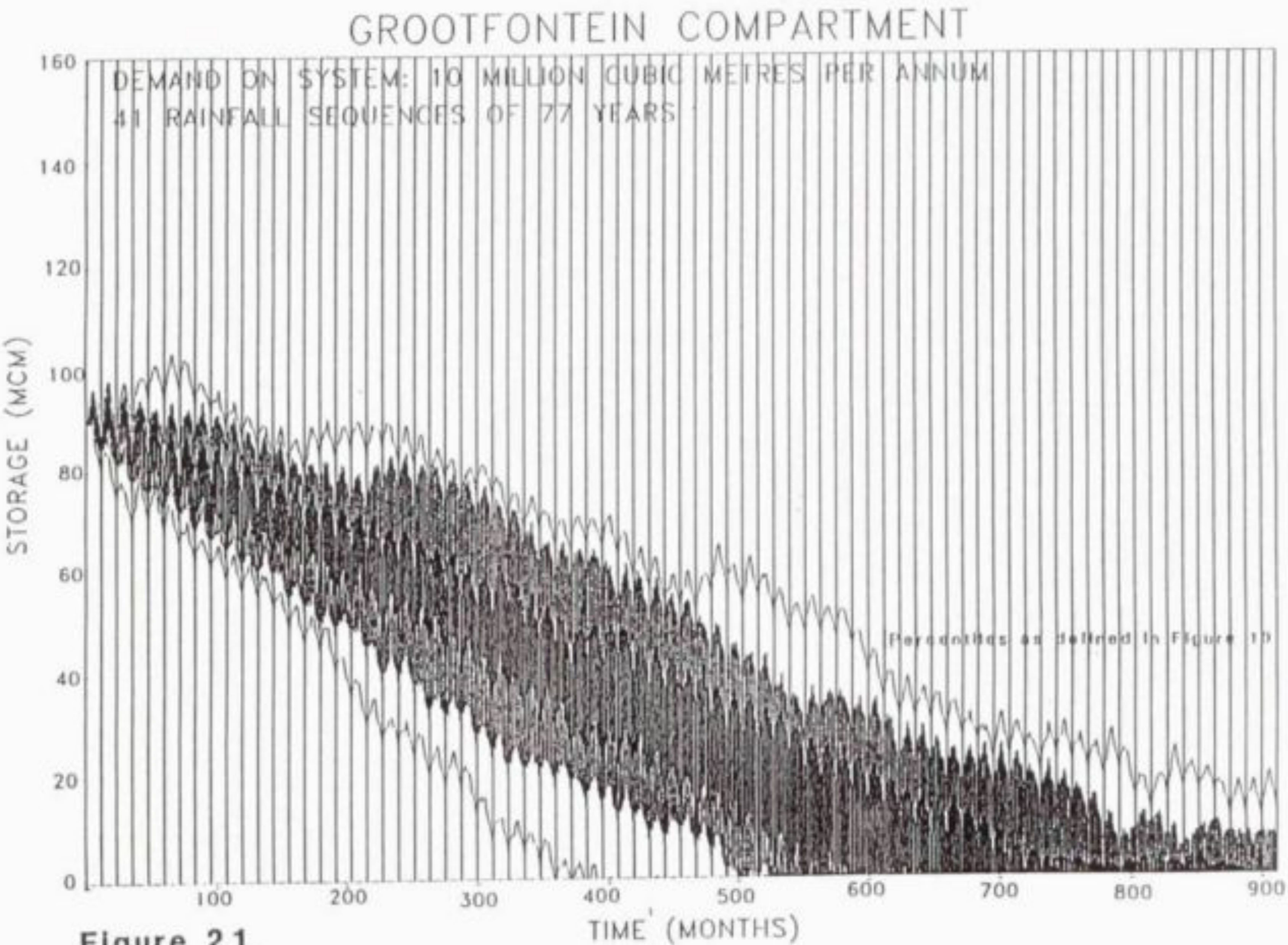

GROOTFONTEIN COMPARTMENT
DEMAND ON SYSTEM: 10 MILLION CUBIC METRES PER ANNUM
41 RAINFALL SEQUENCES OF 77 YEARS
Percentiles as defined in Figure 19
STORAGE (MCM)
160
140
120
100
80
60
40
20
0
100
200
300
400
500
600
700
800
900
TIME (MONTHS)

Figure 21.

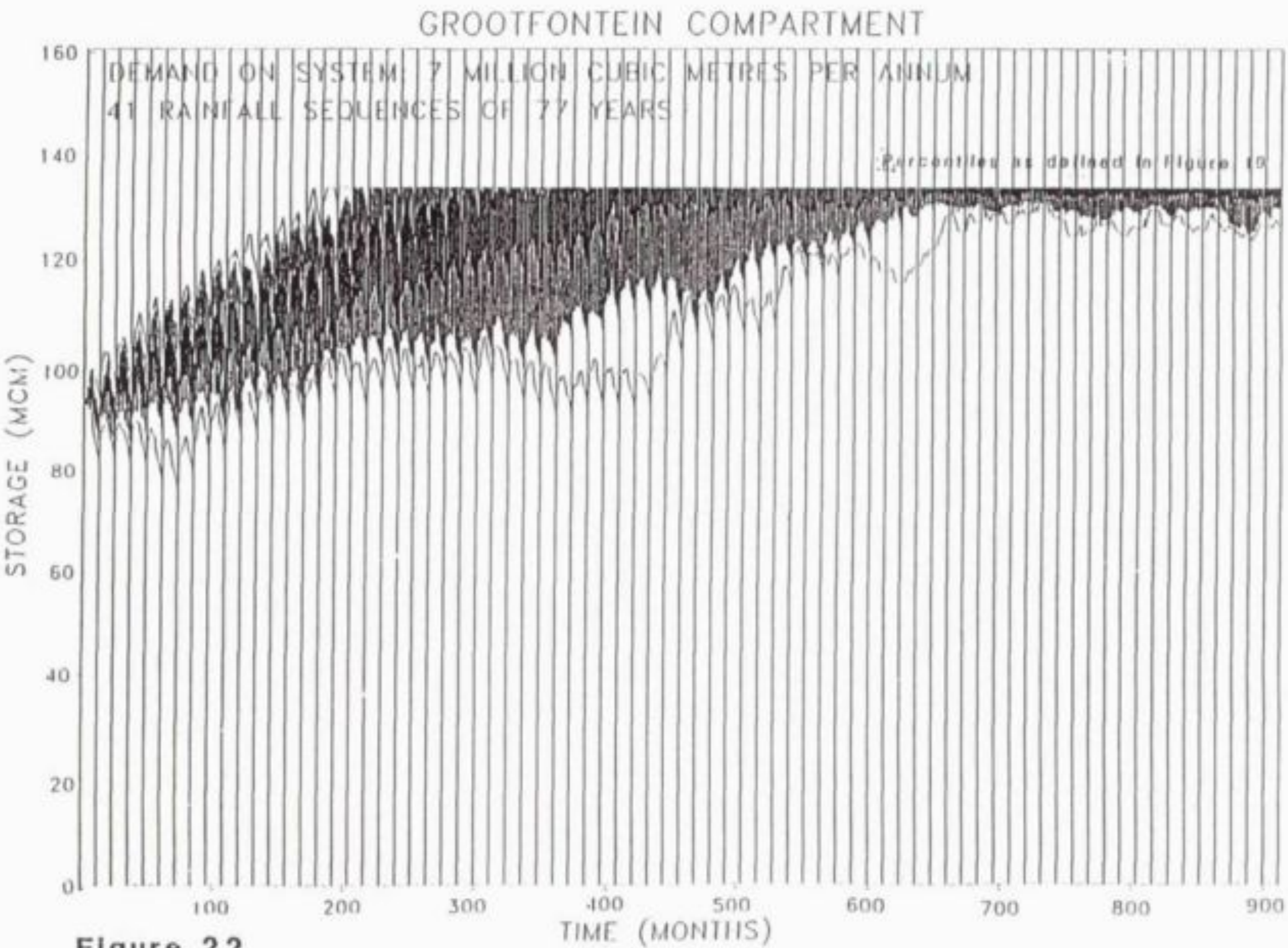

GROOTFONTEIN COMPARTMENT
DEMAND ON SYSTEM: 7 MILLION CUBIC METRES PER ANNUM
41 RAINFALL SEQUENCES OF 77 YEARS
Percentiles as defined in figure 19
STORAGE (MCM)
0
20
40
60
80
100
120
140
160
TIME (MONTHS)
100
200
300
400
500
600
700
800
900

