The Activated Sludge Process Part II – Dynamic Behaviour

G. EKAMA* and G.v.R. MARAIS**
[*RESEARCH OFFICER, WATER RESEARCH COMMISSION OF SOUTH AFRICA, WATER RESOURCES AND PUBLIC HEALTH ENGINEERING, UNIVERSITY OF CAPE TOWN]
[**PROFESSOR, WATER RESOURCES AND PUBLIC HEALTH ENGINEERING, UNIVERSITY OF CAPE TOWN]

Synopsis

A mathematical model describing the dynamic behaviour of the completely mixed activated sludge process is presented. The model includes nitrification.

From the study it has been found that in the activated sludge process the COD is utilized in a two stage mechanism, (i) COD is adsorbed onto the organisms (storage of COD) and (ii) stored COD is metabolized.

From the dynamic modelling of the nitrification aspect of the process it was found that the generally accepted nitrification theory was inadequate. An alternative kinetic theory is proposed which assumes that the influent TKN is split into three fractions (i) an unbiodegradable fraction, (ii) a fraction which is slowly biodegradable (probably proteinaceous nitrogen) and (iii) fraction of nitrogen immediately available for nitrification and synthesis (probably free and saline ammonia).

Experimental data is also presented against which the mathematical model was calibrated. The experimental units were operated at 2,5 and 20 day sludge ages at 20°C. It was found that nitrification occurred in the 2,5 day sludge age units.

A sinesoidal input COD mass load was imposed on the calibrated mathematical model and from the results design charts for sludge generation and peak oxygen consumption rates were constructed.

Design examples for both settled and unsettled municipal sewage are presented.

Introduction

In 1970, Lawrence and McCarthy presented a model describing the kinetic behaviour of the completely mixed activated sludge process. Marais (1973) presented an extension of their work, his model differentiating between active and inert fractions of the sludge mass. Both models included nitrification. Experimental verification of Marais' model was done under steady state conditions only and it was found that the model predicted satisfactorily the experimental behaviour. However, when the model was applied to predict the behaviour of the process under daily cyclic square wave input of nutrient and flow (12 hours on, 12 hours off, Fig. 1), the predictions did not conform to experimental observations:

(1) During the feed period the oxygen consumption was generally much lower and during the no-feed period the oxygen consumption was generally higher than predicted.

(2) Total Kjeldahl Nitrogen (TKN) and nitrate experimental values indicated that less nitrogen was oxidized during the feed period and more was oxidizing during the no-feed period than predicted.

It was evident that there is a deficiency in the model in both the oxidation of the carbonaceous material and the TKN. These defects were not discernable under steady state condi-

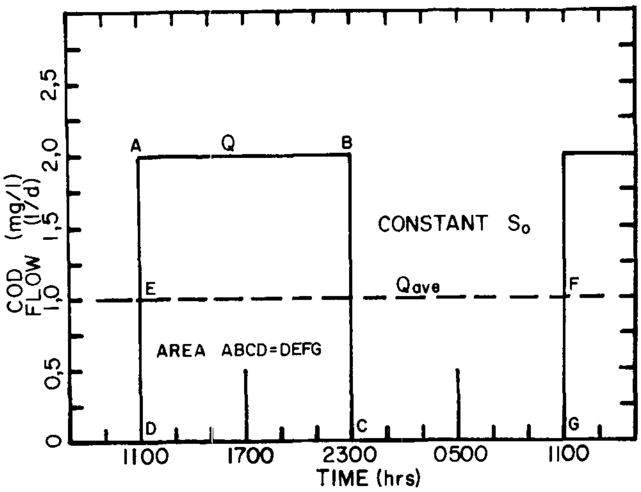

Figure 1
Influent feed pattern of experimental units.

tions. In this state, if a series of sequentially dependent reactions having different rates are present, the results are governed by the lowest rate. Only an investigation under cyclic flow conditions can expose the reactions governed by intermediate rates.

To resolve the inconsistencies exposed in the Marais model under cyclic flow conditions, the following hypotheses on process behaviour were advanced:

(1) There appears to be a two phase process governing the utilization of COD in the wastewater: (i) a phase of disappearance of COD from the liquid onto the organisms, i.e. adsorption or storage of the COD, and (ii) a phase of metabolism of the absorbed (stored) material by the organisms. The rates of the two phases are apparently different. This hypothesis has qualitative support from the behaviour of contact stabilization processes.

(2) Experimental results on nitrification in cyclically loaded completely mixed activated sludge units indicate that (i) a fraction of the TKN (probably proteinaceous nitrogen) is not directly available for nitrification and synthesis of cell mass, but must first be broken down to free and saline ammonia before it can be utilized. This conversion to ammonia seems to be a relatively slow process. (ii) The conversion from ammonia to nitrates appeared to take place at a rate so much faster than that given by the theory based on the Monod function for nitrification, that the maximum rate constant, K_m, in this function must be increased by one to two orders of magnitude to obtain the rates of nitrification observed. Indeed, for the purpose of modelling, the conversion of ammonia to nitrates can be taken as instantaneous.

(3) With COD storage on the organism, a certain mass of nitrogen is also stored. This became apparent when calibrating the dynamic model with the experimental data observed in cyclically fed activated sludge units. No reasonable comparison could be obtained for the TKN and nitrate values by splitting the influent nitrogen only into fractions of slowly biodegradable nitrogen and immediately available ammonia in the bulk solution. It was necessary to include some storage of nitrogen, linked to the COD storage.

Marais and Ekama (1975) presented a model for the completely mixed sludge process under steady state conditions that included the above hypotheses. However, as they were concerned only with steady state behaviour, the storage mechanism was effectively by-passed and could be neglected because in steady state the masses in storage remain constant. Their equations of the model did not, therefore, include storage effects. Their model is an improvement on the model presented by Marais (1973) in that the effluent TKN can be calculated. In the Marais model the effluent TKN had to be estimated empirically.

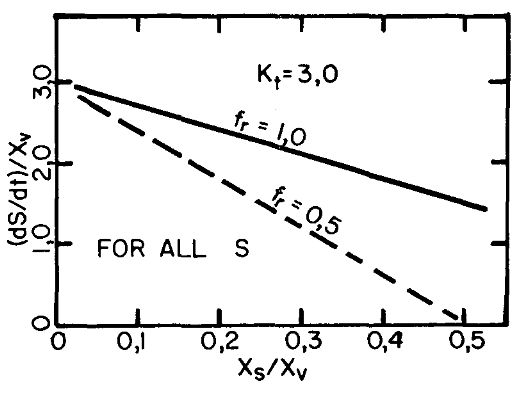

Figure 2
Diagrammatic representation of Eq. (7).

In this paper an extension of the models of Lawrence and McCarty and Marais is presented to describe the behaviour of the activated sludge process under dynamic conditions for both carbonaceous and nitrogen degradation. Experimental data against which the model was calibrated is also presented.

Theory

Carbonaceous Degradation

1. Influent COD fractions

In unsettled municipal sewage the influent COD is composed of three fractions:

(i) A soluble unbiodegradable COD fraction, f_u.

(ii) An inert organic solid fraction, f_i, which accumulates in the process reactor if the sludge age is longer than the hydraulic retention time. The fraction is measured in the reactor as VSS.

(iii) A biodegradable fraction (the balance after (i) and (ii) have been deducted) which serves as the energy source for the biological process.

Let the total influent COD = S_o (mg COD/ℓ)

Then unbiodegradable COD, $S_{ui} = f_u S_o$ (mg COD/ℓ) (1)

Inert solids measured as VSS, $X_{ii} = f_i S_o$ (mg VSS/ℓ) (2)

Equivalent COD of inert solids, $S_{xii} = P\, f_i S_o$ (mg COD/ℓ) (3)

where

P is the COD/VSS ratio equal to 1,42.

The biodegradable COD, S_i, is given by

$$S_i = S_o - f_u S_o - P f_i S_o$$
$$= S_o(1 - f_u - f_i P) \quad \text{(mg COD/ℓ)} \quad (4)$$

In the experimental investigation using unsettled municipal sewage, it was found that $f_u = 0{,}05$ and $f_i = 0{,}09$. Thus:

$$S_i = S_o(1 - 0{,}05 - 1{,}42 * 0{,}09)$$
$$= 0{,}822\ S_o \quad (5)$$

2. Biodegradable COD removal

Andrews and Busby (1973) initially proposed Eq. (6) as the rate function for COD entering into storage onto the organism, i.e. COD disappearing from the liquid phase.

$$dS/dt = -K_t X_v (f_r - X_s/X_v) \quad (6)$$

Multiplying out the right hand side of Eq. (6):

$$dS/dt = -K_t f_r X_v + K_t X_s \quad (7)$$

A diagrammatic plot of Eq. (7) is given in Fig. 2. Equation (7)

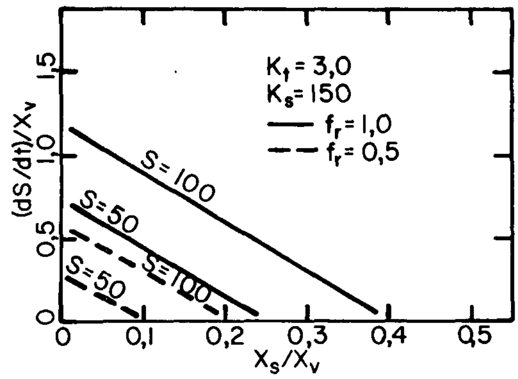

Figure 3
Diagrammatic representation of Eq. (9).

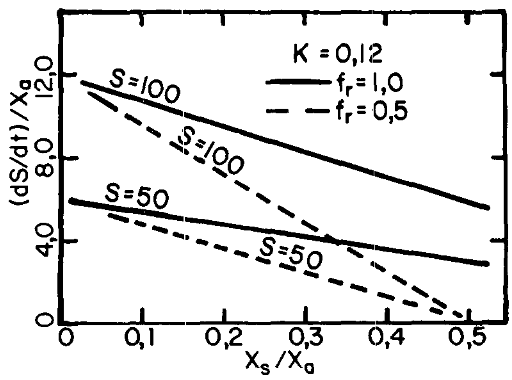

Figure 5
Diagrammatic representation of Eq. (13).

implies that (i) the rate of disappearance of COD from the liquid phase is independent of the biodegradable COD in the liquid phase, (ii) the rate is dependent on the MLVSS and (iii) there is a limit to the COD that can be stored onto the organisms.

Andrews recognized that this model was defective in that it did not incorporate the effect of the COD in the liquid. He suggested that the influence of the COD in the liquid can be incorporated by modifying f_r in Eq. (6) to read:

$$\frac{dS}{dt} = -K_t X_v \left(f_r\left(\frac{S}{K_s + S}\right) - \frac{X_s}{X_v}\right) \qquad (8)$$

Expanding the right hand side of Eq. (8) and dividing by X_v

$$\frac{dS/dt}{X_v} = -K_t f_r\left(\frac{S}{K_s + S}\right) + K_t \frac{X_s}{X_v} \qquad (9)$$

A graphical representation of Eq. (9) is given in Fig. 3. This relationship implies that the maximum mass of COD that can be stored increases as S increases. The rate, dS/dt, increases as S increases but reaches a maximum value.

