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ASSESSING THE RISK OF DEFICIENCIES IN STREAMFLOW

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ASSESSING THE RISK OF DEFICIENCIES IN STREAMFLOW

BY

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1. INTRODUCTION

There are at least two good reasons why it is important to develop methods of assessing the risk of deficiencies in streamflow. Firstly streamflow constitutes the inflow to reservoirs whose storage levels during times of drought are a matter of national concern. The annual inflow to most reservoirs in South Africa varies so much from year to year that it is obviously inadequate to base water resources planning on only the average inflow, the risk of deficiencies simply has to be taken into account.

Secondly as the integral of spatial catchment processes, with rainfall as the driving force, streamflow constitutes a direct measure of spatial drought. The spatial correlation structure of rainfall particularly on a seasonal basis is extremely difficult to adequately reproduce in a model. Although theory for this type of model is available its practical application, for example the simulation of spatial daily rainfall, is unrealistic at the present time. The problem involves the estimation of more parameters than is feasible with present methodology and with the available data base.

The study of droughts in terms of duration, magnitude (mean deficit) and severity (maximum deficit) is one of the most neglected aspects of engineering hydrology. Particularly little attention has been directed towards the quantitative assessment of drought risk compared to, say, the massive body of literature accorded to the study of extreme-values such as floods and storm rainfall. No valid methodology for the frequency analysis of drought is generally available and stochastic models used to generate event series fail to accurately reproduce historical critical periods (Askew *et al*, 1971) unless the appropriate generating model is very carefully identified. Yevjevich of the effort which went into the research described in this report was devoted to developing a statistical theory for model selection, a subject which is not adequately covered in existing statistical literature. The details of this theory are discussed in Appendix 2. The purpose of concentrating on this particular aspect of the overall project is that as with other applications which involve the use of statistical models, so here one's estimate of the risk associated with a given event will vary considerably if different models are fitted to the historical record. In the final analysis the accuracy of one's estimates is directly dependent on the accuracy of the model and therefore on the quality of one's model selection technique.

The available historical data records are typically quite short (for the purpose of assessing drought risk) and consequently one would expect estimates to be accordingly inaccurate. Any realistic assessment of risk must take account of potential unreliability of the estimates. The analytical derivation of confidence limits or even standard errors for the estimates which we need here is, except for a few special models, hopelessly complex. We propose that this problem can be solved by using something of a statistical innovation - the Bootstrap technique.

The proposed approach is adventurous and somewhat contrary to much of the direction of current statistical hydrological research and method. This has to an increasing degree moved forward in terms of theoretical developments based on classical statistical theory, but the practical application of these achievements has been minimal because of their mathematical complexity and the specialist skills needed to implement them. Our intention is to illustrate a scheme for the probabalistic analysis of annual and monthly streamflow which although computer intensive,

(1967) applied the statistical theory of runs to drought analysis but little if any of this approach has found its way into standard hydrological analysis, This is largely confined to the examination of flow-duration curves (NERC, 1980) or the identification of a frequency model for regional analysis. In the latter case Eratakulan (1970) used moment-ratio diagrams to select between competing univariate models but such a procedure fails to associate sufficient weight to the appropriate portion of the distribution function, that is the lower tail.

The statistical problems associated with drought analysis are fairly complex since a drought, unlike a flood or a storm, is not an "instantaneous" event. It has a duration and a critical deficit associated with each level of risk. One needs, therefore, to consider not only modelling a simple sequence of random variables, such as the annual sequence of inflows to a reservoir, but further to consider the distribution of sums of these variables, for example the 2, 3, 4,... year total inflow volume, and the distribution of these sums over, say, an operational horizon of interest such as 5 years. Statistical models which accommodate all of these requirements are in fact available, but they can be complex and analytically intractable. In general, one has little choice but to resort to Monte Carlo methods. In this respect this report offers no new alternative.

The two main issues which arise when one attempts to answer questions by statistical means are the choice of model and the accuracy of the estimates. A substantial proportion requires no specialist mathematical skills to comprehend. The methods have been applied to extreme storm rainfall (Zucchini and Adamson, 1983) and have considerable potential in the field of hydrology in general.

The traditional hydrological measure of risk is the return period, i.e. the reciprocal of the probability that a given event will take place in any given year. For some of the questions which we consider the return period is an inappropriate measure of risk, and we will simply use probability instead. In fact one of the points which we wish to emphasise in this report is that there is a broad diversity of design and operational questions which can be proposed, and that the risk of deficiencies in streamflow can be elaborated beyond the simple assessment of the return period associated with a particular event. Our purpose here is to demonstrate how the proposed methodology can be applied to answer a variety of questions which may be of interest, the four discussed in Chapter 2 should be regarded as examples and not a complete list.

2. FOUR QUESTIONS OF INTEREST

In a drought analysis several questions need to be asked of the data, we will consider four:

(I) What is the probability that the streamflow in a *given* year will be less than 'x' units?

(II) What is the probability that the total streamflow over 'm' *given* years will be less than 'x' units?

(III) Given a time horizon of 'h' years, what is the probability that the lowest streamflow in the 'h' years will be less than 'x' units?

(IV) Given a time horizon of 'h' years, what is the probability that the lowest consecutive 'm' year total streamflow will be less than 'x' units?

One can of course invert the questions and inquire of the streamflow 'x' associated with a given probability 'p'. Questions I and II are probably most pertinent to reservoir design whilst III and IV are more relevant to operational considerations over a fixed interval of time. It is assumed that we have a sequence of annual streamflow totals $x_1, x_2, \ldots, x_n$ which can be regarded as realisations of independently and identically distributed random variables. As can be seen by examining the estimates in Appendix 3 the annual serial correlation coefficient for practically all the rivers in South Africa is not significantly different from zero. From the point of view of persons having to assess drought risk this is indeed fortunate because otherwise rather more sophisticated methodology would be required to answer the above questions.

In theory all four questions can be answered if one knows $F(x)$, the common distribution function of the annual streamflow, but even if this be the case it is rarely possible to provide simple computational formulae. Suppose, however, that $F(x)$ were known then the answer to (I) is simply $F(x)$ itself. Question II is a little more tricky since one has to derive the distribution, $F_m(x)$, of the sum of m independently and identically distributed random variables. Formally, one has that:

$$F_m(x) = \iiint_R \cdots \int f(t_1)\, f(t_2) \ldots f(t_m)\, dt_1\, dt_2 \ldots dt_m$$

where $R = \{(t_1, t_2, t_3 \ldots t_m) : \sum_{i=1}^{m} t_i \leqslant x\}$ and

$f(x)$ is the probability density function corresponding to the distribution function $F(x)$.

For certain families of distributions, that is those that enjoy the so-called "reproductive property" (notably the Normal and the Gamma), $F_m(x)$ can be evaluated explicitly, but for others (the Log-Normal, the Weibull, the Extreme Value etc) this is not possible. McMahon (1982) did in fact find the Gamma model and its reproductive properties appropriate to the computation of $F_m(x)$ for a selection of Australian reservoir inflows. A similar study for South African reservoir inflows by the authors found the model to be inappropriate and one is left with the fact that for a number of well known distributions $F_m(x)$ is simply not available in any utilitarian form. For large m one can of course apply the Central Limit Theorem and approximate $F_m(x)$ using a Normal distribution. In practise though the values of m of interest are rather small, typically $m = 2, 3, 4, 5$, and so such a approximation would be inaccurate, particularly in the tails of the distribution.

To answer Question II then, one has to resort to using numerical techniques to evaluate $F_m(x)$ or, more simply, to using Monte Carlo methods. In the latter case one simply generates many sequences of m random numbers having the distribution function F(x) and approximates $F_m(x)$ by the empirical distribution of the sums of the generated sequences.

Turning now to Question III, if F(x) is known then the answer is easy to come by. The distribution function $F_{1,h}(x)$, of the smallest of h independently and identically distributed random variables is given by:

$$F_{1,h}(x) = 1 - (1 - F(x))^h$$

Finally, in order to answer Question IV, one needs $F_{m,h}(x)$, the distribution function of the minimum total of m consecutive random variables in a sequence of length h. In all but a few very special cases $F_{m,h}(x)$ is very complex and not available in closed form. One has little alternative but to use Monte Carlo techniques to generate sequences of length h, compute the totals over m years and find the minimum. The empirical distribution function of the minimum converges to $F_{m,h}(x)$.

Summarising, the answers to the 4 questions posed above are simply:

(I) $p = F(x)$ (II) $p = F_m(x)$

(III) $p = F_{1,h}(x)$ (IV) $p = F_{m,h}(x)$

The answers to the inverse questions, i.e. if we wished to enquire of the streamflow, x, associated with a probability, p, of not being exceeded, are then

(I)' $x = F^{-1}(p)$ (II)' $x = F_m^{-1}(p)$

(III)' $x = F_{1,h}^{-1}(p)$ (IV)' $x = F_{m,h}^{-1}(p)$

These distribution functions are, of course, unknown. The four steps required to compute the required answers are:

1. Select a suitable model for the distribution function of the annual streamflow. The selection procedure which was used for the 60 rivers analysed in Appendix 3 is discussed in Chapter 3. Details of the theory of this and of univariate model selection is general are given in Appendix 2.

2. Estimate the parameters of the selected model. Detailed algorithms for computing maximum likelihood estimates of the parameters are given in Appendix 1. The parameter estimates for the rivers considered in Appendix 3 are also given there.

3. Use the estimated distribution function, $\hat{F}(x)$, to derive an estimate of the distribution function of interest, e.g. $\hat{F}_{2,5}(x)$. As mentioned this is carried out by simulation (see Chapter 5). Algorithms to generate random deviates for the six distributions which are considered here are given in Appendix 1.

4. Replace the distribution function of interest by its estimator, e.g. replace $F_{2,5}(x)$ by $\hat{F}_{2,5}(x)$ in IV, above, to obtain the desired estimate (see Chapter 5).

5. Apply the Bootstrap method to compute estimates of the confidence limits for the answer obtained in 4. Methods to do this are discussed in Chapter 4.

Steps 1 and 3 are computationally quite intensive but still well within the capababilities of a typical desk-top micro-computer. Step 5 involves the repeated application of steps 2, 3 and 4 and as a rule can only conveniently be fully carried out on a larger computer. Where only a micro-computer is available we nevertheless strongly recommend that step 5 be carried out even if only 10 Bootstrap repetitions are feasible. Even an inaccurate assessment of the variation of the final estimate is far better than none. The main factor determining the computing time is the model which is selected at step 1, the exponential distribution requires the least time and the gamma distribution the most (for questions of the type III and IV).

3. MODEL SELECTION

For the purposes of illustration, and for the analyses summarised in Appendix 3, six families of distributions are considered:

Normal : $f(x) = \frac{1}{\sqrt{2\pi}\ \sigma} e^{-(x-\mu)^2/2\sigma^2}$

Log-Normal : $f(x) = \frac{1}{\sqrt{2\pi}\ \sigma x} e^{-(\ell n\ x\ -\ \mu)^2/2\sigma^2}$, $x > 0$

Gamma : $f(x) = \frac{x^{\alpha-1}\ e^{-x/\beta}}{\beta^{\alpha}\ \Gamma(\alpha)}$, $x > 0$

Exponential : $F(x) = 1 - e^{-x/\theta}$, $x > 0$

Weibull : $F(x) = 1 - e^{-(x/\delta)^{\rho}}$, $x > 0$

Extreme (Type I) : $F(x) = \exp(-e^{-(x-\xi)/\eta})$, $x > 0$

The theory discussed in Appendix 2 is not restricted to these particular families. In fact the theory can be used to compare a parametric model of the above type to "distribution-free" models such as those based on the empirical distribution function using for example the Weibull plotting positions. We have not included the distribution-free models in the above list of candidates because for the typical sample sizes which are available and for the types of distributional shapes which arise in the application considered here, they would very seldom indeed come into contention for selection.

It is quite natural to expect that each model applied to a sample of total annual flows will lead to different answers to our questions. It is therefore of singular importance to select with great care between competing models. The "true" or operating distribution, F(x), is only approximated firstly because F(x) is *extremely* unlikely to belong to the fitted family and secondly even if it did the parameters are unknown and can only be estimated on the basis of a finite (usually small) sample. Proper model selection needs to take account of both sources of error and the object is not to find the operating model but rather the most appropriate approximation for the situation at hand. In our application our attention is directed at the low annual inflows to a reservoir, that is at the lower tail of the distribution. A measure of the discrepancy between the operating model and the approximating model which emphasises the fit in this position of the distribution is

$$\Delta = \max_{x} |F(x)^d - F_\theta(x)^d|$$

where $F_\theta(x)$ is the approximating distribution function having parameter vector θ, e.g. for the normal distribution $\theta = (\mu, \sigma^2)$. The selection constant d determines where the emphasis in the fit should be placed. For emphasis in the lower tail $0 \leqslant d \leqslant 1$ is appropriate. The resulting empirical descrepancy (c.f. Appendix 2 for definitions of this and other terms) is

$$\Delta_n(\theta) = \max_{1 \leqslant i \leqslant n} |\{i/(n+1)\}^d - F_\theta(x_i)^d|$$

where $x_1, x_2, \ldots, x_n$ are the observations. This in turn leads to the following criterion

$$C = E_{\hat{F}} \max_{1 \leqslant i \leqslant n} |\{i/(n+1)\}^d - F_{\hat{\theta}}(x_i)^d|$$

where $\hat{\theta}$ is the maximum likelihood estimator of θ.

The evaluation of C is carried out by the Bootstrap method. (As mentioned above we will again use Bootstrap method to estimate confidence limits, here we use it for the purpose of model selection.) The procedure to compute C is as follows:

Step 1: Select a random sample of size n (with replacement) from the original observations $\{x_1, x_2, ..., x_n\}$ to obtain a Bootstrap sample $\{x_1^*, x_2^*, ..., x_n^*\}$.

Step 2: Compute the maximum likelihood estimate $\hat{\theta}^*$ of the parameter vector θ using the Bootstrap sample as the data.

Step 3: Compute $C^* = \max_{1 \leqslant i \leqslant n} |\{i/(n+1)\}^d - F_{\hat{\theta}^*}(x)^d|$

Step 4: Repeat Steps 1 to 3 a large number of times keeping a record of the criteria C*.

As the number of Bootstrap iterations increases so the average of the generated values C* converges to the required criterion C. In practice about 100 iterations are sufficient to yield reasonable accuracy.

The above procedure is repeated for each family of models. That family which leads to the smallest value of C is selected as most appropriate.

In Appendix 3 the values of the criteria corresponding to d = 1; or 5; and 0,25 are given for 60 rivers and for each of the 6 distribution families considered here. The results for three rivers are discussed in Chapter 6.

To save computing time one can eliminate, at the very beginning, those families which obviously do not fit the data.

4. CONFIDENCE LIMITS

Any answer to the four questions proposed above are estimates which are themselves subject to sampling variations. We need therefore to assess the accuracy of our answers by attaching confidence limits to them. Efron (1980) gives methods of estimation of confidence intervals pertinent to our situation in which the statistical accuracy of our estimate cannot be found numerically. This is the so-called Bootstrap algorithm which can be implemented as follows:

Suppose that a model has been selected for our inflow data on the basis of which, for a given x (inflow), we have estimated a probability $\hat{p}$. Then:

Step 1 : Select a random sample of size n (with replacement) from the set $\{x_1, x_2, \ldots, x_n\}$ to obtain a Bootstrap sample $\{x_1^*, x_2^*, \ldots, x_n^*\}$.

Step 2 : Fit the same model as selected for the original data to the Bootstrap sample using the same estimation procedure and use this to obtain an estimate of the required probability $\hat{p}^*$.

Step 3 : Repeat Steps 1 and 2 a large number of times keeping a record of the estimates $\hat{p}^*$.

As the number of Bootstrap iterations increases so the sample percentage points of the $\hat{p}^*$ converge to estimates of the corresponding percentage points of $\hat{p}$. For example, the estimates of the 90% confidence interval of $\hat{p}$ based on 1000 Bootstrap realisations would be the interval between the 50th and 950th largest values of $\hat{p}^*$.

We recommend the following refinement to the above method which we might term "the smooth Bootstrap". Instead of sampling from the observed flows as in Step 1, that is sampling from the empirical distribution, one can sample from a smoothed version of this distribution. In the examples to follow the smooth Bootstrap was used. The observed inflows $x_1, x_2, \ldots, x_n$ were plotted against their Weibull plotting positions $w_i = i/(n+1)$. A polynomial

$$x_i \approx Q(w_i) = \alpha_0 + \alpha_1 w_i + \alpha_2 w_i^2 + \ldots + \alpha_\ell w_i^\ell$$

was fitted to these points by the method of least squares where the degree of the polynomial was sufficiently high (in the examples $\ell = 9$) so as to fit the observed points fairly closely. Particular care is needed to ensure that the fitted polynomial, $Q(w)$, leads to reasonable values in the neighbourhood of $w = 0$ and $w = 1$. Step 1 in the Bootstrap algorithm is then replaced by:

Step 1* Generate n uniformly and independently distributed random deviates $u_1, u_2, \ldots, u_n$ and set $x_i^* = Q(u_i)$, $i = 1, 2, \ldots, n$.

This modification is particularly recommended when only a small sample is available, as is often the case in South Africa. It enables one to augment the information available in the sample with one's judgement about how the distribution of the streamflow is likely to behave. For sample sizes of about 50 or more this refinement is unlikely to lead to substantial improvement in accuracy and we recommend that the original algorithm be used because it involves less computation.

For completeness we also give the algorithm to compute the confidence interval for $\hat{x}$ for a given p. It is a straightforward modification of the above algorithm.

Step 1: Select a random sample of size n (with replacement) from the set $\{x_1, x_2, \ldots, x_n\}$ to obtain a Bootstrap sample $\{x_1^*, x_2^*, \ldots, x_n^*\}$.

Step 2: Fit the same model as selected for the original data to the Bootstrap sample using the same estimation procedure and use this to obtain an estimate of $\hat{x}^*$.

Step 3: Repeat Steps 1 and 2 a large number of times keeping a record of the estimates of $\hat{x}^*$.

As the number of Bootstrap iterations increases so the sample percentage points of the $\hat{x}^*$ converge to estimates of the corresponding percentage points of $\hat{x}$.

5. ESTIMATING THE RISK OF DEFICIENCIES

We now give in more detail the requirements to answer our four questions. We assume that the model $\hat{F}(x)$ for $F(x)$ has been selected.

Question I

To estimate the probability that the flow in a given year will be less than x units we simply use

$$\text{(I)} \quad \hat{p} = \hat{F}(x)$$

In this and in all other cases which follow the Bootstrap algorithm is then used to estimate the confidence limits for $\hat{p}$. To estimate a deficient inflow associated with a given risk p one uses:

$$\text{(I)}' \quad \hat{x} = \hat{F}^{-1}(p) \; .$$

Detailed algorithms to evaluate $\hat{F}^{-1}(p)$ for the distribution discussed here are given in Appendix 1.

Question II

For the Normal distribution, the distribution of the sum of m independently and identically distributed, $N(\mu,\sigma^2)$ random variables is also Normally distributed as $N(m\mu,m\sigma^2)$. Similarly for the Gamma distribution, $G(\alpha,\beta)$, the sum is distributed as $G(m\alpha,\beta)$. The Exponential distribution is a special case of the Gamma with $\alpha = 1$ and so the sum of m independently and identically distributed Exponential variables, $E(\lambda)$, is distributed as $G(m,\lambda)$. It follows that for these distributions once $F(x)$ has been fitted an estimator for $F_m(x)$ is immediately available.

For the Log-Normal, Weibull and Extreme Value Type 1 distributions, however, the distribution of the sum is not known. Here we recommend that Monte-Carlo methods are used:

Step 1: Generate a random sample $x_1', x_2', \ldots, x_m'$ from $\hat{F}(x)$ and compute $y' = x_1' + x_2' + \ldots + x_m'$.

Step 2: Repeat Step 1 a large number of times and keep a record of the generated y'.

(For details of the generating algorithms see Appendix 1.)

The empirical distribution of the y' converges to the distribution of the sum of m random variables which have the distribution function $\hat{F}(x)$. The required estimate is then given by:

(II) $\hat{p} = \hat{F}_m(x)$

The deficient streamflow corresponding to a given risk p is given by:

(II)' $\hat{x} = \hat{F}_m^{-1}(p)$

<u>Question III</u>

Here $\hat{F}_{1,h}(x)$ is a simple function of $\hat{F}(x)$:

$$\hat{F}_{1,h}(x) = 1 - (1 - \hat{F}(x))^h$$

and the required estimates are given by

(III) $\hat{p} = \hat{F}_{1,h}(x)$ and

(III)' $\hat{x} = \hat{F}^{-1}_{1,h}(x)$

Question IV

Here there is little choice but to use Monte-Carlo methods:

Step 1: Generate a random sample $x_1', x_2', \ldots, x_h'$ from $\hat{F}(x)$, and set

$$y_j' = \sum_{i=j}^{j+m-1} x_i' \quad , \quad j = 1,2,\ldots,h-m+1 ,$$

$$z' = \min (y_1', y_2', \ldots, y_{h-m+1}')$$

Step 2: Repeat Step 1 a large number of times and keep a record of the z'.

The empirical distribution of the z' converges to $\hat{F}_{m,h}(x)$, the distribution of the minimum m year flow in h years, given that the flows are distributed as $\hat{F}(x)$. The required estimates are then given by

(IV) $\hat{p} = \hat{F}_{m,h}(x)$ and

(IV)' $\hat{x} = \hat{F}^{-1}_{m,h}(p)$.

6. MONTHLY STREAMFLOW GENERATION

The above analysis is on an annual basis, however, for the operation of reservoir storage systems in a drought situation decisions need to be made at least on a monthly basis. In this chapter we demonstrate how the above methodology can be expanded to deal with monthly flows. The Method of Fragments, (Svanidze (1969), (1980); Huynh Nqoc Phien and Vithana, (1982)) has been found to be capable of reproducing the mean, standard deviation, skew and serial correlation* of monthly streamflow sequences and is used here. Essentially we generate annual flows as before, find the historical annual flow which is nearest in magnitude and disaggregate the generated annual total in direct proportion to the corresponding historical monthly sequence.

To assess drought risk over an operational horizon of say h years one generates a large number of h-year sequences of monthly flows and accumulates on a monthly basis. The percentiles of the distribution of these running totals are then plotted. The 5% percentile curve, for example, gives the cumulative monthly inflow associated with a 5% risk of occurrence. It is important to note that these percentile curves can only be used to assess risk from the given time origin, they cannot be entered at any arbitrary subsequent time since no cognisance of prior information is taken. This is a weakness of the method, but the practical problems in South Africa is simply this : that inflows to

*In fact the serial correlation structure is only partially preserved. The serial correlation between the last month in a year and the first of the next year produced by this method is zero, whereas in reality it will be different from zero.

reservoirs are highly seasonal and operational considerations require an assessment of the risks associated with possible m-seasonal inflow generally from 1 October.

Figures 1 and 2 illustrate such curves for the Vaal and Midmar dams. These graphs are of course based on average initial conditions. In theory one could account for antecedent flow and models of varying complexity which can do this are available. However one would then need to produce a set of curves for each season and for each possible initial condition. If one wishes to do this then an alternative method of generating monthly sequences would have to be found.

Figure 1 shows the percentiles in the distribution of cumulative monthly inflows into Vaal Dam. Historically the two most severely deficient inflow sequences occurred during the early nineteen thirties and during the present drought which is generally regarded as having started in the 1978/79 season. The three-season inflow starting in October 1930 is more severe than the initial 36 month sequence of the present drought. However the total 60 month inflow of the present drought is less than half of the corresponding inflow in the earlier drought and is associated with a probability of less than 1%.

September 1980 marked a critical point in storage level at Midmar since the reservoir was considerably drawn down. The focus of interest here was to decide on suitable restrictions on consumption. Figure 2 starts at this point in time and provides a means of assessing the risk of further deficiencies in inflow over the ensuing three seasons. It can be seen that the actual sequence of events was particularly severe. At no stage over the 36 month period did the cummulative inflow exceed the 20% percentile level and it ended below the 2,5% level.

FIGURE 1

Vaal Dam: Simulated non-exceedance percentiles from 1 October over a 60 month period for the cumulative monthly inflows. The solid line represents October 1978 to September 1983, the dashed line October 1930 to September 1935.