Figure 22

GROOTFONTEIN COMPARTMENT-CALIBRATED T-VALUES

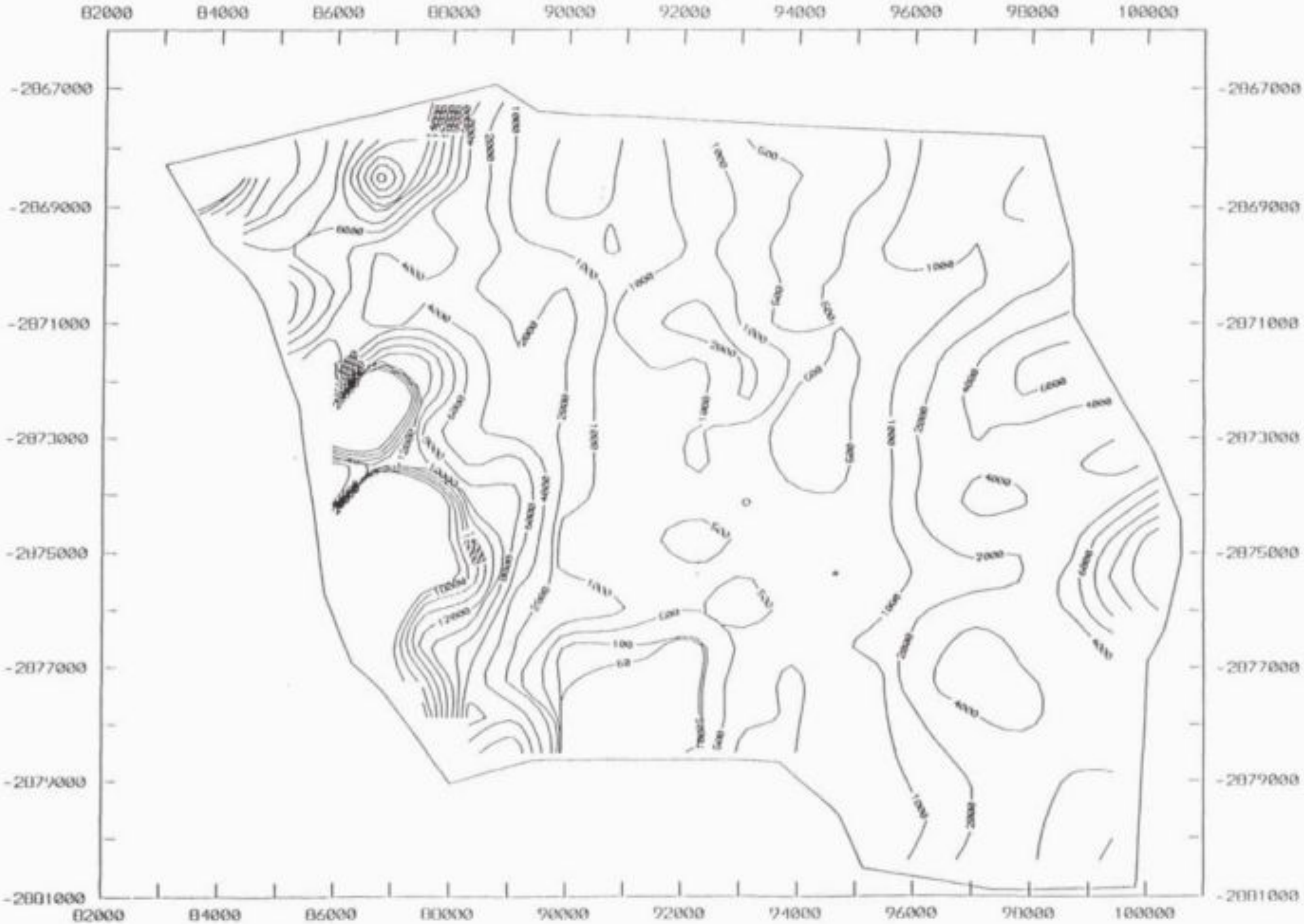


Figure 24

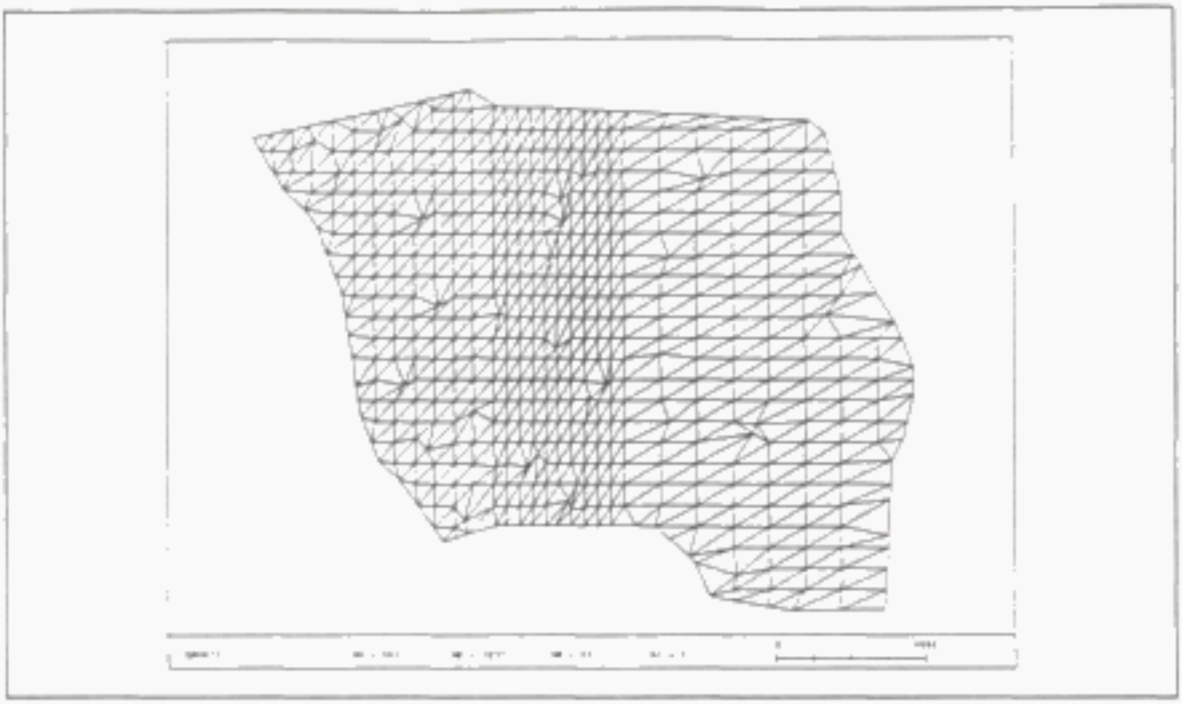


Figure 23 Grootfontein Compartment Finite Element Network

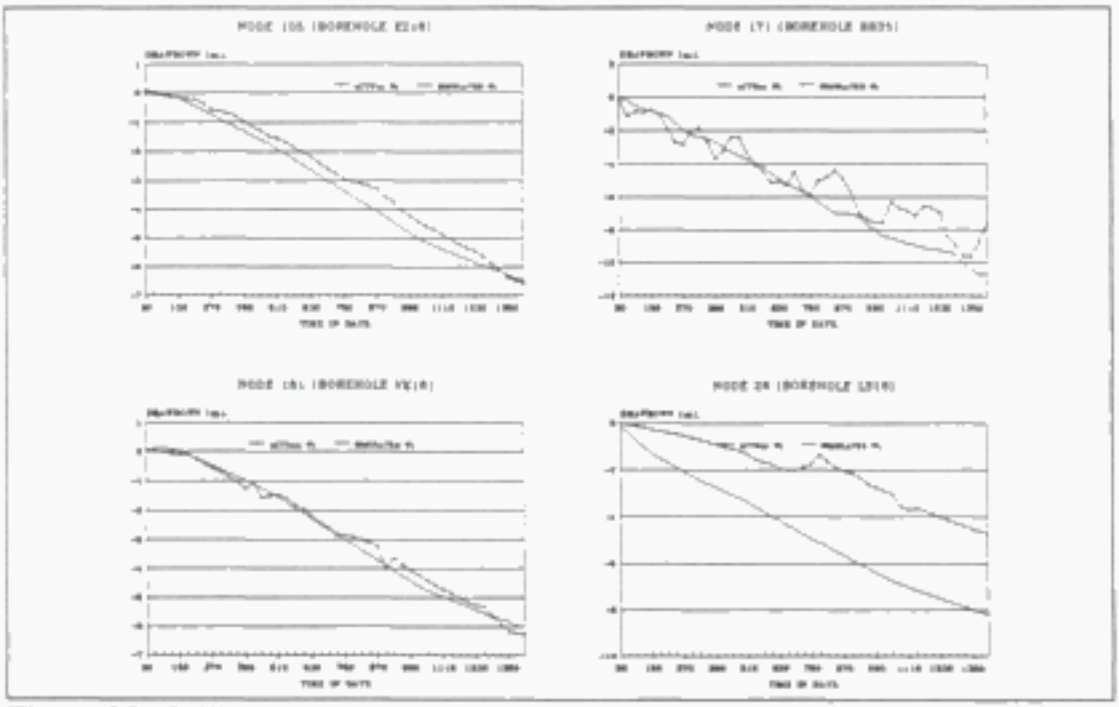


Figure 25 Calibration Results

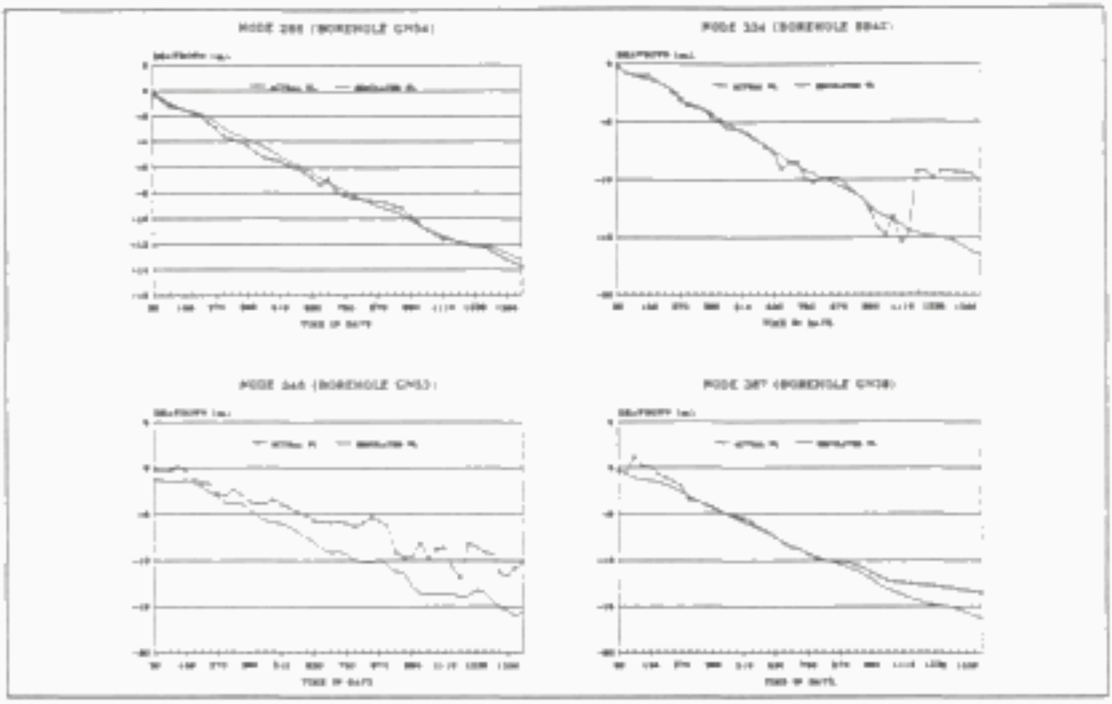


Figure 25 Calibration Results

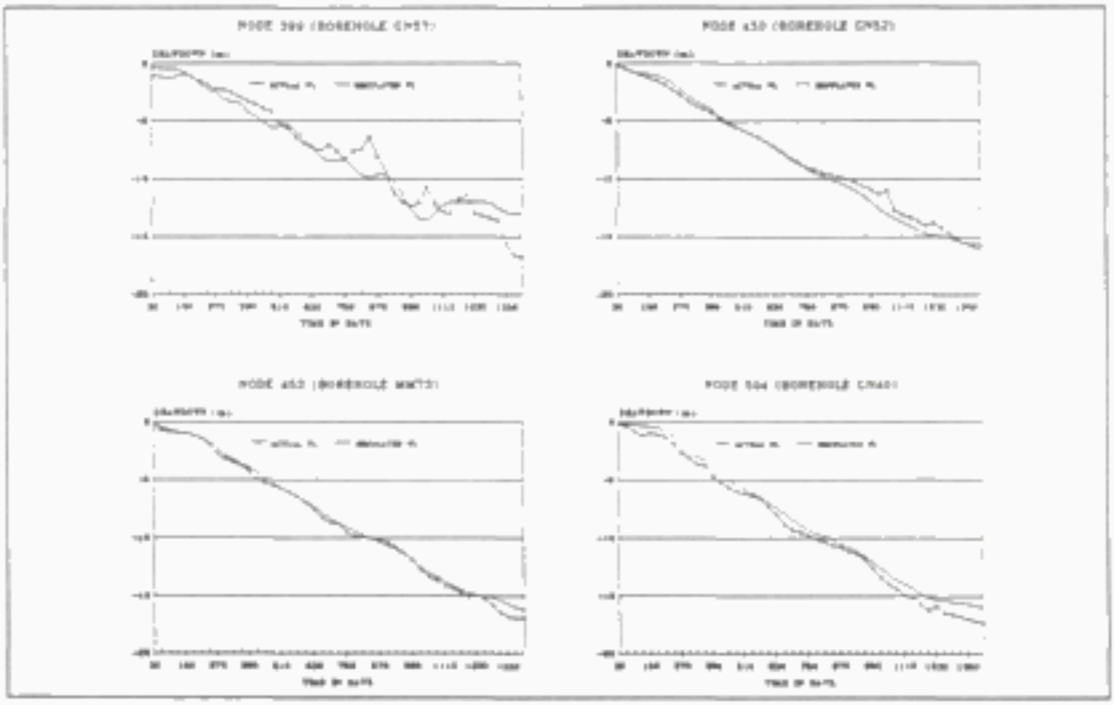


Figure 25 Calibration Results

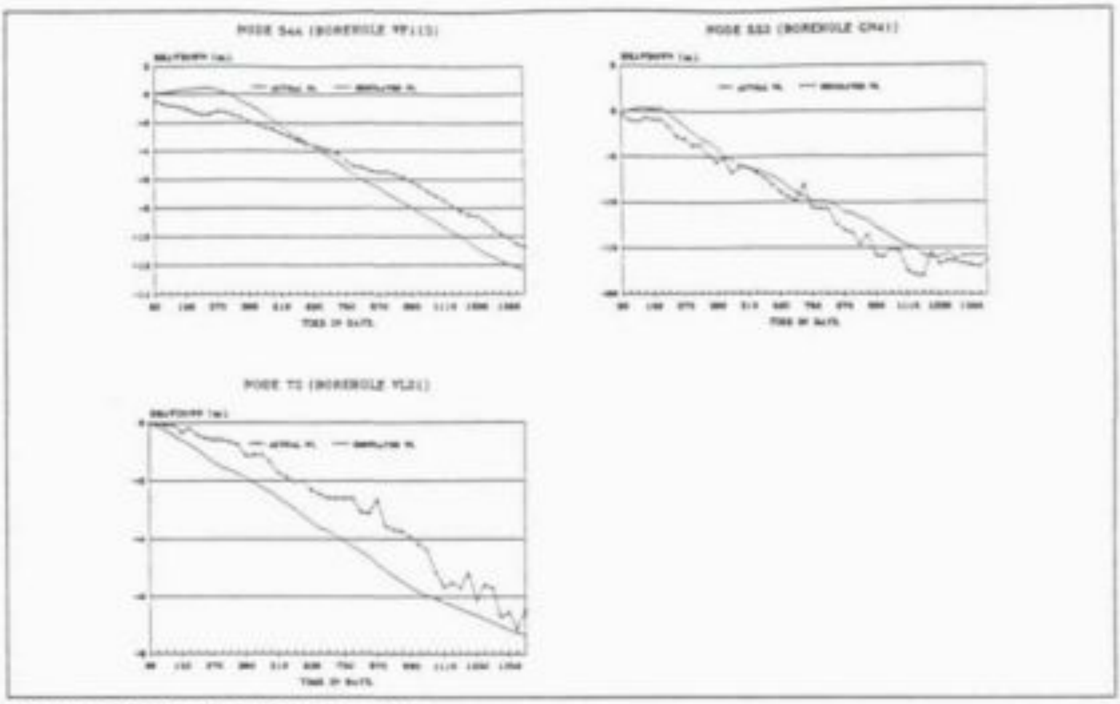


Figure 25 Calibration Results

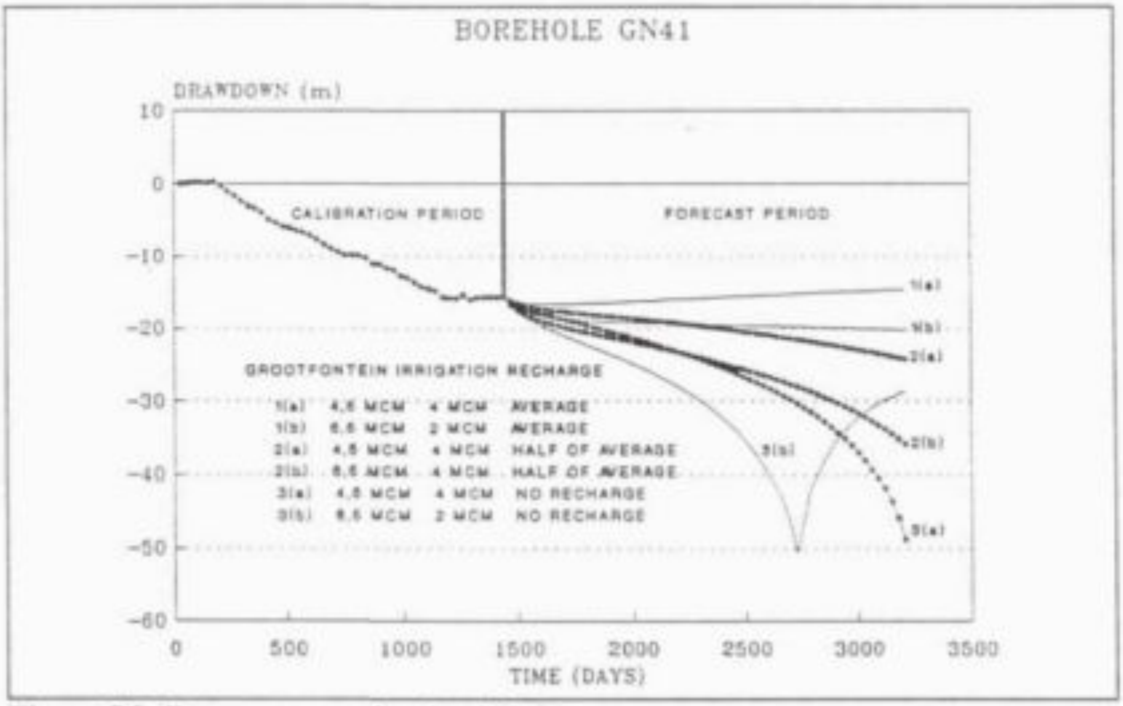


Figure 26 Forecast water level - GN41

11. RESULTS OF THE RISK ANALYSIS TECHNIQUES APPLIED TO THE GROOTFONTEIN AQUIFER CASE STUDY

In order to test the methodology proposed and developed in Section 2 of this report a detailed case study was carried out. The Grootfontein Aquifer was chosen for the case study as it was an aquifer which had been intensively studied in the past and has also been the subject for testing the management procedures developed by the IGS.

To aid the reader the following terminology introduced in paragraph 7 of Section 2 will be briefly restated.

One realization of the risk evaluation system requires

i) a management model, involving the number of discharge boreholes and their pump out rates,

ii) a zonal or geostatistical transmissivity simulation,

iii) a zonal or geostatistical storativity simulation,

iv) a simulated rainfall sequence of a number of consecutive months (in our case study we used 1188 months representing 99 years, each year running from July to June),

which serve as input into the aquifer simulation model (AQUAMOD). This geohydrological simulation model is used to generate the output required to calculate the statistics.

12. CASE STUDY CALIBRATION AND VALIDATION

In this section the calibration and validation of the models will be discussed followed by a presentation of the results obtained for a full simulation study.

12.1 CALIBRATION

The methods chosen to calibrate a model will depend on the data available and could vary from study to study. In the absence of actual observations, soft data (i.e. obtained by expert opinion) could be used. It must be emphasized that the usefulness of the results depend on the accuracy of the calibration and therefore, wherever possible, observed data should be used for calibrating models.

In order to follow the geostatistical approach, actual data must be available to fit variograms. The variogram is a function that models the data in a geostatistical analysis in order to simulate the real situation as close as possible. Storativity data was obtained from Janse van Rensburg et al., (1987). This data were sparse and no variogram could be fitted. In the absence of a variogram usual practice is to simulate a nugget effect equal to the variance of the real data. The data used for transmissivity was the same as that used to calibrate AQUAMOD for the Grootfontein aquifer and from this information a variogram was estimated. From references listed in Appendix A of Section 2, storativity and transmissivity almost always have a lognormal distribution. Therefore the original transmissivity values were transformed to normal data to be able to fit a variogram, normal values were simulated and the simulated data were transformed back to lognormal data. Conditional simulations were applied to the transmissivity data, these simulations are simulations that are conditioned on the values of measured input parameters, and on the specific locations of these measurements. Because of the problems with storativity data, these simulations were not conditional, but when transforming back to lognormal data, the mean and variance of the original data were used, so as to get a range between the smallest and largest values that satisfy the data that were available.

When following the inversion approach, convergence problems with the program developed at IGS were experienced and it was therefore necessary to generate covariance matrices as input to the zonal simulator. The following procedure was developed and applied to generate a covariance matrix for transmissivity. Distances within and between zones were averaged, and then used as the distance parameter in the experimental variogram to generate covariances between and within zones. No experimental variogram could be fitted for the storativity values and thus it was assumed that no covariances existed between zones. The covariance between zones was set to zero and variance within the zones was set to the overall variance.

As discussed in paragraph 7.1.3 of Section 2, the rainfall simulator (corrected for average mean) was used to generate the 1200 month rainfall sequences.

The recharge rate is a function of the physical property of the aquifer and remains constant during all the simulations. The determination of the recharge rate is dealt with in more detail in section 13.

It needs to be emphasised that the final results of this project will be dependent on the calibration of the model components. This calibration was not the focus of the project and only available information was used. It must be emphasized that it was not the objective of this project to provide a well calibrated simulation model, but to test the methodologies and techniques developed given a calibrated model.