In this investigation, experimental observation indicated that the filtered effluent COD, S, remained substantially constant in the units, whether in steady state or under cyclic load conditions. Indeed, in the cyclically fed unit, no significant difference could be picked up in the effluent quality between the feed and non feed period. Incorporating Eq. (8) in the mathematical model, it was found impossible to predict this

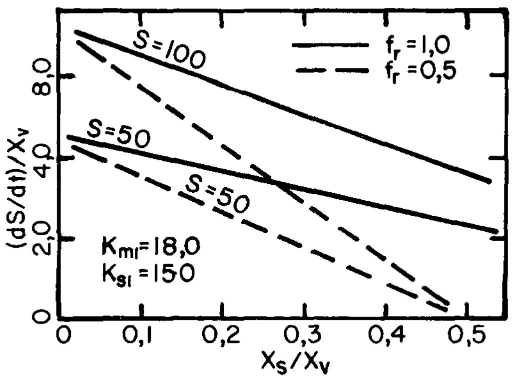

Figure 4
Diagrammatic representation of Eq. (11).

constant effluent COD, S. The following modification was necessary to give a relatively constant effluent COD. In Eq. (10), K_t was replaced by a Monod type function:

$$\frac{dS}{dt} = -\frac{K_{ml} S}{K_{sl} + S} \quad X_v\left(f_r - \frac{X_s}{X_v}\right) \qquad (10)$$

Expanding the right hand side of Eq. (10) and dividing by X_v:

$$\frac{dS/dt}{X_v} = -\frac{K_{ml} S f_r}{(K_{sl} + S)} + \frac{K_{ml} S X_s}{(K_{sl} + S) X_v} \qquad (11)$$

A graphical representation of Eq. (11) is given in Fig. 4. Equation (11) has the following properties: (i) the rate of COD going into storage increases with increasing COD in the liquid, but is limited to a maximum rate K_{ml}, (ii) at low COD, S, (the usual case in wastewater treatment) the rate of COD storage is proportional to S.

Two further modifications to Eq. (11) were necessary:

(i) From the experimental data, the total soluble COD in the liquid was low, about 40 mg/ℓ, irrespective of the sludge age. This meant that the biodegradable COD, S, was even lower. The half saturation constant, K_{sl}, has been reported to be in the order of 150 mg COD/ℓ. Thus K_{sl} is very much larger than S, so that a simplification of the Monod function is justified:

$$\frac{K_{ml} S}{K_{sl} + S} \approx K\,S \qquad (12)$$

where

$$K_{ml}/K_{sl} = K$$

(ii) Andrews accepted that the total volatile mass is included in the COD adsorption mechanism. It is very likely that only the active fraction of the sludge can adsorb COD. In Eq. (10), X_v (MLVSS) was therefore replaced by X_a (MLVASS). Including the two modifications in Eq. (10), (see Fig. 5):

$$dS/dt = -K\,S X_a (f_r - X_s/X_a) \qquad (13)$$

It was only by using Eq. (13) in the mathematical model that reasonable correlation was obtained between experimental data and the mathematical model.

3. Synthesis of cell mass

A basic assumption in the model is that only stored COD can be utilized by the organism for synthesis. The rate at which the stored COD is utilized is assumed to be given by the usual Monod relationship, with the difference that the COD surrounding the organism is now the stored COD, S_s, i.e.

$$\frac{dX_a}{dt} = Y\left(\frac{K_{m2}S_s}{K_{s2} + S_s}\right)X_a \qquad (14)$$

The Monod relationship relates mass synthesis to substrate concentrations surrounding the organism. Mass synthesis is measured as mg VSS/ℓ and substrate as mg COD/ℓ. The substrate surrounding the organism is stored COD, S_s, measured as mg COD/ℓ. However, it can be written in terms of mg VSS/ℓ as follows:

$$S_s = X_sP$$

This is necessary as the stored COD adds to the VSS, i.e. the stored COD is expressed as mg VSS/ℓ and is included in the measured total volatile solids, X_v. Replace S_s by X_sP in Eq. (14):

$$\frac{dX_a}{dt} = Y\frac{K_{m2}X_sP}{K_{s2} + X_sP}X_a \qquad (15)$$

The simplification of the Monod function (Eq. (12)) cannot be applied to Eq. (15). The half saturation constant for cell synthesis, K_{s2}, has been found empirically (by fitting the dynamic model to experimental data) to be about 100 mg COD/ℓ. The stored COD varies over the daily cyclic period, the degree of variation depending on the sludge age. At 2,5 days sludge age, the stored COD varies about 280 mg COD/ℓ (200 mg VSS/ℓ). At 20 days sludge age, the stored COD varies about 70 mg COD/ℓ (50 mg VSS/ℓ). These variations are of the same order as K_{s2} so that the simplified Monod function would be inappropriate. When Eq. (15) was written in the simplified form and included in the model, a poor correlation between experimental and predicted data was achieved; in the 2,5 day sludge age tests, it was impossible to model the oxygen consumption behavioural pattern over the period of seven hours after the feed period terminated. However, when the full Monod relationship, Eq. (15) was included in the model, experimental behaviour could be satisfactorily predicted.

From Eq. (13) and Eq. (15) the rate of change of the stored COD is found, i.e.

$$\left\{\begin{matrix}\text{rate of change}\\ \text{of stored COD}\\ \text{(as VSS)}\end{matrix}\right\} = \left\{\begin{matrix}\text{rate of COD}\\ \text{into}\\ \text{storage}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of utiliza-}\\ \text{tion of stored}\\ \text{COD for synthesis}\end{matrix}\right\}$$

$$\frac{dX_s}{dt} = K\,SX_a\left(f_r - \frac{X_s}{X_a}\right)\frac{1}{P} - \frac{K_{m2}X_sP}{K_{s2} + X_sP}X_a \qquad (16)$$

4. Endogenous respiration

Endogenous respiration is a phenomenon which manifests itself by a nett reduction of active mass per day, when there is no influent feed. There are two approaches in giving an explanation to this phenomenon:

(i) In the first approach, a black box behaviour is proposed in that the nett results only are modelled. A biodegradable fraction disappears which is directly equated to the oxygen consumption for endogenous respiration, and an inert unbiodegradable fraction remains, accumulating as endogenous residue.

(ii) In the second approach, an attempt is made to separate out reactions which take place during endogenous respiration. It is considered that a percentage of the live mass dies and lyse biodegradable nutrient back into the liquid, leaving the balance as unbiodegradable endogenous residue. The lysed COD is added to the COD in the liquid, and passes through the same phases of storage and storage abstraction for synthesis. This approach in effect, implies that endogenous respiration (cell maintenance) *per se* does not exist, and that oxygen is consumed only for synthesis.

Approach (i): Let b be the nett endogenous respiration rate constant per day. Then

$$dX_{a2}/dt = -bX_a \qquad \text{(mg VSS/ℓ/d)} \quad (17)$$

A fraction, f, of the live mass that disappears is unbiodegradable and remains as endogenous residue, i.e.

$$dX_e/dt = fbX_a \qquad \text{(mg VSS/ℓ/d)} \quad (18)$$

The nett volatile mass that disappears from the system per day (mg VSS/ℓ/d) is directly equated to oxygen consumption for endogenous respiration, i.e.

$$dO_e/dt = P(1 - f)bX_a \qquad \text{(mg O/ℓ/d)} \quad (19)$$

Experimentally, Marais and Ekama (1975) found that f = 0,20 and b = 0,24.

Approach (ii): Let the actual die-off rate constant of the actual organisms be b′ per day. Let the actual unbiodegradable fraction be f′. Thus

The active mass lost due to organism die-off

$$= b'X_a \qquad \text{(mg VSS/ℓ/d)} \quad (20)$$

The mass of biodegradable COD released back into the liquid

$$= P(1 - f')b'X_a \qquad \text{(mg COD/ℓ/d)} \quad (21)$$

The mass of organisms synthesized from this COD

$$= Y\,P(1 - f')b'X_a \qquad \text{(mg VSS/ℓ/d)} \quad (22)$$

The equivalent COD of the mass synthesized

$$= PYP(1 - f')b'X_a \qquad \text{(mg COD/ℓ/d)} \quad (23)$$

The oxygen consumption associated with this synthesis of mass is the difference between Eq. (21) and Eq. (23):

$$dO_e/dt = P(1 - f')b'X_a - PYP(1 - f')b'X_a$$
$$= (1 - PY)P(1 - f')b'X_a \qquad \text{(mg O/ℓ/d)} \quad (24)$$

The build up of endogenous residue, X_e, is

$$dX_e/dt = f'b'X_a \qquad \text{(mg VSS/ℓ/d)} \quad (25)$$

Under steady state conditions, (ignoring storage of COD) both approaches must yield the same results, i.e. the endogenous residue given by Eqs. (18) and (25) must be equal. Also, in the second approach the nett mass lost is the difference between Eqs. (20) and (22) and must be equal to that given by Eq. (17). Two equations, with two unknowns, f′ and b′, are obtained:

$$fb = f'b' \qquad (26)$$

$$-bX_a = b'X_a + YP(1 - f')b'X_a \qquad (27)$$

Solving for f′ and b′ in Eq. (26) and Eq. (27):

$$b' = \frac{b(1 - YPf)}{(1 - YP)} \qquad (28)$$

$$f' = \frac{f(1 - YP)}{(1 - YPf)} \qquad (29)$$

Substituting Y = 0,43, P = 1,42, f = 0,2 and b = 0,24 into Eq. (28) and Eq. (29) yields:

$$b' = 0,54$$

$$f' = 0,09$$

The oxygen consumption for both approaches should be equal. Substituting the respective values for b, f, Y and P into Eq. (19), the oxygen consumption for endogenous respiration (Approach (1)) is:

$$dO_e/dt = 0,273\ X_a \qquad \text{(mg O/ℓ/d)}$$

Substituting the respective values for b', f', Y and P into Eq. (24), the oxygen utilization for cell synthesis from lysed COD (Approach (ii)) is:

$$dO_e/dt = 0,273\ X_a \qquad \text{(mg O/ℓ/d)}$$

The two approaches therefore yield identical results when steady state conditions are considered (storage of COD neglected). However, in dynamic modelling when all the COD must first pass through the storage phase, the second approach gives slightly different results from the first. There is little to choose between the two approaches, although the second approach is intuitively more satisfying.

5. Carbonaceous oxygen demand

The rate of oxygen consumption is equivalent to the energy (COD) lost during the synthesis of the cell mass.

The rate of synthesis of cell mass is given by Eq. (14). The equivalent COD of the cell mass synthesized is:

$$\frac{dS_x}{dt} = PY\frac{K_{m2}X_sP}{(K_{s2} + X_sP)}X_a \qquad (30)$$

The COD released from storage is given by the second term of Eq. (16). The difference between this COD and that given by Eq. (30) is the oxygen consumption for synthesis:

$$\frac{dO_c}{dt} = (1 - PY)\ \frac{K_{m2}X_sP}{(K_{s2} + X_sP)}X_a \qquad (31)$$

If Approach (i) is used to model endogenous respiration, then the total oxygen consumption, for carbon removal, is given by the sum of Eq. (19) and Eq. (31).

$$dO_c/dt = (1 - PY)\ \frac{K_{m2}X_sP}{(K_{s2} + X_sP)}X_a + P(1 - f)bX_a \qquad (32)$$

If Approach (ii) is used then only synthesis of cell mass occurs and the total oxygen consumption for carbon removal is given by Eq. (31). However, the values of K, K_{m2} and K_{s2} must be replaced by K', K'_{m2} and K'_{s2} respectively to take into account the extra amount of cell mass that is synthesized from the COD released by the lysing of the dead organisms. In this report the analysis is based on Approach (i). A comparison between Approaches (i) and (ii) can be made by comparing Figs. 6 and 7.