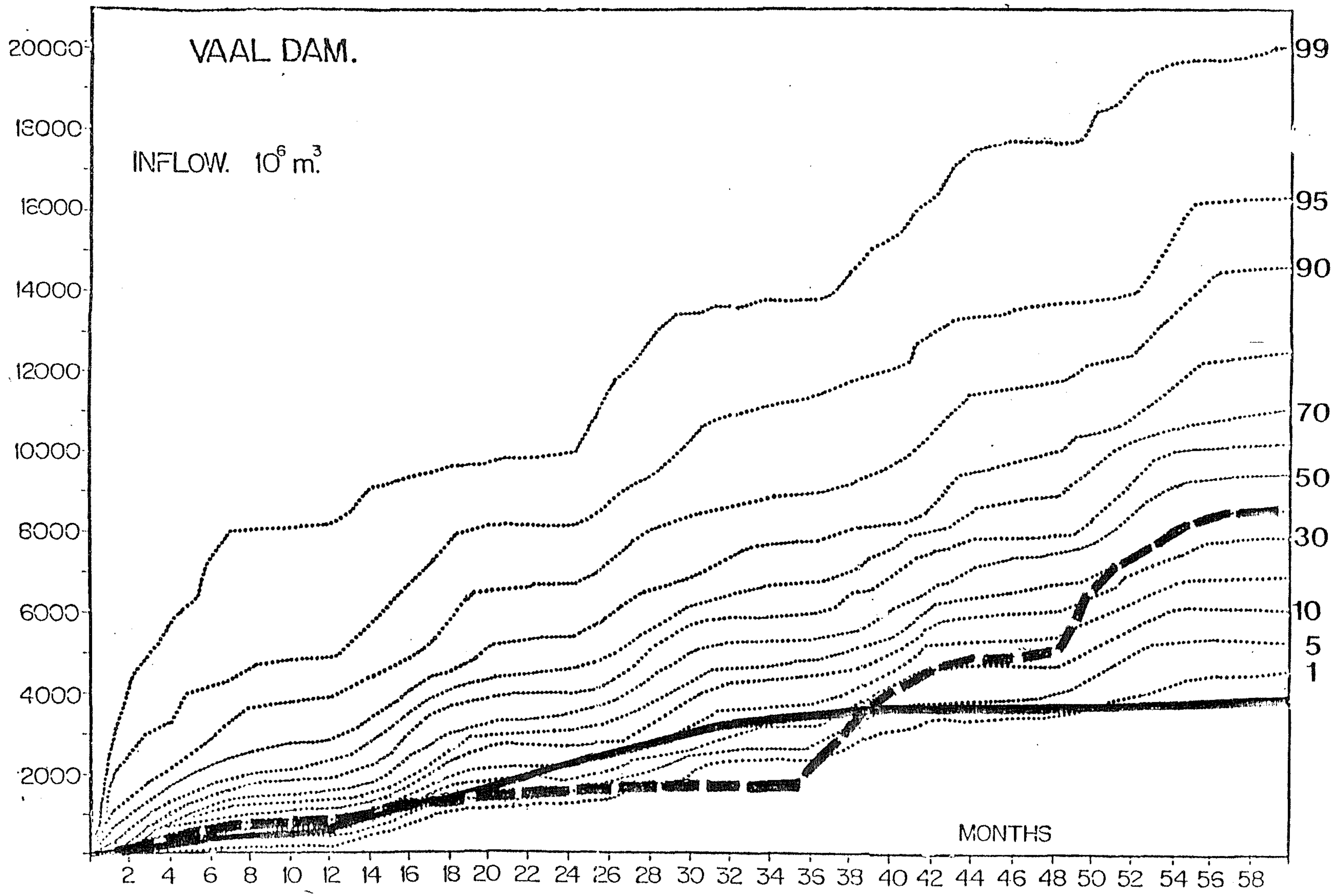
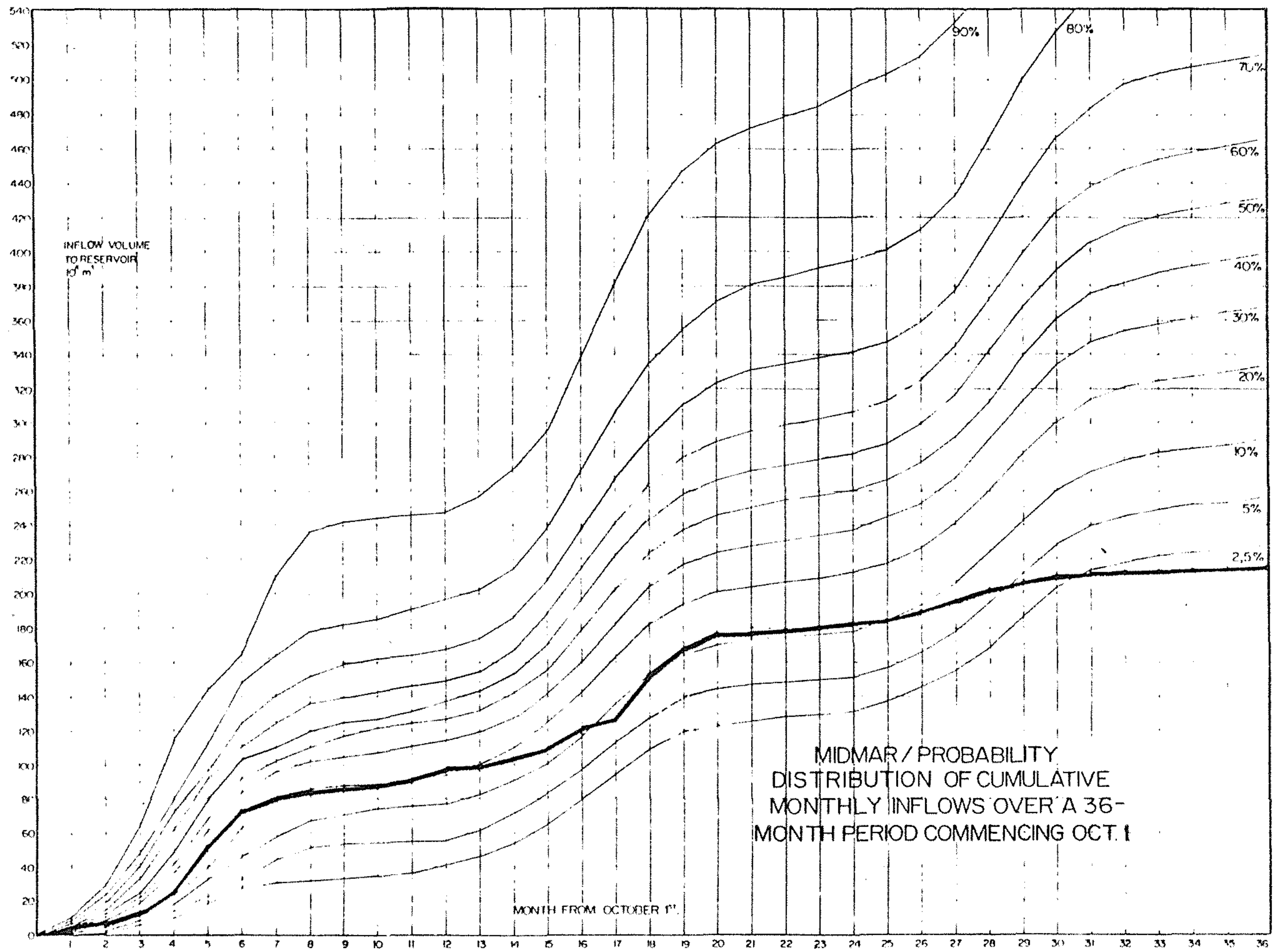


FIGURE 2

Midmar Dam: Simulated non-exceedance percentiles from 1 October over a 36 month period for the cumulative monthly inflows. The solid line represents October 1980 to September 1983.



7. EXAMPLES OF APPLICATION

The annual sequence of inflows to three South African reservoirs are considered to illustrate the proposed method of drought analysis. Vaal Dam, Midmar and Kalkfontein illustrate three distinct types of empirical density functions with regard to the distribution of their sequences of annual inflows, from a 'J' shape at Kalkfontein to an almost symmetrical density at Midmar. These data are given in Table 1 and the maximum likelihood estimates for the parameters of each of the six distributions considered are given in Table 2.

The selection criterion (c.f. Chapter 3) for each distribution and for various values of d are given in Table 3. For Vaal Dam the Log-Normal distribution leads to the lowest criterion for each value of 'd' and consequently this distribution was selected to represent the flows. For Midmar the Extreme Value (Type 1) distribution leads to the smallest criterion for d = 1 and d = 0,5, whereas for d = 0,25 the criterion for the Gamma model is a little lower. Because the difference is small the Extreme Value distribution was selected in this case. For Kalkfontein the choice is a little more complex. At d = 1 the Log-Normal distribution is the obvious choice, but at d = 0,5 and d = 0,25 the Exponential distribution becomes a better candidate. As we are mainly interested in the low flows more weight is given to the criterion at $d < 1$ and so the Exponential model is chosen. However, it is worthy of note that for Kalkfontein none of the six univariate models considered leads to a particularly good fit.

Having completed the process of model selection for the annual inflows to each reservoir we now generate a stochastic sequence on each model and use the explicit algorithms presented earlier in order to answer our four questions. The results for each dam are presented in Tables 4 to 9. These tables are completed by adding the 90% confidence limits to our estimates using the "smooth" Bootstrap procedure.

A detailed examination of the tables reveals some particular features. Firstly, the confidence limits are not symmetrical about our estimates and is as should be expected. The asymmetry of confidence intervals is naturally influenced by the shape of the distribution function about a particular quantile.

The confidence limits are hauntingly wide but reflect the precision with which the estimate of a particular quantile is known and the considerable qualification of results that is required of the engineering hydrologist when making such estimates. We do at least know in our case just how good these are and this information should be incorporated into any subsequent analysis.

Our percentile estimates in Tables 4 to 9 were drawn from a generated sequence of 20 000 years for each reservoir and the confidence limits estimated from 300 replicates of length n (historical sample size). It is of obvious interest to know just how good our Bootstrapped estimates of the estimates of the confidence limits are. Stedinger (1983) gives exact confidence limits for design events drawn from a Log-Normal model. Table 10 reveals that for Vaal Dam (where the Log-Normal was the selected univariate model) our result is comfortably close to the exact result and inspire authenticity to those where the exact result is not in fact available.

Using the material contained in the tables we are now in a position to pose some specific questions and provide answers and confidence (in this case the 90% interval) in our answers.

(I) What is the probability that over the next water year more than the mean annual inflow will discharge into each of the 3 reservoirs?

Vaal	($\bar{x}$ = 1 975.$10^6 m^3$)	(27%)	38%	(52%)
Midmar	($\bar{x}$ = 152.$10^6 m^3$)	(22%)	42%	(56%)
Kalkfontein	($\bar{x}$ = 164.$10^6 m^3$)	(27%)	36%	(45%)

(II)' For each reservoir what is the total inflow volume over the next two water years that can be expected such that there is only a 10% chance of failure to achieve this unknown sum (in units of $10^6 m^3$)?

Vaal	(1 593)	1 783	(2 464)
Midmar	(155)	185	(221)
Kalkfontein	(25)	88	(101)

(III)' For each reservoir let Q be the minimum flow in the next 5 years. Which value, q, is such that the probability that Q is less than q is 20% (in units of $10^6 m^3$)?

Vaal	(361)	457	(587)
Midmar	(33)	53	(80)
Kalkfontein	(0)	6,0	(14,7)

(IV)' Consider an operating horizon of 10 years for Kalkfontein dam and let q be the minimum 3-year inflow which will occur. What levels of q are associated with 5%, 10% and 20% risks of deficiency? Here h = 10, m = 3 and from Table 9 we see that the required levels are (in units of $10^6 m^3$):

risk	estimate	estimated 90% confidence interval
5%	84	(57, 130)
10%	95	(73, 161)
20%	119	(86, 190)

(IV) Over the next four water years at Midmar dam a total inflow of not less than $400.10^6 m^3$ is required. What is the probability that this requirement will be met?

Here h = 4, m = 4 and $\hat{p}$ = 97% with (90%) confidence limits (80%, 99%).

(III) Operation considerations at Midmar require a minimum assured one year inflow of $50.10^6 m^3$ within the next four year period. What is the probability that this is met?
Here h = 4, m = 1 and $\hat{p}$ = 86% with (90%) confidence interval of (63%, 95%).

It is also possible by using the methodology described here to give an estimate of the probability that the last two conditions concerning inflow into Midmar are simultaneously met and also to estimate a confidence interval for this probability.

8. CONCLUSIONS

The diversity of complex problems associated with the study of droughts have to date very largely been approached from a qualitative point of view. The question of drought definition is a well known case in point where the precise level of deficient rainfall, storage or river-flow is a function of climate and major usage. Thus, there are a vast number of drought definitions pertinent to particular climates, crops and seasonal patterns of demand. Where quantitative analyses of drought have been attempted they have been found to be severely restricted by the lack of any precise analytical technique with which to tackle the questions that are undoubtedly of interest.

Monte-Carlo techniques allow us to address these complicated problems and to draw statistical inferences. Furthermore, the Bootstrap algorithm allows us to assess the accuracy of our result. We have posed four specific questions with respect to the risk of deficient annual inflow and sequences of them to a reservoir and have shown how these can be computed given an operational horizon of interest. The provision of extensive tables for each reservoir has permitted us to illustrate how a further broad diversity of design and operational questions can be proposed and how the concept of risk of deficient inflows to storage systems can be elaborated beyond the simple notion of "the T-year event".

The scheme proposed above does require a good deal of computing. However it must be kept in mind that data in such situations is scarce, expensive to collect, code and distribute. Furthermore one has to consider the potential cost of the consequences of incorrect decisions being taken because they are based on unrealistic estimates of risk.

Compared to these the computing cost is negligibly small and there is no excuse for not using the best available estimation methods.

TABLE 1

Water Year	(1)	(2)	(3)	Water Year	(1)	(2)	(3)
1912/13			57	1947/48	1101	150	717
1913/14			58	1948/49	642	87	40
1914/15			363	1949/50	1939	130	385
1915/16			56	1950/51	639	142	53
1916/17			174	1951/52	1167	109	95
1917/18			46	1952/53	1951	132	67
1918/19			19	1953/54	882	119	101
1919/20			554	1954/55	3510	129	335
1920/21			64	1955/56	1546	89	152
1921/22			41	1956/57	5379	313	97
1922/23			389	1957/58	3656	134	119
1923/24	765		194	1958/59	1345	237	46
1924/25	4778		748	1959/60	1449	74	35
1925/26	809	44	55	1960/61	2039	164	57
1926/27	1284	110	111	1961/62	962	126	105
1927/28	863	92	67	1962/63	1316	146	237
1928/29	1612	122	140	1963/64	1136	135	21
1929/30	2755	146	166	1964/65	2890	188	52
1930/31	779	68	114	1965/66	520	111	194
1931/32	698	84	140	1966/67	3393	208	271
1932/33	470	89	74	1967/68	597	120	38
1933/34	3302	258	474	1968/69	687	86	195
1934/35	2550	156	86	1969/70	1173	153	46
1935/36	1689	124	9	1970/71	1008	121	125
1936/37	4361	142	159	1971/72	1977	202	345
1937/38	1146	141	22	1972/73	440	163	5
1938/39	3929	256	69	1973/74	2176	401	1324
1939/40	2178	179	82	1974/75	5727	204	53
1940/41	2535	159	168	1975/76	4803	419	492
1941/42	1039	215	74	1976/77	2395	132	48
1942/43	3598	369	334	1977/78	2367	174	6
1943/44	6864	191	82	1978/79	600	194	15
1944/45	1696	99	20	1979/80	1464	36	85
1945/46	1278	56	100	1980/81	1202	96	7
1946/47	1117	146	42	1981/82	375	86	
				1982/83		38	

Annual inflow records (1) Vaal Dam; (2) Midmar; (3) Kalkfontein

Note: (a) The inflows are nett volumes ($10^6 m^3$).

(b) The figures for Vaal Dam include Sterkfontein.

(c) The record at Midmar prior to 1963/4 was augmented using annual rainfall using the Pitman model based on monthly data.

(d) The figure for 1982/3 at Midmar represents a projected figure at March 83.

TABLE 2

VAAL DAM		
GAMMA	α = 2,153E + 00	β = 9,174E + 02
NORMAL	μ = 1,975E + 03	σ = 1,462E + 03
LOGNORMAL	μ = 7,339E + 00	σ = 7,082E - 01
EXPONENTIAL	θ = 1,975E + 03	
WEIBULL	ρ = 1,467E + 00	δ = 2,200E + 03
EXTREME-1	ξ = 1,353E + 03	η = 9,559E + 02
MIDMAR DAM		
GAMMA	α = 4,216E + 00	β = 3,594E + 01
NORMAL	μ = 1,515E + 02	σ = 7,910E + 01
LOGNORMAL	μ = 4,897E + 00	σ 5,034E - 01
EXPONENTIAL	θ = 1,515E + 02	
WEIBULL	ρ = 2,036E + 00	δ = 1,717E + 02
EXTREME-1	ξ = 1,179E + 02	η = 5,604E + 01
KALKFONTEIN DAM		
GAMMA	α = 9,515E + 01	β = 1,723E + 02
NORMAL	μ = 1,640E + 02	σ = 2,133E + 02
LOGNORMAL	μ = 4,489E + 00	σ = 1,145E + 00
EXPONENTIAL	θ = 1,640E + 02	
WEIBULL	ρ = 9,182E - 01	δ = 1,565E + 02
EXTREME-1	ξ = 8,947E + 01	η = 1,030E + 02

Maximum likelihood estimates of the parameters of each of the six distributions fitted to the annual inflow data

TABLE 3

MODEL	CRITERION C		
	VAAL DAM	MIDMAR DAM	KALKFONTEIN DAM
d = 1			
GAMMA	0,0993	0,0843	0,1059
NORMAL	0,1690	0,1497	0,2273
LOGNORMAL	0,0595	0,0837	0,0739
EXPONENTIAL	0,1776	0,3058	0,1129
WEIBULL	0,0987	0,1144	0,0879
EXTREME-1	0,1187	0,0680	0,1628
d = 0,5			
GAMMA	0,0896	0,0725	0,1190
NORMAL	0,2408	0,1367	0,3576
LOGNORMAL	0,0444	0,1066	0,0968
EXPONENTIAL	0,2867	0,3268	0,0780
WEIBULL	0,1389	0,1103	0,0949
EXTREME-1	0,1197	0,0693	0,2006
d = 0,25			
GAMMA	0,1084	0,0707	0,1427
NORMAL	0,2489	0,1558	0,3450
LOGNORMAL	0,0358	0,1479	0,1231
EXPONENTIAL	0,2855	0,3152	0,0636
WEIBULL	0,1584	0,0874	0,0986
EXTREME-1	0,1395	0,0797	0,2201

Univariate model selection based on the minimisation of the criterion 'C'. As 'd' decreases so the fit of the model at the lower tail is emphasised.

TABLE 4

$P(Q) \leqslant q$	m = 1	m = 2	m = 3	m = 4	m = 5
5%	(333) 475 (598)	(1059) 1470 (2133)	(2049) 2646 (3802)	(3012) 3911 (5172)	(4050) 5320 (6950)
10%	(454) 630 (754)	(1593) 1783 (2464)	(2741) 3133 (4392)	(3626) 4577 (6093)	(4607) 5958 (7817)
20%	(655) 841 (1031)	(1927) 2247 (2932)	(3178) 3760 (4945)	(4217) 5361 (6802)	(5261) 6890 (8662)
30%	(831) 1067 (1288)	(2241) 2655 (3449)	(3611) 4349 (5463)	(4857) 6043 (7554)	(6194) 7801 (9460)
40%	(1048) 1298 (1553)	(2568) 3080 (3923)	(3965) 4821 (6007)	(5400) 6787 (8041)	(6947) 8591 (10096)
50%	(1287) 1538 (1858)	(2822) 3461 (4261)	(4426) 5387 (6489)	(5909) 7390 (9066)	(7464) 9428 (11451)
60%	(1544) 1877 (2230)	(3107) 3960 (4520)	(4842) 6019 (7223)	(6384) 8092 (10082)	(8095) 10222 (12460)
70%	(1865) 2278 (2739)	(3569) 4574 (5461)	(5245) 6779 (8082)	(7092) 8897 (11926)	(8687) 11121 (14264)
80%	(2364) 2818 (3504)	(4112) 5379 (6651)	(6103) 7778 (9871)	(7779) 10085 (12847)	(9733) 12560 (16847)
90%	(3209) 3881 (4815)	(5003) 6698 (8487)	(6927) 9053 (10960)	(9082) 11899 (18611)	(11365) 14708 (21453)
95%	(4048) 4911 (6511)	(6099) 8112 (13114)	(8620) 11433 (16493)	(10881) 13775 (23860)	(13857) 16354 (28232)

VAAL DAM : Percentiles of m year inflow volumes with 90% confidence intervals

TABLE 5

P(Q) ≤ q	m = 1			m = 2			m = 3			m = 4			m = 5		
5%	(43)	57	(74)	(111)	155	(192)	(204)	273	(340)	(288)	391	(474)	(420)	515	(609)
10%	(61)	72	(101)	(155)	185	(221)	(261)	309	(371)	(351)	430	(502)	(443)	556	(650)
20%	(80)	92	(121)	(184)	213	(250)	(315)	349	(411)	(420)	485	(558)	(540)	612	(705)
30%	(92)	107	(139)	(217)	244	(280)	(341)	377	(439)	(468)	524	(607)	(591)	661	(765)
40%	(106)	123	(159)	(237)	267	(311)	(372)	412	(481)	(487)	556	(612)	(615)	709	(801)
50%	(118)	139	(180)	(262)	290	(340)	(394)	443	(508)	(510)	591	(680)	(662)	760	(867)
60%	(132)	157	(200)	(280)	314	(366)	(414)	472	(557)	(550)	640	(722)	(680)	791	(889)
70%	(150)	177	(236)	(305)	343	(403)	(440)	508	(599)	(573)	679	(764)	(709)	850	(962
80%	(167)	202	(270)	(322)	378	(434)	(466)	555	(611)	(603)	727	(836)	(744)	890	(1002)
90%	(188)	246	(317)	(357)	437	(497)	(509)	639	(700)	(626)	790	(882)	(801)	981	(1245)
95%	(211)	285	(354)	(378)	488	(541)	(523)	661	(751)	(700)	878	(1171)	(827)	1041	(1473)

MIDMAR : Percentiles of 'm' year inflow volumes with 90% confidence intervals

TABLE 6

P(Q) ⩽ q	m = 1	m = 2	m = 3	m = 4	m = 5
5%	(0,0) 8,4 (10,2)	(3) 57 (86)	(25) 135 (149)	(98) 227 (256)	(140) 326 (359)
10%	(7,7) 16,9 (21,3)	(25) 88 (101)	(130) 179 (248)	(138) 283 (326)	(221) 411 (495)
20%	(28,1) 37,4 (47,8)	(60) 133 (153)	(201) 256 (340)	(290) 377 (521)	(343) 514 (700)
30%	(46,6) 57,3 (70,0)	(141) 178 (214)	(255) 324 (463)	(377) 452 (670)	(441) 593 (819)
40%	(60,2) 83,2 (98,0)	(202) 230 (391)	(311) 377 (560)	(415) 526 (728)	(526) 682 (1004)
50%	(86,0) 116,0 (149,0)	(261) 285 (463)	(372) 440 (613)	(512) 606 (875)	(606) 770 (1206)
60%	(119,0) 153,0 (188,0)	(314) 339 (520)	(433) 519 (719)	(577) 689 (997)	(680) 869 (1460)
70%	(155,0) 198,0 (244,0)	(371) 406 (611)	(491) 593 (859)	(612) 767 (1157)	(762) 970 (1693)
80%	(212,0) 266,0 (332,0)	(420) 491 (754)	(570) 711 (1011)	(707) 912 (1376)	(869) 1126 (1978)
90%	(301,0) 379,0 (477,0)	(503) 630 (961)	(633) 870 (1432)	(829) 1147 (1655)	(904) 1329 (2311)
95%	(389,0) 494,0 (630,0)	(721) 771 (1160)	(719) 1098 (1782)	(902) 1322 (2019)	(1137) 1561 (2747)

KALKFONTEIN : Percentiles of 'm' year inflow volumes with 90% confidence intervals

TABLE 7

P(Q≤q)	h = 2 m = 1	h = 3 m = 1	h = 4 m = 1	h = 5 m = 1	h = 10 m = 1
5%	(259) 395 (486)	(221) 340 (451)	(190) 313 (429)	(167) 293 (408)	(151) 250 (396)
10%	(309) 482 (570)	(274) 425 (520)	(239) 390 (486)	(206) 360 (444)	(191) 305 (413)
20%	(497) 632 (762)	(466) 556 (711)	(421) 495 (662)	(361) 457 (587)	(329) 370 (559)
30%	(580) 765 (878)	(519) 650 (756)	(457) 582 (690)	(419) 540 (662)	(370) 425 (621)
40%	(706) 870 (988)	(625) 750 (874)	(555) 660 (791)	(492) 612 (733)	(421) 475 (640)
50%	(877) 1045 (1109)	(781) 862 (977)	(679) 755 (872)	(600) 694 (808)	(461) 530 (689)