12.2 VALIDATION

It was important to test that the simulation components in the model were producing simulations which would provide a realistic reflection of the variation inherent in the natural system being modelled. To do this the model components were calibrated for the Grootfontein Aquifer (see section on calibration). Having done this, it was necessary to check that the model components were operating as expected and validation tests were carried out to confirm that the simulations for transmissivity, storativity and rainfall were realistic.

Rainfall

The rainfall simulations were validated against the rainfall records obtained for station Slurry. Statistics, for example mean, median, variance, percentiles of simulated against actual records were compared. For the original data there were 4 years out of a total of 75 years with total rainfall less than 350 millimetres. The simulations compared well with this statistic. The number of years with total rainfall less than 350 millimetres varies from 0 years to 6 years out of a total of 99 sequences. There were also a few simulations with rainfall less than 250mm which would represent a severe drought.

Zonal simulation

The covariance matrix that was given as input to the zonal simulations was tested in the following manner. Ninety nine simulations were generated and the covariance between these simulations was computed and compared to the input covariance matrix. The comparison was found to be satisfactory.

Geostatistical simulation

The geostatistical simulations were tested by computing the experimental variogram for a selection of the individual simulations. These variograms were then compared against the original variograms fitted to real data. The comparison was satisfactory.

13. EVALUATION OF RECHARGE RATE

At the outset it was considered necessary to investigate the sensitivity of the aquifer simulation to different recharge values. Ignoring the possibility of overflows, the total available recharge was absorbed by the aquifer, while the total extraction was 8 492 499 cubic metres per year. A simple balance analysis shows that an 8.8% recharge to the aquifer will lead to global equilibrium.

In order to accommodate an overflow situation, AQUAMOD was adapted to set nodal water levels to a defined maximum whenever they increase above this maximum as a result of insufficient discharge.

To gain more insight five realizations of the risk evaluation system were investigated. Each realization used the same management model, storativity and transmissivity simulation and rainfall sequence but 5 recharge values of 8.8%, 9%, 10%, 10.5% and 11% were used. Each realization commenced with nodes set to maximum water levels. The water level was monitored at the node with the highest pump extraction rate (node 552). The water levels at 30 day intervals for a period of 110 years for each of the realizations are shown in Figure 27. It is clear that for recharge values of 8.8% and 9% recharge, the aquifer level will slowly drop. For a recharge between 10% and 10.5% it appears to stabilize after some time and for 11% the recharge is consistently greater than discharge. It is important to note that the recharge is a physical property but since it is unknown will sometimes be set at different values to test system sensitivity. From the figure it is clear that the process is highly dependent on the (unknown) recharge.

In Figure 28 the yearly maximum, minimum and average water levels over all nodes is shown for one realization using 10% recharge. It can be seen that the maximum level of at least one node rises to its overflow value and remains there even although the nodal minimum and average levels reflects a strong cyclical effect. This can be explained by the fact that at least one node was situated outside the influence sphere of the abstraction points thus could reach and remain at its maximum value. From this is may be inferred that the maximum statistic will have limited value in practice. The average and minimum statistics are in this case very closely correlated. There is no reason to believe that this will always be the case.

Based on these results and after discussion with IGS it was decided to use a recharge value of 10% of mean annual rainfall in all further analyses. While the study above was important to obtain a realistic value for this investigation it needs

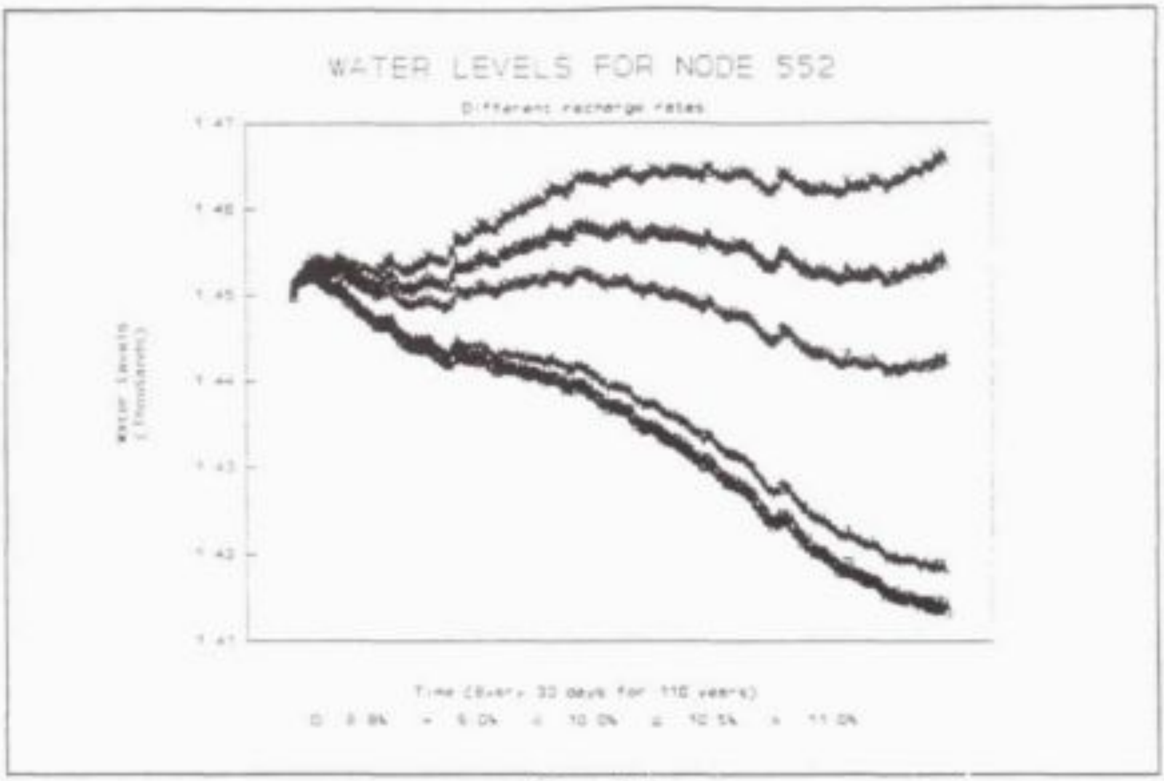


Figure 27

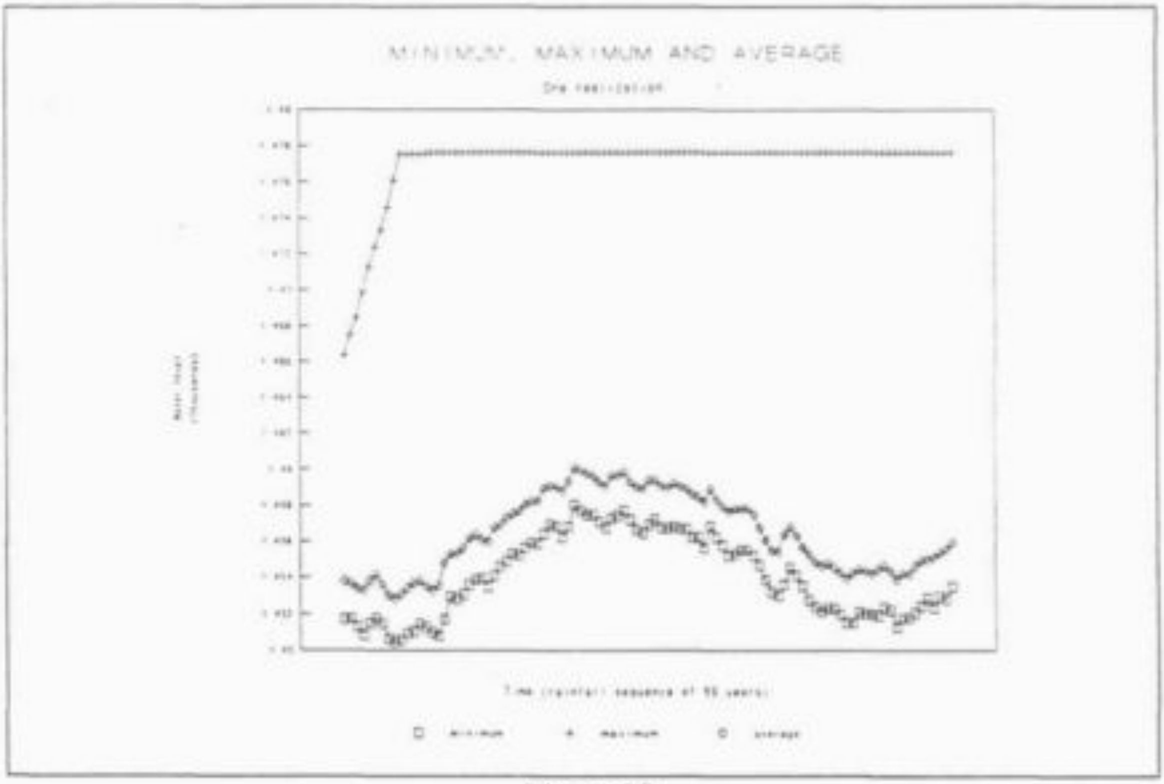


Figure 28

be emphasized that in practice recharge involves geological factors and needs to be determined geohydrologically.

14. COMBINATION OF RISK EVENTS

For the purposes of the case study a risk event will be defined in the terms in which a manager might specify the requirements of a pumping policy. For example, 'it is unacceptable if x% or more boreholes fail a specified tolerance test during a particular time period'. Thus the five events used to illustrate the methods will be 8% or more, 16% or more, 25% or more, 33% or more and 42% or more boreholes failing the tolerance test. Note that a failure defined as 8% or more boreholes below the tolerance will occur more frequently than one requiring 42% or more below. In other words the probability to obtain failures in 8% or more is greater than the probability of 42% or more boreholes failing.

Four tolerance levels were considered for each of the five risk events.

- Tolerance 1 = 1/6(maximum water level - base water level)
- Tolerance 2 = 1/3(maximum water level - base water level)
- Tolerance 3 = 1/2(maximum water level - base water level)
- Tolerance 4 = 2/3(maximum water level - base water level)

15. ZONAL SIMULATIONS

As described in paragraph 6.1 of Section 2 the aquifer is divided into zones. The number of simulated values will depend on the number of zones allocated to this region. Every node in a specific zone will be set to the value that was simulated for that zone. The assumption in the zonal approach is that the aquifer can be accurately represented by a few homogeneous regions. Later when considering the geostatistical simulations the variations on a finer grid are modelled.

15.1 SUMMARY STATISTICS FOR ZONAL SIMULATIONS

Next the risk evaluation system was used in a Monte Carlo study to demonstrate the generation of summary statistics (see paragraph 8.1 of Section 2) for zonal simulations. Ninety nine realizations were computed each using a different storativity and transmissivity simulation and a 99 year rainfall sequence. The statistics defined in paragraph 8.1 of Section 2 were then computed. In addition the study was repeated for a number of different pumping volumes chosen as a percentage of 8.5 million cubic metres per annum. A pumping rate of 8.5 million cubic metres has been determined in other studies as a sustainable yield of water which can be extracted. The statistics were computed twice, once using all nodes and once using extraction boreholes only. For discussion purposes the E[MinMin], E[MaxMax] and E[AvgAvg] are displayed graphically. The statistics related to all nodes and those relating to boreholes only are displayed in figures 29 and 30 respectively.

The results in the two figures are essentially self explanatory. The values remain fairly constant until a pumping rate of $1.1*8.5 = 9.35$ million cubic metres is reached, thereafter it declines steadily. Comparing the two figures the E[AvgAvg] remains essentially the same. As to be expected the E[MaxMax] for all nodes is slightly higher than for boreholes only reflecting the fact that some nodes are less affected by pumpout than those at borehole locations. On the other hand the E[MinMin] is slightly lower for all nodes than at boreholes only. The likely explanation is that nodes can exist which are affected by more than borehole.

From these observations it appears that the proposed summary statistics react as expected to different discharge values and are thus suitable as measures for evaluating a management plan.

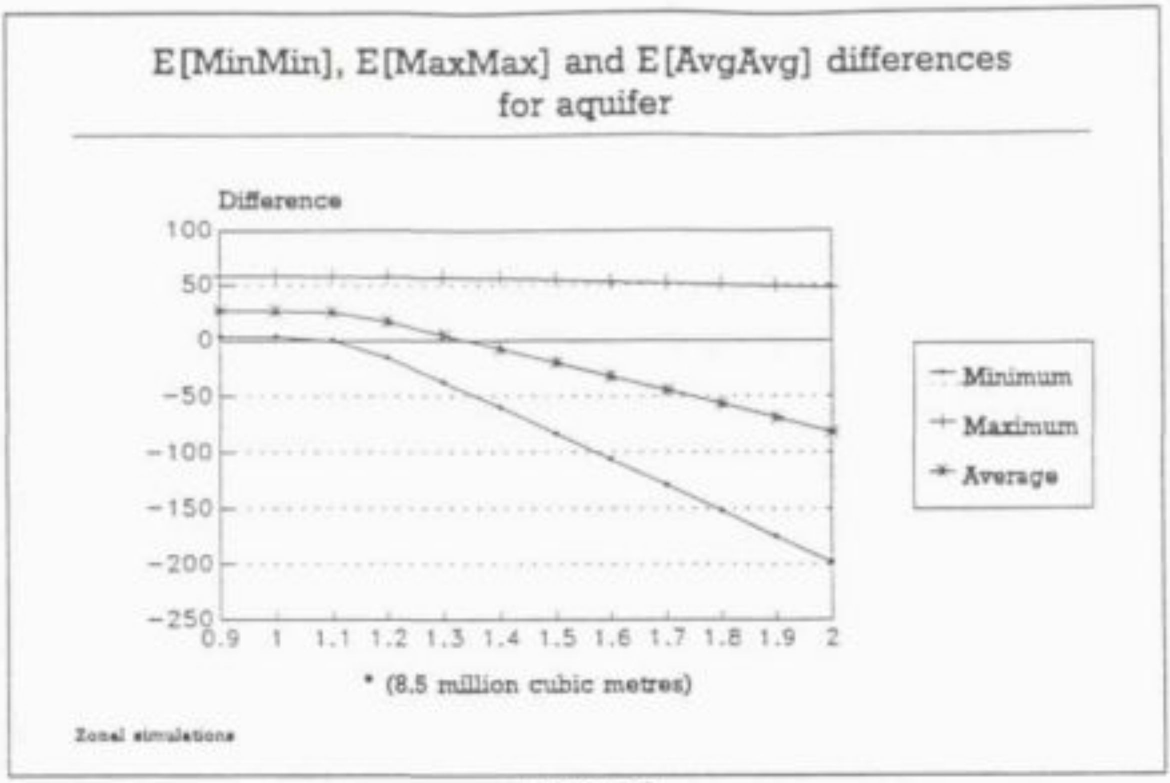


Figure 29

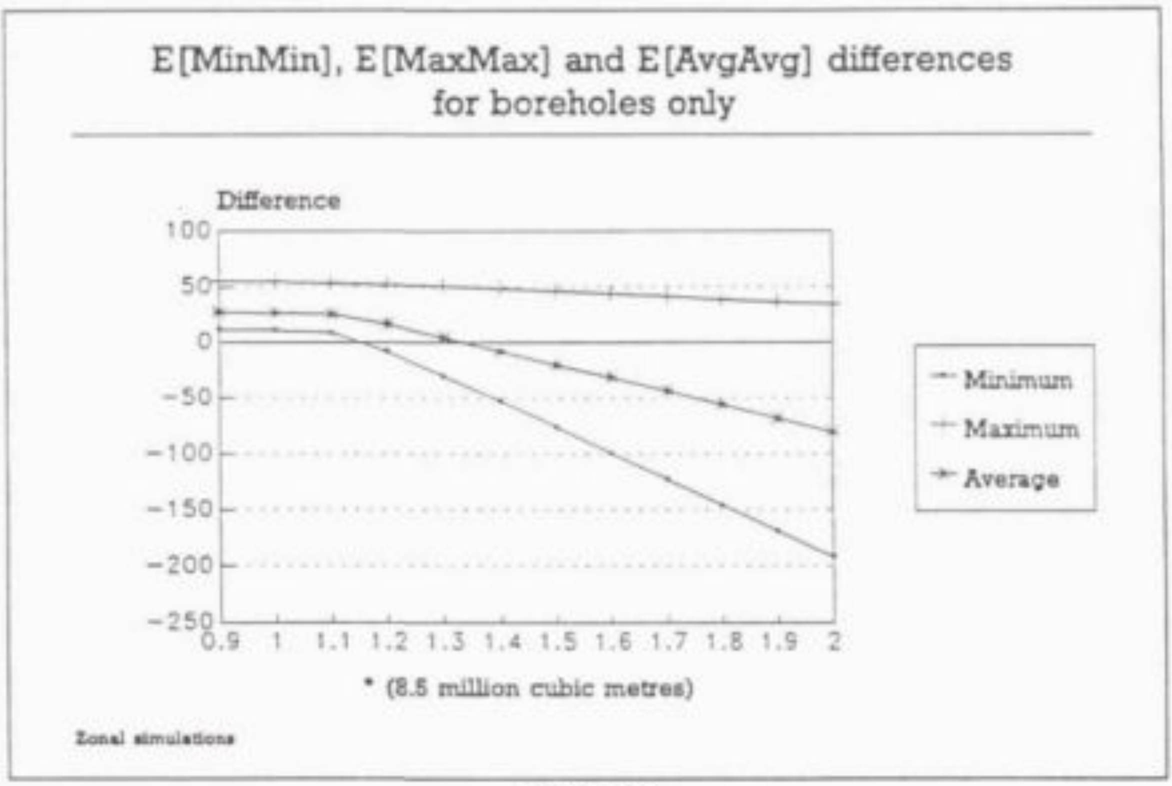


Figure 30

15.2 RISK STATISTICS FOR ZONAL SIMULATIONS

In this section a Monte Carlo study is reported which demonstrates the use of the risk statistics (see paragraph 8.3 of Section 2) to evaluate a particular management scheme using zonal simulations. Ninety nine realizations were computed each using a different storativity and transmissivity simulation and 99 year rainfall sequence. The study was repeated for a number of different pumping volumes chosen as a percentage of 8.5 million cubic metres per annum.