Nitrogen Removal

1. Influent TKN fractions

Marais and Ekama (1975) suggested that the influent TKN can be divided into three fractions:

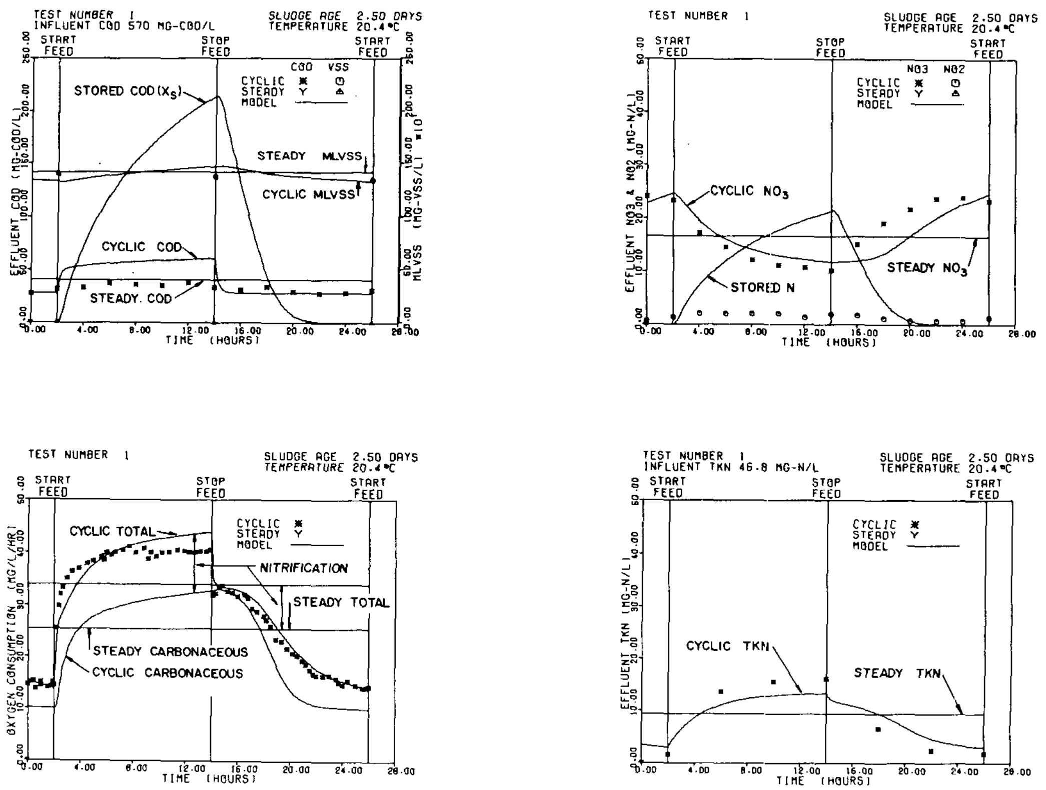

Figure 6
Comparison of 26 hour test number 1, 2, 5 day sludge age units, and theoretical model.

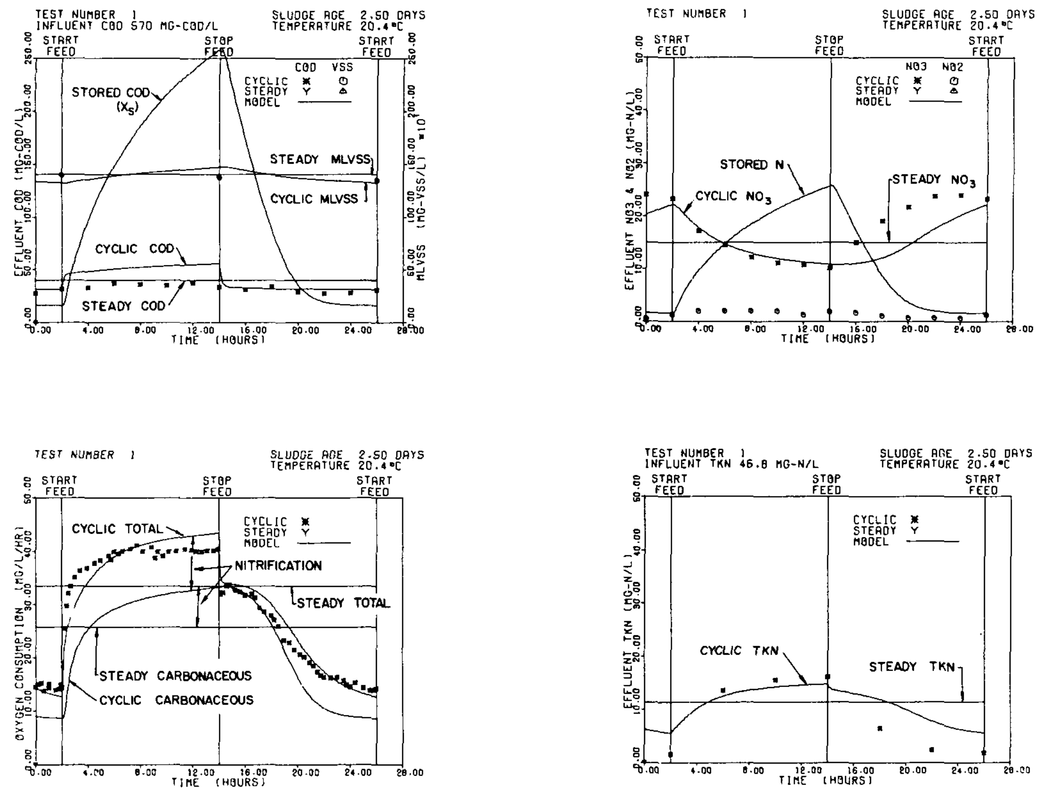

Figure 7
Comparison of experimental data of 26 hour test 1 and the theoretical model for 2,5 days sludge age and modelling endogenous respiration as in Approach (ii).

(i) A fraction, f_{nu}, (about 2 per cent) is unbiodegradable in terms of the time the TKN is exposed to the degradation process.

(ii) A fraction, f_{sd}, (about 80 per cent) is slowly biodegradable. This fraction is probably proteinaceous nitrogen which must be changed by the active organisms to free and saline ammonia before it can be utilized for synthesis or nitrification.

(iii) The remaining fraction $(1 - f_{nu} - f_{sd})$, is immediately available for synthesis and nitrification. This fraction is probably influent free and saline ammonia.

The following relationship can now be developed:

Let the total influent TKN by N_i mg N/ℓ. The unbiodegradable soluble, TKN, N_{ui}, in the influent is

$$N_{ui} = f_{nu}N_i \quad \text{(mg N/}\ell\text{)} \tag{33}$$

A fraction, f_n, (about 10 per cent) with respect to the inert solids of the influent, X_{ii} is nitrogen. Let N_{xii} be the concentration of nitrogen in the inert solids of the influent:

$$N_{xii} = f_n X_{ii} \tag{34}$$

The remaining nitrogen, N_{ai}, is available to the process as slowly degradable TKN and immediately available ammonia

$$N_{ai} = N_i - N_{ui} - N_{xii} \tag{35}$$

A fraction, f_{sd}, of N_{ai} is slowly degradable influent TKN, N_{si}, and the balance $(1 - f_{sd})$, is immediately available ammonia, N_{bi}, viz:

$$N_{si} = f_{sd}N_{ai} \tag{36}$$

$$N_{bi} = (1 - f_{sd})N_{ai} \tag{37}$$

Marais and Ekama (1975) proposed a first order decay rate for the degradation of the slowly biodegradable TKN in the reactor, N_s, as follows:

$$dN_s/dt = K_r X_a N_s \tag{38}$$

2. Nitrogen requirements for sludge

A fraction, f_n, of the sludge is nitrogen as TKN. The value of f_n ranges between 9 and 12 per cent with an average of about 10 per cent. Nitrogen requirements for cell mass synthesis is abstracted from the TKN available to the process, N_{ai}. Let, f_{ncs}, be the fraction of nitrogen required for cell synthesis taken from the slowly biodegradable nitrogen, and the remaining requirement $(1 - f_{ncs})$ taken from the immediately available ammonia fraction. (From the calibration of the mathematical model with experimental data it was found that $f_{ncs} = 0$, i.e. all nitrogen required for synthesis is abstracted from the free and saline ammonia.)

When endogenous respiration occurs, a concentration of nitrogen, $f_n(1 - f)bX_a$, (see Eq. (17)), is released back into the liquid. Assume that a fraction of the lysed nitrogen, f_{ens}, is slowly biodegradable, the remaining fraction, $(1 - f_{ens})$, is free and saline ammonia. The nitrogen incorporated in the endogenous residue, $f_n f b X_a$, remains as inert. Empirically, by fitting the dynamic model to experimental data, it was found that 80 per cent of the nitrogen released back into the liquid, was slowly biodegradable nitrogen, the division being the same as for the influent TKN, i.e. $f_{ens} = 0{,}80$.

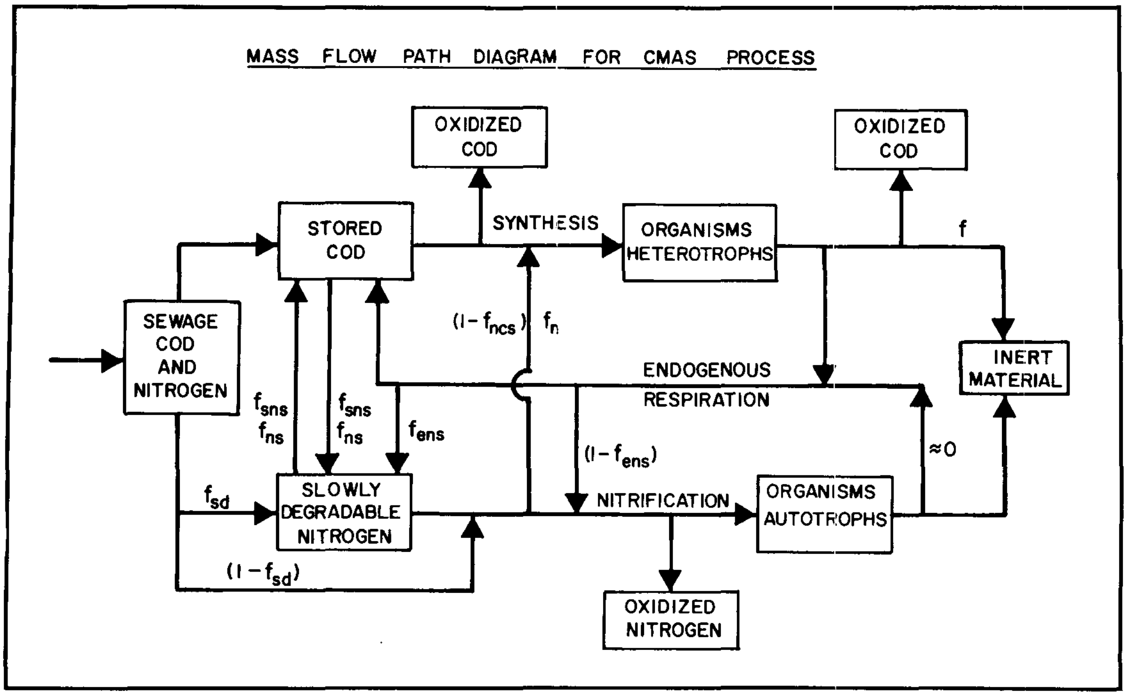

Figure 8
Mass flow diagram for the activated sludge process including nitrification.