P(Q≤q)		h = 3 m = 2	h = 4 m = 2	h = 5 m = 2	h = 10 m = 2
5%		(881) 1225 (1721)	(760) 1175 (1645)	(688) 1115 (1519)	(565) 950 (1366)
10%		(990) 1480 (1961)	(864) 1355 (1804)	(750) 1275 (1662)	(669) 1085 (1581)
20%		(1287) 1860 (2301)	(1089) 1665 (2130)	(944) 1535 (1927)	(821) 1270 (1762)
30%		(1542) 2175 (2692)	(1377) 1925 (2483)	(1245) 1765 (2246)	(1000) 1415 (1964)
40%		(1881) 2475 (3007)	(1621) 2170 (2863)	(1460) 1975 (2629)	(1308) 1565 (2108)
50%		(2211) 2805 (3462)	(1931) 2400 (3198)	(1699) 2175 (2888)	(1463) 1705 (2511)

P(Q≤q)			h = 4 m = 3	h = 5 m = 3	h = 10 m = 3
5%			(1990) 2360 (2602)	(1883) 2200 (2502)	(1511) 1920 (2320)
10%			(2469) 2720 (2975)	(2295) 2530 (2882)	(1854) 2150 (2545)
10%			(2822) 3300 (3499)	(2481) 3040 (3333)	(1921) 2460 (2820)
30%			(3303) 3790 (3961)	(3006) 3450 (3824)	(2490) 2730 (3261)
40%			(3727) 4250 (4318)	(3390) 3870 (4270)	(2554) 2970 (3487)
50%			(3965) 4670 (4821)	(3549) 4230 (4598)	(2843) 3190 (3581)

VAAL DAM : Percentiles of the 'm' year minimum annual inflow over a time horizon 'h' years (with 90% confidence intervals)

TABLE 8

P(Q≤q)	h = 2 m = 1	h = 3 m = 1	h = 4 m = 1	h = 5 m = 1	h = 10 m = 1
5%	(28) 44 (68)	(23) 36 (61)	(22) 35 (57)	(19) 32 (50)	(18) 25 (44)
10%	(39) 56 (83)	(32) 46 (70)	(30) 45 (66)	(26) 41 (65)	(24) 32 (60)
20%	(58) 72 (96)	(42) 63 (91)	(38) 57 (86)	(33) 53 (80)	(29) 42 (72)
30%	(71) 89 (118)	(56) 74 (99)	(47) 66 (98)	(44) 62 (89)	(37) 50 (83)
40%	(85) 102 (128)	(63) 83 (113)	(55) 75 (103)	(49) 70 (100)	(43) 56 (94)
50%	(93) 106 (136)	(74) 93 (127)	(60) 83 (111)	(53) 77 (106)	(49) 62 (101)

P(Q≤q)		h = 3 m = 2	h = 4 m = 2	h = 5 m = 2	h = 10 m = 2
5%		(94) 140 (186)	(81) 132 (170)	(76) 125 (164)	(69) 111 (150)
10%		(106) 161 (208)	(96) 151 (192)	(81) 143 (181)	(76) 124 (170)
20%		(129) 190 (228)	(105) 175 (211)	(94) 165 (196)	(83) 143 (184)
30%		(153) 214 (241)	(119) 195 (236)	(103) 182 (227)	(99) 156 (218)
40%		(177) 233 (266)	(138) 212 (259)	(127) 199 (139)	(117) 168 (227)
50%		(203) 254 (285)	(166) 227 (271)	(141) 215 (260)	(131) 180 (240)

P(Q≤q)		h = 4 m = 3	h = 5 m = 3	h = 10 m = 3
5%		(205) 249 (291)	(200) 239 (279)	(191) 206 (259)
10%		(222) 280 (330)	(216) 268 (321)	(203) 232 (311)
20%		(259) 317 (364)	(229) 300 (348)	(224) 264 (329)
30%		(288) 348 (399)	(243) 327 (377)	(238) 280 (360)
40%		(317) 377 (431)	(270) 352 (400)	(249) 299 (391)
50%		(350) 403 (363)	(291) 377 (441)	(265) 313 (427)

MIDMAR DAM : Percentiles of the 'm' year minimum annual inflow over a time horizon of 'h' years (with 90% confidence intervals)

TABLE 9

P(Q≤q)	h = 2, m = 1	h = 3, m = 1	h = 4, m = 1	h = 5, m = 1	h = 10, m = 1
5%	(3,1) 11,0 (21,3)	(0,6) 2,5 (15,6)	(0) 1,8 (10,9)	(0) 1,0 (8,4)	(0) 0,3 (6,2)
10%	(6,9) 13,8 (29,6)	(0,2) 5,5 (19,0)	(0) 3,8 (15,2)	(0) 2,5 (9,9)	(0) 1,3 (8,6)
20%	(10,0) 19,0 (40,2)	(5,1) 12,0 (29,7)	(1,8) 8,8 (21,3)	(0) 6,0 (14,7)	(0) 3,8 (11,7)
30%	(18,9) 28,8 (62,6)	(9,6) 19,0 (44,4)	(5,2) 14,0 (36,0)	(2,1) 11,0 (24,1)	(0) 5,8 (15,9)
40%	(29,0) 41,3 (89,0)	(14,0) 27,8 (53,2)	(9,9) 21,0 (42,2)	(6,0) 16,3 (29,0)	(1) 7,7 (20,0)
50%	(42,0) 55,5 (107,7)	(22,0) 39,0 (75,0)	(14,0) 27,0 (62,0)	(9,0) 22,0 (41,0)	(1) 10,0 (29,0)

P(Q≤q)		h = 3, m = 2	h = 4, m = 2	h = 5, m = 2	h = 10, m = 2
5%		(7) 47 (81)	(5) 40 (70)	(4) 35 (62)	(3) 21 (55)
10%		(19) 63 (96)	(15) 52 (84)	(12) 45 (76)	(9) 27 (70)
20%		(34) 96 (121)	(27) 76 (107)	(23) 65 (83)	(19) 40 (76)
30%		(66) 130 (159)	(50) 101 (126)	(40) 86 (108)	(28) 54 (92)
40%		(102) 163 (192)	(79) 129 (159)	(62) 104 (130)	(40) 68 (119)
50%		(137) 196 (230)	(105) 160 (202)	(91) 128 (179)	(68) 81 (154)

P(Q≤q)			h = 4, m = 3	h = 5, m = 3	h = 10, m = 3
5%			(87) 128 (150)	(72) 104 (141)	(57) 84 (130)
10%			(118) 153 (199)	(99) 116 (176)	(73) 95 (161)
20%			(137) 204 (240)	(120) 143 (206)	(86) 119 (190)
30%			(159) 255 (302)	(136) 165 (261)	(102) 143 (228)
40%			(206) 303 (357)	(145) 180 (330)	(120) 167 (307)
50%			(248) 352 (411)	(169) 186 (386)	(138) 195 (340)

KALKFONTEIN : Percentiles of the 'm' year minimum annual inflow over a time horizon of 'h' years (with 90% confidence intervals)

TABLE 10

T (years)	Q(T) $10^6 m^3$	90% confidence interval		
		Stedinger (1983)	Bootstrap	
			300 replicates	10000 replicates
100	292	378-207	392-193	378-204
50	355	449-259	466-270	444-291
10	616	743-487	754-454	743-487

VAAL DAM : Exact and Bootstrapped 90% confidence intervals for the T year annual drought inflow estimated from a lognormal model

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APPENDIX 1 SIX DISTRIBUTIONS

The six distributions considered in the main text are extensively discussed in the statistical literature, an excellent source being Johnson and Kotz (1970). The main reason for repeating this material here is to provide a convenient summary of only those properties of these distributions which are required in order to apply the methods described in the text. It is hoped that this will make the methods more accessible to non-statisticians who will be spared the necessity of extracting the relevant properties from statistical textbooks. In particular several explicit algorithms (which are seldom given in such books) will be given. Some of these are available as parts of packages on larger computer systems but not yet for micro-computers. The algorithms described here have been implemented on an IBM PC micro-computer.

Throughout this appendix we will use $x_1, x_2, \ldots, x_n$ to denote n observations which are assumed independently and identically distributed.

NORMAL DISTRIBUTION

Parameters

μ (location)

σ^2 (dispersion)

Moments

Expectation:	μ
Variance:	σ^2
Coefficient of variation:	σ/μ
Skew:	0

Probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2} , \qquad -\infty < x < \infty$$

Distribution function

$$F(z) = \int_{-\infty}^{z} f(x)\,dx , \qquad -\infty < z < \infty$$

There is no closed expression for F(z), but one has that

$$F(z) = \Phi\left(\frac{z-\mu}{\sigma}\right) ,$$

where Φ is the standard normal distribution function for which approximations are available. The approximation employed in the following algorithm is due to Hastings (1955); c.f. Abromowitz and Stegun, 1972):

```
STEP 1  INPUT    Z

STEP 2  SET      S = 0,3989423*EXP(-0,5*Z*Z)
                 T = 1/(1+0,2316419*ABS(Z))

STEP 3  SET      PHI = S*T*(0,3193815 - T*(0,3565638
                       -T*(1,781478 - T*(1,821256
                       -T*1,330274))))

STEP 4  TEST IF  Z > 0  THEN SET  PHI = 1 - PHI

STEP 5  OUTPUT   PHI
```

The absolute magnitude of the approximation error is less than 10^{-6}.

Percentage points

$Z(p) = F^{-1}(p) \quad , \quad 0 < p < 1$

Again no closed expression is available, but Z(p) may be computed using

$Z(p) = \Phi^{-1}(p)\sigma + \mu \quad , \quad 0 < p < 1$

The following algorithm for $\Phi^{-1}(p)$ (called INVPHI below) is from Abromowitz and Stegun (1972):

```
STEP 1  INPUT    P

STEP 2  TEST IF  P > 0,5  THEN SET  S = 1,  SET  T = 1-P
                          ELSE SET  S =-1,  SET  T = P

STEP 3  SET      U = SQR(-2*LOG(T))

STEP 4  SET      V = U - (2,515517 + U*(0,802853
                   + U*0,01328))/(1+U*(1,432788
                   + U*(0,189269 + U*0,001308)))

STEP 5  SET      INVPHI = S*V

STEP 6  OUTPUT   INVPHI
```

The absolute magnitude of the approximation error is less than $4,5*10^{-4}$.

Maximum likelihood estimators

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} x_i^2 - \hat{\mu}^2$$

Generating normal deviates

It is assumed that a random number generator which generates uniformly distributed random deviates between 0 and 1 is available. Several methods to generate normal deviates (using uniform deviates) are available. For example one can simply add a certain number (usually 10) uniform deviates and transform these using a suitable linear transformation. A second, and more accurate method is to generate a uniform deviate, R, and use $\Phi^{-1}(R)$ as normal deviate. The algorithm described below, due to Box and Muller (1958) is exact in the sense that the transformation from uniform to normal is exact.

The following algorithm can be used to generate pairs of standard normal random deviates.

STEP 1 *GENERATE* two uniform random deviates R1, R2

STEP 2 *SET* T = SQR(-2*LOG(R1))

STEP 3 *SET* X1 = T*SIN(6,283185*R2)
X2 = T*COS(6,283185*R2)

STEP 4 *OUTPUT* X1, X2

To generate deviates with mean μ and variance σ^2 one simply multiplies X1 and X2 by σ and adds μ.

LOGNORMAL DISTRIBUTION

Parameters

μ (location in log-space)
σ^2 (dispersion in log-space)

Moments

Expectation: $e^{\mu}e^{\sigma^2/2}$

Variance: $e^{2\mu}e^{\sigma^2}(e^{\sigma^2}-1)$

Coefficient of variation: $(e^{\sigma^2}-1)^{\frac{1}{2}}$

Skew: $(e^{\sigma^2}-1)^{\frac{1}{2}}(e^{\sigma^2}+3)$

Probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma x}\, e^{-(\ell n\, x-\mu)^2/2\sigma^2} \quad , \qquad x > 0 .$$

Distribution function

$$F(z) = \int_0^z f(x)\, dx \quad , \qquad z > 0.$$

There is no closed expression for F(x) but one has

$$F(z) = \Phi\left(\frac{\ell n\, z-\mu}{\sigma}\right) \quad , \qquad z > 0,$$

where Φ is the standard normal distribution for which an approximation was given above.

Percentage points

$$Z(p) = F^{-1}(p) \quad , \qquad 0 < p < 1$$

No closed expression is available for Z(p) but one has that

$$Z(p) = \exp(\Phi^{-1}(p)\sigma+\mu)$$

and an approximation for $\Phi^{-1}(p)$ was given above.

Maximum likelihood estimators

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} \ell n(x_i)$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} \ell n(x_i)^2 - \hat{\mu}^2$$

Generating lognormal deviates

To generate a lognormal deviate, X, with parameters μ and σ^2 one first generates a standard normal deviate, D, and sets

$$X = \exp(D\sigma+\mu)$$

An algorithm to generate standard normal deviates has already been given above.

EXPONENTIAL DISTRIBUTION

Parameter

$\theta > 0$ (scale)

Moments

Expectation:	θ
Variance:	θ^2
Coefficient of variation:	1
Skew:	2

Probability density function

$$f(x) = \frac{1}{\theta} e^{-x/\theta} \quad , \quad x > 0$$

Distribution function

$$F(z) = 1 - e^{-z/\theta} \quad , \quad z > 0$$

Percentage points

$$Z(p) = -\theta \, \ell n(1-p) \quad , \quad 0 < p < 1$$

Maximum likelihood estimator

$$\hat{\theta} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

Generating exponential deviates

$$X = -\theta \, \ell n(R)$$

where R is a uniform (0,1) random deviate.

WEIBULL DISTRIBUTION

Parameters

$\delta > 0$ (scale)
$\rho > 0$ (shape)

Moments

Expectation: $\delta\Gamma(1+1/\rho)$

Variance: $\delta^2\{\Gamma(1+2/\rho) - \Gamma(1+1/\rho)^2\}$

Coefficient of variation: $\{\Gamma(1+2/\rho)/\Gamma(1+1/\rho) - 1\}^{\frac{1}{2}}$

Skew $\{\Gamma(1+3/\rho) - 3\Gamma(1+2/\rho)\Gamma(1+1/\rho)^2 +$
$+ 2\Gamma(1+1/\rho)^3\}/\{\Gamma(1+2/\rho) - \Gamma(1+1/\rho)^2\}^{3/2}$

Probability density function

$$f(x) = (\tfrac{\rho}{\delta})(\tfrac{x}{\delta})^{\rho-1} e^{-(x/\delta)^{\rho}} \quad , \quad x > 0$$

Distribution function

$$F(x) = 1 - e^{-(x/\delta)^{\rho}} \quad , \quad x > 0$$

Percentage points

$$Z(p) = \delta[-\ell n(1-p)]^{1/\rho} \quad , \quad 0 < p < 1$$

Maximum likelihood estimators

The maximum likelihood estimation of ρ is the solution to the equation

$$\hat{\rho} = \{(\sum_{i=1}^{n} x_i^{\hat{\rho}} \ell n\, x_i)/(\sum_{i=1}^{n} x_i^{\hat{\rho}}) - \frac{1}{n} \sum_{i=1}^{n} \ell n\, x_i\}$$

This equation can only be solved numerically, an algorithm to do this is given below. Once $\hat{\rho}$ is available one can compute

$$\hat{\delta} = \{\frac{1}{n} \sum_{i=1}^{n} x_i^{\hat{\rho}}\}^{1/\hat{\rho}}$$

In the following algorithm to estimate ρ (RHO) and δ (DELTA) it is assumed that at least N = 3 observations are available. The notation *SUM* [ ] denotes the sum of the term in the brackets for I = 1,2,...,N.

```
STEP 1  INPUT     N, X(1), X(2),...,X(N), where the observa-
                  tions are arranged in increasing order of
                  magnitude.

STEP 2  SET        I1 = INT((N+1)/4), I2 = INT((N+1)*3/4)
                  RHO = LOG(LOG(1-I2/N)/LOG(1-I1/N))/
                                       LOG(X(I2)/(X(I1))
                    T = SUM [LOG(X(I))]/N

STEP 3  SET        T0 = SUM [X(I)**RHO]
                   T1 = SUM [LOG(X(I))*X(I)**RHO]
                   T2 = SUM [LOG(X(I)**2*X(I)**RHO]
                   T3 = RHO*(T1/T0-T) - 1
                   T4 = (T1/T0-T) + RHO*(T2*T0-T1*T1)/T0**2

                  RHO = RHO - T3/T4

STEP 4  TEST IF   ABS(T3) > 0,0001  THEN GO TO  STEP 3

STEP 5  SET       DELTA = (T0/N)**(1/RHO)

STEP 6  OUTPUT    RHO, DELTA
```

Notes:

(1) To enhance efficiency the computation of T0,T1,T2 and T3 in STEP 3 should be carried out in a single loop.

(2) If storage is available the quantities LOG(X(I)), should be computed once only in STEP 2 rather than repeatedly computed in the iteration.

(3) For some data sets no real maximum likelihood estimate exists. It is therefore advisable to limit the number of iterations, e.g. to stop if there is no convergence after 50 iterations.

Generating Weibul random deviates

$$X = \delta[-\ell n(R)]^{1/\rho}$$

where R is a Uniform (0,1) random deviate.

EXTREME (TYPE I) DISTRIBUTION

Parameters

ξ (location)

$\eta > 0$ (scale)

Moments

Expectation:	$\xi + \gamma\eta \approx \xi + 0{,}577216\eta$
Variance	$\pi^2\eta^2/6 \approx 1{,}644934\eta^2$
Coefficient of variation:	$1{,}282550\eta/(\xi + 0{,}577216\eta)$
Skew:	$1{,}13955$

Probability density function

$$f(x) = \frac{1}{\eta} \exp[-e^{-(x-\xi)/\eta}]e^{-(x-\xi)/\eta} \quad , \quad x > 0$$

Distribution function

$$F(z) = \exp[-e^{-(z-\xi)/\eta}] \quad , \quad z > 0$$

Percentage points

$$Z(p) = \xi - \eta\, \ell n(-\ell n\ (p)) \quad , \quad 0 < p < 1$$

Maximum likelihood estimators

The maximum likelihood estimator of η is the solution to the equation

$$\hat{\eta} = \frac{1}{n} \sum_{i=1}^{n} x_i - \left\{\sum_{i=1}^{n} x_i\ e^{-x_i/\hat{\eta}}\right\} / \left\{\sum_{i=1}^{n} e^{-x_i/\hat{\eta}}\right\}$$

This equation can only be solved numerically, an algorithm to do this is given below. Once $\hat{\eta}$ is available one can compute

$$\hat{\xi} = -\hat{\eta}\,\ell n\left(\frac{1}{n}\sum_{i=1}^{n} e^{-x_i/\hat{\eta}}\right)$$

In the following algorithm to estimate η (ETA) and ξ (XI) the notation *SUM* [ ] denotes the sum of the term in the bracket for I = 1,2,...,N

```
STEP 1  INPUT   N, X(1), X(2),...,X(N)

STEP 2  SET      XM = SUM [X(I)]/N
                 XV = SUM [X(I)*X(I)]/N - XM*XM
                ETA = 0,779698*SQR(XV)

STEP 3  FOR I = 1 TO N  SET Y(I) = EXP(-X(I)/ETA)

STEP 4  SET      T0 = SUM [Y(I)]
                 T1 = SUM [Y(I)*X(I)]
                 T2 = SUM [Y(I)*X(I)*X(I)]
                 T3 = ETA + T1/T0 - XM
                 T4 = 1 + (T2*T0-T1*T1)/(T0*ETA)**2
                ETA = ETA - T3/T4

STEP 5  TEST IF ABS(T3) > 0,0001  THEN GO TO  STEP 3

STEP 6  SET      XI = -ETA*LOG(T0/N)

STEP 7  OUTPUT  ETA, XI
```

Notes:

(1) To enhance efficiency the computation STEPS 3 and 4 should be carried out in a single loop. It is then also unnecessary to store the Y(I) values in an array.

(2) For some data sets no real maximum likelihood estimate exists. It is therefore advisable to limit the number of iterations, e.g. to stop after 50 iterations if there is no convergence by then.

Generating Extreme (Type I) deviates

$$X = \xi - \eta \, \ell n(-\ell n\,(R))$$

where R is a uniform (0,1) random deviate.

GAMMA DISTRIBUTION

Parameters

$\alpha > 0$ (shape)

$\beta > 0$ (scale)

Moments

Expectation:	$\alpha\beta$
Variance:	$\alpha\beta^2$
Coefficient of variation:	$\alpha^{\frac{1}{2}}$
Skew:	$2/\alpha^{\frac{1}{2}}$

Probability density function

$$f(x) = \frac{x^{\alpha-1}\ e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)} \quad , \qquad x > 0$$

Distribution function

$$F(z) = \int_0^z f(x)\ dx \quad , \qquad z > 0$$

There is no closed expression for $F(z)$, one has that

$$F(z) = \Gamma_{z/\beta}(\alpha)/\Gamma(\alpha)$$

where $\Gamma_x(\alpha)$ is the incomplete gamma function defined by

$$\Gamma_x(\alpha) = \int_0^x t^{\alpha-1}\ e^{-t}\ dt \quad , \qquad x > 0$$

Algorithms for this function, the complete gamma function, $\Gamma(\alpha)$, the digamma function, $\psi(\alpha)$, and the trigamma function, $\psi'(\alpha)$, are given at the end of this appendix. The latter two functions are needed for maximum likelihood estimation of the parameters.

Percentage points

$Z(p) = F^{-1}(p)$, $0 < p < 1$

No closed expression is available for Z(p), nor do we know of any accurate direct approximation. One way to solve the problem is to solve the equation

$F(Z) - p = 0$

for Z using an iterative method. An approximation for F was given above. The Newton Rapson method works well in this case, but of course one has to evaluate F a number of times and consequently there is a good deal of computation which needs to be carried out. This computation is nevertheless well within the capabilities of a typical microcomputer.

In the following algorithm it is assumed that subprograms to compute the incomplete gamma function $\Gamma_x(\alpha)$ and the gamma function, $\Gamma(\alpha)$ are available. We use the notation GAMINC (X,ALPHA) and GAMMA (ALPHA) for these subprograms.

```
STEP 1  INPUT      ALPHA, BETA, P

STEP 2  SET        Z = ALPHA*BETA
                   G = GAMMA (ALPHA)

STEP 3  SET        D = EXP(-Z/BETA)*(Z/BETA)**(ALPHA-1)/
                                          (BETA*G)
                   F = (GAMINC(C/BETA, ALPHA)-P)/D

STEP 4  TEST IF    F > Z  THEN SET  Z = Z/2,  GO TO STEP 3
                          ELSE SET  Z = Z - F

STEP 5  TEST IF  ABS(F) > 10^-6  THEN GO TO  STEP 3

STEP 6  OUTPUT   Z
```

Maximum likelihood estimation

The maximum likelihood estimation of α and β solutions to the equations

$$\psi(\hat{\alpha}) - \ell n\ (\hat{\alpha}) + \ell n\ (\tfrac{1}{n} \textstyle\sum_{i=1}^{n} x_i) - \tfrac{1}{n}(\textstyle\sum_{i=1}^{n} \ell n\ x_i) = 0$$

$$\hat{\beta} = \tfrac{1}{n}(\textstyle\sum_{i=1}^{n} x_i)/\hat{\alpha}$$

where ψ denotes the digamma function. In the following algorithm to solve these equations by iteration it is assumed that subprograms PSI( ) and PSID( ) to compute the digamma and trigamma functions are available. For explicit algorithms see the end of this appendix.

```
STEP 1  INPUT     N, X(1), X(2),...,X(N)

STEP 2  SET          XM = SUM [X(I)]/N
                    XLM = SUM [LOG(X(I)))]/N
                      R = LOG(XM) - XLM
                  ALPHA = (1 + SQR(1 + 4*R/3))/(4*R)

STEP 3  SET           H = (PSI(ALPHA) - LOG(ALPHA) + R)/
                              (PS1D(ALPHA) - 1/ALPHA)
                  ALPHA = ALPHA - H

STEP 4  TEST IF  ABS(H) > 0,0001  THEN GO TO  STEP 3

STEP 5  SET      BETA = XM/ALPHA

STEP 6  OUTPUT   ALPHA, BETA
```

Generating gamma deviates

Until recently no simple "exact" algorithm for generating gamma random deviates was available except for the case where α is an integer or half-integer. Direct solution of the equation

R = F(X)

where R is a uniform (0,1) random deviate using the algorithm described for computing the percentage points is not recommended because it involves substantial computing. In a typical simulation program hundreds of even thousands of random deviates are required. The well-known Wilson-Hilforty transformation (c.f. Johnson and Kotz, 1970) requires less computation but is rather inaccurate for small values of α. The following algorithm is due to Whittaker (1973).

```
STEP 1  INPUT     ALPHA, BETA

STEP 2  SET       IA = INT(ALPHA)
                  FA = ALPHA-IA
                  X1 = 0
                  X2 = 0

STEP 3  TEST IF   IA = 0  THEN GO TO  STEP 5

STEP 4  GENERATE  uniform random deviates R(1), R(2),...,R(IA)
                  X1 = SUM [-LOG(R(I))]

STEP 5  TEST IF   FA = 0  THEN GO TO  STEP 9

STEP 6  GENERATE  uniform random deviates R1, R2
        SET       S1 = R1**(1/FA)
                  S2 = R2**(1/(1-FA))

STEP 7  TEST IF   S1 + S2 > 1  THEN GO TO  STEP 6

STEP 8  GENERATE  a uniform random deviate R
        SET       X2 = -S1*LOG(R)/(S1 + S2)

STEP 9  SET        X = (X1 + X2)*BETA

STEP 10 OUTPUT     X
```

Notes:

(1) It is not necessary to actually store the array R(1), R(2),...,R(IA), above; they can be used directly and then discarded.