The results of the simulation are displayed graphically and discussed below. One figure is used for each risk event and includes results for all four tolerances.

EXPECTED PERIODS

The results for each of the five risk events are shown in figures 31 to 35.

The period is the average length of time a failure is expected to continue for consecutive time steps. It should be noted that a period cannot be longer than the total length of the experiment when pumping under severe conditions and therefore the period tends to the maximum although the correct period should be longer than the experimental time. This is seen in the figures as a convergence to the maximum period and is a function of the experimental setup and should be ignored. It is interesting to observe that for all five risk events the expected periods (measured in months) of failure remain insignificant until the discharge reaches $8.5*1.1 = 9.35$ million cubic metres. Thereafter the expected periods increase linearly, ignoring the levelling effect referred to earlier. As expected for the more severe pumping rates the expected periods are ranked in the order of their corresponding tolerance stringency. The difference between the tolerances become more marked the more severe the pumping rates. These figures can be used to show that for a pump rate of 8.5 million cubic metres no significant periods of failure are expected while for $1.1*8.5 = 9.35$ million cubic metres only short periods of failure are expected. The aquifer is unable to sustain higher pump rates for significant periods.

When comparing the results for different risk events a slight drop is seen in the expected period for the less frequent events (for example the periods for 42% or more, shown in figure 33, are slightly less than for 8% or more shown in figure 31.