3. Storage of nitrogen

In order to obtain a reasonable comparison between experimental data and the mathematical model, it was necessary to include some storage of nitrogen. It is likely that a certain fraction of the stored COD is linked to proteinaceous matter, and in this fashion links the storage of nitrogen to the storage of COD. Let the mass of stored nitrogen be a fraction, f_{ns}, of the stored COD. Of this stored nitrogen, assume a fraction, f_{sns}, is slowly biodegradable nitrogen and the balance, $(1 - f_{sns})$, is stored free and saline ammonia. Empirically, by comparing the mathematical model predictions with experimental data, the fraction f_{ns} was found to be 0,05, i.e. the storage of nitrogen was 5 per cent of the stored COD, and f_{sns} was found to be 1, i.e. all the stored nitrogen was slowly biodegradable and is assumed to be unchanged when the nitrogen comes out of storage.

A diagrammatic mass flow path for the complete model described above, i.e. for carbon removal and nitrification, is given in Fig. 8.

Process Kinetics

1. The mathematical model

The basic differential equations for sludge generation, COD utilization and nitrification derived above are applied to the particular case of the completely mixed activated sludge process (Fig. 9). The process equations are developed by doing a mass balance on each particular variable of the process.

The general form of the mass balance equation is –

$$\left\{\begin{array}{c}\text{rate of}\\\text{mass}\\\text{change}\end{array}\right\} = \left\{\begin{array}{c}\text{rate of}\\\text{mass}\\\text{input}\end{array}\right\} - \left\{\begin{array}{c}\text{rate of}\\\text{mass}\\\text{output}\end{array}\right\} \pm \left\{\begin{array}{c}\text{rate of mass}\\\text{change due to}\\\text{process reaction}\end{array}\right\} \quad (39)$$

(i) The biodegradable COD, S, in the reactor liquid:

$$\left\{\begin{array}{c}\text{rate of change of}\\\text{biodegradable}\\\text{COD}\end{array}\right\} = \left\{\begin{array}{c}\text{rate of biodegradable}\\\text{COD}\\\text{input}\end{array}\right\} - \left\{\begin{array}{c}\text{rate of biodegradable}\\\text{COD}\\\text{output}\end{array}\right\}$$

$$- \left\{\begin{array}{c}\text{rate of}\\\text{biodegradable COD}\\\text{going into storage}\end{array}\right\} + \left\{\begin{array}{c}\text{rate of release of}\\\text{biodegradable COD}\\\text{from the lysing of}\\\text{dead cells}\end{array}\right\}$$

$$\frac{dS}{dt} = Q(S_i - S)/V - K\ X_a S(f_r - X_s/X_a) + \{P(1 - f')b'\ X_a\}^* \quad (40)$$

(ii) Unbiodegradable COD in the reactor

$$\left\{\begin{array}{c}\text{rate of change of}\\\text{unbiodegradable}\\\text{COD in reactor}\end{array}\right\} = \left\{\begin{array}{c}\text{rate of gain}\\\text{from influent}\\\text{flow}\end{array}\right\} - \left\{\begin{array}{c}\text{rate of loss}\\\text{through}\\\text{effluent flow}\end{array}\right\}$$

$$dS_u/dt = Q(S_{ui} - S_u)/V \quad (41)$$

(iii) Active mass concentration, X_a, in the reactor

$$\left\{\begin{array}{c}\text{rate of change}\\\text{of active}\\\text{mass}\end{array}\right\} = \left\{\begin{array}{c}\text{rate of gain}\\\text{from synthesis}\\\text{of stored COD}\end{array}\right\}$$

***If endogenous respiration is modelled as in Approach (ii), then only is the term in brackets{} included and f, b, K, K_{m2}, and K_{m2} are replaced by f′, b′, K′, K'_{m2} and K'_{s2} respectively in all kinetic equations.**

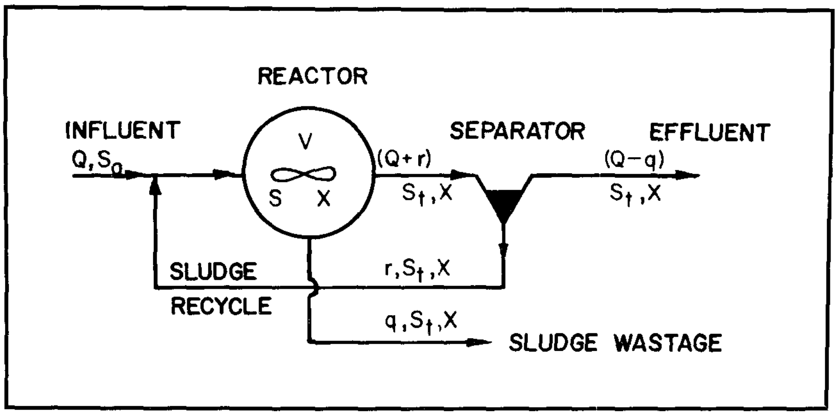

Figure 9
Diagrammatic layout of the completely mixed activated sludge process.

$$- \left\{ \begin{array}{c} \text{rate of loss} \\ \text{due to} \\ \text{cell die-off} \end{array} \right\} - \left\{ \begin{array}{c} \text{rate of loss} \\ \text{in sludge} \\ \text{draw off} \end{array} \right\}$$

$$\frac{dX_a}{dt} = Y \frac{K_{m2} X_s P}{(K_{s2} + X_s P)} X_a - bX_a - X_a \frac{V}{q} \qquad (42)$$

(iv) The stored COD (as VSS) accumulated in the sludge

$$\left\{ \begin{array}{c} \text{rate of change} \\ \text{of stored} \\ \text{COD} \end{array} \right\} = \left\{ \begin{array}{c} \text{rate of COD} \\ \text{going into} \\ \text{storage} \end{array} \right\}$$

$$- \left\{ \begin{array}{c} \text{rate of COD} \\ \text{released from} \\ \text{storage for} \\ \text{synthesis} \end{array} \right\} - \left\{ \begin{array}{c} \text{rate loss of} \\ \text{stored COD} \\ \text{in sludge} \\ \text{draw-off} \end{array} \right\}$$

$$\frac{dX_s}{dt} = K\, SX_a(f_r - \frac{X_s}{X_a}) \frac{1}{P} - \frac{K_{m2} X_s}{(K_{s2} + X_s P)} X_a - X_s \frac{V}{q} {*} \qquad (43)$$

(v) Inert material accumulating through endogenous residue:

$$\left\{ \begin{array}{c} \text{rate of change} \\ \text{of endogenous} \\ \text{residue} \end{array} \right\} = \left\{ \begin{array}{c} \text{gain from} \\ \text{cell} \\ \text{die-off} \end{array} \right\} - \left\{ \begin{array}{c} \text{loss in} \\ \text{sludge} \\ \text{wastage} \end{array} \right\}$$

$$dX_e/dt = fbX_a - X_e V/q \qquad (44)$$

(vi) Inert material accumulating from the inert material in the influent:

$$\left\{ \begin{array}{c} \text{rate of change} \\ \text{of inert} \\ \text{material} \end{array} \right\} = \left\{ \begin{array}{c} \text{rate of increase} \\ \text{through} \\ \text{influent} \end{array} \right\}$$

$$- \left\{ \begin{array}{c} \text{rate of loss} \\ \text{due to sludge} \\ \text{wastage} \end{array} \right\}$$

$$dX_i/dt = QX_{ii}/V - X_i V/q \qquad (45)$$

(vii) Rate of change of total mass (MLVSS) is the sum of the rate of changes of active, stored, endogenous and inert masses.

$$dX_v/dt = dX_a/dt + dX_s/dt + dX_e/dt + dX_i/dt \qquad (46)$$

(viii) Slowly biodegradable TKN in reactor

$$\left\{ \begin{array}{c} \text{rate of change of} \\ \text{slowly degradable} \\ \text{nitrogen} \end{array} \right\} = + \left\{ \begin{array}{c} \text{rate of gain in} \\ \text{slowly degradable} \\ \text{N into reactor} \end{array} \right\}$$

$$- \left\{ \begin{array}{c} \text{rate of loss of} \\ \text{slowly degradable} \\ \text{N out of reactor} \end{array} \right\} + \left\{ \begin{array}{c} \text{rate of gain} \\ \text{from organism} \\ \text{die-off} \end{array} \right\}$$

$$- \left\{ \begin{array}{c} \text{rate of loss} \\ \text{due to organism} \\ \text{synthesis} \end{array} \right\} + \left\{ \begin{array}{c} \text{rate of gain from} \\ \text{N being released} \\ \text{from storage} \end{array} \right\}$$

$$- \left\{ \begin{array}{c} \text{rate of loss due} \\ \text{to N going into} \\ \text{storage} \end{array} \right\} - \left\{ \begin{array}{c} \text{rate of loss due to N} \\ \text{being converted to} \\ \text{immediately available N} \end{array} \right\}$$

$$dN_s/dt = Q(f_{sd}N_{ai} - N_s)/V + f_{ens}f_n(1 - f)bX_a$$

$$- f_{ncs}f_n\, Y \frac{K_{m2} X_s P X_a}{(K_{s2} + X_s P)} + f_{sns}f_{ns} \frac{K_{m2} X_s}{(K_{s2} + X_s P)} X_a$$

$$- f_{sns}f_{ns}\, K\, SX_a(f_r - X_s/X_a)/P - K_r N_s X_a \qquad (47)$$

(ix) Unbiodegradable nitrogen in the reactor

$$\left\{ \begin{array}{c} \text{rate of change of} \\ \text{unbiodegradable} \\ \text{nitrogen} \end{array} \right\} = \left\{ \begin{array}{c} \text{rate of gain} \\ \text{from} \\ \text{influent flow} \end{array} \right\}$$

$$- \left\{ \begin{array}{c} \text{rate of loss} \\ \text{from effluent} \\ \text{flow} \end{array} \right\}$$

$$dN_u/dt = Q(N_{ui} - N_u)/V \qquad (48)$$

(x) The immediately available nitrogen in the reactor

$$\left\{ \begin{array}{c} \text{rate of change} \\ \text{of immediately} \\ \text{available N} \end{array} \right\} = \left\{ \begin{array}{c} \text{rate of gain} \\ \text{from} \\ \text{influent flow} \end{array} \right\}$$

***X V/q = mass drawn off for sludge wastage and is only correct if sludge is drawn off continuously over the whole day. However, it can be adapted for discrete interval draw-off in the mathematical model by**

$$X_a V/q = \sum_{i=1}^{n} W_i {*} X_a \Delta t/V$$

where
W_i = waste flow (ℓ/d) over the interval i
n = number of draw-off intervals
Δt = length of interval

$$+\left\{\begin{array}{c}\text{rate gain from}\\ \text{slowly changed}\\ \text{degradable N}\end{array}\right\}+\left\{\begin{array}{c}\text{rate of gain}\\ \text{from organism}\\ \text{die-off}\end{array}\right\}$$

$$-\left\{\begin{array}{c}\text{rate of loss}\\ \text{due to organism}\\ \text{synthesis}\end{array}\right\}+\left\{\begin{array}{c}\text{rate of gain from}\\ \text{N being released}\\ \text{from storage}\end{array}\right\}$$

$$-\left\{\begin{array}{c}\text{rate of loss due}\\ \text{to N going into}\\ \text{storage}\end{array}\right\}$$

$$dN_{h3}/dt = Q(1 - f_{sd})N_{ai}/V + K_rN_sX_a$$

$$+ (1 - f_{ens})f_n(1 - f)bX_a - (1 - f_{ncs})f_nY \frac{K_{m2}X_sP}{(K_{s2} + X_sP)} X_a$$

$$+ (1 - f_{sn})\, f_{ns} \frac{K_{m2}X_s}{(K_{s2} + X_sP)} X_a$$

$$- (1 - f_{sns})f_{ns}K\, SX_a(f_r - X_s/X_a)/P \qquad (49)$$

(xi) Stored nitrogen in reactor:

$$\left\{\begin{array}{c}\text{rate of change}\\ \text{of stored}\\ \text{nitrogen}\end{array}\right\} = \text{fraction*} \left\{\begin{array}{c}\text{rate of change}\\ \text{of stored}\\ \text{COD}\end{array}\right\}$$

$$dN_{st}/dt = f_{ns}{*}dX_s/dt \qquad (50)$$

(xii) Nitrates in the reactor liquid:

Once the organisms have taken or returned all the nitrogen from whichever fraction they desire, the remaining free and saline ammonia (immediately available nitrogen) is directly and instantly oxidized to nitrates.

$$\left\{\begin{array}{c}\text{rate of change}\\ \text{of nitrates in}\\ \text{reactor liquid}\end{array}\right\} = \left\{\begin{array}{c}\text{rate of } NO_3\\ \text{gain from}\\ \text{influent flow}\end{array}\right\}$$

$$-\left\{\begin{array}{c}\text{rate of loss of}\\ NO_3 \text{ in effluent}\\ \text{flow}\end{array}\right\}$$

$$N_{o3}/dt = Q(N_{o3i} - N_{o3})/V + dN_{h3}/dt \qquad (51)$$

In general, for normal domestic sewage, the influent nitrate concentration, N_{o3i}, is negligible.