(2) This algorithm employs a rejection technique, that is unless S1 + S2 $\leq$ 1 STEP 6 has to be repeated. It can be shown that on average the number of rejections is not large, in fact the worst case (with FA = 0,5) involves about 21,5% rejections.

GAMMA FUNCTION

The algorithm given below is based on an asymptotic expansion of the gamma function:

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt \quad , \quad \alpha \neq 0, -1, -2, \ldots$$

For $\alpha \geqslant 10$ a 4-term Sterling approximation is used (Abromowitz and Stegun page 257). For $\alpha < 10$ the following recurrence relationship is applied in order to increase the argument of the gamma function to a number greater than or equal to 10.

$$\Gamma(\alpha+1) = \alpha\Gamma(\alpha) \quad , \quad \alpha \neq 0, -1, -2, \ldots$$

```
STEP 1  INPUT      ALPHA

STEP 2  SET        A = ALPHA
                   G = 1

STEP 3  TEST IF    A > 10   THEN GO TO   STEP 5

STEP 4  SET        G = G*A
                   A = A + 1
        GO TO      STEP 3

STEP 5  SET        T = (1+(0,0833333 + 3,47222E-3
                       -2,681327E-3/A)/A)/A
                   GAMMA = EXP(-A + (A-0,5)*LOG(A)+0,918939)*T/G

STEP 6  OUTPUT     GAMMA
```

DIGAMMA FUNCTION

The algorithm given below is based on an asymptotic expansion of the digamma function (Abromowitz and Stegun (1972), page 258):

$$\psi(\alpha) = \frac{d\ \ell n\ \Gamma(\alpha)}{dx} \quad , \quad \alpha \neq 0, -1, -2, \ldots$$

For $\alpha < 4$ the recurrence relationship

$$\psi(\alpha+1) = \psi(\alpha) + 1/\alpha \quad , \quad \alpha \neq 0, -1, -2, \ldots$$

is used repeatedly until the argument is greater than or equal to 4.

```
STEP 1  INPUT     ALPHA

STEP 2  SET       A = ALPHA
                  P = 0

STEP 3  TEST IF   A ≥ 4  THEN GO TO  STEP 5

STEP 4  SET       P = P - 1/A
                  A = A + 1
        GO TO     STEP 3

STEP 5  SET       T = 1/(A*A)
                  U = T*(8,333333E-2 - T*(8,333333E-3
                      - T*3,968254E-3))
                DIG = P + LOG(A) - 0,5/A - U

STEP 6  OUTPUT    DIG
```

TRIGAMMA FUNCTION

The algorithm given below is based on an asymptotic expansion (Abramowitz and Stegun (1972), page 260);

$$\psi'(\alpha) = \frac{d^2 \ \ell n \ \Gamma(\alpha)}{d\alpha^2} \quad , \qquad \alpha \neq 0, -1, -2, \ldots$$

For $\alpha < 4$ the recurrence relationship

$$\psi'(\alpha+1) = \psi'(\alpha) - 1/\alpha^2$$

is used repeatedly until the argument is greater than or equal to 4.

```
STEP 1  INPUT     ALPHA

STEP 2  SET       A = ALPHA
                  P = 0

STEP 3  TEST IF   A >= 4  THEN GO TO  STEP 5

STEP 4  SET       P = P - 1/(A*A)
                  A = A + 1
        GO TO     STEP 3

STEP 5  SET       T = 1/(A*A)
                  U = T*(1,666667E-1 - T*(3,333333E-2
                    - T*2,380953E-2))
               TRIG = P + 1/A + 0,5*T + U/A

STEP 6  OUTPUT    TRIG
```

INCOMPLETE GAMMA FUNCTION

The incomplete gamma function

$$\Gamma_x(\alpha) = \int_0^x x^{\alpha-1} e^{-t}\, dt \qquad , \qquad \alpha > 0, \quad x > 0$$

is required for the computation of probabilities associated with the gamma distribution. Several methods of approximating this function have been suggested, e.g. series expansions, a continued fraction representation, asymptotic expansions, etc. While each of these representations are suitable for approximating $\Gamma_x(\alpha)$ within particular ranges of the parameter α and the argument x, the programmer should beware of using any single representation for all α and x because for certain combinations of values nonsensical results are obtained on a digital computer.

The algorithm given below is based on a confluent hypergeometric representation of $\Gamma_x(\alpha)$ and is suitable for $\alpha \leqslant 50$ and for "reasonable" x values, i.e. x values which are such that $\varepsilon \leqslant \Gamma_x(\alpha)/\Gamma(\alpha) \leqslant 1-\varepsilon$ where $\varepsilon = 5\times10^{-7}$ (approximately). In other words, it is suitable for computing probabilities of the gamma distribution for x lying between the ε and $1-\varepsilon$ percentiles. This range easily contains the region of interest in practical applications.

```
STEP 1  INPUT     ALPHA, X

STEP 2  SET       T = 1
                  S = 1
                  L = 0

STEP 3  SET       T = T*((1+L)/(1+ALPHA+L))*(X/(L+1))
                  S = S + T

STEP 4  TEST IF   ABS(T/S) > 10^-7  THEN SET  L = L + 1
                                        GO TO STEP 3

STEP 5  SET       GAMINC = S*EXP(-X)*(X**ALPHA)/ALPHA

STEP 6  OUTPUT    GAMINC
```

Notes:

(1) The order of multiplication and division at STEP 3 is important to avoid "overflow" on the computer.

(2) For all but certain extreme combinations of α and x convergence is quite rapid (less than 20 terms).

(3) The accuracy of the above algorithm is dependent on the type of machine used. On a typical micro-computer one can expect 6 figure accuracy for $\Gamma_x(\alpha)/\Gamma(\alpha)$.

APPENDIX 2 SELECTING AN APPROXIMATING DISTRIBUTION

The theory described in this appendix was developed in collaboration with Professor H. Linhart of the Institut für Statistik und Ökonometrie, University of Göttingen, West Germany. The results of this research have been submitted for publication in the form of a paper with the above title. For the most part this appendix contains those extracts from the paper which are relevant to applications in hydrology.

We have included theory relating to the selection of discrete distributions. Although this material is not required for the methods described in the main text it is relevant to the study of the occurrence of storms, as is demonstrated in an example to follow, and can be applied to any other of the many discrete variables which arise in the context of hydrology.

1. DISCREPANCIES

Suppose that the observations $x_1, x_2, \ldots, x_n$ can be regarded as realisations of n independently and identically distributed random variables and that we wish to estimate their common distribution. As a rule a number of different models can be fitted to the data and the question arises as to which one should be selected.

Traditionally selection has been based on naive methods comparing the values of certain goodness-of-fit statistics. Cox (1961, 1962) considered the problem more systematically from the classical point of view. A bibliography of subsequent developments along these lines is given in Pereira (1977). We use a general approach (Linhart and Zucchini, 1982a, 1982b) which differs from the above. A well-known example which emerges in this framework is Akaike's (1973) Information Criterion.

Briefly we suppose that the operating model has some unknown distribution function F, and that a number of different (parametric) approximating families are available. Let F_θ, $\theta \in \Theta$, denote the distribution function corresponding to one of these approximating families.

Whichever approximating family of models is used there will be, as a rule, a number of discrepancies between the operating model and the model which is fitted to the data.

Each discrepancy describes some particular aspect of the "lack of fit". The relative importance of these discrepancies will differ according to the application at hand, and consequently it is suggested that the user should decide which discrepancy is to be minimized. In what follows we suppose that this decision has been made and we will refer to the selected discrepancy as *the* discrepancy.

It is a mapping

$$\Delta : M \times M \to \mathbb{R} \quad ,$$

where M is the space of all distribution functions, and should be such that for any F, G $\in$ M,

$$\Delta(F,G) \geqslant (F,F) \; .$$

For simplicity we write

$$\Delta(\theta) = \Delta(\theta,F) = \Delta(F_\theta,F) \quad .$$

Some examples of discrepancies are:

Kullback-Leibler: $\Delta(\theta) = -E_F \log f_\theta(x)$

Gauss: $\Delta(\theta) = \Sigma_x(f(x) - f_\theta(x))^2$

Kolmogorov: $\Delta(\theta) = \max\limits_x |F(x) - F_\theta(x)|$

Neyman-chisquared: $\Delta(\theta) = \Sigma_x(f(x) - f_\theta(x))^2/f(x)$, $f(x) \neq 0$

Pearson-chisquares: $\Delta(\theta) = \Sigma_x(f(x) - f_\theta(x))^2/f_\theta(x)$, $f_\theta(x) \neq 0$

Here f and f_θ are the probability functions or probability density functions corresponding to F and F_θ, respectively. In the case of continuous random variables the sums must be replaced by integrals.

We assume that each approximating family, F_θ, $\theta \in \Theta$, has a best member, F_{θ_o}, in the sense that

$$\theta_o = \arg\min_{\theta\in\Theta} \Delta(F_\theta,F)$$

exists and is unique. The corresponding discrepancy

$\Delta(\theta_o) = \Delta(F_{\theta_o}, F)$

is called the discrepancy due to approximation.

Let $\hat{\theta}$ be an estimator of θ_o, then $F_{\hat{\theta}}$ is the fitted model and

$\Delta(\hat{\theta}) = \Delta(F_{\hat{\theta}}, F)$,

the overall discrepancy, is a measure of the eventual lack of fit. It is a random variable whose distribution depends, inter alia, on what we call the fitting procedure, namely the approximating family and the associated method of estimation.

In principal any method of estimation could be combined with a given approximating family to provide a fitting procedure, but the most natural method would seem to be minimum discrepancy estimation:

If $\Delta_n(\theta)$ is an asymptotically unbiased estimator of $\Delta(\theta)$ then

$\hat{\theta}_n = \arg\min_{\theta \in \Theta} \Delta_n(\theta)$

is a minimum discrepancy estimator.

Such estimators are also known as minimum distance estimators. (Wolfowitz, 1952. See also the bibliography of Parr, 1981.) They are also M-estimators (see, e.g. Serfling, 1980).

As a basis for model selection one can either use $\Delta(\theta_o)$ or the distribution of $\Delta(\hat{\theta})$, (perhaps some characteristic of this distribution, typically the expectation). The latter takes errors due to estimation into account, and indicates how much can be achieved for the given sample size. On the other hand $\Delta(\theta_0)$ indicates what could be achieved in principle, not just for the given sample size.

2. ESTIMATING THE EXPECTED DISCREPANCY

To implement the above procedure one needs to derive the expected (overall) discrepancy, $E_F\Delta(\hat{\theta})$, and to find an estimator for it. The derivation is often, though not always, straightforward but the estimation presents difficulties. In many cases it is practically impossible to obtain a reasonable estimator with finite sample methods and one has to resort to asymptotic methods.

Under certain regularity conditions one can prove (Linhart and Volkers, 1983):

If $\hat{\theta}_n$ is a sequence of minimum discrepancy estimators then

$$E_F\Delta(\hat{\theta}_n) \approx \Delta(\theta_o) + \text{tr } \Omega^{-1}\Sigma/2n \quad ,$$

where

$$\Omega = \{\Omega_{ij}\} = \{\partial^2\Delta(\theta_o)/\partial\theta_i\partial\theta_j\}$$

and Σ is the asymptotic covariance matrix of

$$\sqrt{n}\{\partial\Delta_n(\theta_o)/\partial\theta_i\}, \qquad i = 1,2,\ldots,p,$$

where p = dimension (θ) is the number of free parameters,

$$E_F\Delta_n(\hat{\theta}) + \text{tr } \Omega^{-1}\Sigma/n \approx E_F\Delta(\hat{\theta}) \quad ,$$

$$E_F\Delta_n(\hat{\theta}) + \text{tr } \Omega^{-1}\Sigma/2n \approx \Delta(\theta_o) \quad .$$

The approximations are obtained by replacing the covariance matrix of $\sqrt{n}(\hat{\theta}_n-\theta_o)$ by its asymptotic covariance matrix and by omitting an $O_p(1/n)$ term. Under additional conditions on uniform integrability of certain sequences the error in the approximation is $O(1/n)$, and

$$\Sigma = (\lim_{n\to\infty} E_F n(\partial\Delta_n(\theta_o)/\partial\theta_i)(\partial\Delta_n(\theta_o)/\partial\theta_j)) \quad .$$

The results lead to asymptotically unbiased estimators of $E_F\Delta(\hat{\theta})$ and of $\Delta(\theta_o)$, i.e. to criteria.

If tr $\Omega^{-1}\Sigma$ is not known it is estimated by tr $\Omega_n^{-1}\Sigma_n$, where Ω_n and Σ_n are estimators of Ω and Σ and one uses

(I) $\quad \Delta_n(\hat{\theta}) + \text{tr } \Omega_n^{-1}\Sigma_n/n$

as criterion based on $E_F\Delta(\hat{\theta})$ and

(II) $\quad \Delta_n(\hat{\theta}) + \text{tr } \Omega_n^{-1}\Sigma_n/2n$

as criterion based on $\Delta(\theta_o)$.

For some discrepancies (including the Kullback-Leibler, Pearson-chisquared and Neyman-chisquared discrepancies) it can be shown that if the approximating family contains the operating model then tr $\Omega^{-1}\Sigma$ depends only on $p = \dim(\theta)$ (in the mentioned cases the trace is p and 2p respectively) and so simpler criteria can be given. For the Kullback-Leibler discrepancy for example this leads to

(I)* $\quad \Delta_n(\hat{\theta}) + p/n \quad ,$

(II)* $\quad \Delta_n(\hat{\theta}) + p/2n \quad ,$

i.e. Akaike's (1973) Information Criterion.

There are many situations where even the asymptotic methods outlined above lead to difficulties. Then one could use Bootstrap estimation methods which are particularly suitable in this context. These give directly estimators of $E_F\Delta(\hat{\theta})$ and it is not necessary to derive this expectation at all.

In our context the Bootstrap method introduced by Efron (1979) can be briefly explained as follows.

Let F be the operating distribution function and F_n the empirical distribution function. The expected discrepancy is in fact of the form $E_F\Delta(\hat{\theta},F)$ and a natural estimator of it is $E_{F_n}\Delta(\hat{\theta},F_n) = E_{F_n}\Delta_n(\hat{\theta})$. Note that F_n and Δ_n are fixed now, the only random variable is $\hat{\theta}$. The essential trick is to evaluate this last expectation by Monte Carlo methods. One generates repeated samples of size n from the now given and fixed distribution F_n. Each sample leads to another (Bootstrap) estimate $\hat{\theta}^*$ (usually obtained by minimizing another (Bootstrap) Δ_n^*) and a corresponding $\Delta_n(\hat{\theta}^*)$. The average of the generated $\Delta_n(\hat{\theta}^*)$ converges to $E_{F_n}\Delta_n(\hat{\theta})$. Furthermore the observed distribution of the generated $\Delta_n(\hat{\theta}^*)$ converges to the distribution of $\Delta_n(\hat{\theta})$ under F_n which is an estimate of the distribution of $\Delta(\hat{\theta})$ under F. So by means of Bootstrap methods one can not only estimate the expectation of the overall discrepancy but in fact its complete distribution.

An additional advantage of the Bootstrap method is the ease with which it is possible to switch from one discrepancy to another without the need for tedious derivations of theoretical results. The discrepancy discussed in Chapter 3 is a case in point. It would be very difficult indeed to obtain even approximate expressions for the expected discrepancy. Bootstrap methods allow us to circumvent this theoretical difficulty entirely. This is fortunate because otherwise we would not have been able to focus attention on the part of the distribution which is of interest, namely the lower tail.

3. AN EXAMPLE OF THE ASYMPTOTIC METHODS: THE KULLBACH-LEIBLER DISCREPANCY

The main advantage which asymptotic methods enjoy over Bootstrap methods is that they do not require much computational effort. There are two purposes to this section, firstly to illustrate how asymptotic results are obtained for a particular discrepancy and secondly to give the results for some of the important univariate distributions. The Kullback-Leibler discrepancy is the natural discrepancy if one uses maximum likelihood estimation and as such is clearly important. It is defined by

$$\Delta(\theta) = -E_F \log f_\theta(x) \quad ,$$

where f_θ is the probability density function of the approximating family, and where the expectation is taken with respect to the operating model. Here

$$\Delta_n(\theta) = -(1/n) \sum_{i=1}^{n} \log f_\theta(x_i) \quad ,$$

where x_i, $i = 1,2,...,n$, are the observations, and one has that the minimum discrepancy estimator is the maximum likelihood estimator.

This discrepancy was used by Akaike (1973) to develop his Information Criterion for model selection under the assumption that the operating model is a member of the approximating family. White (1982) analysed this problem rigorously and in his development allows for the possibility of misspecification.

For this discrepancy one obtains

$$\{\Omega\}_{rs} = -E_F\{\partial^2 \log f_{\theta_0}(x)/\partial\theta_r\partial\theta_s\}$$

$$\{\Sigma\}_{rs} = E_F\{(\partial \log f_{\theta_0}(x)/\partial\theta_r)(\partial \log f_{\theta_0}(x)/\partial\theta_s)\}$$

$$\{\Omega_n\}_{rs} = -(1/n)\left\{\sum_{i=1}^{n} \partial^2 \log f_{\hat{\theta}}(x_i)/\partial\theta_r\partial\theta_s\right\}$$

$$\{\Sigma_n\}_{rs} = (1/n)\left\{\sum_{i=1}^{n} (\partial \log f_{\hat{\theta}}(x_i)/\partial\theta_r)(\partial \log f_{\hat{\theta}}(x_i)/\partial\theta_s)\right\}$$

The criterion is

$$\Delta_n(\hat{\theta}_n) + \text{tr}\ \Omega_n^{-1}\Sigma_n/n$$

which can be approximated by $\Delta_n(\hat{\theta}) + p/n$ unless p, the number of parameters, is small, say 1 or 2.

We now give some of the intermediate results in the derivation of the criterion for the lognormal distribution and then simply list the criteria for a number of other important distributions.

The lognormal distribution

For this distribution the logarithm of the density function is

$$\log f_\theta(x) = -\log x - (1/2)\log 2\pi - (1/2)\log\lambda - (1/2\lambda)(\log x-\mu)^2 , \quad x > 0,$$

where, for convenience we have written λ instead of the usual σ^2.

One has

$$\partial \log f_\theta(x)/d\lambda = (1/2\lambda)((\log x - \mu)^2/\lambda - 1) ,$$

$$\partial \log f_\theta(x)/d\mu = (1/\lambda)(\log x - \mu) ,$$

and the maximum likelihood estimators are $\hat{\mu} = m_1'$ and $\hat{\lambda} = m_2$ where m_h' and m_h, $h = 1,2,\ldots,$ are the sample moments and sample moments about the sample mean respectively except that in this case (i.e. lognormal distribution) they are computed using the log x_i instead of the x_i, $i = 1,2,\ldots,n$. It follows that

$$\{\Sigma\}_{11} = (1/4\lambda_0^2)(E(\log x - \mu_0)^4/\lambda_0^2 - 2E(\log x - \mu_0)^2/\lambda_0+1),$$

$$\{\Sigma\}_{12} = \{\Sigma\}_{21} = (1/2\lambda_0^2)(E(\log x - \mu_0)^3/\lambda_0 - E(\log x - \mu_0)),$$

$$\{\Sigma\}_{22} = (1/\lambda_0^2)E(\log x - \mu_0)^2 \quad ,$$

$$\{\Sigma_n\}_{11} = (m_4-m_2^2)/4m_2^4 \quad ,$$

$$\{\Sigma_n\}_{12} = \{\Sigma_n\}_{21} = m_3/2m_2^3 \quad ,$$

$$\{\Sigma_n\}_{22} = 1/m_2 \quad .$$

Also

$$\partial^2 \log f_\theta(x)/\partial\mu^2 = -1/\lambda \quad ,$$

$$\partial^2 \log f_\theta(x)/\partial\mu\partial\lambda = -(\log x - \mu)/\lambda^2 \quad ,$$

$$\partial^2 \log f_\theta(x)/\partial\lambda^2 = 1/2\lambda^2 - (\log x - \mu)^2/\lambda^3 \quad .$$

The elements of $-\Omega$ are the expectations of these derivatives with respect to the operating model taken at the values λ_0 and μ_0 and it follows immediately that

$$\{\Omega_n\}_{11} = 1/2m_2^2 \quad ,$$

$$\{\Omega_n\}_{12} = \{\Omega_n\}_{21} = 0 \quad ,$$

$$\{\Omega_n\}_{22} = 1/m_2 \quad ,$$

thus $\mathrm{tr}\,\Omega_n^{-1}\Sigma_n = (m_4+m_2^2)/2m_2^2$ and the criterion is

$$-\textstyle\sum_{i=1}^{n} \log f_{\hat\theta}(x_i)/n + \mathrm{tr}\,\Omega_n^{-1}\Sigma_n/n$$

$$= m_1' + (1/2)(\log 2\pi m_2 + 1) + (m_4 + m_2^2)/2m_2^2 n \quad .$$

For the <u>normal</u> distribution the criterion is

$$(1/2)(\log 2\pi m_2 + 1) + (m_4 + m_2^2)/2m_2^2 n \quad ,$$

where the sample moments are computed using the original data rather than their logarithms.

The criterion for the <u>gamma</u> distribution with density

$$f_\theta(x) = \beta^\alpha x^{\alpha-1} e^{-\beta x}/\Gamma(\alpha) \quad , \qquad x \geq 0 \quad ,$$

is given by

$$\log \Gamma(\hat\alpha) - \hat\alpha(\log \hat\beta - 1) - (\hat\alpha - 1)m_1'[\log x] + \mathrm{tr}\,\Omega_n^{-1}\Sigma_n/n \quad ,$$

where $\hat\alpha$ and $\hat\beta$ are the maximum likelihood estimators of α and β, i.e. the solutions to the equations

$$\hat\alpha\hat\beta = m_1'[x]$$

$$\log \hat\beta - \psi(\hat\alpha) = -m_1'[\log x] \quad .$$

Here $\psi(z) = \partial \log \Gamma(z)/\partial z$ is Euler's ψ-function and $m_h'[\ ]$ and $m_h[\ ]$ denote the sample moments of the variables in the brackets. One also needs:

$\{\Sigma_n\}_{11} = m_2[x]$,

$\{\Sigma_n\}_{12} = -m_{11}[x, \log x]$,

$\{\Sigma_n\}_{22} = m_2[\log x]$,

$\{\Omega_n\}_{11} = m_1'[x]^2/\hat{\alpha}$,

$\{\Omega_n\}_{12} = -m_1'[x]/\hat{\alpha}$,

$\{\Omega_n\}_{22} = \psi'(\hat{\alpha})$.