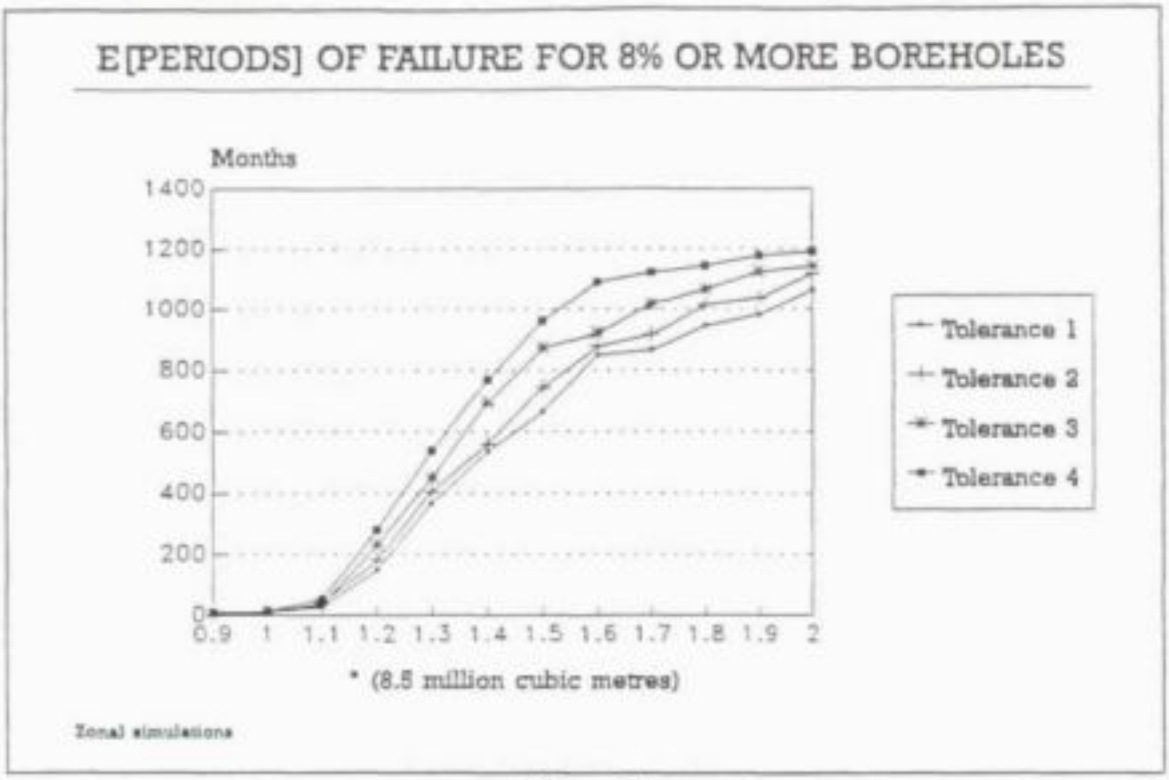


Figure 31

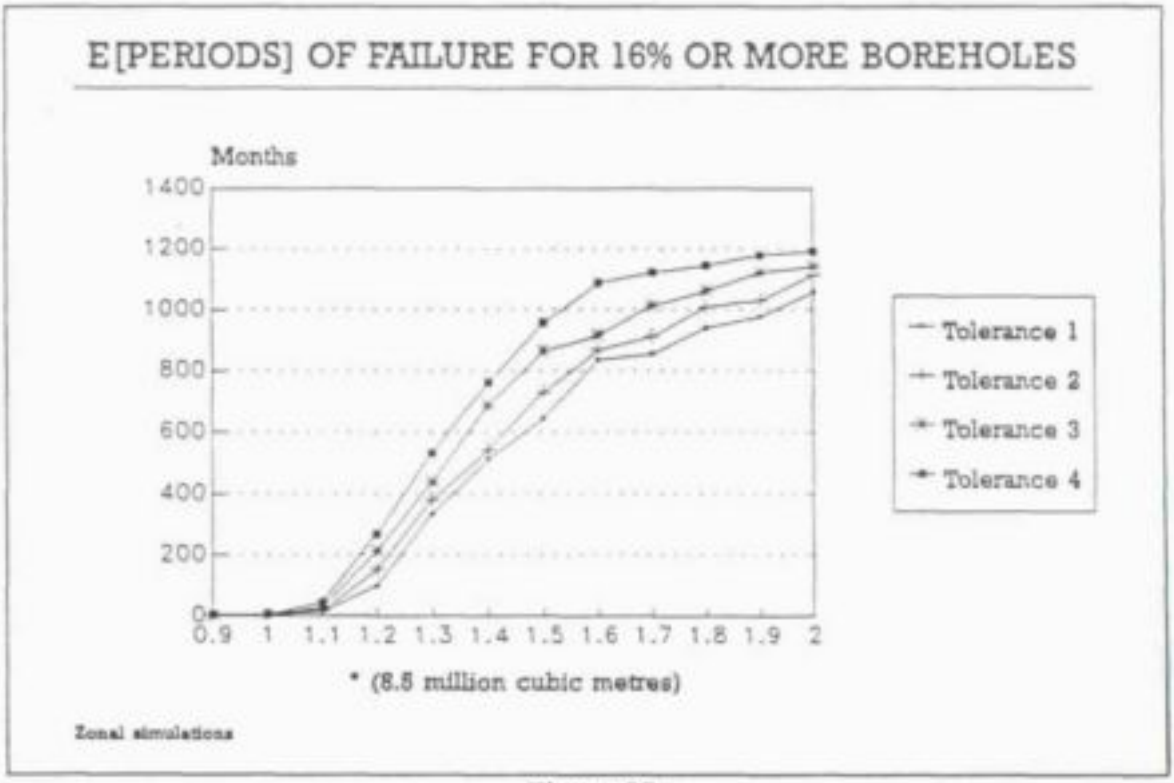


Figure 32

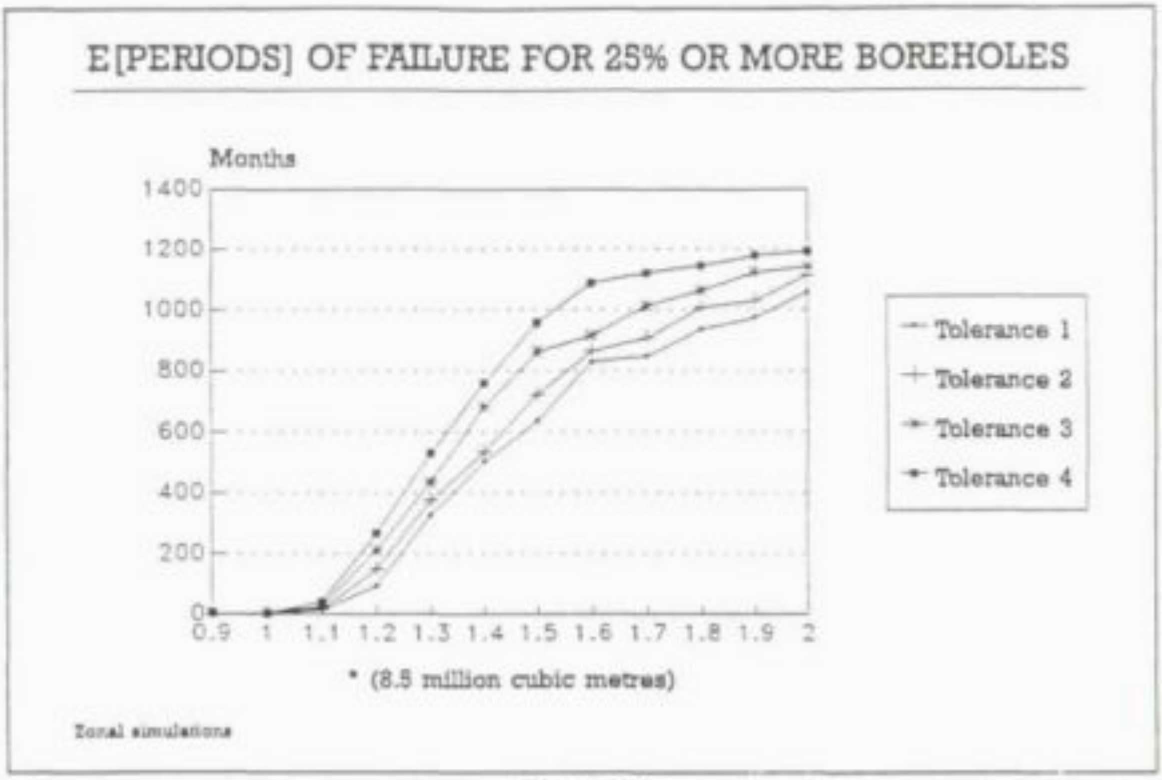


Figure 33

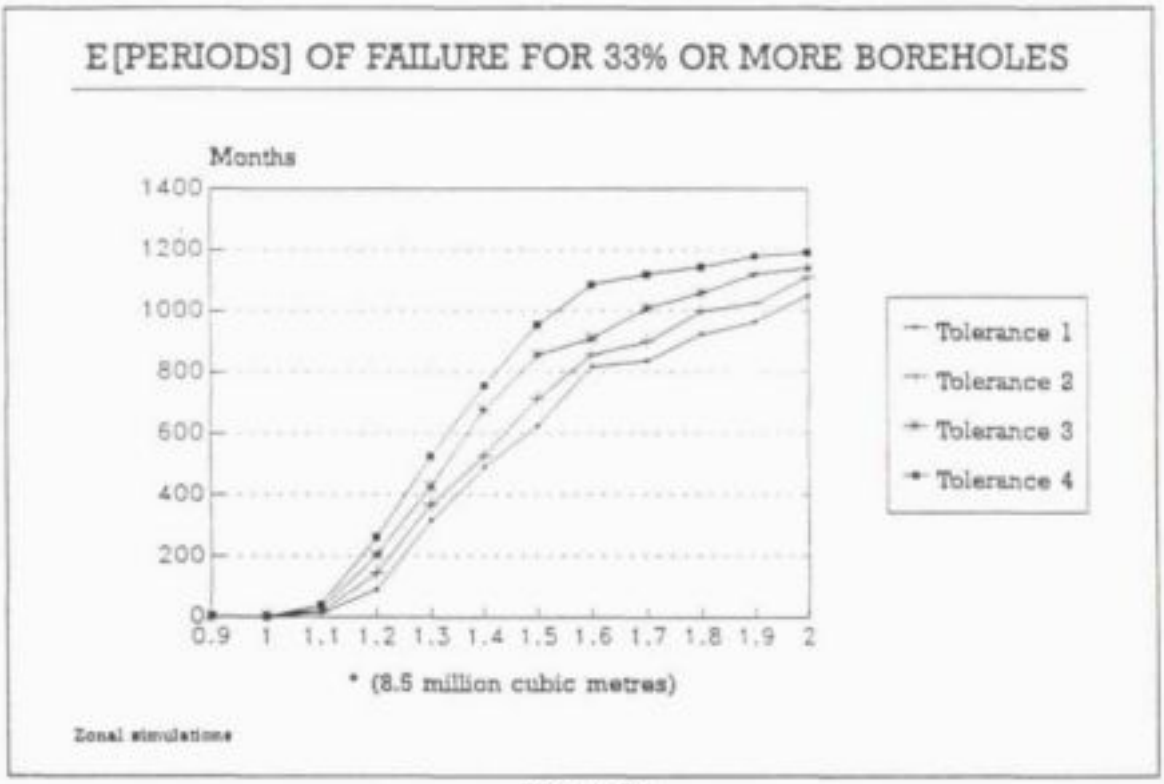


Figure 34

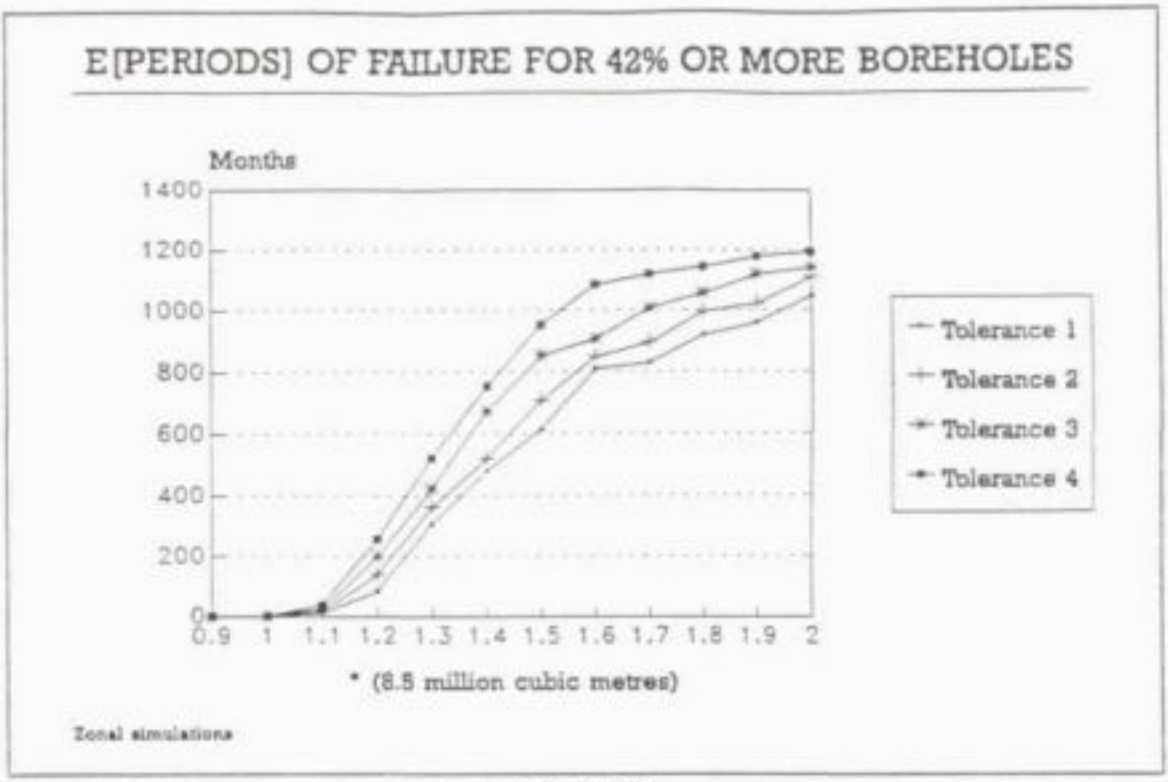


Figure 35

EXPECTED PROBABILITIES

The results for each of the five different risk events are shown in figures 36 to 40.

Note that a probability value close to one would indicate that the aquifer is being over exploited while a value close to zero will indicate that it is operating well within the specified conditions.

In all the figures it is observed that for a pump rate of 8.5 million cubic metres no probabilities significantly greater than zero are recorded while for 1.1*8.5 million cubic metres only small probabilities of failure are found. Thereafter the expected probabilities increase sharply thus indicating the extent to which the aquifer is unable to sustain these high pump rates.

It is seen that the probabilities in figure 36 are generally higher than in figure 40. This reflects the fact that 8% or more boreholes includes the event 42% or more boreholes.

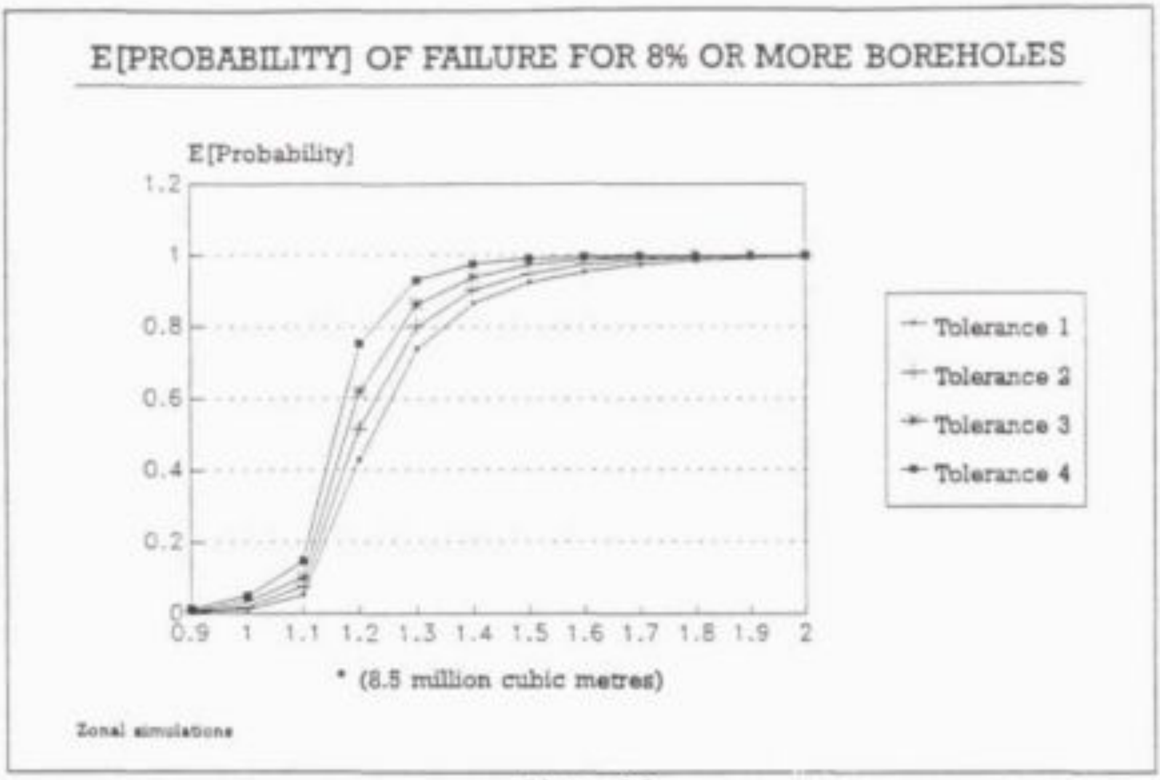


Figure 36

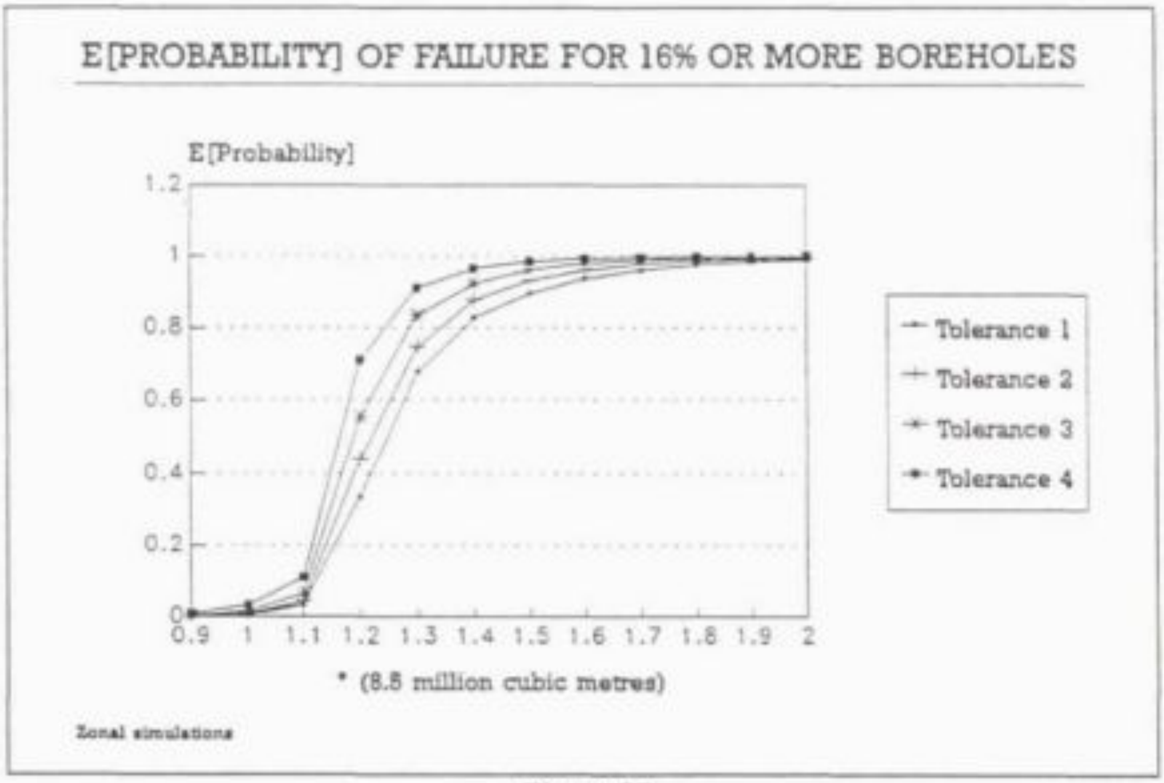
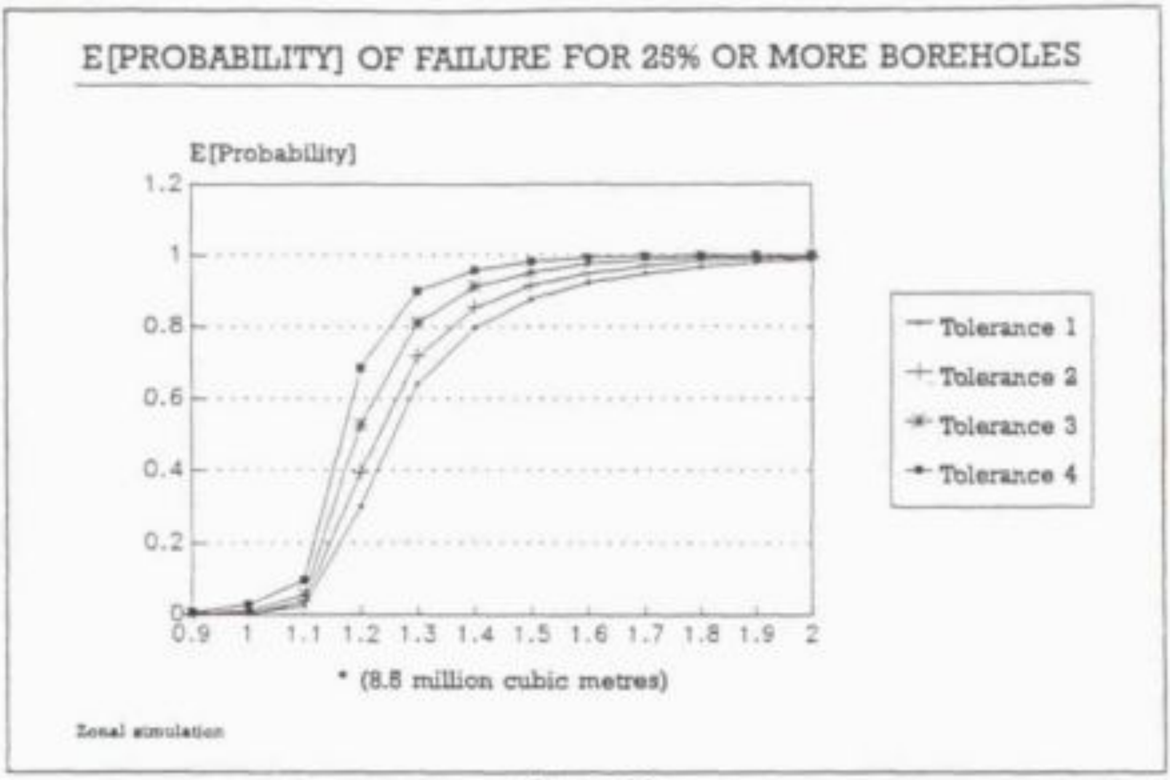


Figure 37



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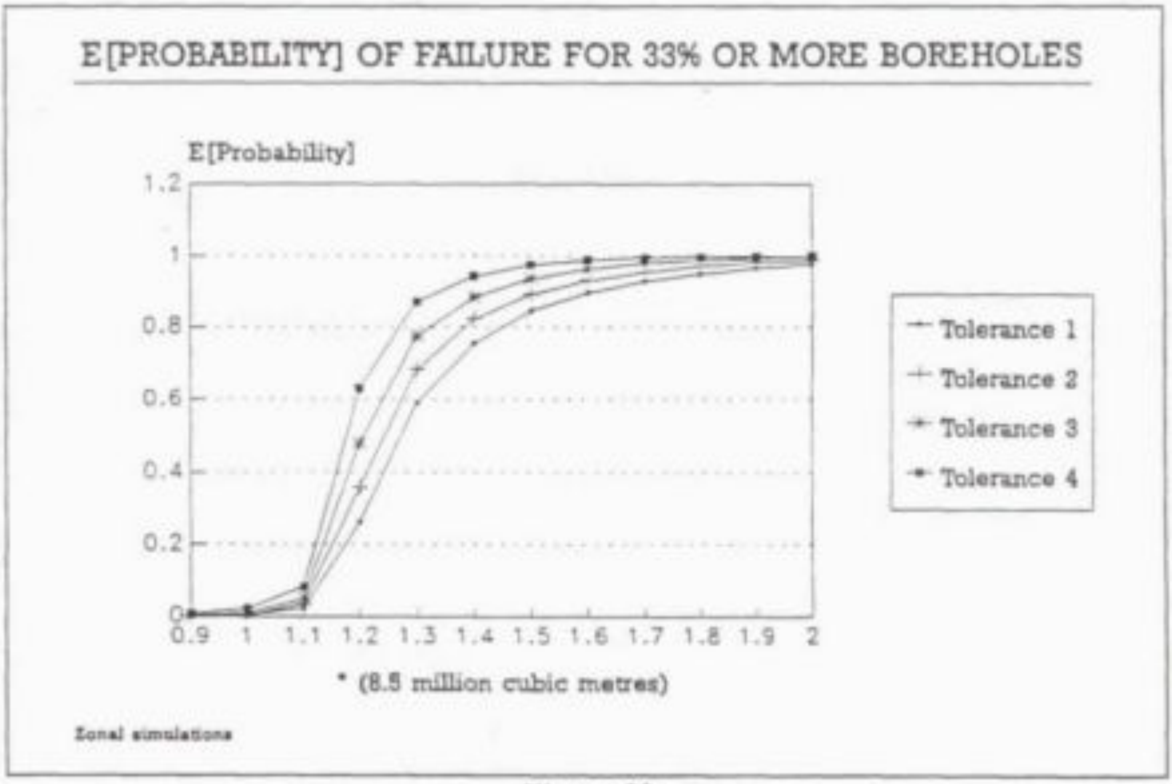


Figure 39

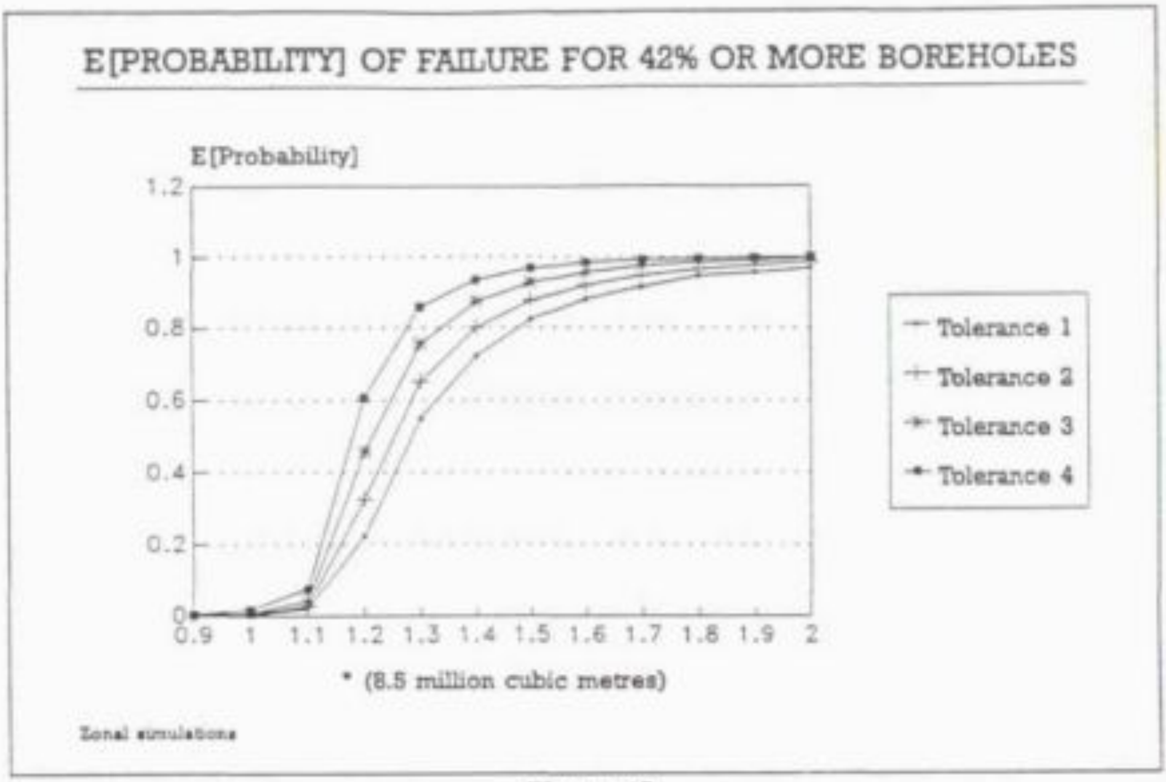


Figure 40

16. GEOSTATISTICAL SIMULATIONS

The geostatistical approach as described in paragraph 6.2 of Section 2 simulates transmissivity and storativity values on regular, rectangular grids. Each node will be allocated the value of the simulated grid node closest to it. As mentioned when considering the zonal simulations, the geostatistical method models local variation and thus could be a closer representation to reality than the zonal case unless the aquifer is composed of a few very homogeneous regions.