(xiii) Oxygen utilization

(a) Carbonaceous oxygen demand

If endogenous respiration is considered as in Approach (i), i.e. considering nett loss of active organism mass, then the oxygen utilization rate for COD removal is given by Eq. (32).

If the actual loss of active mass is considered (Approach ii) then the oxygen utilization rate is given by Eq. (31), but K_{m2} and K_{s2} must be replaced by K'_{m2} and K'_{s2} respectively.

(b) Nitrification oxygen demand

Stoichiometrically every 1 mg/ℓ ammonia as N (1 mg NH_3 – N/ℓ) converted to nitrates requires 4,6 mg/ℓ of oxygen.

The free and saline ammonia oxidized to nitrates is given by Eq. (49). Oxygen utilization for nitrification is directly related to Eq. (49) by the factor 4,6 mg/ℓ oxygen required per mg (NH_3 – N/ℓ) converted.

$$\frac{dO_n}{dt} = 4{,}6\ \frac{dN_{h3}}{dt} \qquad (mgO_2/\ell/d) \qquad (52)$$

The equations above, Eqs. (40 to 52) and Eqs. (31 or 32) describe the kinetic behaviour of the completely mixed activated sludge process including nitrification.

The total oxygen consumption, which corresponds to that measured experimentally, is the sum of Eqs. (31) or (32) and Eq. (52).

$$dO_t/dt = dO_c/dt + dO_n/dt \qquad (53)$$

2. Method of solution

The set of equations describing the process is solved numerically by a discrete step by step method. A finite time interval, Δt, is selected and the equations written in discrete form, linking the beginning and end of each interval, for example, Eq. (41):

$$dS_u/dt = (S_u(i + 1) - S_u(i))/\Delta t \qquad (54)$$

Equating Eq. (41) to Eq. (54) and solving $S_u(i + 1)$, the unbiodegradable COD in the reactor at time (i + 1) is given by:

$$S_u(i + 1) = S_u(i) + \Delta t[Q(i)\{S_{ui}(i) - S_u(i)\}/V] \qquad (55)$$

The values of each of the other unknowns at time (i + 1) are found in a similar fashion by using their respective discrete approximations of the differential equations. These values in turn serve as starting values to calculate the values at (i + 2). The solution of the set of equations (one for each unknown) is advanced, step by step, over the whole day, then repeated over the next day until two consecutive days' results differ by a preselected amount. The system is then considered to be in "dynamic steady state".

An attempt was made to utilize more sophisticated methods of integration (such as the 4th order Runge-Kutta method) to allow larger integration steps, Δt. However, whatever the method of integration used, the process showed instability when the step Δt was greater than 1/240 day (6 min). Consequently, the simplest integration procedure (1st order method) was used.

3. Description of the computer model

Two computer models have been developed for the completely mixed activated sludge process, one which includes the concept of COD storage and the other not. The model that does not include COD storage is that presented by Marais and Ekama (1975).

The computer models are versatile in that each can execute the following as desired:

(a) Sludge wastage over any one or more intervals.
(b) Input data values can be read in individually or interpolated linearly from hourly values read in.
(c) The length of the time interval of integration is preselective.
(d) Storage of nitrogen can be any fraction of the stored COD ($0 < f_{ns} < 1{,}0$)
(e) Nitrogen from the influent, endogenous respiration storage and synthesis can be split into any proportion of slowly biodegradable and immediately available ammonia ($0 < f_{sns}, f_{ens}, f_{ncs}, f_{sd} < 1{,}0$).

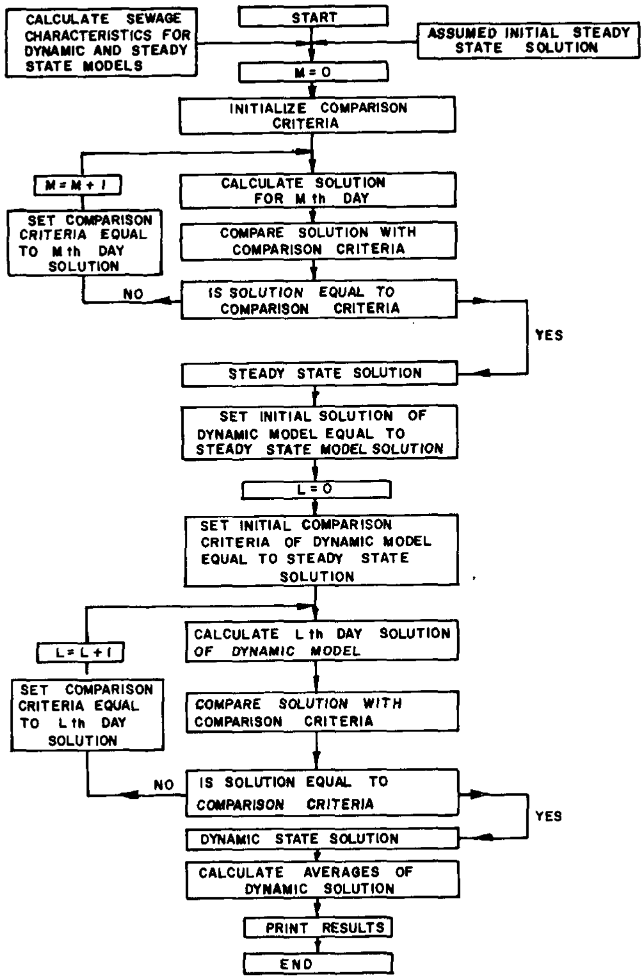

Figure 10
Digital computer calculation procedure for the mathematical model.

The calculation procedure is given in the form of an algorithm Fig. 10.

Calcomp subroutine plot programmes were written to plot the response waves of the experimental data and the mathematical models.

All programmes above were written in FORTRAN and executed on a UNIVAC 1106 system.

Experimental Procedure

The experimental set consisted of two pairs of laboratory scale completely mixed activated sludge units. One pair operated at 2,5 days sludge age, the other at 20 days. The units in each pair had identical design parameters (Table 1). One unit of a pair received a daily cyclic square wave feed (12 hours on and 12 hours off), the other unit received a constant feed rate over 24 hours (Fig. 1).

In each pair the units received the same mass of COD per day. This was assured by filling the individual influent containers with the required volume of feed for the day. The feed pumps of the cyclic units were calibrated to deliver this volume of feed in 12 hours so that no feeding takes place over the remaining 12 hours. In the steady state units, the volume of feed was delivered over 24 hours. For the cyclic units, the error in the 12 hour feed period was at most a half an hour, due to calibration error in pumping rates.

TABLE 1

DESIGN PARAMETERS FOR LABORATORY SCALE COMPLETELY MIXED ACTIVATED SLUDGE UNITS 1, 2, 3 AND 4

Parameter	Unit 1	Unit 2	Unit 3	Unit 4
State	Cyclic	Steady	Steady	Cyclic
Sludge age (d)	2,5	2,5	20	20
Volume of reactor	6,73 ℓ	6,73 ℓ	12,0 ℓ	12,0 ℓ
Volume of feed (ℓ/d)	18,0	18,0	14,0	14,0
Sludge wastage (ℓ/d)	2,7	2,7	0,6	0,6

Unsettled municipal sewage was used as influent. In the first 3 sets of data, the influent COD was set at 600 mg/ℓ and at 500 mg/ℓ for the remaining 5 sets of data. The daily influent batches were prepared from a stronger COD diluted to the required concentration. The influent containers were kept at 5°C to reduce degradation action. It was found necessary to warm the influent by coiling the feed pipe through a heating bath before discharging to the unit, to prevent a daily temperature variation in the cyclically fed unit (up to 4°C temperature variation was observed when preheating was omitted). The units were operated in a temperature controlled room at 20°C.

In the cyclically fed units, sludge was wasted from the reactors during the feed period only; if sludge was abstracted during the non-feed period, the settling tanks were emptied. In the cyclically fed 2,5 day sludge age unit, sludge was wasted on a continuous basis over the 12 hours feed period by pumping every five minutes to abstract every day 40 per cent (1/2,5) of the reactor liquor. Sludge was wasted in a similar fashion in the steady state unit but over 24 hours. In the 20 day sludge age units, the total volume of mixed liquor required for sludge wastage was drawn off at one fixed time per day as the waste volume constituted only 5 per cent (1/20) of the reactor volume so that single batch wastage per day had little effect on the process variables.

The units were monitored for at least 1–1/2 sludge ages to assure that steady state conditions were operative. Once steady state was achieved, regular testing at one or two day intervals was performed on all the units. The programme of the tests is listed in Table 2. It was found that the VSS and effluent COD were relatively insensitive to cyclic feed conditions and gave similar results to the steady state tests irrespective of the time the tests were taken, i.e. during the feed or non-feed period. These tests, therefore, give an overall view of the behaviour of the units over the entire testing period (Figs. 11 to 18). SVI tests were done at intervals to compare the settling properties of sludges from cyclic and steady state units (Figs. 19 to 23).

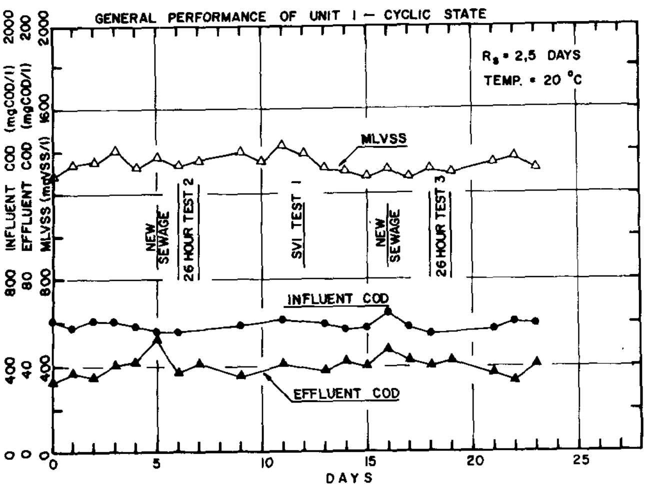

Figure 11
General performance of cyclic 2,5 day sludge age unit (1) over 1st testing period.