For the Poisson distribution with parameter λ it is easy to show that tr $\Omega_n^{-1}\Sigma_n = m_2/m_1'$ and the criterion is

$$m_1'(1 - \log m_1') + m_1'[\log \Gamma(x+1)] + m_2/m_1'n \quad .$$

The geometric distribution with probability function

$$f_\theta(x) = \theta(1-\theta)^x \quad , \qquad x = 0,1,2,...,$$

has tr $\Omega_n^{-1}\Sigma_n = m_2/m_1'(1+m_1')$ and criterion given by

$$(1+m_1') \log (1+m_1') - m_1' \log m_1' + m_2/m_1' (1+m_1')n \quad .$$

For the negative binomial distribution with probability function

$$f_\theta(x) = \Gamma(\beta+x)(1-\alpha)^\beta\alpha^x/(\Gamma(\beta)\Gamma(x+1)) \quad , \qquad x = 0,1,2,...,$$

the maximum likelihood estimators are the solutions to the equations

$$\textstyle\sum_{k=1}^{\infty} F(k)/(\beta+k-1) + n \log (\beta/(\beta+m_1')) = 0 \quad ,$$

$$\hat{\alpha} = m_1'/(\hat{\beta}+m_1') \quad ;$$

where F(k) is the number of observations greater or equal to k, k = 1,2,... . It can be shown that:

$$\{\Sigma_n\}_{11} = m_2[x]/\hat{\alpha}^2 ,$$

$$\{\Sigma_n\}_{12} = m_{11}[x,\psi(\hat{\beta}+x)]/\hat{\alpha} ,$$

$$\{\Sigma_n\}_{22} = m_2[\psi(\hat{\beta}+x)] ,$$

$$\{\Omega_n\}_{11} = \hat{\beta}/\hat{\alpha}(1-\hat{\alpha})^2 ,$$

$$\{\Omega_n\}_{12} = 1/(1-\hat{\alpha}) ,$$

$$\{\Omega_n\}_{22} = \psi'(\hat{\beta}) - m_1[\psi'(\hat{\beta}+x)] .$$

The criterion is

$$-m_1'[\log \Gamma(x+\hat{\beta})] - \hat{\beta} \log (1-\hat{\alpha}) ,$$

$$-m_1'[x] \log \hat{\alpha} + m_1'[\log \Gamma(x+1)] + \log \Gamma(\hat{\beta}) + \mathrm{tr}\, \Omega_n^{-1}\Sigma_n/n .$$

4. NEYMAN'S AND PERSON'S CHISQUARED DISCREPANCIES

If one has n independent observations on a discrete variable the frequencies of the k different possible values are multinomially distributed. The operating model is the saturated model: the k-1 free parameters are simply the probabilities of the possible values of the variable. The same holds if data (on discrete or continuous variables) are grouped into k categories.

For such situations we developed (1983) model selection criteria which use the classical chisquared goodness of fit statistics. The two discrepancies, empirical discrepancies and criteria are the following.

	Discrepancy	Empirical discrepancy	Criterion
Neyman-chisq.	$\Sigma_i(\pi_i - f_i(\theta))^2/\pi_i$	$\Sigma_i(p_i - f_i(\theta))^2/p_i$	$\chi^2_N + 2p$
Pearson-chisq.	$\Sigma_i(\pi_i - f_i(\theta))^2/f_i(\theta)$	$\Sigma_i(p_i - f_i(\theta))^2/f_i(\theta)$	$\chi^2_p + 2p$

Here π_i and $f_i(\theta)$ are the probabilities under the operating and the approximating model and Neyman's (χ^2_N) and Pearson's (χ^2_p) chisquared are the empirical discrepancies at $\theta = \hat{\theta}$.

The estimator $\hat{\theta}$ can either be maximum likelihood or minimum (Neyman and Pearson) chisquared estimator. As a rule one should use in the fitting procedure the minimum discrepancy. $\hat{\theta}$ will thus usually be minimum chisquared estimator (Neyman or Pearson, respectively) if the Neyman-chisquared or Pearson-chisquared discrepancy is used.

For the saturated model the criterion is for both discrepancies equal to 2(k-1).

5. A DISCREPANCY WHICH EMPHASISES THE FIT IN THE LOWER TAIL OF A DISTRIBUTION

The discrepancy which was discussed in Chapter 3 of the main text is:

$$\Delta(F_\theta, F, d) = \max_x |F(x)^d - F_\theta(x)^d| \quad .$$

Now both F_θ and F are distribution functions and are therefore non-decreasing and take on values in the interval [0, 1]. For any distribution function, say G(x), and any d in the interval (0, 1) one has that

$$K(x,d) = G(x)^d - G(x) \geq 0$$

and as d is decreased so this difference becomes smaller (except for G(x) = 0 or 1). Also it is easy to show that

$$K(x_1,d) \geq K(x_2,d) \quad \text{for} \quad x_1 < x_2 \quad ,$$

and that for all x which are such that G(x) > 0

$$\lim_{d \to o} G(x)^d = 1 \quad .$$

It follows from these two properties that by selecting d-small enough

$$|F(x)^d - F_\theta(x)^d|$$

in the above discrepancy will be relatively unaffected by differences between F(x) and $F_\theta(x)$ in the upper tail of the distribution whereas for those values of x for which F(x) and $F_\theta(x)$ are small (i.e. the lower tail) the differences between these two functions will contribute more substantially to the discrepancy.

An asymptotically unbiased estimator of this discrepancy, i.e. an empirical discrepancy is

$$\Delta_n(\theta,d) = \max_{1 \leq i \leq n} |i/(n+1)^d - F_\theta(x_i)^d|$$

The expected overall discrepancy, i.e. $E_F\Delta_n(\hat{\theta},d)$ is intractable. However we can estimate it by means of Bootstrap methods. An algorithm to do this was outlined in Chapter 3.

6. EXAMPLES OF APPLICATION

6.1 Annual flow of the Vaal river at Standerton

The table below gives the annual flow of the Vaal river at Standerton in the years 1905 to 1969. The flow is in millions of cubic meters measures over "water years", i.e. October to September.

1905	222	1927	235	1949	534
1906	1095	1928	346	1950	129
1907	452	1929	778	1951	317
1908	1298	1930	95	1952	640
1909	882	1931	111	1953	291
1910	889	1932	78	1954	1461
1911	276	1933	554	1955	611
1912	216	1934	364	1956	809
1913	103	1935	460	1957	637
1914	490	1936	1151	1958	336
1915	446	1937	286	1959	245
1916	386	1938	1401	1960	686
1917	2580	1939	651	1961	319
1918	408	1940	746	1962	365
1919	258	1941	224	1963	306
1920	606	1942	568	1964	479
1921	715	1943	1593	1965	42
1922	1539	1944	217	1966	683
1923	183	1945	496	1967	250
1924	696	1946	256	1968	324
1925	110	1947	295	1969	556
1926	193	1948	274		

Annual flow [million m^3] of the Vaal river at Standerton

We consider three approximating families; the normal, lognormal and gamma. The fact that the normal distribution yields a non-zero probability for the event that the flow is negative is not of practical importance unless this probability is not sufficiently small.

For the purposes of comparing the asymptotic results given in section 3 with the results of Bootstrap methods we will use firstly the Kullback-Leibler discrepancy.

The criteria for the three models using the results of section 4 are:

Normal:	7,517 + 4,763/65 = 7,590
Lognormal:	7,184 + 2,110/65 = 7,216
Gamma:	7,199 + 2,227/65 = 7,233

The lognormal distribution has the smallest value of the criterion and is judged to be the most appropriate.

The results of 100 Bootstrap simulations yielded:

	mean	standard deviation
Normal	7,552	0,060
Lognormal	7,202	0,021
Gamma	7,216	0,020

With 100 simulations the given means estimate $E_{F_n} \Delta(\hat{\theta}, F_n)$ (which in turn estimates $E_F \Delta(\hat{\theta}, F)$) with a standard error of 0,006 (normal) and 0,002 (lognormal and gamma).

These means, the Bootstrap criteria, are very close to the values of the asymptotic criteria. The rank order is preserved, the lognormal distribution is deemed to be the most appropriate.

When the Kolmogorov discrepancy was applied in connection with maximum likelihood estimation the following Bootstrap estimates were obtained.

	mean	standard deviation
Normal	0,164	0,040
Lognormal	0,092	0,032
Gamma	0,102	0,032

The lognormal distribution emerges also here as the most appropriate.

6.2 Annual number of storms in Pretoria

The table below gives the annual number of storms observed in Pretoria (Station 513/404) for the 71 years 1906 to 1976.

Number of Storms	1	2	3	4	5	6	7	8	
Frequency	2	2	3	4	10	8	10	6	
Number of Storms	9	10	11	12	13	14	15	16	17
Frequency	5	7	5	3	0	2	2	1	1

Annual number of storms

For the purposes of deriving the distribution of large storms it is of interest to hydrologists to estimate the distribution of the annual number of storms. In this context the two approximating families of interest are the Poisson and negative binomial.

Using the asymptotic results for the Kullback-Leibler criterion one obtains the following criteria.

Poisson: $2,714 + 1,606/71 = 2,737$

Negative Binomial: $2,593 + 1,937/71 = 2,620$

The negative binomial distribution is preferable.

By means of Bootstrap methods (100 replications) we obtained

	mean	standard deviation
Poisson	2,837	0,825
Negative Binomial	2,608	0,451

There is again good agreement between the Bootstrap means (standard errors 0,08 and 0,05) and the criteria.

For the Pearson-chisquared discrepancy and fitting procedures using minimum chisquared estimators the criterion of section 4 gives:

Poisson: 45,5
Negative Binomial: 13,6

The Bootstrap results, based on 100 replications, were

	mean	standard deviation
Poisson	45,7	61,8
Negative Binomial	13,9	37,6

The standard deviation of the Bootstrap distribution is exceptionally large. The reason is the strong dependence of the chisquared discrepancy on the tail of the distribution and the tails of repeated small samples from long tailed distributions vary considerably.

6.3 Annual maximum storms at Vryheid

The standard statistical method of estimating design storms (and other design events such as floods), from observed annual maximum storms can be described briefly as follows:

Let F be the distribution function of annual maximum storms. Then under the assumption that these storms are independently and identically distributed, the distribution function of the largest storm in h years is F^h. So the storm associated with a design horizon of h years and risk (i.e. probability) of occurrence r, is the solution to the equation

$$F^h(x) = 1-r \quad .$$

The design storm, x, is estimated by fitting a model to F, say $F_{\hat{\theta}}$, where θ are the parameters, and then using

$$\hat{x} = F_{\hat{\theta}}^{-1}((1-r)^{1/h}) \quad .$$

The risk, r, is given and is seldom more than 0,20 in practice, so one is dealing with the upper tail of the distribution function F. As h is increased so the design storm becomes associated with increasingly extreme values of this tail.

Clearly for this application it is important to select approximating models which fit the upper tail of the distribution function F well. The lower tail is not so important. Also, in order to take account of the relevant portion of the distribution, the discrepancy should be a function of design horizon. Since it is F^h which is finally of interest (rather than F), a discrepancy which seems reasonable and which satisfies the above desiderata is

$$\Delta(F_\theta, F; h) = \max_x |F(x)^h - G_\theta(x)^h| \quad .$$

This discrepancy was applied to select an approximating model for the data given below, viz. the annual maximum one-day storms at Vryheid.

Year	depth	year	depth	year	depth
1951	45,2	1961	52,5	1971	84,5
1952	66,5	1962	50,0	1972	74,5
1953	142,0	1963	170,0	1973	94,0
1954	83,9	1964	62,0	1974	80,0
1955	61,1	1965	43,5	1975	74,0
1956	60,6	1966	60,0	1976	64,0
1957	84,5	1967	60,0	1977	60,0
1958	80,0	1968	53,5	1978	51,5
1959	79,0	1969	58,0	1979	58,5
1960	137,5	1970	93,0	1980	88,0

Annual maximum 24-hour storm depths [mm] at Vryheid, 1951-1980.

The following families were considered: gamma, normal, lognormal, exponential, Weibull, Extreme (type I).

The following empirical discrepancy, based on the "Weibull plotting position", was used:

$$\Delta_n(\theta) = \max_{1 \leq i \leq n} |(i/(n+1))^h - F_\theta(x_i)^h| \quad .$$

Maximum likelihood estimation was used throughout.

Gamma:	$\hat{\alpha} = 8,97$	$\hat{\beta} = 8,44$
Normal:	$\hat{\mu} = 75,7$	$\hat{\sigma} = 28,6$
Lognormal:	$\hat{\mu} = 4,27$	$\hat{\sigma} = 0,321$
Exponential:	$\hat{\theta} = 75,7$	
Weibull:	$\hat{\rho} = 2,68$	$\hat{\delta} = 85,1$
Extreme (I):	$\hat{\xi} = 64,1$	$\hat{\eta} = 17,9$

The table below gives the Bootstrap estimates of the mean and standard deviation of the overall discrepancy.

Design horizon, h		Gamma	Normal	Lognorm.	Expon.	Weibull	Extr.I	Smallest mean
1	mean	0,1676	0,1922	0,1560	0,4200	0,1801	0,1545	Extr. I
	std.dev.	0,0352	0,0352	0,0349	0,0250	0,0288	0,0363	
5	mean	0,3131	0,3318	0,3008	0,3829	0,3168	0,3261	Lognorm.
	std.dev.	0,0696	0,0657	0,0701	0,0786	0,0489	0,0882	
10	mean	0,4493	0,4697	0,4258	0,4569	0,4249	0,4593	Weibull
	std.dev.	0,0931	0,0934	0,0865	0,0727	0,0713	0,1065	

Estimates of expectation and standard deviation overall discrepancy based on 200 Bootstrap replications.

On the basis of this criterion the extreme (type I) distribution should be used for h = 1, the lognormal for h = 5 and the Weibull for h = 10. However, for h = 1 and h = 10 the lognormal distribution leads to criteria which are quite close to those minima and consequently it would not be unreasonable to use the lognormal distribution for each case.

7. CONCLUDING REMARKS

The universal applicability of the Bootstrap method and the ease with which it can be implemented makes it particularly attractive for the purpose of selecting a univariate distribution family. Although we have not discussed this aspect here, the method can also be applied to compare parametric models (such as the normal, gamma etc ..) to "distribution-free" models (histogram-type densities) which are frequently used by hydrologists. It has however been our experience that for the sample sizes which are usually available parametric models are preferable by far. In the terminology of section 1 they lead to significantly lower discrepancies due to estimation, distribution-free models may fit the observed sample better but in general they fit the operating model, i.e. the underlying distribution, less well.

APPENDIX 3

APPLICATION OF THE MODEL SELECTION CRITERIA TO 67 ANNUAL INFLOW RECORDS

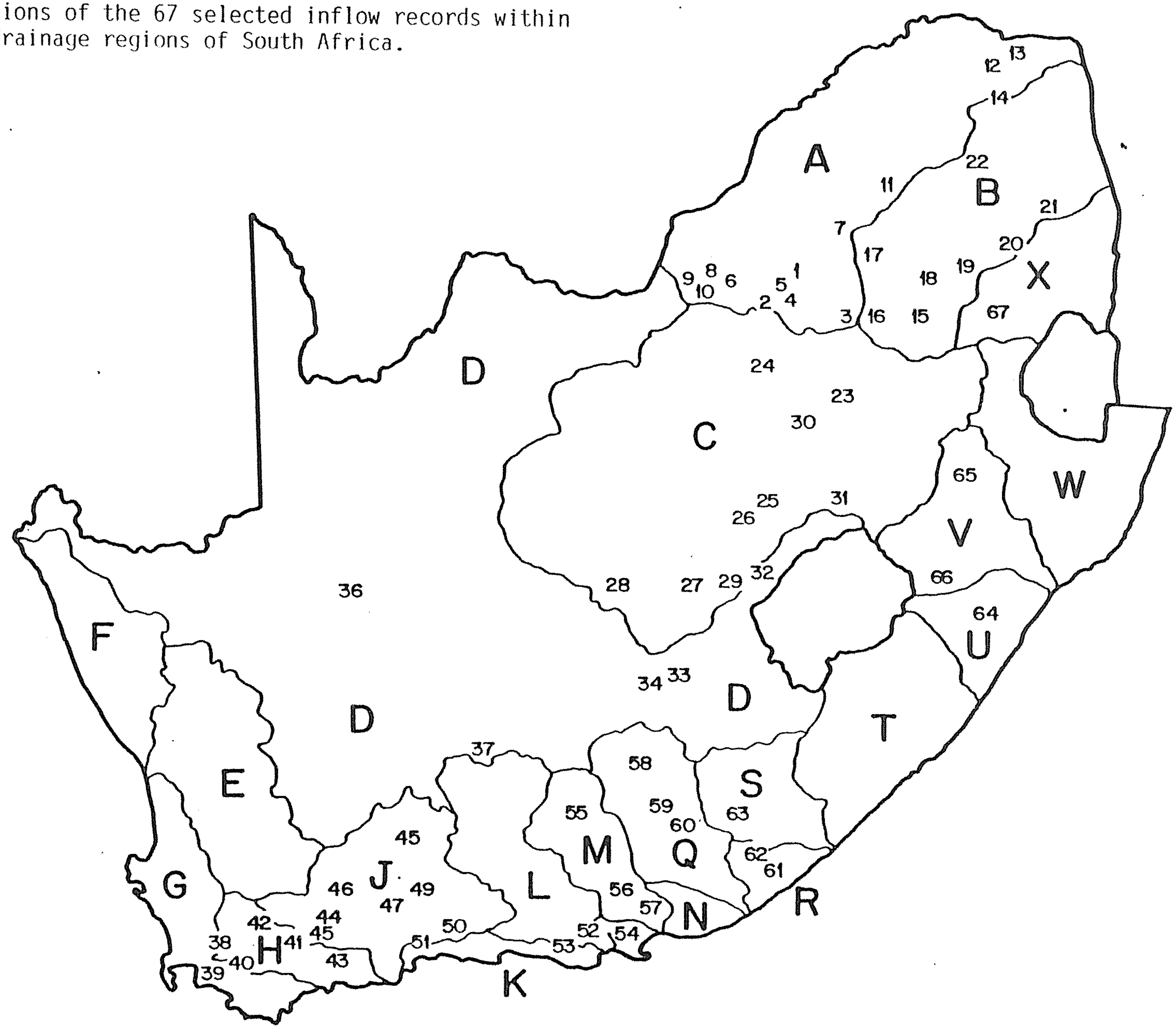
The model selection methods described in Appendix 2 were applied to 67 selected reservoir annual inflow records whose positions are indicated in Figure A3.1. Each of the six models discussed in Appendix 1 was fitted to each record and the corresponding parameter estimates are given, as are the original data.

Three selection criteria were computed for each model based on the discrepancy described in Chapter 3. These correspond to values of d = 1,00 ; 0,50 and 0,25 (represented on the printout by the letter H). The model leading to the smallest value of the criterion should be selected. For strong emphasis on the fit in the lower tail of the distribution H = 0,25 should be used, whereas for an overall best fit H = 1,00 is more appropriate.

Also given are estimates of the serial correlation coefficients of lag 1 together with approximate critical points for the null hypothesis that the population values are equal to zero (at the 95% level of significance). This hypothesis is rejected in only 7 out of the 67 cases. However it should be kept in mind that at this level of significance and with 67 independent tests one would expect 3 out of the 67 to be rejected even if the null hypothesis were true in all the cases. Some of the high estimates are probably due to non-stationarity rather than "ordinary" serial correlation (c.f. Nzhele Dam).

FIGURE A3.1

Positions of the 67 selected inflow records within the drainage regions of South Africa.



ALPHABETICAL LIST OF DAM INFLOWS ANALYSED IN THIS APPENDIX.
(.....). STATION NO.; (..) MAP KEY.

Albasini	(A9R01)	(14)	Lindley's Poort	(A2R07)	(6)
Allenmanskraal	(CHR01)	(25)	Loskop	(B3R02)	(18)
Armenia	(D2R02)	(32)	Luphephe/Nwanedze	(A8R02)	(13)
Beervlei	(L3R01)	(52)	Marico-Bosveldt	(A3R01)	(8)
Bellair	(JIR02)	(45)	Menin	(C8R01)	(31)
Bethulie	(D3R01)	(33)	Mentz	(N2R01)	(56)
Boskop	(C2R01)	(24)	Midmar	(U2R01)	(64)
Bospoort	(A2R06)	(5)	Nooitgedacht	(X1R01)	(67)
Bronkhorstspruit	(B2R01)	(16)	Nuwe Doringpoort	(B1R01)	(15)
Buffelspoort	(A2R05)	(4)	Nzelele	(A8R01)	(12)
Calitzdorp	(J2R01)	(47)	Ohrigstad	(B6R01)	(20)
Chelmsford	(V3R01)	(65)	Olifantsnek	(A2R03)	(3)
Doorndraai	(A6R01)	(11)	Oukloof	(J2R03)	(49)
Duiwenhok	(H8R01)	(43)	Paul Sauer	(L8R01)	(53)
Ebenezer	(B8R01)	(22)	Poortjieskloof	(H3R01)	(41)
Erfenis	(C4R02)	(26)	Prinzrivier	(J1R01)	(44)
Floriskraal	(J1R03)	(46)	Rietvlei	(A2R04)	(3)
Grassridge	(Q1R01)	(58)	Rooiberg	(D5R01)	(36)
Groendal	(M1R01)	(54)	Rooikrans	(R2R02)	(62)
Hartebeespoort	(A2R01)	(1)	Rust de Winter	(B3R01)	(17)
H.F. Verwoerd	(A3R02)	(34)	Rustfontein	(C5R03)	(29)
Kalkfontein	(C5R02)	(28)	Slagboom	(N4R01)	(57)
Kommanassie	(J3R01)	(50)	Steenbras	(GAR01)	(39)
Keerom	(H4R02)	(42)	Stettynskloof	(H1R01)	(40)
Klasierie	(B7R01)	(21)	Tierpoort	(C5R01)	(27)
Klein Marico	(A3R02)	(9)	Tonteldoos	(B4R01)	(19)
Kommandodrif	(Q2R02)	(60)	Vaal	(C1R01)	(23)
Koppies	(C7R01)	(30)	Vanryneveldtspas	(N1R01)	(55)
Krommellenboog	(A3R03)	(10)	Victoria West	(D6R01)	(37)
Kromrivier	(K9R01)	(51)	Wagendrif	(V7R01)	(66)
Laing	(R2R01)	(61)	Warmbad	(A2R08)	(7)
Lake Arthur	(Q1R04)	(59)	Waterdown	(S3R01)	(63)
Leeubos	(D4R01)	(35)	Wemmershoek	(G1R02)	(38)
Leeugamka	(J2R02)	(48)			

(1) HARTEBEESPOORT DAM. A2R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920				66.40	270.00	95.90	72.20	121.00	141.00	152.00
1930	73.50	54.50	32.80	235.00	80.10	145.00	182.00	98.60	155.00	113.00
1940	156.00	86.40	221.00	621.00	194.00	164.00	88.50	127.00	74.30	170.00
1950	98.50	63.90	91.30	72.90	350.00	148.00	204.00	178.00	126.00	79.20
1960	99.90	50.30	60.80	79.20	70.10	43.70	342.00	82.20	52.80	79.00
1970	177.10	166.30	89.30	152.00	418.00	475.70	413.40	644.80	154.40	280.00
1980	233.70	147.30								

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.255	0.361	-0.255

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	164.6441	SIGMA=	131.0841
LOGNORMAL	MU=	4.8711	SIGMA=	0.6612
GAMMA	ALPHA=	2.3021	BETA=	71.5183
WEIBULL	RHO=	1.4406	DELTA=	183.5580
EXTREME-1	XI=	113.9954	ETA=	74.4778
EXPONENTIAL	THETA=	164.6441		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2042	0.2740	0.2751
LOG NORMAL	0.1140	0.0995	0.0966
GAMMA	0.1407	0.1365	0.1429
WEIBULL	0.1392	0.1906	0.1978
EXTREME TYPE-1	0.1495	0.1385	0.1439
EXPONENTIAL	0.2279	0.3289	0.3105

(2) OLIFANTSNEK DAM. A2R03.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920									13.00	8.07
1930	7.33	2.79	1.49	23.20	11.00	14.70	16.00	9.50	14.40	8.72
1940	17.90	7.97	14.10	57.20	15.70	9.99	5.95	10.70	3.92	9.06
1950	7.84	4.65	7.37	2.95	25.10	4.98	13.40	28.50	11.00	3.28
1960	9.00	2.79	1.27	2.86	3.56	11.30	62.90	2.75	2.47	2.07
1970	13.80	8.70	4.90	7.50	16.00	111.90	24.50	57.30	5.19	
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.274	0.094	-0.274

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	14.5592	SIGMA=	19.2939
LOGNORMAL	MU=	2.1842	SIGMA=	0.9590
GAMMA	ALPHA=	1.1500	BETA=	12.6598
WEIBULL	RHO=	0.9903	DELTA=	14.4895
EXTREME-1	XI=	8.3429	ETA=	8.4174
EXPONENTIAL	THETA=	14.5592		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2605	0.3394	0.3181
LOG NORMAL	0.1005	0.0969	0.0889
GAMMA	0.1489	0.1426	0.1489
WEIBULL	0.1381	0.1708	0.1767
EXTREME TYPE-1	0.1638	0.1756	0.1856
EXPONENTIAL	0.1521	0.1796	0.1843

(3) RIETVLEI DAM. A2R04.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930				12.10	2.81	14.60	16.20	1.96	9.87	9.09
1940	12.10	6.89	13.40	37.50	14.40	8.72	4.11	6.05	4.36	11.40
1950	5.12	3.63	8.58	6.17	17.90	10.60	14.40	9.66	6.38	5.78
1960	4.97	1.58	1.28	3.55	5.70	5.56	34.70	7.35	4.76	9.97
1970	15.49	28.13	8.48	17.30	82.93	48.69	18.99	96.51	14.34	
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.289	0.259	-0.289