16.1 SUMMARY STATISTICS FOR GEOSTATISTICAL SIMULATIONS

The risk evaluation system was used in a Monte Carlo study to demonstrate the generation of summary statistics (see paragraph 8.1 of Section 2) for geostatistical simulations. As in the case of the zonal simulations ninety nine realizations were computed each using a different storativity and transmissivity simulation and 99 year rainfall sequence. The study was repeated for a number of different pumping rates chosen as a percentage of 8.5 million cubic metres per annum. Because of higher computing cost fewer pumping rates where used than in the case of the zonal simulations. The statistics described in section 8.1 of Volume 2 were generated. As in the zonal simulations the statistics were computed twice, once using all nodes and once using extraction boreholes only. For discussion purposes the E[MinMin], E[MaxMax] and E[AvgAvg] are displayed graphically. The statistics related to all nodes and those relating to boreholes only, are displayed in figures 41 and 42 respectively.

These results, as in the zonal simulations, are essentially self explanatory. The same conclusion can be made that the proposed summary statistics react as expected to different discharge values and are thus suitable as measures for measuring a management plan. It is noteworthy that trends are the same as for zonal simulations but less severe. This tendency will be seen for the risk results as well.

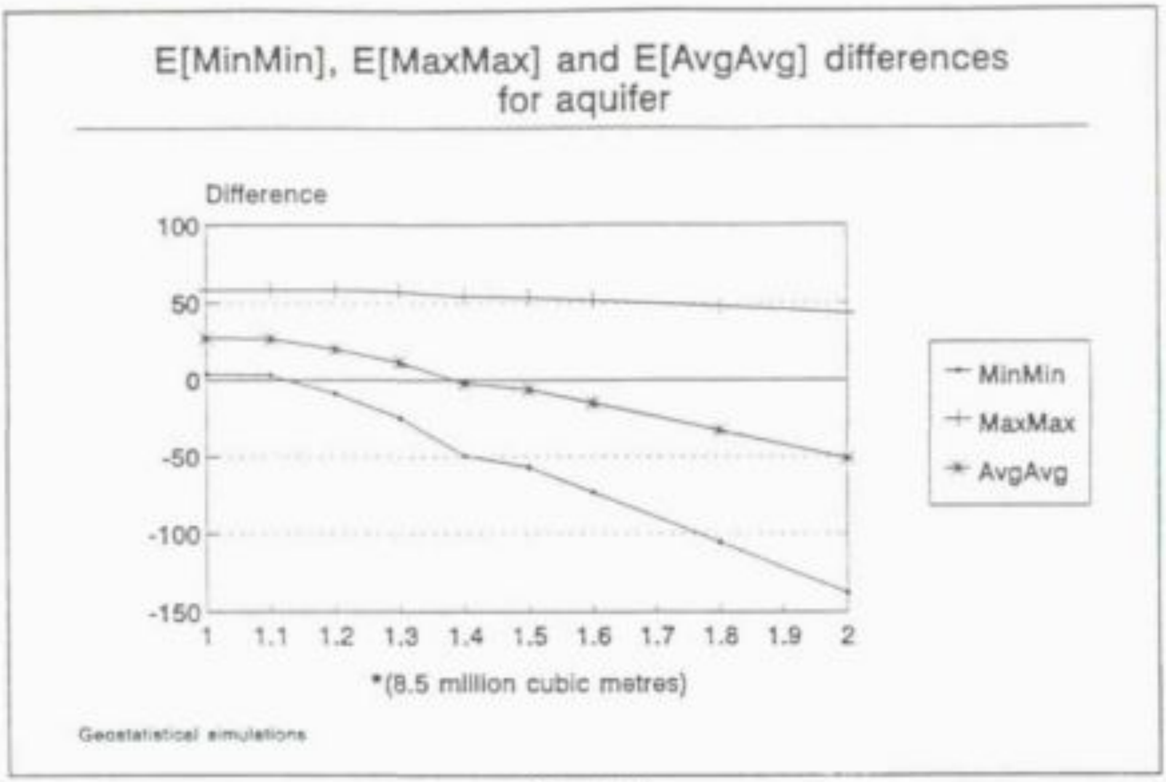


Figure 41

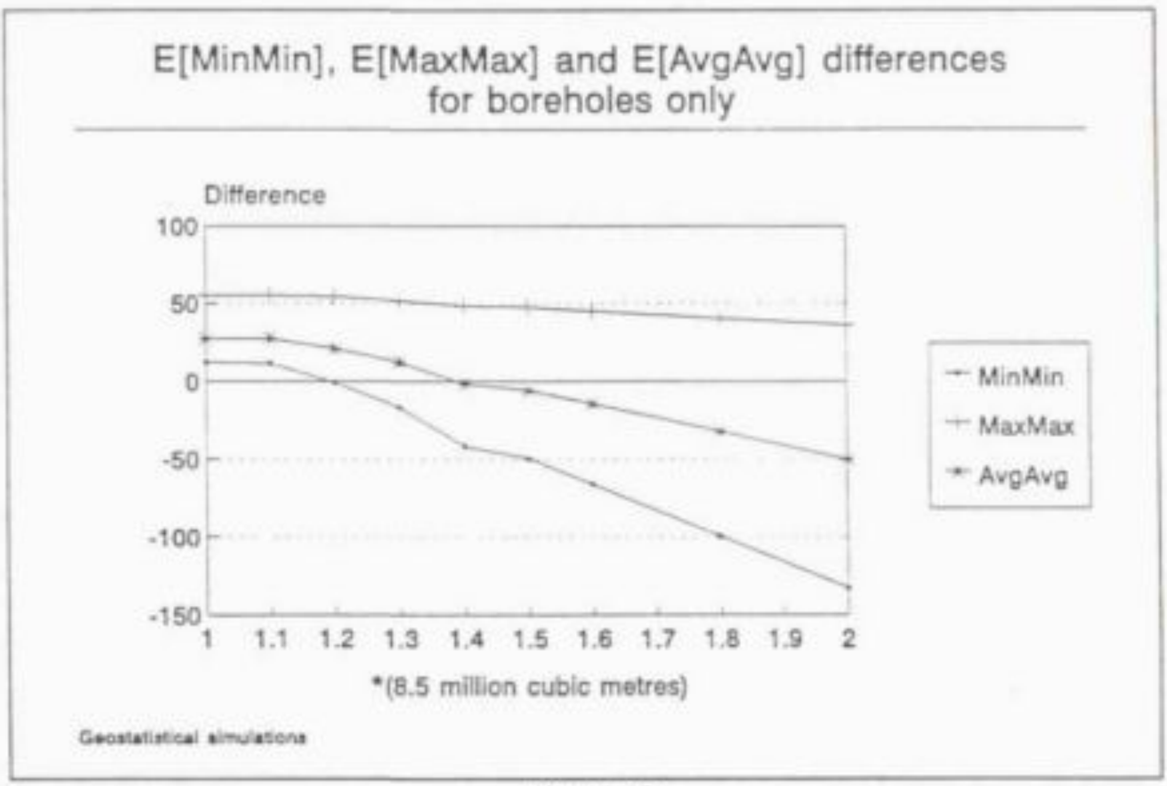


Figure 42

16.2 RISK STATISTICS FOR GEOSTATISTICAL SIMULATIONS

In this section a Monte Carlo study is reported which demonstrates the use of the risk statistics (see paragraph 8.3 of Section 2) to evaluate a particular management scheme using geostatistical simulations. As in the case of the zonal simulations ninety nine realizations were computed each using a different storativity and transmissivity simulation and 99 year rainfall sequence. The study was repeated for a number of different pumping volumes chosen as a percentage of 8.5 million cubic metres per annum. Because of computing cost considerations, fewer pumping rates where used than for the zonal.

The results of the simulation are displayed graphically and discussed below. One figure is used for each of the five risk events and includes results for all four tolerances.

EXPECTED PERIODS

The results are shown in figures 43 to 47. As in the zonal simulation case the converging of the curves for high periods should be ignored.

It can be noted that the general form of the results are similar to those obtain for the zonal case. However the geostatistical results are markedly more tolerant of the various pumping schemes. This can be seen by comparing the periods for various pumping volumes and noting that the geostatistical value is generally lower than the value obtained for the zonal simulation.