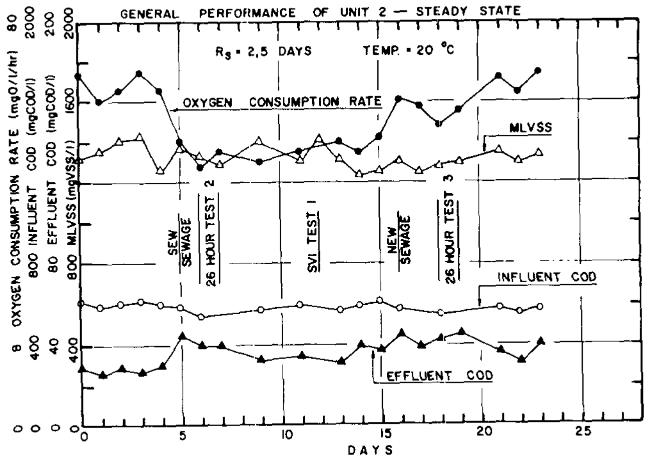

Figure 12
General performance of steady 2,5 day sludge age unit (2) over 1st testing period.

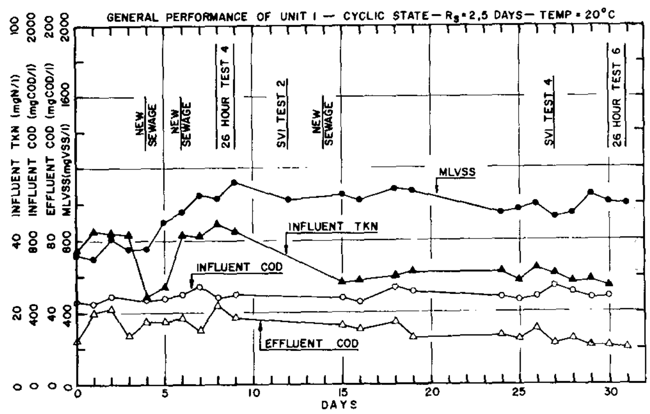

Figure 13
General performance of cyclic 2,5 day sludge age unit (1) over 2nd testing period.

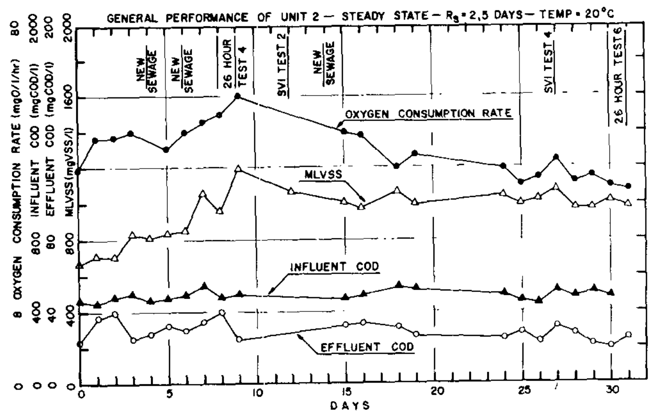

Figure 14
General performance of steady 2,5 day sludge age unit (2) over 2nd testing period – COD removal.

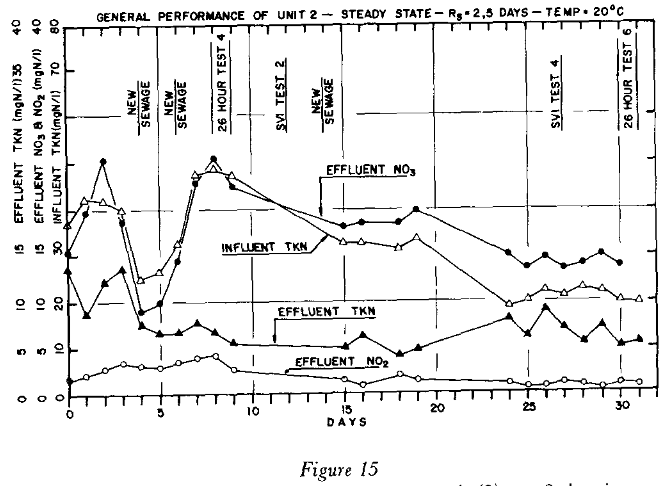

Figure 15
General performance of steady 2,5 day sludge age unit (2) over 2nd testing period – Nitrogen removal.

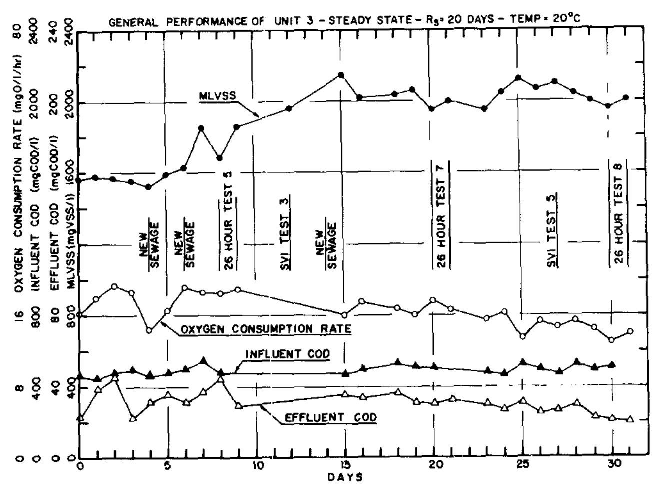

Figure 16
General performance of steady 20 day sludge age unit (3) over 2nd testing period – COD removal.

TABLE 2

PROGRAMME OF TESTS PERFORMED ON CMAS UNITS 1, 2, 3 AND 4 GIVING GENERAL PERFORMANCE OF UNITS OVER TEST PERIOD

Test	Cyclic	Steady
1	Influent COD	Influent COD
2	Influent TKN	Influent TKN
3	Effluent COD	Effluent COD
4	—	Effluent TKN
5	VSS	VSS
6	—	NO_3 NO_2
7	—	O_2 Uptake Rate

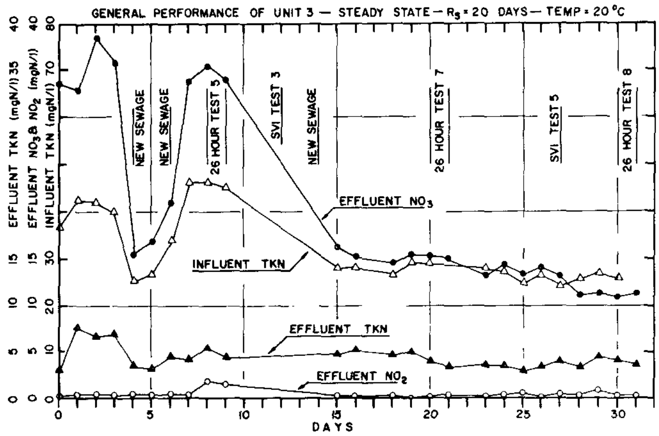

Figure 17
General performance of steady 20 day sludge age unit (3) over 2nd testing period – Nitrogen removal.

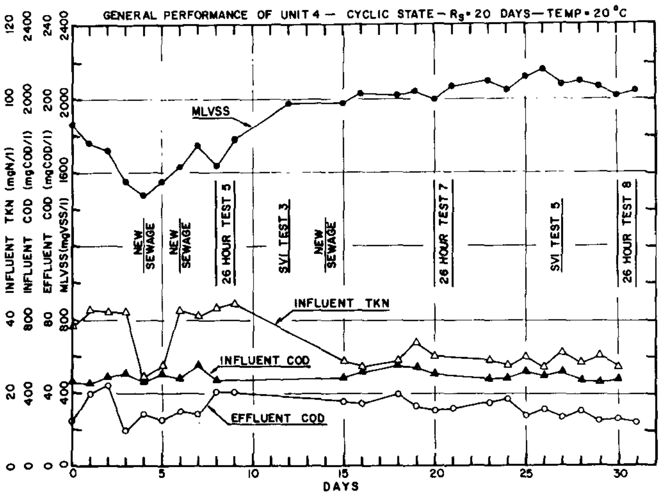

Figure 18
General performance of cyclic 20 day sludge age unit (4) over 2nd testing period.

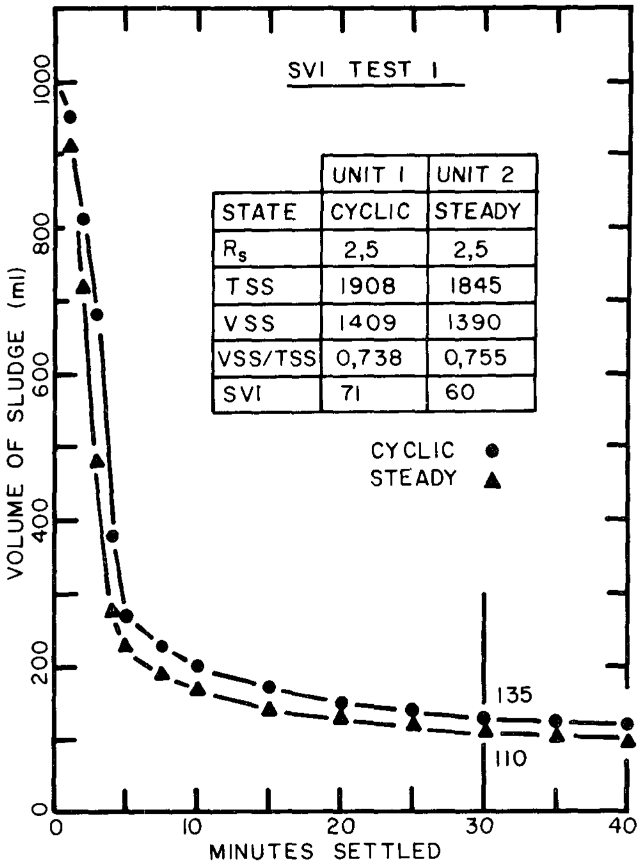

Figure 19
Results of SVI test 1 on sludges from 2,5 day sludge age steady and cyclic state units.

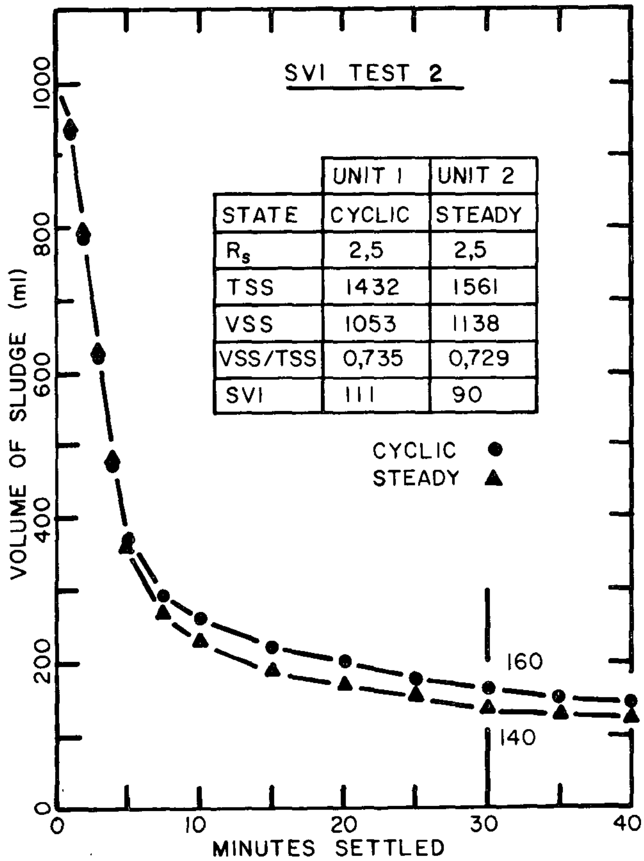

Figure 20
Results of SVI test 2 on sludges from 2,5 day sludge age steady and cyclic state units.