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	14.8709	SIGMA=	18.7862
LOGNORMAL	MU=	2.2481	SIGMA=	0.9125
GAMMA	ALPHA=	1.2483	BETA=	11.9125
WEIBULL	RHO=	1.0303	DELTA=	15.0931
EXTREME-1	XI=	8.7413	ETA=	8.2852
EXPONENTIAL	THETA=	14.8709		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2697	0.3190	0.2987
LOG NORMAL	0.1096	0.1056	0.1072
GAMMA	0.1633	0.1263	0.1221
WEIBULL	0.1567	0.1595	0.1542
EXTRÉME TYPE-1	0.1696	0.1479	0.1502
EXPONENTIAL	0.1759	0.1913	0.1801

(4) BUFFELSPOORT DAM. A2R05.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930					4.90	15.60	26.30	9.50	22.70	13.50
1940	17.90	9.10	21.30	51.70	18.10	16.50	8.70	16.40	9.90	18.00
1950	13.40	9.20	12.50	8.80	25.10	14.70	15.20	13.00	11.20	5.70
1960	11.90	5.90	3.70	2.50	4.50	3.00	25.80	6.60	5.30	3.70
1970	13.50	12.10	8.20	19.20	19.90	27.60	26.70	25.60	6.20	13.50
1980	12.80									

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.286	0.253	-0.286

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	14.1936	SIGMA=	9.0027
LOGNORMAL	MU=	2.4585	SIGMA=	0.6580
GAMMA	ALPHA=	2.7289	BETA=	5.2012
WEIBULL	RHO=	1.7017	DELTA=	15.9832
EXTREME-1	XI=	10.3539	ETA=	6.4005
EXPONENTIAL	THETA=	14.1936		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1329	0.1712	0.1793
LOG NORMAL	0.1152	0.1115	0.1130
GAMMA	0.0945	0.0936	0.0848
WEIBULL	0.0940	0.1011	0.0998
EXTREME TYPE-1	0.0987	0.1021	0.0904
EXPONENTIAL	0.2094	0.2797	0.2665

(5) BOSPOORT DAM. A2R06.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930							32.50	4.04	30.20	12.30
1940	22.50	7.30	31.10	97.00	34.20	37.00	4.19	17.90	11.10	10.70
1950	6.06	6.81	3.57	1.23	52.10	13.90	14.50	28.90	6.21	2.22
1960	10.20	0.31	0.89	0.47	5.33	0.66	142.00	0.44	1.01	0.62
1970	6.40	12.60	1.38	10.70	44.20	254.60	53.30	146.00		
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.302	0.159	-0.302

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	28.0771	SIGMA=	49.0701
LOGNORMAL	MU=	2.1739	SIGMA=	1.6981
GAMMA	ALPHA=	0.5406	BETA=	51.9397
WEIBULL	RHO=	0.6552	DELTA=	19.9877
EXTREME-1	XI=	12.2508	ETA=	20.7312
EXPONENTIAL	THETA=	28.0771		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2806	0.3720	0.3335
LOG NORMAL	0.1072	0.1167	0.1020
GAMMA	0.1485	0.1709	0.1767
WEIBULL	0.1117	0.1180	0.1200
EXTREME TYPE-1	0.2339	0.2559	0.2481
EXPONENTIAL	0.2566	0.2468	0.2144

(6) LINDLEY'S POORT DAM. A2R07.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										2.90
1940	4.30	7.90	18.20	96.90	10.70	16.90	7.30	6.10	3.90	6.20
1950	3.90	2.50	8.00	4.50	35.40	29.80	33.10	26.40	8.00	4.10
1960	41.20	6.70	2.70	4.10	3.60	10.80	115.00	8.60	3.30	0.80
1970	21.90	22.20	7.00	46.50	22.00	115.40	41.40	117.70	4.30	
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.310	0.054	-0.310

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	23.3050	SIGMA=	32.2359
LOGNORMAL	MU=	2.4161	SIGMA=	1.2091
GAMMA	ALPHA=	0.8090	BETA=	28.8083
WEIBULL	RHO=	0.8301	DELTA=	20.7344
EXTREME-1	XI=	11.5261	ETA=	15.9395
EXPONENTIAL	THETA=	23.3050		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2475	0.3346	0.3052
LOG NORMAL	0.1501	0.1304	0.1272
GAMMA	0.2023	0.1789	0.1697
WEIBULL	0.1704	0.1870	0.1766
EXTREME TYPE-1	0.2462	0.2257	0.2217
EXPONENTIAL	0.2323	0.1916	0.1438

(7) WARMBAD DAM. A2R08.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940						0.30	2.50	1.40	1.30	2.60
1950	1.80	0.80	9.10	5.00	23.30	13.60	4.60	2.60	5.70	2.10
1960	22.00	1.50	1.10	0.50	0.80	0.20	15.60	2.30	4.50	3.70
1970	19.30	13.30	2.30	15.30	35.30	38.40	10.70	13.10	2.70	0.00
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.336	0.403	-0.336

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	8.2147	SIGMA=	9.8481
LOGNORMAL	MU=	1.3496	SIGMA=	1.3568
GAMMA	ALPHA=	0.7865	BETA=	10.4448
WEIBULL	RHO=	0.8372	DELTA=	7.4371
EXTREME-1	XI=	4.2685	ETA=	5.7147
EXPONENTIAL	THETA=	8.2147		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2393	0.2779	0.2566
LOG NORMAL	0.1379	0.1115	0.1046
GAMMA	0.1716	0.1398	0.1168
WEIBULL	0.1531	0.1257	0.1051
EXTREME TYPE-1	0.2280	0.1989	0.1884
EXPONENTIAL	0.2086	0.1751	0.1237

(8) MARICO-BOSVELD DAM. A3R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930						92.90	36.10	19.60	45.90	24.80
1940	25.60	19.30	52.30	160.00	35.80	61.10	17.00	21.30	12.90	8.80
1950	12.00	5.10	10.90	13.80	45.40	27.50	29.00	20.70	14.20	8.10
1960	33.70	16.40	8.40	15.80	4.70	19.90	52.80	10.40	10.20	3.60
1970	35.60	38.70	12.20	30.50	45.10	95.70	31.80			
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.302	0.228	-0.302

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	30.6095	SIGMA=	29.2549
LOGNORMAL	MU=	3.0883	SIGMA=	0.8243
GAMMA	ALPHA=	1.6485	BETA=	18.5680
WEIBULL	RHO=	1.2311	DELTA=	33.0408
EXTREME-1	XI=	19.8184	ETA=	15.9769
EXPONENTIAL	THETA=	30.6095		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1742	0.2311	0.2266
LOG NORMAL	0.0990	0.1071	0.1066
GAMMA	0.1150	0.1064	0.0984
WEIBULL	0.1113	0.1258	0.1214
EXTREME TYPE-1	0.1296	0.1205	0.1141
EXPONENTIAL	0.1658	0.2239	0.2144

(9) KLEIN-MARICO DAM. A3R02.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930						9.19	12.20	3.90	9.54	4.32
1940	2.76	7.08	18.80	45.50	14.60	40.10	11.60	5.10	1.45	1.58
1950	1.17	0.73	3.11	1.47	16.70	1.90	5.72	7.48	2.44	0.32
1960	6.53	6.27	0.66	0.74	0.48	9.17	32.90	1.93	1.63	0.63
1970	7.60	4.60	2.36	9.87	19.80	24.61	51.31	82.79	29.53	
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.295	0.545	-0.295

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	11.8675	SIGMA=	16.5761
LOGNORMAL	MU=	1.6459	SIGMA=	1.3790
GAMMA	ALPHA=	0.7262	BETA=	16.3409
WEIBULL	RHO=	0.7887	DELTA=	10.2088
EXTREME-1	XI=	5.8803	ETA=	8.2664
EXPONENTIAL	THETA=	11.8675		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2427	0.3349	0.3092
LOG NORMAL	0.0951	0.0996	0.0886
GAMMA	0.1245	0.1392	0.1492
WEIBULL	0.1023	0.1204	0.1277
EXTREME TYPE-1	0.1823	0.2186	0.2198
EXPONENTIAL	0.1687	0.1601	0.1216

(10) KROMELLENBOOG DAM. A3R03.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950					21.80	7.40	7.10	5.50	2.10	10.10
1960	9.20	4.80	5.80	4.70	10.20	49.80	3.90	4.30	1.10	11.40
1970	5.10	3.20	9.90	19.00	94.60	39.10				
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.418	0.289	-0.418

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	15.0045	SIGMA=	21.4361
LOGNORMAL	MU=	2.1236	SIGMA=	1.0341
GAMMA	ALPHA=	0.9885	BETA=	15.1789
WEIBULL	RHO=	0.9120	DELTA=	14.2092
EXTREME-1	XI=	7.9047	ETA=	9.2616
EXPONENTIAL	THETA=	15.0045		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2961	0.2826	0.2432
LOG NORMAL	0.1548	0.1255	0.1066
GAMMA	0.2190	0.1563	0.1216
WEIBULL	0.1948	0.1647	0.1417
EXTREME TYPE-1	0.2435	0.1743	0.1404
EXPONENTIAL	0.2266	0.1823	0.1407

(11) DOORNDRAAI DAM. A6R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950			30.90	83.10	53.80	26.80	10.80	34.30	8.10	67.30
1960	4.90	4.80	4.90	35.30	4.30	12.00	11.30	19.50	34.30	1.50
1970	45.70	76.50	56.80	29.20	19.10					
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.409	0.207	-0.409

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	29.3565	SIGMA=	24.3930
LOGNORMAL	MU=	2.9328	SIGMA=	1.0899
GAMMA	ALPHA=	1.2597	BETA=	23.3038
WEIBULL	RHO=	1.1723	DELTA=	31.0070
EXTREME-1	XI=	18.4781	ETA=	17.6311
EXPONENTIAL	THETA=	29.3565		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1694	0.1554	0.1386
LOG NORMAL	0.1659	0.1581	0.1621
GAMMA	0.1510	0.1398	0.1122
WEIBULL	0.1445	0.1441	0.1201
EXTREME TYPE-1	0.1589	0.1507	0.1136
EXPONENTIAL	0.1619	0.1491	0.1190

(12) NZEHELE DAM. A8R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950										
1960				15.60	4.60	6.60	22.80	38.70	2.70	17.70
1970	4.50	22.10	73.90	5.70	86.50	116.50	95.60	230.20	228.50	
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.490	0.611	-0.490

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	60.7625	SIGMA=	75.2655
LOGNORMAL	MU=	3.2563	SIGMA=	1.4597
GAMMA	ALPHA=	0.7092	BETA=	85.6808
WEIBULL	RHO=	0.7836	DELTA=	52.5112
EXTREME-1	XI=	30.5218	ETA=	43.9208
EXPONENTIAL	THETA=	60.7625		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2394	0.2127	0.1783
LOG NORMAL	0.1616	0.1469	0.1184
GAMMA	0.1815	0.1448	0.1080
WEIBULL	0.1663	0.1456	0.1121
EXTREME TYPE-1	0.2284	0.1801	0.1298
EXPONENTIAL	0.2203	0.2065	0.1581

(13) LUPHEPHE/NWANEDSI DAMS (COMBINED). 1964/5 TO 1975/6.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950										
1960					5.50	11.70	23.20	8.90	21.30	11.30
1970	11.20	50.40	6.40	46.60	63.70	48.00				
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.566	0.236	-0.566

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	25.6833	SIGMA=	20.6490
LOGNORMAL	MU=	2.9198	SIGMA=	0.8637
GAMMA	ALPHA=	1.6808	BETA=	15.2808
WEIBULL	RHO=	1.3335	DELTA=	28.0969
EXTREME-1	XI=	16.6168	ETA=	14.2102
EXPONENTIAL	THETA=	25.6833		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2420	0.1904	0.1358
LOG NORMAL	0.2118	0.1619	0.1151
GAMMA	0.2349	0.1818	0.1244
WEIBULL	0.2246	0.1780	0.1274
EXTREME TYPE-1	0.2440	0.1894	0.1276
EXPONENTIAL	0.2021	0.1917	0.1542

(14) ALBASINI DAM. A9R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950			32.40	39.80	55.70	63.70	17.50	109.00	44.00	16.10
1960	36.80	8.50	8.90	2.40	5.70	0.80	8.60	7.20	0.30	23.50
1970	10.20	12.70	13.30	64.90	13.70	7.10	41.00	6.10	4.00	
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.377	0.238	-0.377

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	24.2185	SIGMA=	25.7123
LOGNORMAL	MU=	2.5543	SIGMA=	1.3438
GAMMA	ALPHA=	0.9212	BETA=	26.2908
WEIBULL	RHO=	0.9437	DELTA=	23.5818
EXTREME-1	XI=	13.8110	ETA=	15.5084
EXPONENTIAL	THETA=	24.2185		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2257	0.2342	0.2131
LOG NORMAL	0.1311	0.1615	0.2022
GAMMA	0.1349	0.1086	0.0883
WEIBULL	0.1268	0.1237	0.1151
EXTREME TYPE-1	0.1925	0.1545	0.1258
EXPONENTIAL	0.1507	0.1591	0.1492

(15) NUWE DOORINGPOORT DAM. B1R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950				136.00	495.00	398.00	256.00	108.00	91.80	122.00
1960	73.40	38.30	31.70	44.60	68.80	5.10	183.00	41.30	45.30	57.70
1970	48.30	25.00	30.50	100.60	287.30	187.90	114.70	96.90		
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.392	0.574	-0.392

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	123.4880	SIGMA=	120.8977
LOGNORMAL	MU=	4.3939	SIGMA=	0.9966
GAMMA	ALPHA=	1.3260	BETA=	93.1278
WEIBULL	RHO=	1.1327	DELTA=	129.6934
EXTREME-1	XI=	75.9820	ETA=	69.7481
EXPONENTIAL	THETA=	123.4880		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2204	0.2168	0.1981
LOG NORMAL	0.1225	0.1464	0.1765
GAMMA	0.1453	0.1222	0.1073
WEIBULL	0.1404	0.1372	0.1270
EXTREME TYPE-1	0.1668	0.1358	0.1051
EXPONENTIAL	0.1499	0.1929	0.1668

(16) BRONKHORSTSPRUIT DAM. B2R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										43.40
1950	6.70	13.20	15.80	64.10	58.80	73.90	38.80	80.10	18.00	33.40
1960	15.50	18.80	18.80	46.80	10.20	85.50	32.90	15.40	46.20	69.90
1970	113.80	6.30	14.90	161.40	110.20	47.60	268.00	10.10	47.00	
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.358	-0.037	-0.358

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	52.8500	SIGMA=	54.9751
LOGNORMAL	MU=	3.5401	SIGMA=	0.9531
GAMMA	ALPHA=	1.3116	BETA=	40.2940
WEIBULL	RHO=	1.1072	DELTA=	55.1402
EXTREME-1	XI=	32.1893	ETA=	30.5656
EXPONENTIAL	THETA=	52.8500		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2116	0.2518	0.2309
LOG NORMAL	0.1439	0.1271	0.0998
GAMMA	0.1594	0.1398	0.1173
WEIBULL	0.1442	0.1404	0.1302
EXTREME TYPE-1	0.1811	0.1652	0.1433
EXPONENTIAL	0.1394	0.1734	0.1662

(17) RUST DE WINTER DAM. B3R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930			..	8.10	63.90	147.00	23.00	47.00	50.40	51.10
1940	12.70	55.60	144.00	15.60	46.50	8.20	19.10	11.80	15.90	10.10
1950	5.50	13.00	18.50	61.50	206.00	99.70	22.20	12.40	15.60	25.20
1960	12.20	8.20	5.40	13.20	3.10	166.00	38.30	47.80	83.00	44.90
1970	234.00	6.00	24.40	138.50	110.40	80.90	95.20	5.00	26.40	30.70
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.286	0.104	-0.286

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	50.7064	SIGMA=	55.7597
LOGNORMAL	MU=	3.3576	SIGMA=	1.1131
GAMMA	ALPHA=	1.0138	BETA=	50.0138
WEIBULL	RHO=	0.9682	DELTA=	49.9382
EXTREME-1	XI=	28.3258	ETA=	32.5167
EXPONENTIAL	THETA=	50.7064		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2187	0.2995	0.2861
LOG NORMAL	0.1170	0.1036	0.0857
GAMMA	0.1524	0.1409	0.1446
WEIBULL	0.1336	0.1414	0.1464
EXTREME TYPE-1	0.1915	0.1984	0.2050
EXPONENTIAL	0.1453	0.1457	0.1377

(18) LOSKOP DAM. B3R02.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930							21.50	379.00	693.00	877.00
1940	848.00	221.00	733.00	1334.00	227.00	678.00	211.00	239.00	239.00	339.00
1950	133.00	103.00	300.00	339.00	1150.00	1089.00	697.00	316.00	445.00	405.00
1960	439.00	127.00	121.00	132.00	192.00	43.00	572.00	306.00	327.00	560.00
1970	318.00	339.00	111.00	341.00	1143.00	680.00	285.00	643.00		
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.302	0.306	-0.302

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	445.1310	SIGMA=	326.0901
LOGNORMAL	MU=	5.7967	SIGMA=	0.8717
GAMMA	ALPHA=	1.8061	BETA=	246.4629
WEIBULL	RHO=	1.4200	DELTA=	490.4806
EXTREME-1	XI=	302.4076	ETA=	229.7365
EXPONENTIAL	THETA=	445.1310		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1963	0.1764	0.1764
LOG NORMAL	0.1244	0.1402	0.2047
GAMMA	0.1248	0.1076	0.1021
WEIBULL	0.1292	0.1146	0.1094
EXTREME TYPE-1	0.1464	0.1180	0.0916
EXPONENTIAL	0.1680	0.2004	0.1677

(19) TONTELDOOS DAM. B4R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950										
1960		2.60	2.50	7.60	0.60	4.30	3.10	1.50	2.70	0.90
1970	7.20	2.10	11.00	6.40	5.60	2.80	2.00	0.90	3.10	4.40
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.450	-0.059	-0.450

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	3.7526	SIGMA=	2.7204
LOGNORMAL	MU=	1.0629	SIGMA=	0.7770
GAMMA	ALPHA=	2.0779	BETA=	1.8059
WEIBULL	RHO=	1.4957	DELTA=	4.1774
EXTREME-1	XI=	2.5896	ETA=	1.8629
EXPONENTIAL	THETA=	3.7526		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2106	0.1659	0.1322
LOG NORMAL	0.1394	0.1424	0.1272
GAMMA	0.1590	0.1331	0.1031
WEIBULL	0.1633	0.1323	0.0971
EXTREME TYPE-1	0.1693	0.1383	0.1020
EXPONENTIAL	0.1899	0.2128	0.1796

(20) OHRIGSTAD DAM. B6R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950							4.60	9.50	26.70	16.40
1960	21.90	28.40	9.70	10.80	8.90	16.10	8.70	14.80	8.20	10.30
1970	6.10	7.30	29.90	12.00	27.50	14.60	33.40	23.70	28.60	
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.409	0.137	-0.409

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	16.4391	SIGMA=	8.9801
LOGNORMAL	MU=	2.6480	SIGMA=	0.5753
GAMMA	ALPHA=	3.4550	BETA=	4.7581
WEIBULL	RHO=	2.0106	DELTA=	18.6623
EXTREME-1	XI=	12.3145	ETA=	6.7978
EXPONENTIAL	THETA=	16.4391		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1982	0.1546	0.1241
LOG NORMAL	0.1619	0.1248	0.0992
GAMMA	0.1761	0.1340	0.0966
WEIBULL	0.1803	0.1382	0.1051
EXTREME TYPE-1	0.1774	0.1355	0.0951
EXPONENTIAL	0.2646	0.3234	0.2712

(21) KLASERIE DAM. B7R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950										
1960		16.20	17.10	8.70	48.90	14.50	27.00	9.60	20.60	8.30
1970										
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.653	-0.431	-0.653

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	18.9889	SIGMA=	12.7535
LOGNORMAL	MU=	2.7834	SIGMA=	0.5768
GAMMA	ALPHA=	3.2731	BETA=	5.8014
WEIBULL	RHO=	1.7361	DELTA=	21.5211
EXTREME-1	XI=	14.0490	ETA=	7.4134
EXPONENTIAL	THETA=	18.9889		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1990	0.1625	0.1158
LOG NORMAL	0.1727	0.1611	0.1206
GAMMA	0.1884	0.1713	0.1262
WEIBULL	0.1841	0.1585	0.1156
EXTREME TYPE-1	0.1891	0.1780	0.1338
EXPONENTIAL	0.2864	0.2981	0.2208

(22) EBENEZER DAM. B8R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950									45.60	53.20
1960	25.30	27.30	17.80	36.00	26.30	44.70	18.60	31.30	15.50	18.20
1970	86.70	17.80	101.60	60.40	68.50	78.00	98.20	38.70		
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.438	0.154	-0.438

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	45.4850	SIGMA=	27.9461
LOGNORMAL	MU=	3.6382	SIGMA=	0.6181
GAMMA	ALPHA=	2.9458	BETA=	15.4406
WEIBULL	RHO=	1.7904	DELTA=	51.5018
EXTREME-1	XI=	33.0873	ETA=	19.8206
EXPONENTIAL	THETA=	45.4850		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1863	0.1736	0.1500
LOG NORMAL	0.1547	0.1479	0.1179
GAMMA	0.1664	0.1519	0.1183
WEIBULL	0.1637	0.1595	0.1325
EXTREME TYPE-1	0.1693	0.1561	0.1163
EXPONENTIAL	0.2630	0.3355	0.2760

(23) VAAL DAM. C1R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920				765.00	4778.00	809.00	1284.00	863.00	1612.00	2755.00
1930	779.00	698.00	470.00	3302.00	2550.00	1689.00	4361.00	1146.00	3929.00	2178.00
1940	2535.00	1039.00	3598.00	6864.00	1696.00	1278.00	1117.00	1101.00	642.00	1939.00
1950	639.00	1167.00	1951.00	882.00	3510.00	1546.00	5379.00	3656.00	1345.00	1449.00
1960	2039.00	962.00	1316.00	1136.00	2890.00	520.00	3393.00	597.00	687.00	1173.00
1970	1008.00	1977.00	440.00	2176.00	5727.00	4803.00	2395.00	2367.00	600.00	1464.00
1980	1202.00	375.00								

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.255	0.118	-0.255

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	1975.3898	SIGMA=	1474.3916
LOGNORMAL	MU=	7.3387	SIGMA=	0.7143
GAMMA	ALPHA=	2.1535	BETA=	917.2808
WEIBULL	RHO=	1.4673	DELTA=	2201.6486
EXTREME-1	XI=	1353.3556	ETA=	955.9212
EXPONENTIAL	THETA=	1975.3898		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1791	0.2470	0.2536
LOG NORMAL	0.0937	0.0852	0.0787
GAMMA	0.1235	0.1187	0.1319
WEIBULL	0.1225	0.1565	0.1740
EXTREME TYPE-1	0.1376	0.1346	0.1506
EXPONENTIAL	0.1913	0.3079	0.3015

(24) BOSKOP DAM. C2R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950								58.00	39.50	46.70
1960	54.60	53.50	54.10	58.70	91.20	53.70	52.50	51.70	83.40	78.50
1970	69.10	80.40	74.40	140.50	129.00	209.10				
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.450	0.469	-0.450

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	77.8211	SIGMA=	41.4201
LOGNORMAL	MU=	4.2575	SIGMA=	0.4213
GAMMA	ALPHA=	5.3213	BETA=	14.6245
WEIBULL	RHO=	2.0660	DELTA=	88.3731
EXTREME-1	XI=	62.1981	ETA=	22.5083
EXPONENTIAL	THETA=	77.8211		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2214	0.2023	0.1757
LOG NORMAL	0.1798	0.1481	0.1149
GAMMA	0.1961	0.1595	0.1263
WEIBULL	0.1979	0.2052	0.1775
EXTREME TYPE-1	0.1934	0.1560	0.1082
EXPONENTIAL	0.3785	0.4259	0.3328

(25) ALLEMANSKRAAL DAM. C4R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950									52.80	57.70
1960	62.60	67.50	53.30	49.70	60.60	256.00	33.10	44.50	28.00	95.40
1970	111.00	16.50	104.00	174.00	296.00	92.50	79.50	19.10	18.30	125.00
1980	20.50									