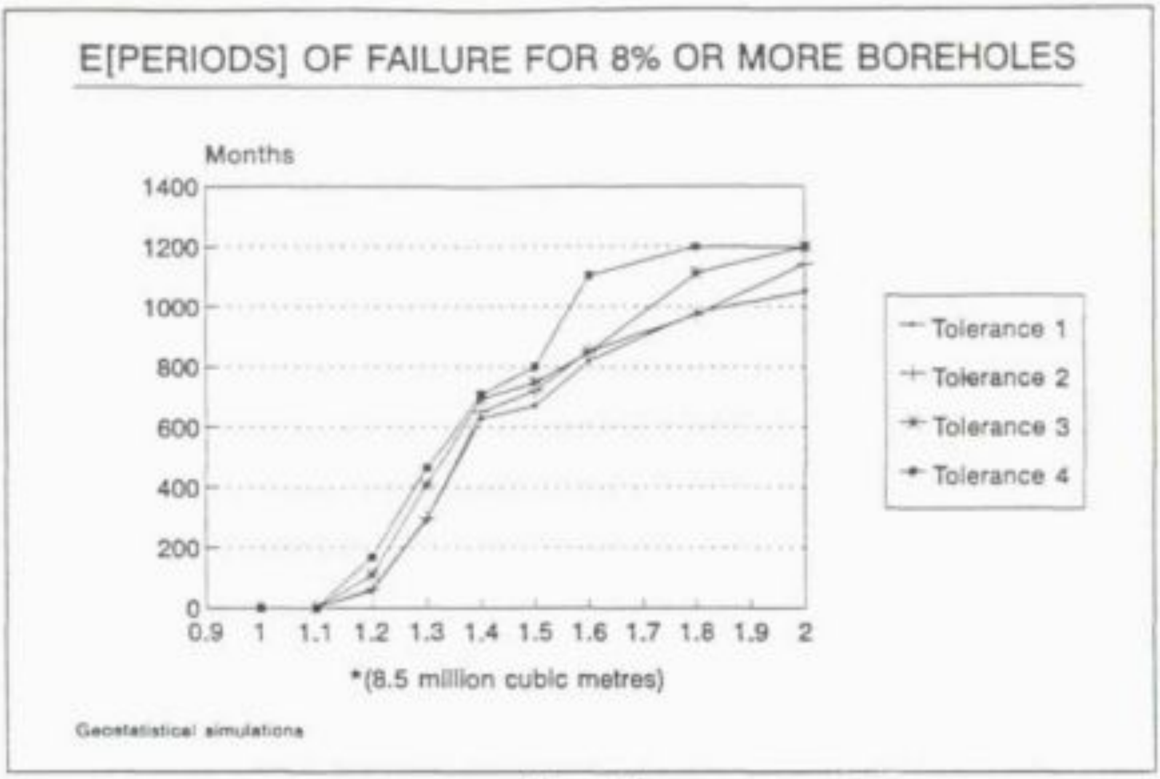


Figure 43

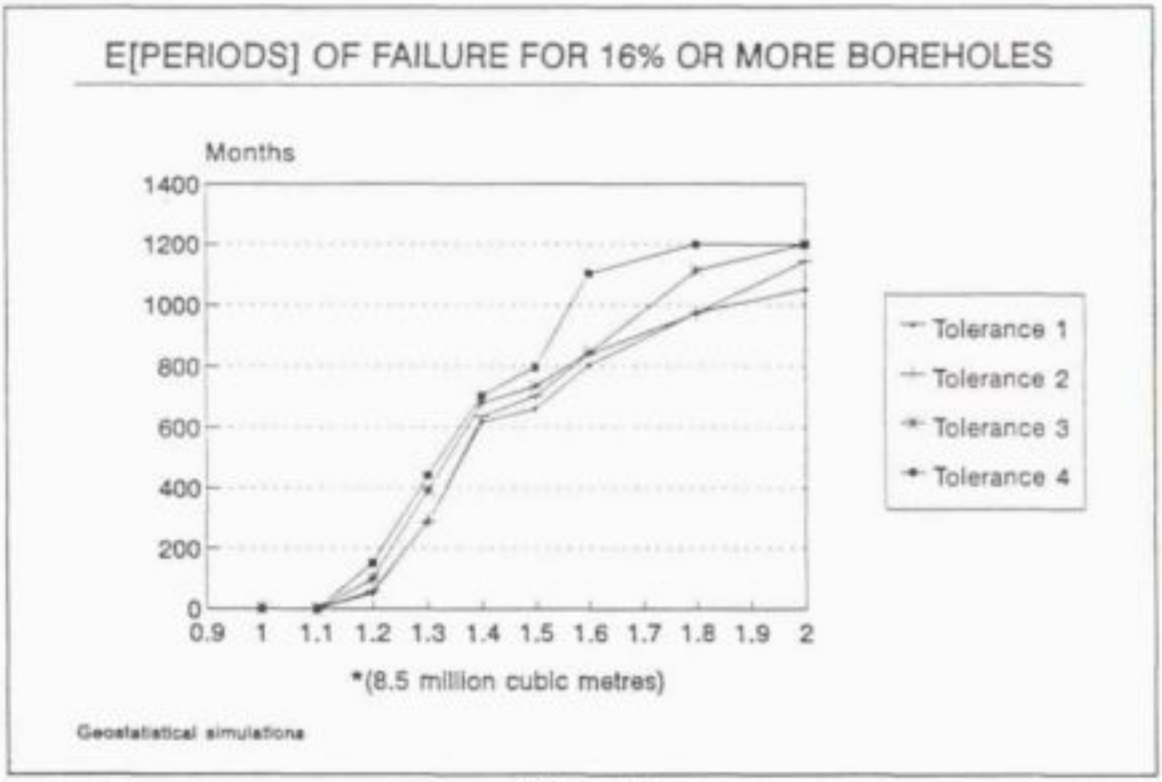


Figure 44

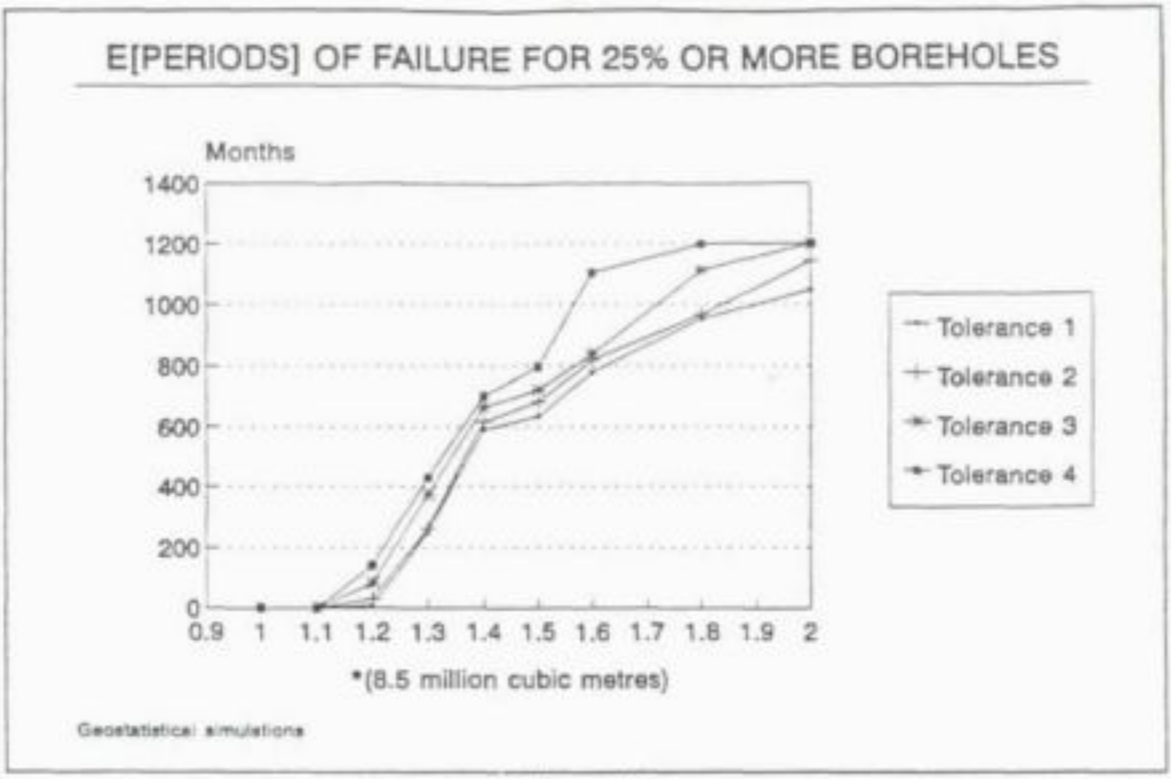
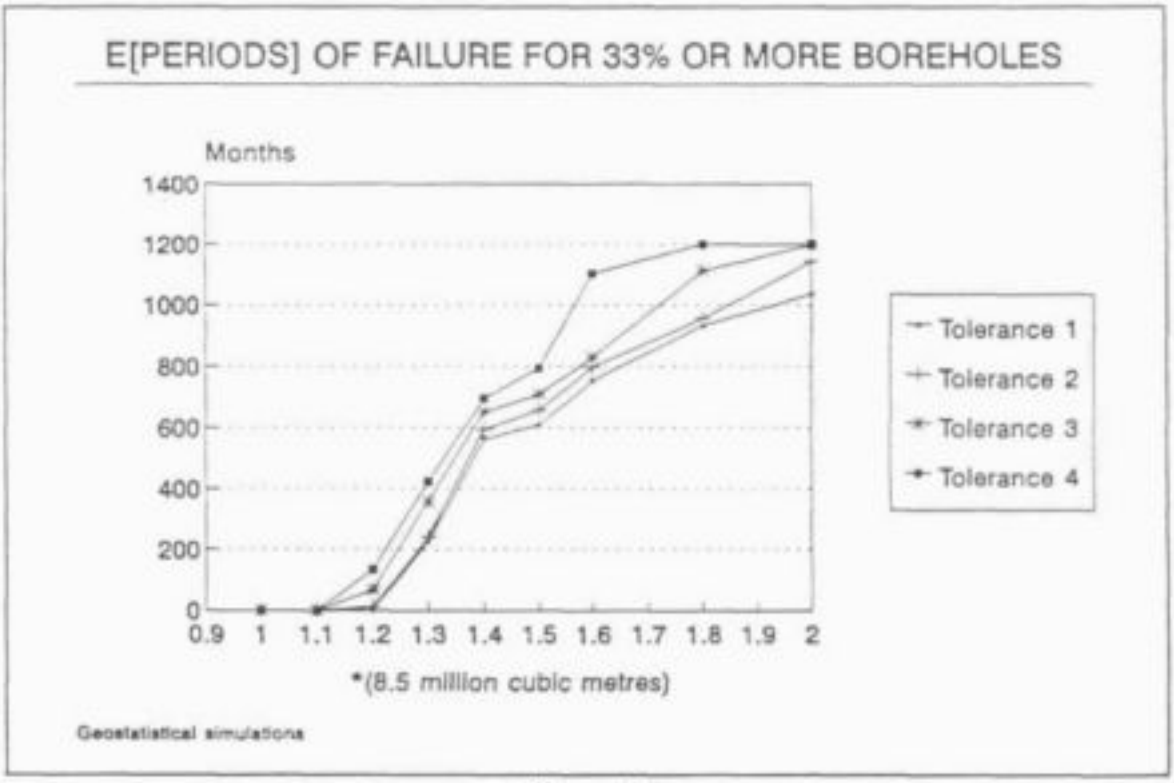


Figure 45



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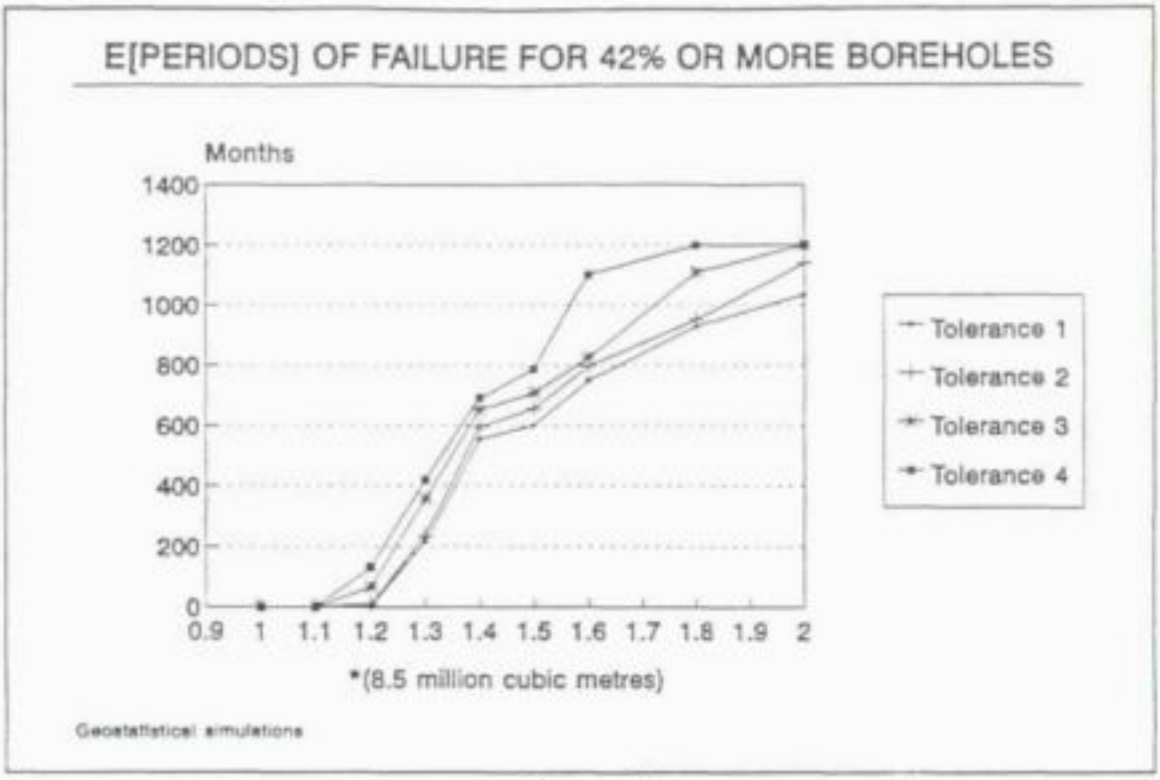

Figure 47

EXPECTED PROBABILITIES

The results are shown in figures 48 to 52.

Once again the general form of the results is similar to those obtained when using zonal simulation. However for the lower pumping rates (less than or equal 1.1*8.5 million cubic metres) the probability is essentially zero whereafter is rises sharply and resembles the performance when using the zonal approach.

Thus it was seen that the zonal approach appears to be more conservative than the geostatistical method. The zonal uses less computing time and could thus be preferred in practice.