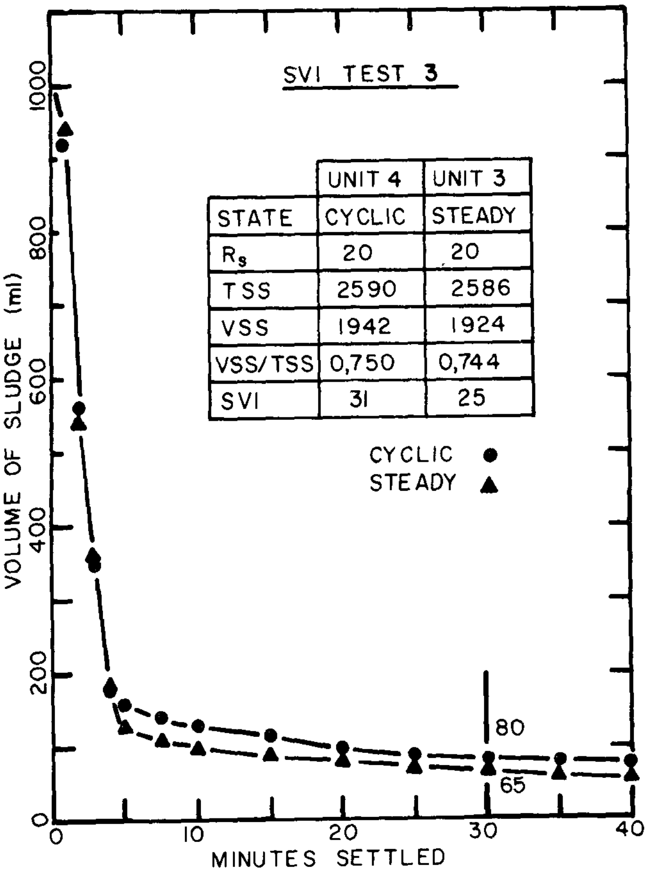

Figure 21
Results of SVI test 3 on sludges from 20 day sludge age steady and cyclic state units.

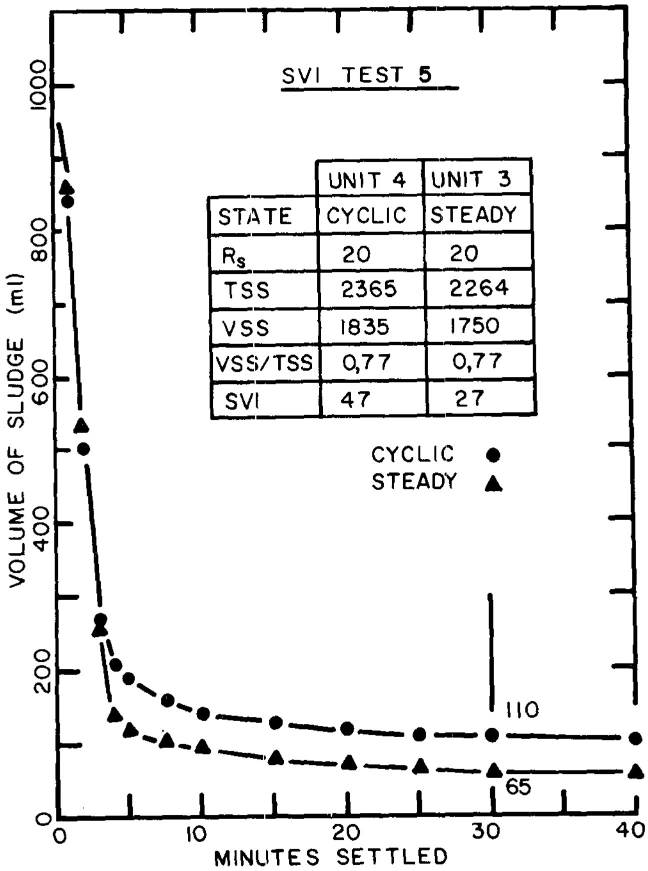

Figure 23
Results of SVI test 5 on sludges from 20 day sludge age steady and cyclic state units.

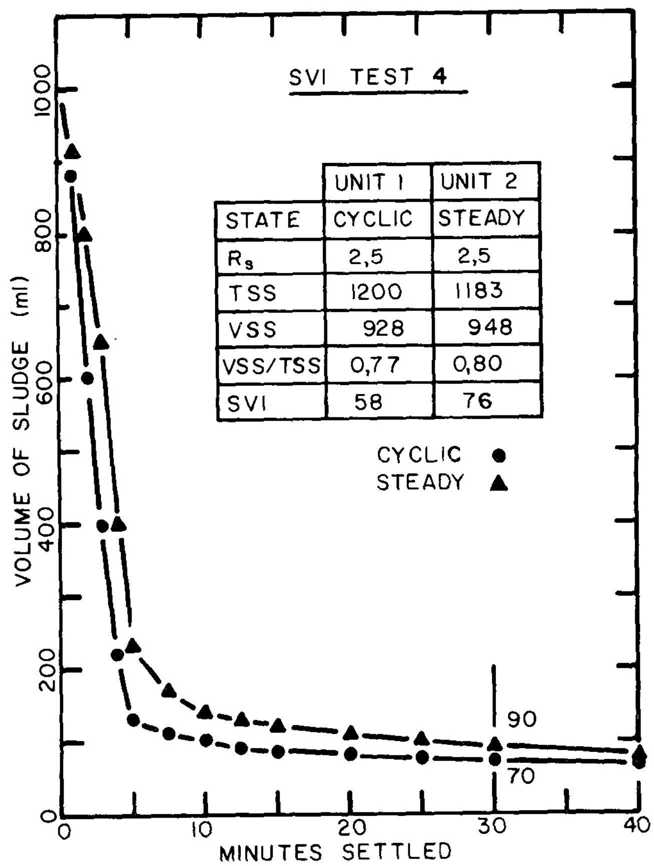

Figure 22
Results of SVI test 4 on sludges from 2,5 day sludge age steady and cyclic state units.

At intervals, both units of a pair were tested continuously for a 26 hour period. The programme of the tests during the 26 hour monitoring period is given in Table 3. Oxygen consumption rates were measured at approximately half hour intervals and at shorter intervals during the start and termination of the feed period. pH was monitored during some of the 26 hour tests, also the temperature of the mixed liquor was recorded. (Fig. 24 and 25.)

Five 26 hour intensive tests were done on the 2,5 day sludge age units and three on the 20 day units. The results of these tests are given in Figs. 6 and 26 to 32.

Test methods

All tests, other than influent tests, were conducted on centrifuged supernatant samples, except in the measurement of MLSS and MLVSS. The methods for measuring the COD, TKN, MLSS, MLVSS concentrations and SVI are those described in "Standard Methods for the Examination of Water and Wastewater" (1971). The nitrate and nitrite concentrations were measured by the auto-analyser automated method. The testing methods followed are given in Technicon Auto-Analyser Methodology; these are Industrial Methods 33.69 and 35.69W.

The pH was measured within 0,05 pH units with a pH meter Type 29 manufactured by Radiometer of Copenhagen, Denmark. Temperature was measured with a mercury bulb thermometer graduated in 0,1°C divisions. The dissolved oxygen concentration was measured with a Yellow Springs Oxygen Probe.

TABLE 3

PROGRAMME OF TESTS PERFORMED ON CMAS UNITS 1, 2, 3 AND 4 DURING THE 26 HOUR CONTINUOUS MONITORING PERIOD

State	Cyclic					Steady				
Time	COD		VSS	TKN	NO_2 NO_3	COD		VSS	TKN	NO_2 NO_3
	Rea.	Eff.				Rea.	Eff.			
1100	x		x	x	x	x		x	x	x
1300					x					
1500				x	x					
1700	x	x			x	x	x		x	x
1900				x	x					
2100					x					
2300	x	x	x	x	x	x	x	x	x	x
0100					x					
0300				x	x					
0500	x				x	x	x		x	x
0700				x	x					
0900					x					
1100	x		x	x	x	x	x	x	x	x

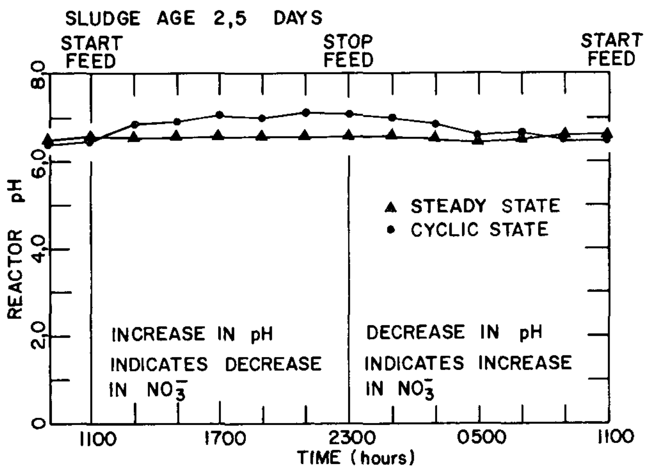

Figure 24
pH variation over a 24 hour period in the 2,5 day sludge age units.

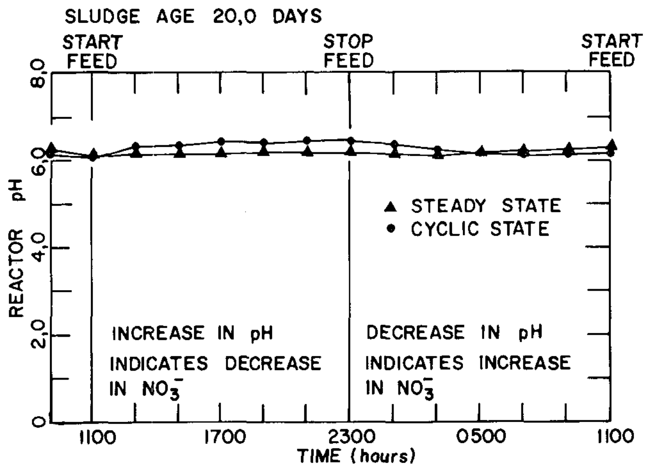

Figure 25
pH variation over a 24 hour period in the 20 day sludge age units.