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.409	0.122	-0.409

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	83.3739	SIGMA=	72.3368
LOGNORMAL	MU=	4.1175	SIGMA=	0.8012
GAMMA	ALPHA=	1.7834	BETA=	46.7506
WEIBULL	RHO=	1.3078	DELTA=	91.1779
EXTREME-1	XI=	54.8819	ETA=	42.7059
EXPONENTIAL	THETA=	83.3739		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2013	0.2048	0.1861
LOG NORMAL	0.1247	0.1312	0.1107
GAMMA	0.1396	0.1231	0.1007
WEIBULL	0.1384	0.1288	0.1165
EXTREME TYPE-1	0.1487	0.1299	0.1065
EXPONENTIAL	0.1806	0.2381	0.2111

(26) ERFENIS DAM. C4R02.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950									85.50	67.50
1960	103.00	123.00	145.00	136.00	182.00	687.00	60.40	114.00	34.30	125.70
1970	189.90	34.10	199.00	144.30	427.30	101.40	227.10	20.70		
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.438	-0.124	-0.438

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	160.3600	SIGMA=	152.8762
LOGNORMAL	MU=	4.7556	SIGMA=	0.8266
GAMMA	ALPHA=	1.7014	BETA=	94.2512
WEIBULL	RHO=	1.2494	DELTA=	173.7662
EXTREME-1	XI=	105.1988	ETA=	81.1102
EXPONENTIAL	THETA=	160.3600		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2313	0.2012	0.1771
LOG NORMAL	0.1415	0.1412	0.1279
GAMMA	0.1587	0.1261	0.0992
WEIBULL	0.1570	0.1259	0.1030
EXTREME TYPE-1	0.1628	0.1324	0.1033
EXPONENTIAL	0.1825	0.1829	0.1540

(27) TIERPOORT DAM. C5R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920			7.00	100.00	9.40	22.00	8.40	9.30	2.70	13.70
1930	11.40	3.20	33.50	7.80	2.50	21.40	0.50	2.90	11.00	14.70
1940	6.50	32.70	19.00	7.80	12.60	3.80	137.00	2.60	38.30	4.90
1950	3.10	5.30	5.20	52.70	45.20	14.80	9.90	5.30	14.70	6.20
1960	4.50	18.20	10.10	2.60	49.90	9.40	12.70	5.20	13.00	58.20
1970	3.20	40.50	5.70	118.10	23.00					
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.269	-0.168	-0.269

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	20.4396	SIGMA=	28.2043
LOGNORMAL	MU=	2.3837	SIGMA=	1.1196
GAMMA	ALPHA=	0.9199	BETA=	22.2200
WEIBULL	RHO=	0.8894	DELTA=	19.0858
EXTREME-1	XI=	10.7054	ETA=	13.1010
EXPONENTIAL	THETA=	20.4396		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2641	0.3462	0.3250
LOG NORMAL	0.0966	0.1332	0.1626
GAMMA	0.1613	0.1737	0.1712
WEIBULL	0.1409	0.1949	0.1868
EXTREME TYPE-1	0.1992	0.2086	0.2145
EXPONENTIAL	0.1704	0.1712	0.1631

(28) KALKFONTEIN DAM. C5R02.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910			57.00	58.00	363.00	56.00	174.00	46.00	19.00	554.00
1920	64.00	41.00	389.00	194.00	748.00	55.00	111.00	67.00	140.00	166.00
1930	114.00	140.00	74.00	474.00	86.00	9.00	159.00	22.00	69.00	82.00
1940	168.00	74.00	334.00	82.00	20.00	100.00	42.00	717.00	40.00	385.00
1950	53.00	95.00	67.00	101.00	335.00	152.00	97.00	119.00	46.00	35.00
1960	57.00	105.00	237.00	21.00	52.00	194.00	271.00	38.00	195.00	46.00
1970	125.00	345.00	5.00	1324.00	53.00	492.00	48.00	6.00	15.00	85.00
1980	7.00									

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.236	-0.216	-0.236

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	163.9710	SIGMA=	214.8925
LOGNORMAL	MU=	4.4906	SIGMA=	1.1506
GAMMA	ALPHA=	0.9532	BETA=	172.0302
WEIBULL	RHO=	0.9188	DELTA=	156.5901
EXTREME-1	XI=	89.4787	ETA=	102.9869
EXPONENTIAL	THETA=	163.9710		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2498	0.3517	0.3403
LOG NORMAL	0.0916	0.1131	0.1303
GAMMA	0.1409	0.1207	0.1320
WEIBULL	0.1216	0.1233	0.1255
EXTREME TYPE-1	0.1858	0.1998	0.2176
EXPONENTIAL	0.1465	0.1227	0.0995

(29) RUSTFONTEIN DAM. C5R03.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950					110.00	21.40	10.50	14.10	6.90	13.80
1960	7.60	33.30	17.80	8.80	56.50	52.80	29.70	14.60	11.70	9.60
1970	58.60	1.20	70.70	3.90	139.20	26.60	13.10	2.30	1.10	
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.392	-0.132	-0.392

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	29.4320	SIGMA=	34.7765
LOGNORMAL	MU=	2.7316	SIGMA=	1.2651
GAMMA	ALPHA=	0.8989	BETA=	32.7438
WEIBULL	RHO=	0.9073	DELTA=	27.9887
EXTREME-1	XI=	16.0498	ETA=	19.0764
EXPONENTIAL	THETA=	29.4320		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2368	0.2441	0.2191
LOG NORMAL	0.1373	0.1475	0.1479
GAMMA	0.1546	0.1220	0.0913
WEIBULL	0.1418	0.1236	0.0967
EXTREME TYPE-1	0.2048	0.1602	0.1291
EXPONENTIAL	0.1582	0.1509	0.1247

(30) KOPPIES DAM. C7R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920	172.00	212.00	82.60	47.90	284.00	22.90	18.20	25.70	40.50	84.30
1930	34.80	17.60	19.70	324.00	173.00	32.50	239.00	109.00	261.00	24.10
1940	72.40	75.60	116.00	334.00	28.80	13.70	247.00	52.80	16.40	110.00
1950	14.50	37.00	464.00	14.10	113.00	10.90	193.00	105.00	15.20	38.30
1960	99.60	83.50	55.00	48.80	139.00	46.60	161.00	8.60	8.20	30.70
1970	84.10	85.50	26.30	76.40	126.60	90.80	50.40	86.50	2.30	
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.255	-0.145	-0.255

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	95.0237	SIGMA=	96.1761
LOGNORMAL	MU=	4.0454	SIGMA=	1.0973
GAMMA	ALPHA=	1.1202	BETA=	84.8282
WEIBULL	RHO=	1.0417	DELTA=	96.7011
EXTREME-1	XI=	56.1446	ETA=	58.0711
EXPONENTIAL	THETA=	95.0237		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1876	0.2742	0.2754
LOG NORMAL	0.1194	0.1156	0.1523
GAMMA	0.1035	0.0986	0.1029
WEIBULL	0.0981	0.1036	0.1097
EXTREME TYPE-1	0.1359	0.1556	0.1725
EXPONENTIAL	0.1018	0.1308	0.1336

(31) MENIN DAM. C8R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920		1.40	0.71	3.68	0.25	0.33	0.70	1.84	1.79	0.18
1930	1.29	0.28	3.32	4.93	0.20	7.11	0.47	1.68	1.08	1.13
1940	0.76	3.06	3.05	0.83	0.66	1.51	0.92	0.08	1.45	3.02
1950	0.06	0.99	0.22	2.21	0.23	3.80	4.18	1.32	0.82	1.18
1960	1.15	1.48	0.49	1.52	0.31	1.76	0.61	0.49	0.26	0.15
1970	0.58	0.10	1.16	1.22	1.11	0.81	1.10	0.20		
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.260	-0.063	-0.260

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	1.3547	SIGMA=	1.3661
LOGNORMAL	MU=	-0.1806	SIGMA=	1.0745
GAMMA	ALPHA=	1.1712	BETA=	1.1567
WEIBULL	RHO=	1.0683	DELTA=	1.3925
EXTREME-1	XI=	0.8174	ETA=	0.8050
EXPONENTIAL	THETA=	1.3547		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1998	0.2818	0.2798
LOG NORMAL	0.1090	0.1057	0.1127
GAMMA	0.1011	0.0990	0.1022
WEIBULL	0.0983	0.0980	0.0997
EXTREME TYPE-1	0.1259	0.1534	0.1689
EXPONENTIAL	0.1059	0.1206	0.1204

(32) ARMENINIA DAM. D2R02.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950					28.80	17.70	37.80	3.70	4.50	10.10
1960	17.30	28.80	14.90	7.30	37.10	50.50	11.00	23.80	5.40	6.60
1970	45.80	1.40	39.90	41.70	103.00	46.40	44.40	11.10		
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.400	0.230	-0.400

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	26.6250	SIGMA=	22.9101
LOGNORMAL	MU=	2.8674	SIGMA=	1.0358
GAMMA	ALPHA=	1.3488	BETA=	19.7396
WEIBULL	RHO=	1.1987	DELTA=	28.3339
EXTREME-1	XI=	16.9979	ETA=	15.4830
EXPONENTIAL	THETA=	26.6250		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1771	0.1761	0.1570
LOG NORMAL	0.1541	0.1341	0.1379
GAMMA	0.1494	0.1183	0.0920
WEIBULL	0.1443	0.1190	0.0986
EXTREME TYPE-1	0.1666	0.1403	0.1038
EXPONENTIAL	0.1502	0.1422	0.1209

(33) BETHULIE DAM. D3R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										12.20
1930	2.80	3.60	1.50	13.50	1.60	0.10	10.60	1.50	4.80	1.90
1940	4.60	0.90	7.10	1.50	0.10	7.80	1.70	14.70	1.80	7.30
1950	2.90	5.00	7.50	4.10	4.60	2.50	2.30	3.70	1.50	0.30
1960	2.40	2.20	6.70	4.10	1.50	0.20	15.60	2.00	6.00	3.10
1970	4.50	3.60	2.00	80.00	6.20					
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.289	-0.061	-0.289

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	6.0022	SIGMA=	11.7853
LOGNORMAL	MU=	1.0870	SIGMA=	1.2427
GAMMA	ALPHA=	0.8368	BETA=	7.1729
WEIBULL	RHO=	0.8372	DELTA=	5.3161
EXTREME-1	XI=	3.1766	ETA=	3.7064
EXPONENTIAL	THETA=	6.0022		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2548	0.3238	0.2988
LOG NORMAL	0.1520	0.1842	0.2227
GAMMA	0.2736	0.2809	0.2695
WEIBULL	0.1460	0.1786	0.1530
EXTREME TYPE-1	0.1695	0.1640	0.1612
EXPONENTIAL	0.1808	0.1865	0.1585

(34) HENDRICK VERWOERD DAM. D3R02.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950										
1960										
1970	5619.00	9024.00	2254.00	13455.00	7334.00	20372.00	10285.00	8197.00	5132.00	3464.00
1980	7004.00	3598.00	2709.00							

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.544	0.074	-0.544

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU= 7572.8462	SIGMA= 5025.3879
LOGNORMAL	MU= 8.7447	SIGMA= 0.6404
GAMMA	ALPHA= 2.8208	BETA= 2684.6054
WEIBULL	RHO= 1.6948	DELTA= 8548.6930
EXTREME-1	XI= 5492.1870	ETA= 3311.4452
EXPONENTIAL	THETA= 7572.8462	

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1668	0.1367	0.1053
LOG NORMAL	0.1327	0.1350	0.1128
GAMMA	0.1395	0.1338	0.1051
WEIBULL	0.1379	0.1187	0.0896
EXTREME TYPE-I	0.1435	0.1405	0.1101
EXPONENTIAL	0.2317	0.2538	0.2108

(35) LEEUBOS DAM. D4R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940									5.22	0.85
1950	0.14	0.43	2.48	3.42	5.57	0.45	0.04	0.29	0.85	1.37
1960	0.82	1.51	0.08	1.18	0.88	2.88	1.37	0.53	1.03	
1970										
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.428	0.176	-0.428

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	1.4948	SIGMA=	1.5756
LOGNORMAL	MU=	-0.1940	SIGMA=	1.2795
GAMMA	ALPHA=	0.9718	BETA=	1.5382
WEIBULL	RHO=	0.9716	DELTA=	1.4757
EXTREME-1	XI=	0.8639	ETA=	0.9310
EXPONENTIAL	THETA=	1.4948		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2305	0.1942	0.1693
LOG NORMAL	0.1504	0.1562	0.1800
GAMMA	0.1398	0.1134	0.0927
WEIBULL	0.1360	0.1206	0.1072
EXTREME TYPE-1	0.1796	0.1350	0.1003
EXPONENTIAL	0.1581	0.1525	0.1235

(36) ROOIBERG DAM. D5R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930				92.80	1.40	2.10	23.30	8.90	47.60	2.90
1940	63.10	1.80	64.30	5.60	28.40	10.20	24.30	1.20	9.00	4.20
1950	17.00	5.70	0.30	911.00	2.00	20.90	14.10	34.00	1.20	77.00
1960	106.00	37.40	20.60	1.30	152.40	96.00	876.50			
1970										
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.336	-0.024	-0.336

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	81.3088	SIGMA=	209.4879
LOGNORMAL	MU=	2.6976	SIGMA=	1.9000
GAMMA	ALPHA=	0.3887	BETA=	209.1679
WEIBULL	RHO=	0.5326	DELTA=	38.3184
EXTREME-1	XI=	26.3336	ETA=	64.8567
EXPONENTIAL	THETA=	81.3088		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.3248	0.3882	0.3343
LOG NORMAL	0.1171	0.1160	0.1048
GAMMA	0.1701	0.2072	0.1981
WEIBULL	0.1233	0.1526	0.1416
EXTREME TYPE-1	0.2537	0.2779	0.2562
EXPONENTIAL	0.3304	0.3194	0.2654

(37) VICTORIA WEST DAM. D6R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920									2.76	0.02
1930	0.11	0.77	1.68	0.24	0.89	0.01	0.11	1.05	2.96	0.40
1940	0.42	2.50	0.98	0.65	1.19	0.09	1.43	0.71	1.41	1.00
1950	0.59	1.54	0.60	2.41	0.94	1.17	1.23	1.66	0.53	0.84
1960	0.24	0.36	0.45	0.51	14.10	0.93	1.32	1.36	1.02	0.20
1970	0.57	0.01	0.72	0.07	0.01	0.01	0.97	2.67	3.59	3.17
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.272	0.004	-0.272

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	1.2533	SIGMA=	2.0208
LOGNORMAL	MU=	-0.6024	SIGMA=	1.6091
GAMMA	ALPHA=	0.7261	BETA=	1.7261
WEIBULL	RHO=	0.8071	DELTA=	1.1047
EXTREME-1	XI=	0.6876	ETA=	0.7974
EXPONENTIAL	THETA=	1.2533		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2297	0.3266	0.3076
LOG NORMAL	0.1835	0.1901	0.2374
GAMMA	0.1280	0.1142	0.1152
WEIBULL	0.1259	0.1455	0.1479
EXTREME TYPE-1	0.1409	0.1652	0.1759
EXPONENTIAL	0.1464	0.1952	0.2159

(38) WEMMERSHOEK DAM. G1R02.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950							80.60	70.30	46.40	65.50
1960	106.00	69.00	60.40	74.20	67.40	63.00	82.10	56.00	76.50	51.30
1970	36.50	41.40	92.00	82.60	79.00	162.20	42.10	57.70	54.80	68.90
1980	51.10									

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.392	-0.027	-0.392

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	69.4800	SIGMA=	25.4655
LOGNORMAL	MU=	4.1884	SIGMA=	0.3208
GAMMA	ALPHA=	9.6557	BETA=	7.1958
WEIBULL	RHO=	2.7601	DELTA=	77.7619
EXTREME-1	XI=	59.1839	ETA=	17.1065
EXPONENTIAL	THETA=	69.4800		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1619	0.1397	0.1269
LOG NORMAL	0.1205	0.1067	0.0928
GAMMA	0.1308	0.1100	0.0942
WEIBULL	0.1744	0.1713	0.1571
EXTREME TYPE-1	0.1166	0.1148	0.1073
EXPONENTIAL	0.3954	0.4563	0.3648

(39) STEENBRAS DAM. G4R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920			76.10	49.60	64.20	54.20	42.60	26.00	24.90	28.40
1930	46.30	71.80	47.50	29.40	35.30	23.80	51.70	37.50	30.90	42.50
1940	77.00	52.10	33.40	75.50	70.60	45.70	37.50	32.90	36.60	32.60
1950	64.70	37.00	71.70	49.30	54.80	62.60	50.90	54.00	31.10	35.10
1960	54.20	44.50	34.40	36.30	40.10	38.80	47.20	29.70	43.30	28.10
1970	31.50	22.00	44.90	42.40	32.40	59.60	27.20	36.00	43.80	43.6[illegible]
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.257	0.212	-0.257

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	44.2724	SIGMA=	14.4179
LOGNORMAL	MU=	3.7402	SIGMA=	0.3184
GAMMA	ALPHA=	10.1269	BETA=	4.3718
WEIBULL	RHO=	3.2855	DELTA=	49.3952
EXTREME-1	XI=	37.6626	ETA=	11.2141
EXPONENTIAL	THETA=	44.2724		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL.	0.1173	0.1246	0.1406
LOG NORMAL	0.0918	0.0795	0.0726
GAMMA	0.0991	0.0866	0.0823
WEIBULL	0.1133	0.1426	0.1592
EXTREME TYPE-1	0.0935	0.0814	0.0741
EXPONENTIAL	0.4003	0.5081	0.4380

(40) STETTYNSKLOOF DAM. H1R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950								58.50	36.70	36.40
1960	72.20	44.50	35.20	39.20	41.10	35.90	61.60	35.60	52.50	31.20
1970	25.40	33.10	73.30	53.80	47.80	114.10	23.80			
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.438	-0.195	-0.438

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	47.5950	SIGMA=	21.0326
LOGNORMAL	MU=	3.7875	SIGMA=	0.3831
GAMMA	ALPHA=	6.8084	BETA=	6.9906
WEIBULL	RHO=	2.4115	DELTA=	53.7673
EXTREME-1	XI=	39.1095	ETA=	13.3146
EXPONENTIAL	THETA=	47.5950		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1837	0.1548	0.1322
LOG NORMAL	0.1423	0.1184	0.0897
GAMMA	0.1593	0.1270	0.0941
WEIBULL	0.1666	0.1568	0.1348
EXTREME TYPE-1	0.1488	0.1288	0.1008
EXPONENTIAL	0.3739	0.4232	0.3335

(41) POORTJESKLOOF DAM. H3R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950							10.10	47.60	3.73	1.88
1960	15.80	17.70	3.52	2.49	5.76	22.60	3.15	0.52	0.66	6.90
1970	6.00	0.40	7.10	1.50	0.25	9.20	0.66	3.63	0.75	17.70
1980	13.43									

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.392	0.026	-0.392

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	8.1212	SIGMA=	10.4112
LOGNORMAL	MU=	1.3175	SIGMA=	1.4077
GAMMA	ALPHA=	0.7680	BETA=	10.5748
WEIBULL	RHO=	0.8269	DELTA=	7.2895
EXTREME-1	XI=	4.2839	ETA=	5.5411
EXPONENTIAL	THETA=	8.1212		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2078	0.2521	0.2258
LOG NORMAL	0.1452	0.1476	0.1199
GAMMA	0.1419	0.1310	0.1092
WEIBULL	0.1361	0.1335	0.1072
EXTREME TYPE-1	0.1811	0.1672	0.1499
EXPONENTIAL	0.1797	0.1906	0.1504

(42) KEEROM DAM. H4R02.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950				8.32	6.81	12.10	7.59	6.16	4.93	6.09
1960	470.00	17.10	4.70	6.63	3.07	5.17	4.54	3.78	5.56	6.30
1970	3.40	3.60								
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.450	-0.035	-0.450

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	30.8342	SIGMA=	106.4005
LOGNORMAL	MU=	1.9935	SIGMA=	1.0923
GAMMA	ALPHA=	0.4500	BETA=	68.5250
WEIBULL	RHO=	0.5934	DELTA=	14.1369
EXTREME-1	XI=	7.5726	ETA=	24.7887
EXPONENTIAL	THETA=	30.8342		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.3791	0.3226	0.2614
LOG NORMAL	0.2499	0.2151	0.1820
GAMMA	0.3323	0.2539	0.2105
WEIBULL	0.2764	0.2911	0.2408
EXTREME TYPE-1	0.3590	0.2661	0.2160
EXPONENTIAL	0.5278	0.4481	0.3073

(43) DUIWENSHOK DAM. H8R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950										
1960				16.40	39.20	50.10	21.60	22.50	11.40	37.30
1970	19.80	15.10	29.60	18.10	28.40	42.60	21.20			
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.524	-0.090	-0.524

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	26.6643	SIGMA=	11.6040
LOGNORMAL	MU=	3.1968	SIGMA=	0.4328
GAMMA	ALPHA=	5.9399	BETA=	4.4890
WEIBULL	RHO=	2.5736	DELTA=	30.1541
EXTREME-1	XI=	21.4870	ETA=	8.5907
EXPONENTIAL	THETA=	26.6643		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1885	0.1430	0.1001
LOG NORMAL	0.1619	0.1325	0.1038
GAMMA	0.1742	0.1402	0.1032
WEIBULL	0.1803	0.1405	0.1015
EXTREME TYPE-1	0.1697	0.1405	0.1082
EXPONENTIAL	0.3316	0.3624	0.2793

(44) PRINSRIVIER DAM. J1R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910							3.30	0.01	1.25	33.50
1920	6.60	0.43	0.18	4.94	0.20	0.09	0.47	2.72	2.35	1.59
1930	3.61	1.41	0.87	2.61	0.55	14.80	3.43	0.96	2.14	2.86
1940	2.06	2.31	1.66	7.24	0.56	2.29	1.73	2.37	1.29	5.03
1950	0.78	3.00	11.20	1.67	6.26	12.80	0.79	2.01	0.79	2.80
1960	5.17	3.28	1.06	1.31	0.08	10.40	0.90	0.50	0.81	2.70
1970	1.90	4.20	5.60	2.50	3.40	4.60	1.30	0.40		
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.249	0.002	-0.249

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	3.3810	SIGMA=	4.9391
LOGNORMAL	MU=	0.5340	SIGMA=	1.3377
GAMMA	ALPHA=	0.8595	BETA=	3.9338
WEIBULL	RHO=	0.8747	DELTA=	3.1330
EXTREME-1	XI=	1.8491	ETA=	2.1151
EXPONENTIAL	THETA=	3.3810		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2462	0.3515	0.3350
LOG NORMAL	0.1149	0.1380	0.2316
GAMMA	0.1172	0.0992	0.1004
WEIBULL	0.1028	0.1082	0.1184
EXTREME TYPE-1	0.1553	0.1832	0.1988
EXPONENTIAL	0.1342	0.1265	0.1491

(45) BELLAIR DAM. J1R02.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										5.30
1920	1.51	0.24	0.57	1.31	0.02	0.83	0.31	1.28	10.10	0.62
1930	1.07	0.12	0.48	4.54	0.64	4.53	0.20	0.95	0.69	1.30
1940	0.33	1.74	4.70	5.09	1.66	2.18	1.05	9.33	1.28	3.74
1950	1.96	5.65	2.43	5.01	1.63	0.97	1.81	0.69	0.82	0.01
1960	7.91	6.37	1.55	5.43	0.67	4.53	0.98	0.13	0.18	0.50
1970	3.10									
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.272	-0.029	-0.272

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	2.3085	SIGMA=	2.4667
LOGNORMAL	MU=	0.1452	SIGMA=	1.4304
GAMMA	ALPHA=	0.8515	BETA=	2.7110
WEIBULL	RHO=	0.9014	DELTA=	2.1965
EXTREME-1	XI=	1.2885	ETA=	1.5098
EXPONENTIAL	THETA=	2.3085		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2326	0.2778	0.2734
LOG NORMAL	0.1224	0.1436	0.2280
GAMMA	0.1268	0.0911	0.0845
WEIBULL	0.1204	0.1012	0.1160
EXTREME TYPE-1	0.1804	0.1735	0.1855
EXPONENTIAL	0.1357	0.1237	0.1444

(46) FLORISKRAAL DAM. J1R03.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950							16.00	18.80	3.10	54.10
1960	37.70	20.60	17.90	8.20	1.30	28.00	9.60	5.00	3.00	70.20
1970	4.50	6.00	34.70	10.40	58.70					
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.450	-0.213	-0.450

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	21.4632	SIGMA=	20.6567
LOGNORMAL	MU=	2.5550	SIGMA=	1.1261
GAMMA	ALPHA=	1.1150	BETA=	19.2500
WEIBULL	RHO=	1.0537	DELTA=	21.9363
EXTREME-1	XI=	12.7125	ETA=	13.3850
EXPONENTIAL	THETA=	21.4632		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2037	0.1862	0.1626
LOG NORMAL	0.1502	0.1335	0.1205
GAMMA	0.1573	0.1305	0.0964
WEIBULL	0.1499	0.1282	0.0992
EXTREME TYPE-1	0.1825	0.1523	0.1111
EXPONENTIAL	0.1602	0.1443	0.1102