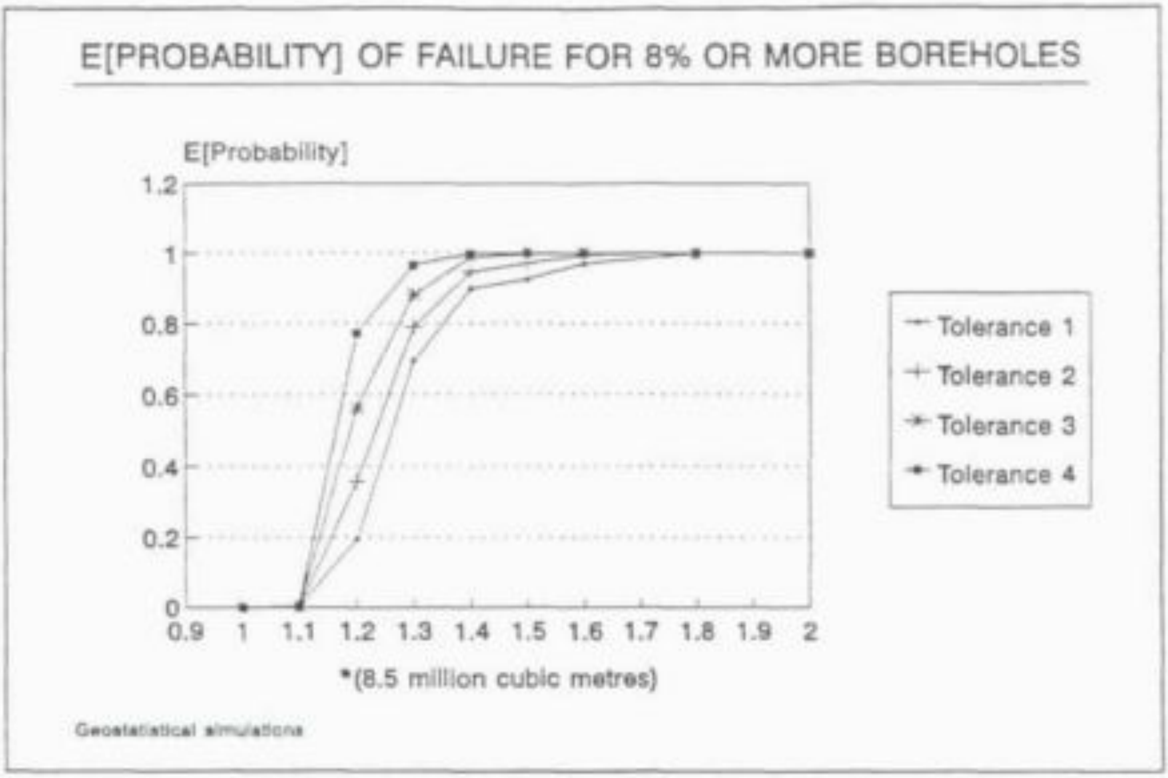

Figure 48

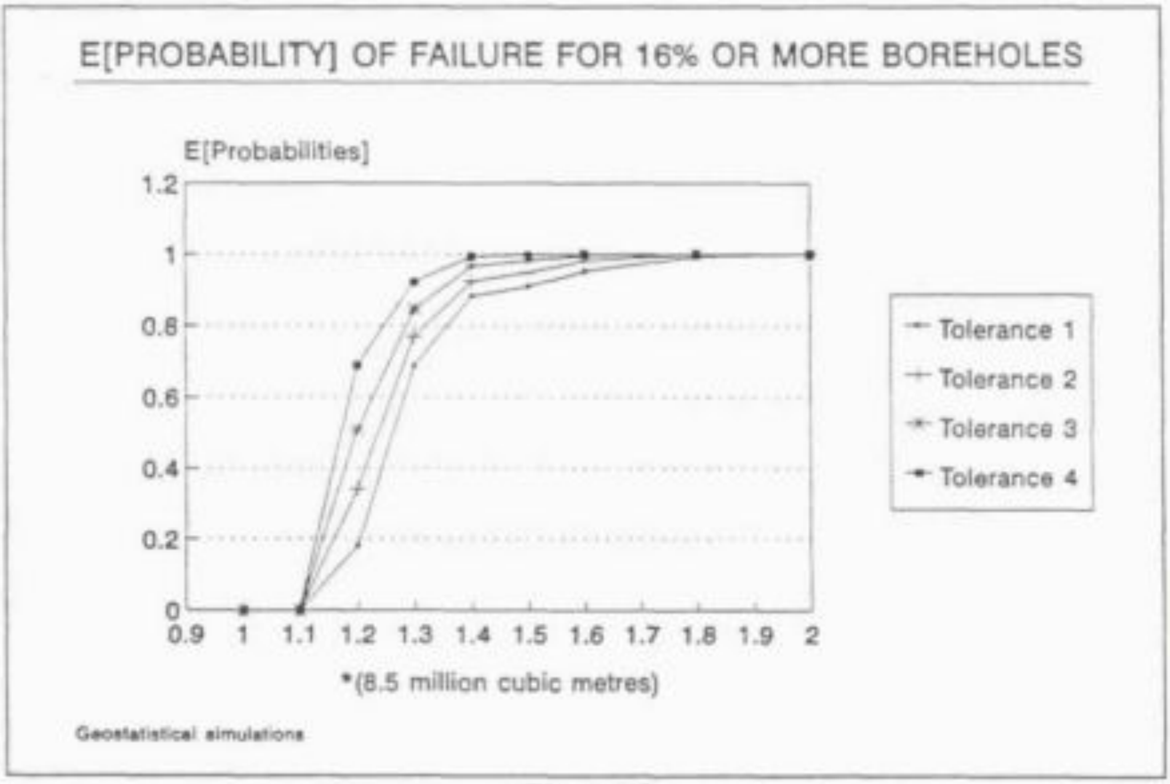

Figure 49

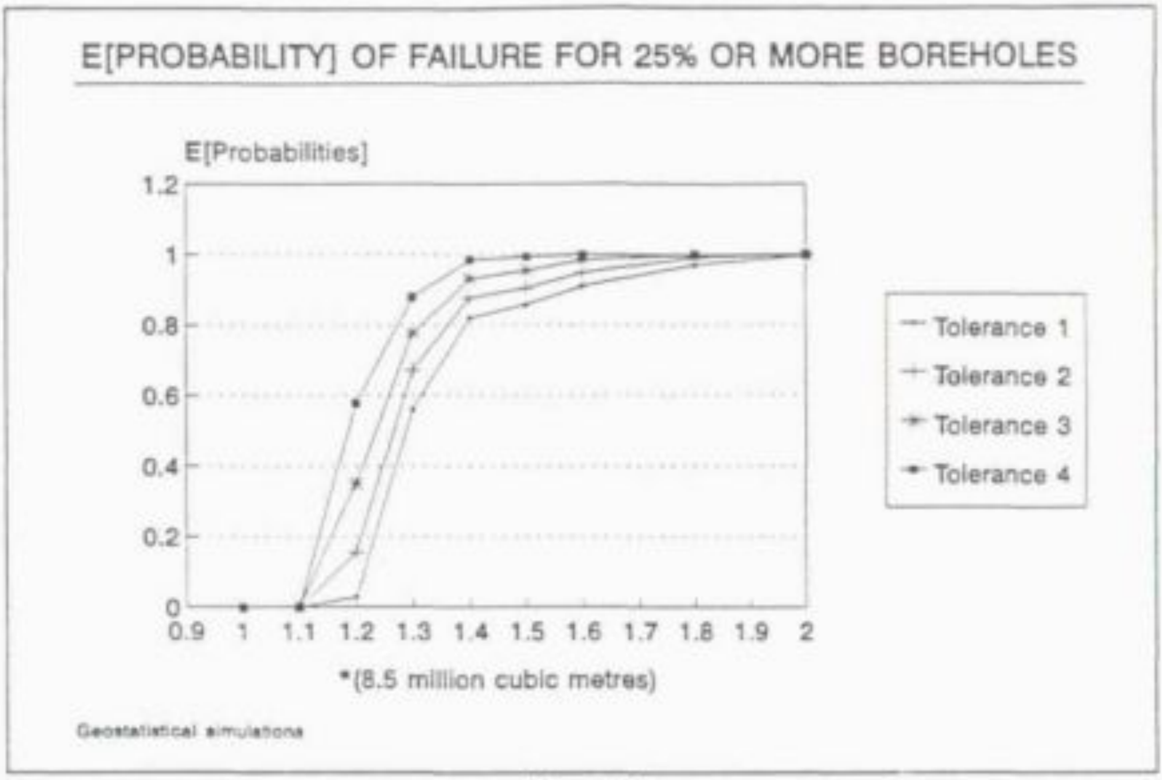

Figure 50

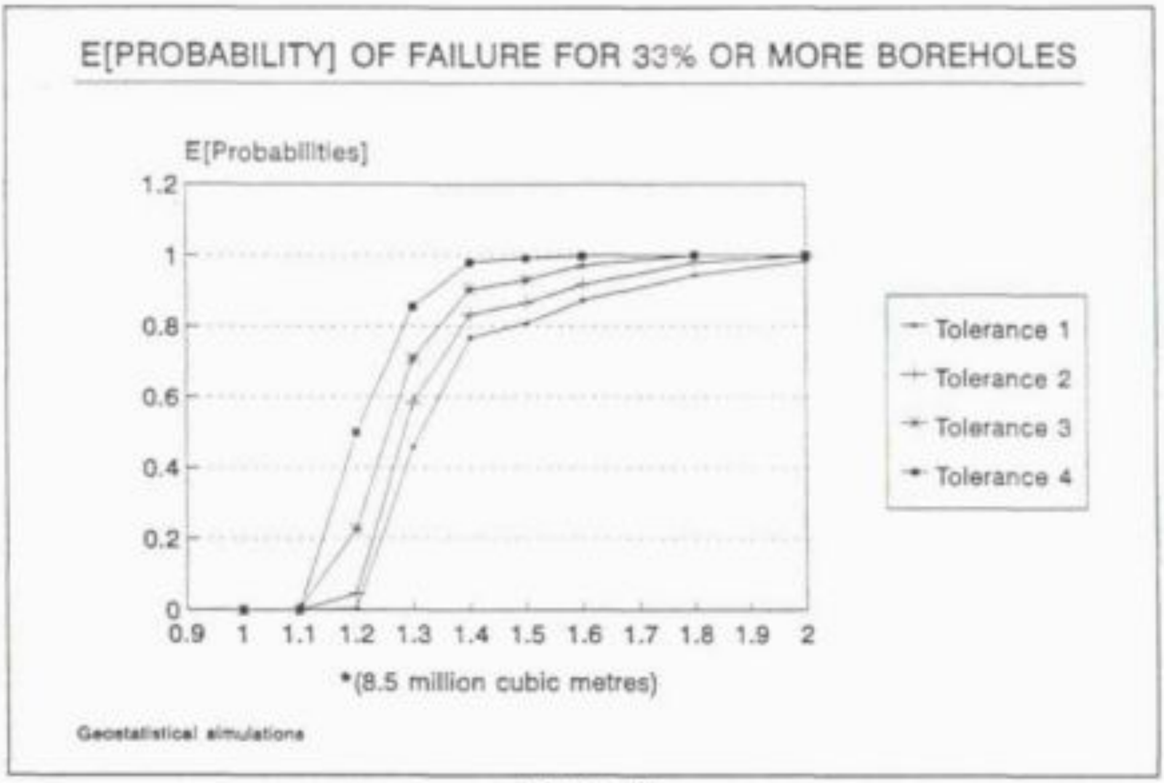

Figure 51

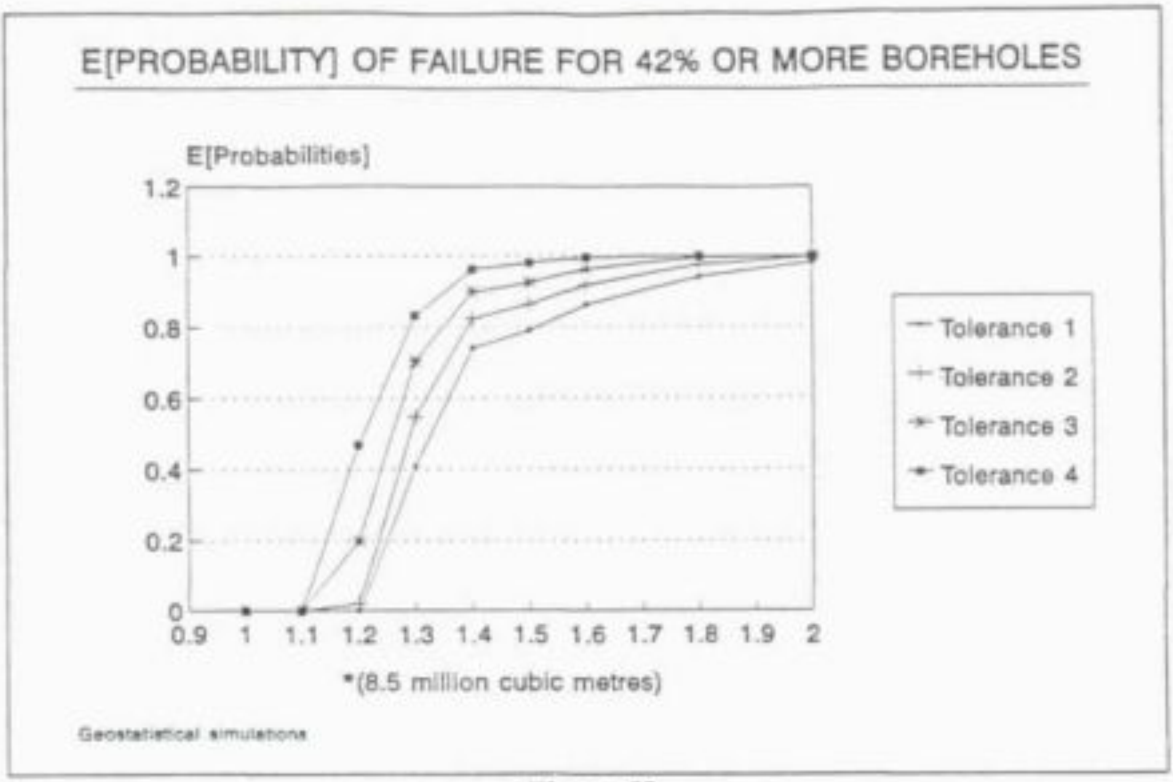

Figure 52

17. CASE STUDY SENSITIVITY

As this study has progressed it has become clear that a major limitation is the tremendous amount of computing required. This project has concentrated on methodology and it was not possible to address the computing issues in any depth. Despite this a small scale sensitivity analysis was undertaken and the results are outlined in this section.

In the current risk evaluation procedures two sources of variation are identified namely that associated with the uncertainty of the S and T values and that linked to the unpredictability of the rainfall.

What is of interest is to assess whether the risk being evaluated is dominated by either one of these.

Thus, for example, if the variation arising from rainfall was such that the uncertainty relating to S and T became insignificant then one might be able to reduce computing by taking the expected values only over different rainfall simulations for a fixed S and T realization.

In order to superficially investigate this aspect the system (99 realizations of 1200 months) was repeated for four different storativity and transmissivity value. For each of these runs the expected value of probability was computed. These are compared to the full results obtained when S, T and rainfall are simultaneously varied.

The results are presented in figure 53. It can be seen that the individual pictures differ greatly from the expected value. Thus had a particular S and T value been chosen one mights have obtained highly misleading results.

It must be remembered that the uncertainty in estimating S and T will determine the amount of variation incorporated in the model. Therefore in the future there could conceivably be cases where S and T was so well determined that it would be pointless to vary them.

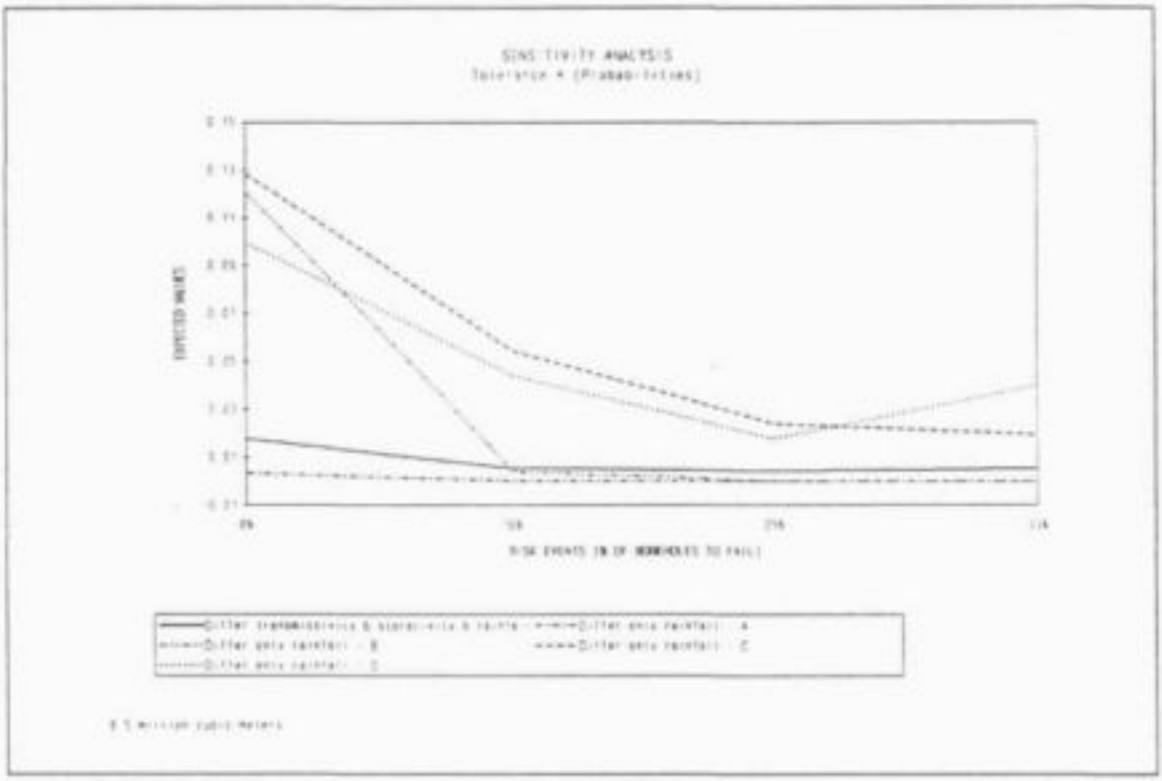

Figure 53

18. CONCLUSIONS AND RECOMMENDATIONS

18.1 MANAGEMENT

Water balance calculations have conclusively indicated that the long-term recharge to the aquifer is in the order of 8,5 million cubic metres per year and that the S-value is in the order 2,15% and 2,45%. With the present water allocations amounting to about 15 million cubic metres per year, it is clear that permits should be reduced to ensure that the aquifer is not depleted in the near future.

Forecasts of water-level reactions using stochastic rainfall record sets and different abstraction from the system indicate that the long-term potential of the aquifer is 8,5 million cubic metres per annum. It can be inferred from the simulations that it is unlikely that the water levels will recover due to normal rainfall pattern; abstraction would have to be cut back to achieve this.

A dynamic model was constructed to simulate the reaction of the aquifer by using the results obtained with the water balance study. A good fit between the actual and simulated water levels was obtained which confirms that the parameter values obtained from the water balance study, are acceptable. The model can be used to predict groundwater flow dynamics in the aquifer at any selected location of interest.

The most important practical result is that from the scenarios performed, it is clear that an abstraction up to 6,5 million cubic meters per annum from the Grootfontein Spring is possible without lowering the water levels, should average recharge occur.

18.2 RISK EVALUATION

The risk analysis methodology developed in the course of the project was tested and successfully implemented in the case study of the Grootfontein aquifer. The conclusions arrived at during the implementation phase of the methodology are summarized below:

The methodology developed adequately describes the risks involved when managing an aquifer under different management plans.

- For this study no geohydrologically derived recharge value was available. A realistic value for the average recharge to the aquifer was estimated using the developed simulated methodology for a fixed abstraction rate. In the case of Grootfontein, pumping facilities for 8.5 million cubic metres are operational and had to be honoured. When a new aquifer is developed the recharge would be established independently (preferably through mass balance techniques) and then our methodology could be applied to determine the safe (acceptable risk) abstraction rate. **Because of the way in which the recharge rate was obtained the results are useful for demonstration purposes but should not be taken as justification for the 8.5 million cubic metre per annum abstraction rate.**

- A risk event was defined in terms of a specified abstraction policy. By combining this with different tolerance levels, visual presentation clearly indicate when a specified tolerance level is exceeded under different management options. Both periods and probability of failure are included in this technique. In this way it was established that above a pumping rate of 1.1 times the current available abstraction rate of 8,5 million cubic metres, water levels decline steadily. It is concluded that the proposed summary statistics are suitable for assessing the management plan.

- Zonal and geostatistical simulation methods were compared and the zonal method is preferred due to the more conservative results it produces. It also requires less input information and in practice would be the preferred way. It is therefore concluded that the zonal methodology be followed.

- The concepts of period of failure and probability of failure were found to be stable and results in easily interpretable statistics for assessing risk of a management plan.

- The often used concept of return time statistics was found not to be suited to this type of risk study.

- The risk analysis system developed is sufficiently flexible to allow for additional statistics to be computed.

- Because of the restriction imposed by the aquifer modelling software's inability to switch pumps on and off as water level varies, the negative results when over exploitation takes place is seen to be over emphasised. This should not be problem in practice as it is evident in situations which should not be realised in practice.

- The techniques demonstrated were designed to test the long term viability of a management plan and this was successfully achieved. The computing time involved was found to be very great and virtually rules out their use as a planning tool on a routine basis. The methodology is now at a stage where it could be extended to address the shorter term needs of an aquifer manager on a routine basis. The computer time to achieve this is expected to be kept manageable by limiting the number of Monte Carlo realizations considering only one or two time periods.

Recommendations

In the light of the successful implementation of the long term risk assessment methodology it is recommended that the work be extended to include the short term risk assessment in a form which could be routinely applied by an aquifer manager.

In order to produce more realistic results aquifer modelling software should be modified or set up so as to allow for the managerial decisions regarding the switching on and off of pumps based on the water level conditions and variable demand.

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