(47) CALITZDORP DAM. J2R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										3.67
1920	10.30	11.20	15.70	1.68	12.00	4.01	1.82	2.68	14.00	6.94
1930	3.21	19.50	9.81	4.54	18.00	4.91	13.00	6.49	10.40	10.70
1940	4.48	2.14	5.88	10.30	2.02	4.18	3.06	4.73	3.80	3.77
1950	15.00	2.86	7.18	11.70	5.80	5.62	5.88	2.23	5.14	1.71
1960	11.00	2.94	7.50	4.85	2.32	5.72	7.59	3.00	5.23	1.75
1970	9.00	5.00	1.90	3.80	4.30	15.50	13.90	5.00		
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.255	-0.068	-0.255

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	6.8193	SIGMA=	4.5546
LOGNORMAL	MU=	1.7015	SIGMA=	0.6773
GAMMA	ALPHA=	2.4448	BETA=	2.7893
WEIBULL	RHO=	1.6125	DELTA=	7.6659
EXTREME-1	XI=	4.8087	ETA=	3.1898
EXPONENTIAL	THETA=	6.8193		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1889	0.2299	0.2393
LOG NORMAL	0.1073	0.1001	0.1032
GAMMA	0.1259	0.1243	0.1389
WEIBULL	0.1327	0.1608	0.1776
EXTREME TYPE-1	0.1316	0.1334	0.1505
EXPONENTIAL	0.2137	0.3430	0.3276

(48) LEEUGAMKA DAM. J2R02.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										90.90
1920	163.00	11.30	9.60	37.80	2.20	0.70	33.00	15.70	11.70	3.60
1930	9.80	48.00	0.70	3.30	128.00	88.10	36.20	48.40	29.80	1.20
1940	22.00	3.00	8.90	5.60	108.00	2.60	76.20	32.40	6.30	2.80
1950	69.90	4.50	12.10	53.40	0.60	1.40	4.30	144.00	5.20	9.10
1960	24.30	15.00	5.20	16.00	3.50	12.30				
1970										
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.286	0.044	-0.286

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	30.2468	SIGMA=	40.4547
LOGNORMAL	MU=	2.4797	SIGMA=	1.5047
GAMMA	ALPHA=	0.6560	BETA=	46.1066
WEIBULL	RHO=	0.7430	DELTA=	24.8930
EXTREME-1	XI=	14.5251	ETA=	22.0114
EXPONENTIAL	THETA=	30.2468		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2492	0.3340	0.3115
LOG NORMAL	0.1060	0.0956	0.0876
GAMMA	0.1523	0.1512	0.1583
WEIBULL	0.1264	0.1227	0.1230
EXTREME TYPE-1	0.2189	0.2399	0.2396
EXPONENTIAL	0.2242	0.2013	0.1453

(49) OUKLOOF DAM. J2R03.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930	1.50	7.80	2.06	0.29	7.57	0.56	1.84	0.21	5.18	1.97
1940	1.07	3.21	1.67	4.02	5.94	0.86	0.68	0.85	1.78	0.97
1950	9.60	0.66	4.97	19.60	2.24	1.13	1.82	0.65	1.61	2.26
1960	9.23	1.18	5.33	2.49	1.46	4.34	13.50	7.44	1.04	0.14
1970	10.51	3.20	1.00	1.00	2.00	4.61	50.44	1.70	1.83	0.36
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.277	-0.030	-0.277

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	4.3474	SIGMA=	7.6725
LOGNORMAL	MU=	0.7523	SIGMA=	1.1690
GAMMA	ALPHA=	0.8242	BETA=	5.2747
WEIBULL	RHO=	0.8284	DELTA=	3.8225
EXTREME-1	XI=	2.1882	ETA=	2.8546
EXPONENTIAL	THETA=	4.3474		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2800	0.3752	0.3425
LOG NORMAL	0.1106	0.0950	0.0976
GAMMA	0.2178	0.1856	0.1837
WEIBULL	0.1494	0.1527	0.1480
EXTREME TYPE-1	0.2376	0.2187	0.2214
EXPONENTIAL	0.2135	0.1632	0.1137

(50) KAMMANASIE DAM. J3R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920		30.30	20.00	75.40	34.20	19.00	9.30	32.60	58.20	19.50
1930	206.00	31.20	12.80	171.00	20.60	27.60	11.30	32.80	22.50	7.40
1940	23.70	13.70	19.10	18.20	14.90	16.90	26.40	110.00	17.70	41.80
1950	170.00	33.60	16.60	23.60	14.60	26.80	18.30	11.40	39.50	43.10
1960	45.60	20.60	27.30	81.20	45.20	13.70	3.60	111.00	22.80	3.60
1970	25.50	19.30	17.80							
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.272	-0.090	-0.272

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	38.0538	SIGMA=	42.6293
LOGNORMAL	MU=	3.2568	SIGMA=	0.8347
GAMMA	ALPHA=	1.4525	BETA=	26.1997
WEIBULL	RHO=	1.1148	DELTA=	39.9372
EXTREME-1	XI=	23.3298	ETA=	19.8628
EXPONENTIAL	THETA=	38.0538		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2891	0.3182	0.3040
LOG NORMAL	0.1280	0.1219	0.1369
GAMMA	0.1913	0.1324	0.1196
WEIBULL	0.1826	0.1710	0.1595
EXTREME TYPE-1	0.1954	0.1419	0.1425
EXPONENTIAL	0.1889	0.2123	0.1949

(51) KROMRIVIER DAM. K9R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940									19.60	103.00
1950	66.50	33.70	150.00	59.40	92.80	171.00	45.10	49.20	60.40	31.80
1960	24.90	47.80	57.80	30.90	53.90	47.80	142.00	25.00	6.90	390.00
1970	48.10	9.50	93.00	34.00	26.30	79.40	12.90	128.30	11.30	170.00
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.346	-0.248	-0.346

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	72.5719	SIGMA=	74.3140
LOGNORMAL	MU=	3.8954	SIGMA=	0.9127
GAMMA	ALPHA=	1.4286	BETA=	50.8009
WEIBULL	RHO=	1.1515	DELTA=	76.7607
EXTREME-1	XI=	45.4512	ETA=	39.9438
EXPONENTIAL	THETA=	72.5719		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2164	0.2280	0.2125
LOG NORMAL	0.1186	0.1150	0.1164
GAMMA	0.1384	0.1082	0.0911
WEIBULL	0.1333	0.1127	0.0981
EXTREME TYPE-1	0.1597	0.1251	0.1057
EXPONENTIAL	0.1601	0.1708	0.1565

(52) BEERVLEI DAM. L3R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950								3.06	0.34	520.00
1960	5.07	83.90	16.00	21.40	15.20	26.40	2.11	63.70	14.70	202.10
1970	4.50	14.00	184.40	9.20	142.00	59.00	6.30			
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.438	-0.267	-0.438

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	69.6690	SIGMA=	122.5073
LOGNORMAL	MU=	2.9764	SIGMA=	1.7912
GAMMA	ALPHA=	0.5010	BETA=	139.0529
WEIBULL	RHO=	0.6214	DELTA=	46.4001
EXTREME-1	XI=	28.7032	ETA=	54.1992
EXPONENTIAL	THETA=	69.6690		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2886	0.2965	0.2494
LOG NORMAL	0.1399	0.1394	0.1399
GAMMA	0.1899	0.1448	0.1165
WEIBULL	0.1556	0.1272	0.1036
EXTREME TYPE-1	0.2739	0.2113	0.1802
EXPONENTIAL	0.3156	0.2771	0.2361

(53) PAUL SAUER DAM. L8R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950										
1960		79.90	218.00	154.00	90.40	172.00	194.00	290.00	130.00	34.00
1970	580.50	161.00	38.60	272.00	115.70	108.40	124.70	54.60	280.60	
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.462	-0.280	-0.462

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	172.1333	SIGMA=	128.9355
LOGNORMAL	MU=	4.9105	SIGMA=	0.7280
GAMMA	ALPHA=	2.2558	BETA=	76.3078
WEIBULL	RHO=	1.5003	DELTA=	192.0599
EXTREME-1	XI=	120.7036	ETA=	81.5017
EXPONENTIAL	THETA=	172.1333		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1878	0.1604	0.1384
LOG NORMAL	0.1221	0.1368	0.1265
GAMMA	0.1324	0.1186	0.0957
WEIBULL	0.1337	0.1116	0.0923
EXTREME TYPE-1	0.1394	0.1231	0.0940
EXPONENTIAL	0.1998	0.2301	0.1919

(54) GROENDAL DAM. M1R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930									4.70	5.50
1940	4.80	7.30	45.70	4.10	8.40	2.90	12.80	5.50	17.20	23.20
1950	29.30	8.70	52.00	18.10	4.80	29.60	12.10	6.50	2.40	5.90
1960	3.20	42.70	14.10	5.70	10.20	28.70	21.00	4.00	5.20	72.80
1970	5.20	3.40	41.30	24.10	7.00	16.50	7.20			
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.314	-0.155	-0.314

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	15.9949	SIGMA=	16.1672
LOGNORMAL	MU=	2.3510	SIGMA=	0.9124
GAMMA	ALPHA=	1.3287	BETA=	12.0377
WEIBULL	RHO=	1.1101	DELTA=	16.7158
EXTREME-1	XI=	9.5538	ETA=	9.3344
EXPONENTIAL	THETA=	15.9949		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2266	0.2865	0.2688
LOG NORMAL	0.1527	0.1278	0.1097
GAMMA	0.1881	0.1657	0.1629
WEIBULL	0.1696	0.1880	0.1881
EXTREME TYPE-1	0.2112	0.1906	0.1895
EXPONENTIAL	0.1585	0.2374	0.2304

(55) VANRYNEVELDSPAS DAM. N1R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920					7.20	14.80	58.00	26.60	26.90	34.50
1930	127.00	15.80	54.50	34.20	13.40	24.20	30.60	44.10	46.90	14.60
1940	9.60	6.80	64.20	18.30	4.70	4.30	80.80	9.90	43.50	38.70
1950	38.00	17.50	24.50	5.20	6.30	9.20	4.20	1.50	2.70	117.00
1960	7.20	107.00	10.80	7.70	1.70	10.20	5.40	12.00	29.80	65.80
1970	6.20	13.20	330.40	20.30						
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.277	-0.109	-0.277

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	34.1580	SIGMA=	51.9672
LOGNORMAL	MU=	2.8751	SIGMA=	1.1496
GAMMA	ALPHA=	0.8923	BETA=	38.2827
WEIBULL	RHO=	0.8740	DELTA=	31.4909
EXTREME-1	XI=	18.0408	ETA=	22.0507
EXPONENTIAL	THETA=	34.1580		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2285	0.3401	0.3177
LOG NORMAL	0.0914	0.0933	0.0952
GAMMA	0.1313	0.1362	0.1412
WEIBULL	0.1088	0.1433	0.1439
EXTREME TYPE-1	0.1757	0.1991	0.2072
EXPONENTIAL	0.1399	0.1395	0.1235

(56) MENTZ DAM. N2R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920			79.60	105.00	28.30	21.30	502.00	92.80	139.00	116.00
1930	610.00	152.00	218.00	193.00	98.40	138.00	161.00	232.00	206.00	41.00
1940	320.00	59.00	374.00	49.50	62.40	46.90	406.00	48.00	149.00	119.00
1950	202.00	56.20	541.00	63.40	37.50	69.30	29.30	43.30	36.40	462.00
1960	87.50	217.00	85.30	39.90	35.60	41.80	52.30	19.40	22.50	230.00
1970	32.10	53.00	733.00	22.60	234.00	508.00	23.70	191.90	57.30	
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.260	-0.229	-0.260

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	157.7982	SIGMA=	167.2098
LOGNORMAL	MU=	4.5780	SIGMA=	0.9899
GAMMA	ALPHA=	1.1732	BETA=	134.5065
WEIBULL	RHO=	1.0410	DELTA=	160.7759
EXTREME-1	XI=	91.2409	ETA=	96.0470
EXPONENTIAL	THETA=	157.7982		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2076	0.3230	0.3114
LOG NORMAL	0.1136	0.1183	0.1296
GAMMA	0.1482	0.1815	0.1963
WEIBULL	0.1294	0.2036	0.2157
EXTREME TYPE-1	0.1788	0.2177	0.2281
EXPONENTIAL	0.1266	0.2242	0.2335

(57) SLAGBOOM DAM. N4R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950										
1960	0.31	8.73	12.40	1.42	0.49	35.20	0.12	3.25	0.06	0.54
1970	1.80	0.20			18.90	11.70	7.13	2.12	2.96	38.10
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.462	-0.237	-0.462

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	8.0794	SIGMA=	11.6829
LOGNORMAL	MU=	0.7866	SIGMA=	1.9589
GAMMA	ALPHA=	0.4892	BETA=	16.5146
WEIBULL	RHO=	0.6112	DELTA=	5.5366
EXTREME-1	XI=	3.5992	ETA=	6.2551
EXPONENTIAL	THETA=	8.0794		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2548	0.2586	0.2190
LOG NORMAL	0.1463	0.1338	0.1148
GAMMA	0.1575	0.1354	0.1080
WEIBULL	0.1477	0.1322	0.0993
EXTREME TYPE-1	0.2384	0.1911	0.1556
EXPONENTIAL	0.2819	0.3067	0.2589

(58) GRASSRIDGE DAM. Q1R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920					103.00	22.80	13.70	27.30	60.60	26.00
1930	22.60	740.50	33.70	123.00	43.40	6.10	22.60	14.10	87.30	28.60
1940	39.80	26.90	30.70	103.00	5.60	19.00	17.80	50.00	5.70	190.00
1950	39.40	17.10	29.30	40.70	11.10	6.00	11.50	13.50	10.30	1.80
1960	9.40	23.00	20.10	10.10	0.60	6.90	80.10	5.10	7.40	20.20
1970	10.30	22.00	7.10							
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.280	-0.030	-0.280

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	46.2612	SIGMA=	107.4894
LOGNORMAL	MU=	3.0209	SIGMA=	1.1925
GAMMA	ALPHA=	0.7376	BETA=	62.7175
WEIBULL	RHO=	0.7719	DELTA=	37.3717
EXTREME-1	XI=	21.7558	ETA=	30.5360
EXPONENTIAL	THETA=	46.2612		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2907	0.3733	0.3391
LOG NORMAL	0.1047	0.1264	0.1646
GAMMA	0.3178	0.3093	0.3048
WEIBULL	0.1484	0.1953	0.1812
EXTREME TYPE-1	0.2191	0.2158	0.2162
EXPONENTIAL	0.2091	0.1868	0.1568

(59) LAKE ARTHUR DAM. Q4R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920					39.00	34.60	57.50	83.80	88.30	61.10
1930	151.00	67.10	224.00	98.60	44.70	64.90	40.70	143.00	99.60	89.00
1940	65.80	93.20	285.00	30.20	45.40	33.30	51.20	5.40	249.00	93.10
1950	66.90	27.10	75.30	34.80	22.40	60.90	16.70	44.90	13.20	35.70
1960	37.30	64.00	51.40	28.80	25.20	34.20	44.70	46.90	31.10	38.40
1970	47.90	33.60								
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.283	0.095	-0.283

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	67.0813	SIGMA=	57.3258
LOGNORMAL	MU=	3.9445	SIGMA=	0.7260
GAMMA	ALPHA=	2.0639	BETA=	32.5020
WEIBULL	RHO=	1.3646	DELTA=	74.0918
EXTREME-1	XI=	45.8592	ETA=	31.1134
EXPONENTIAL	THETA=	67.0813		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2170	0.2373	0.2333
LOG NORMAL	0.1166	0.1283	0.1777
GAMMA	0.1453	0.1217	0.1139
WEIBULL	0.1508	0.1597	0.1434
EXTREME TYPE-1	0.1494	0.1135	0.0959
EXPONENTIAL	0.2344	0.2819	0.2393

(60) KOMMANDODRIF DAM. Q2R02.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950						53.90	21.70	41.30	13.20	37.30
1960	34.80	110.00	17.90	12.60	22.30	102.00	6.80	47.50	43.10	72.60
1970	35.70	15.10	32.80							
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.462	-0.288	-0.462

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	40.0333	SIGMA=	29.2052
LOGNORMAL	MU=	3.4438	SIGMA=	0.7432
GAMMA	ALPHA=	2.1857	BETA=	18.3164
WEIBULL	RHO=	1.5094	DELTA=	44.6855
EXTREME-1	XI=	27.7753	ETA=	19.3641
EXPONENTIAL	THETA=	40.0333		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1858	0.1494	0.1271
LOG NORMAL	0.1460	0.1371	0.1241
GAMMA	0.1459	0.1259	0.0970
WEIBULL	0.1448	0.1187	0.0923
EXTREME TYPE-1	0.1538	0.1316	0.0973
EXPONENTIAL	0.2090	0.2361	0.1922

(61) LAIING DAM. R2R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										40.10
1950	18.60	28.90	156.00	16.90	15.70	76.70	14.40	83.40	13.20	18.40
1960	25.10	157.00	93.80	7.50	18.80	46.50	23.50	4.20	181.00	76.00
1970	28.10	5.30	89.90	27.20						
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.392	-0.128	-0.392

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	50.6480	SIGMA=	51.0234
LOGNORMAL	MU=	3.4427	SIGMA=	1.0361
GAMMA	ALPHA=	1.1756	BETA=	43.0826
WEIBULL	RHO=	1.0601	DELTA=	51.9501
EXTREME-1	XI=	29.6549	ETA=	30.8883
EXPONENTIAL	THETA=	50.6480		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2372	0.2161	0.1961
LOG NORMAL	0.1552	0.1224	0.1078
GAMMA	0.1859	0.1398	0.0955
WEIBULL	0.1750	0.1333	0.1004
EXTREME TYPE-1	0.2171	0.1684	0.1265
EXPONENTIAL	0.1672	0.1471	0.1212

(62) ROOIKRANSDAM. R2R02.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950	11.30	11.60	10.50	3.40	15.50	5.20	6.80	5.80	12.20	13.50
1960	28.80	19.80	2.80	7.40	10.70	2.90	5.80	14.00	22.90	7.90
1970	4.60	20.40	5.90	16.30	5.30	9.00	17.30	0.80	14.40	1.10
1980	0.50									

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.352	-0.011	-0.352

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	10.1419	SIGMA=	6.9589
LOGNORMAL	MU=	1.9902	SIGMA=	0.9671
GAMMA	ALPHA=	1.6788	BETA=	6.0412
WEIBULL	RHO=	1.4426	DELTA=	11.1352
EXTREME-1	XI=	6.9705	ETA=	5.4025
EXPONENTIAL	THETA=	10.1419		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1315	0.1280	0.1159
LOG NORMAL	0.1578	0.1717	0.2094
GAMMA	0.1302	0.1357	0.1391
WEIBULL	0.1202	0.1473	0.1485
EXTREME TYPE-1	0.1234	0.1215	0.1051
EXPONENTIAL	0.1906	0.1831	0.1367

(63) WATERDOWN DAM. S3R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950							13.90	77.90	126.80	50.10
1960	30.90	69.30	42.50	19.40	39.50	52.90	43.10	15.50	35.30	88.10
1970	71.10	20.50	276.70	31.10	166.90	83.50	38.00			
1980										

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.428	-0.197	-0.428

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	66.3333	SIGMA=	61.2899
LOGNORMAL	MU=	3.8973	SIGMA=	0.7653
GAMMA	ALPHA=	1.8305	BETA=	36.2386
WEIBULL	RHO=	1.2851	DELTA=	72.4350
EXTREME-1	XI=	43.6469	ETA=	32.9834
EXPONENTIAL	THETA=	66.3333		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2190	0.2059	0.1807
LOG NORMAL	0.1303	0.1200	0.0995
GAMMA	0.1596	0.1247	0.0970
WEIBULL	0.1562	0.1405	0.1185
EXTREME TYPE-1	0.1691	0.1324	0.0994
EXPONENTIAL	0.1989	0.2411	0.2077

(64) MIDMAR DAM. U2R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920						44.00	110.00	92.00	122.00	146.00
1930	68.00	84.00	89.00	258.00	156.00	124.00	142.00	141.00	256.00	179.00
1940	159.00	215.00	369.00	191.00	99.00	56.00	146.00	150.00	87.00	130.00
1950	142.00	109.00	132.00	119.00	129.00	89.00	313.00	134.00	237.00	74.00
1960	164.00	126.00	146.00	135.00	188.00	111.00	208.00	120.00	86.00	153.00
1970	121.00	202.00	163.00	401.00	204.00	419.00	132.00	174.00	194.00	36.00
1980	96.00	86.00	38.00							

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.257	0.159	-0.257

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	151.6207	SIGMA=	79.8140
LOGNORMAL	MU=	4.8982	SIGMA=	0.5076
GAMMA	ALPHA=	4.2187	BETA=	35.9403
WEIBULL	RHO=	2.0367	DELTA=	171.7742
EXTREME-1	XI=	118.0040	ETA=	56.0628
EXPONENTIAL	THETA=	151.6207		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1589	0.1428	0.1536
LOG NORMAL	0.1149	0.1223	0.1547
GAMMA	0.1094	0.0980	0.1005
WEIBULL	0.1276	0.1215	0.1120
EXTREME TYPE-1	0.1012	0.0961	0.1024
EXPONENTIAL	0.3106	0.3559	0.3273

(65) CHELMSFORD DAM. V3R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950										
1960	80.70	46.50	43.10	174.00	58.00	208.00	34.20	68.90	73.00	104.10
1970	156.00	27.50	182.90	228.10	253.90	125.60	189.50	31.00	76.50	88.00
1980	10.00	6.20								

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.418	0.164	-0.418

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	102.9864	SIGMA=	74.8343
LOGNORMAL	MU=	4.2838	SIGMA=	0.9839
GAMMA	ALPHA=	1.5714	BETA=	65.5376
WEIBULL	RHO=	1.3666	DELTA=	112.3705
EXTREME-1	XI=	68.8247	ETA=	56.7487
EXPONENTIAL	THETA=	102.9864		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1722	0.1428	0.1193
LOG NORMAL	0.1476	0.1697	0.1831
GAMMA	0.1382	0.1340	0.1246
WEIBULL	0.1379	0.1458	0.1370
EXTREME TYPE-1	0.1477	0.1252	0.0983
EXPONENTIAL	0.1650	0.1590	0.1267

(66) WAGENDRIFT DAM. V7R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950										
1960			166.00	230.00	170.00	450.00	77.30	167.00	169.00	108.00
1970	179.90	143.10	395.30	250.00	440.00	173.80	245.80	155.30	243.30	128.50
1980	59.60									

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.450	-0.115	-0.450

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	207.9947	SIGMA=	111.3493
LOGNORMAL	MU=	5.2085	SIGMA=	0.5278
GAMMA	ALPHA=	4.0360	BETA=	51.5346
WEIBULL	RHO=	2.0541	DELTA=	235.9769
EXTREME-1	XI=	160.2194	ETA=	78.3816
EXPONENTIAL	THETA=	207.9947		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.2151	0.1506	0.1074
LOG NORMAL	0.1644	0.1453	0.1359
GAMMA	0.1779	0.1390	0.1099
WEIBULL	0.1886	0.1375	0.1021
EXTREME TYPE-1	0.1713	0.1401	0.1207
EXPONENTIAL	0.3122	0.3351	0.2651

(67) NOOITGEDACHT DAM. X1R01.

DATA. (10**6.M**3).

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/0
1900										
1910										
1920										
1930										
1940										
1950										
1960	64.20	30.10	33.00	31.40	61.20	9.30	69.90	78.40	92.20	85.00
1970	18.80	37.80	32.10	173.00	149.00	165.00	26.20	41.60	12.00	59.00
1980	56.60	18.00								

CORRELATION ANALYSIS.

LAG	CONF. INT.(1)	CORRELATION	CONF. INT.(2)
1	0.418	0.365	-0.418

UNIVARIATE MODEL PARAMETERS.

NORMAL	MU=	61.0818	SIGMA=	47.4836
LOGNORMAL	MU=	3.8277	SIGMA=	0.7976
GAMMA	ALPHA=	1.9075	BETA=	32.0226
WEIBULL	RHO=	1.4035	DELTA=	67.4763
EXTREME-1	XI=	41.1177	ETA=	31.0773
EXPONENTIAL	THETA=	61.0818		

SELECTION CRITERIA

	H=1.0	H=0.5	H=0.25
NORMAL	0.1675	0.1588	0.1468
LOG NORMAL	0.1299	0.1190	0.1128
GAMMA	0.1332	0.1114	0.0861
WEIBULL	0.1259	0.1055	0.0842
EXTREME TYPE-1	0.1436	0.1222	0.0900
EXPONENTIAL	0.1806	0.2091	0.1815