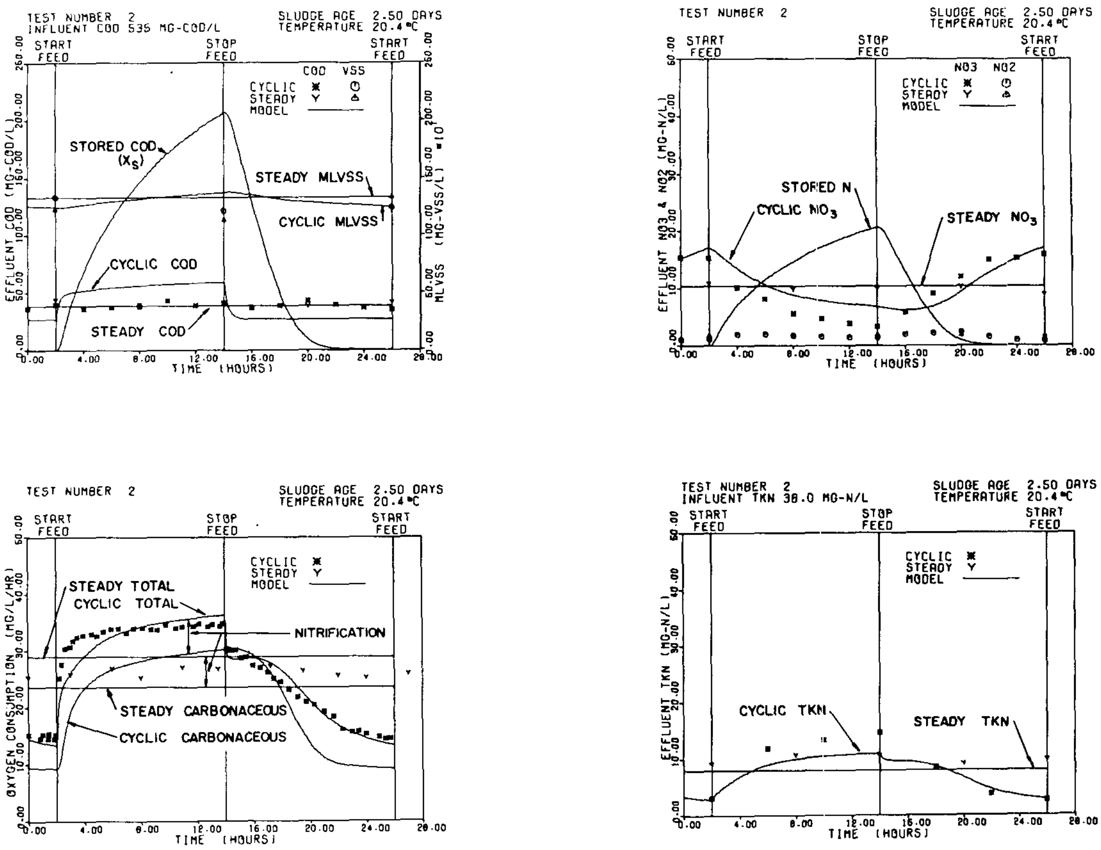

Figure 26
Comparison of 26 hour test number 2, 2,5 day sludge age units, and theoretical model.

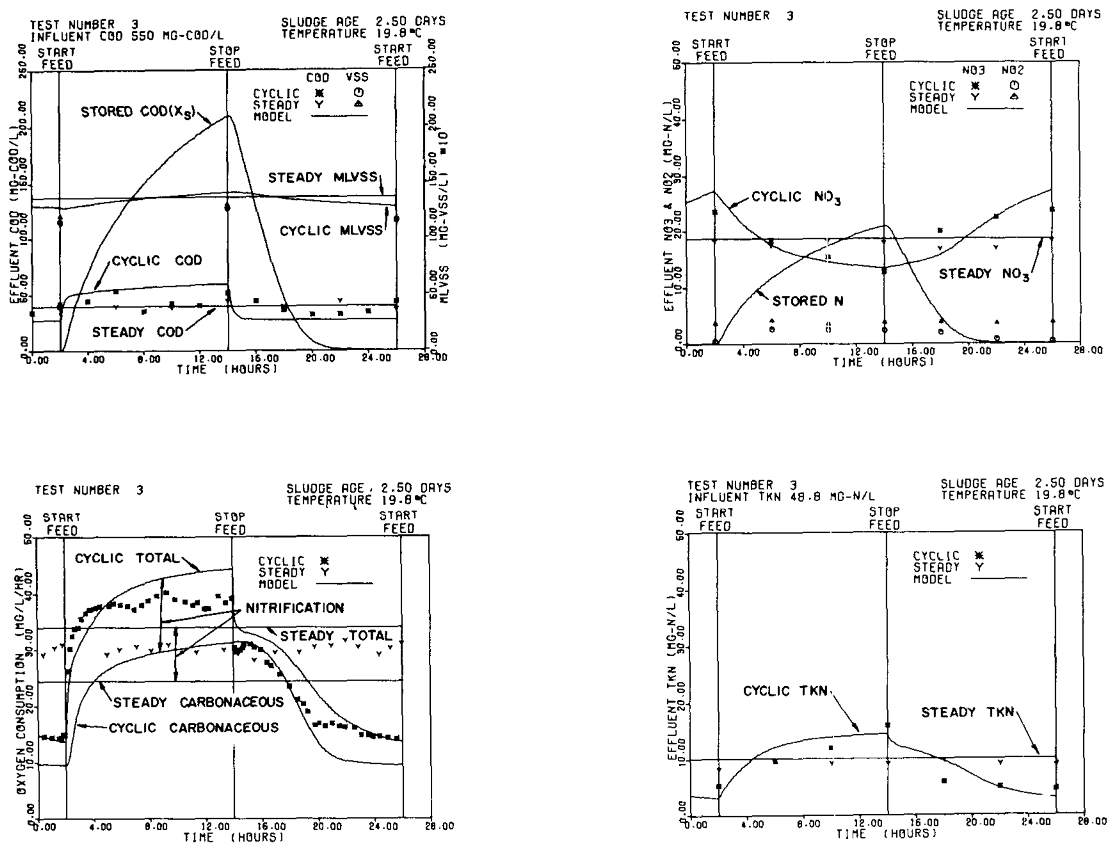

Figure 27
Comparison of 26 hour test number 3, 2,5 day sludge age units, and theoretical model.

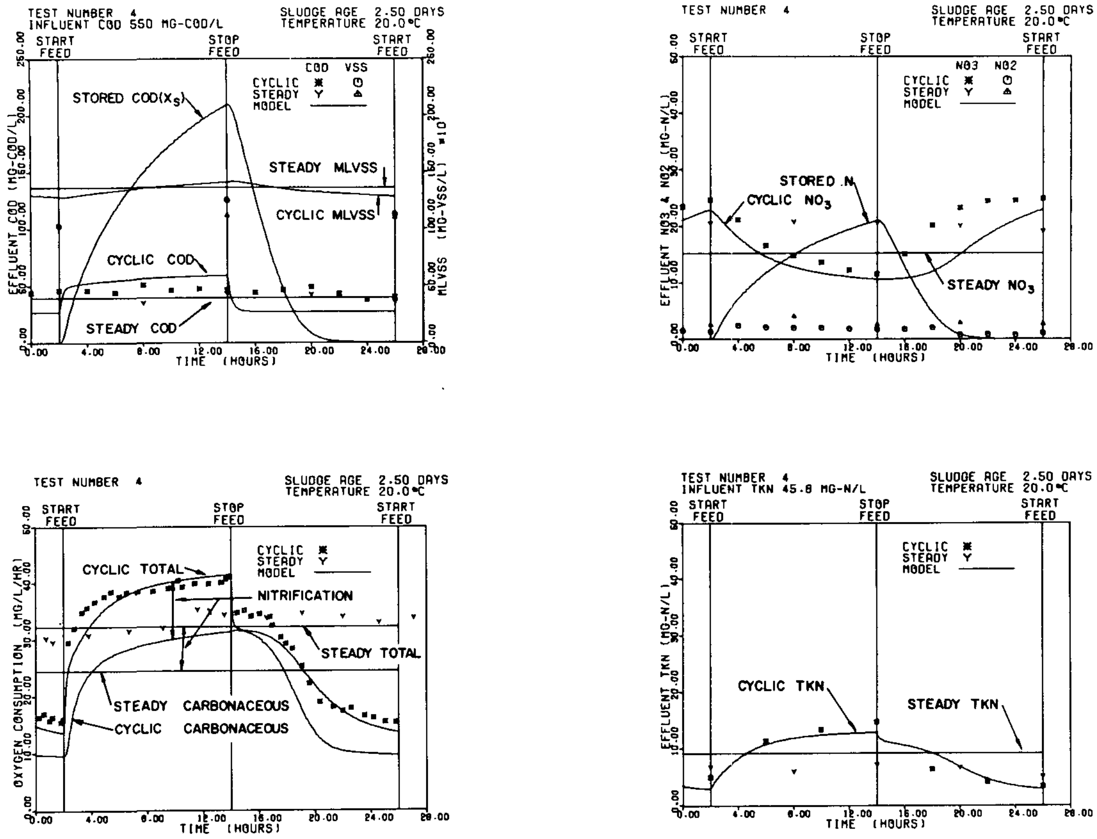

Figure 28
Comparison of 26 hour test number 4, 2,5 days sludge age units, and theoretical model.

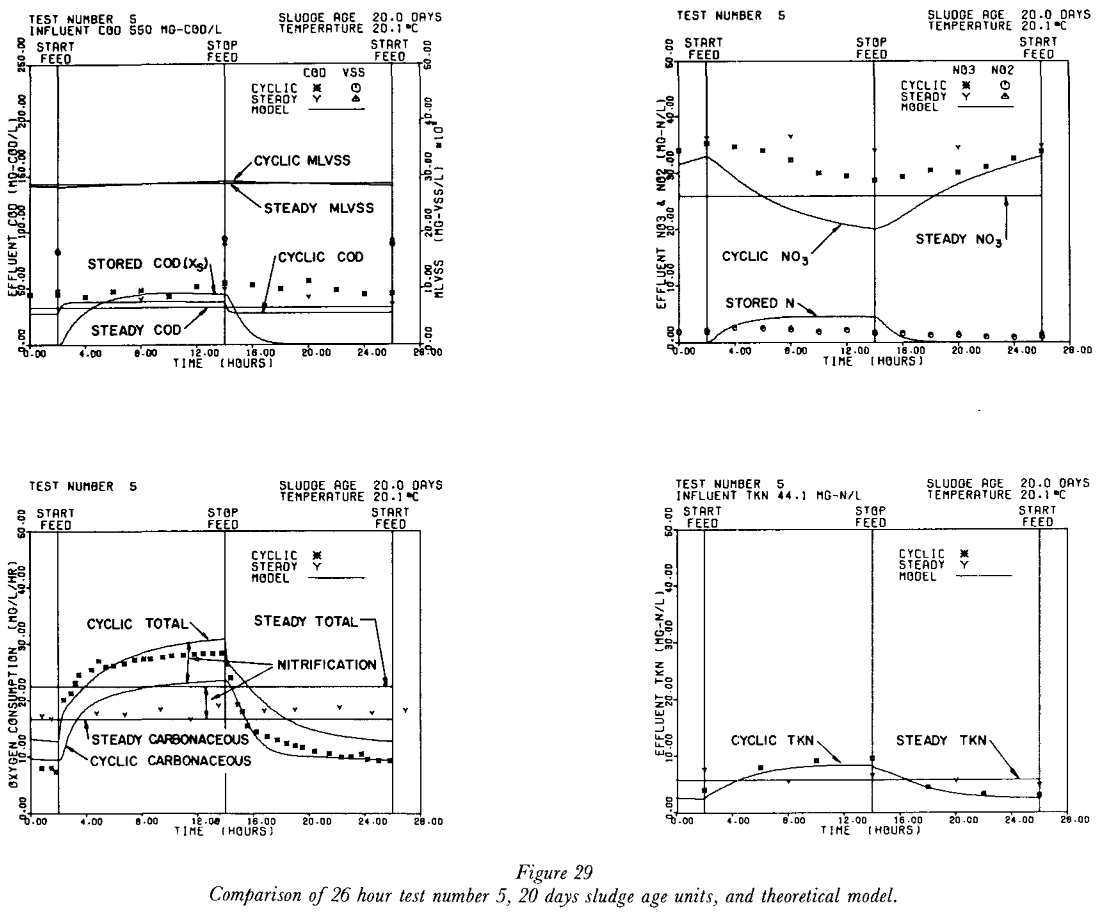

Figure 29
Comparison of 26 hour test number 5, 20 days sludge age units, and theoretical model.

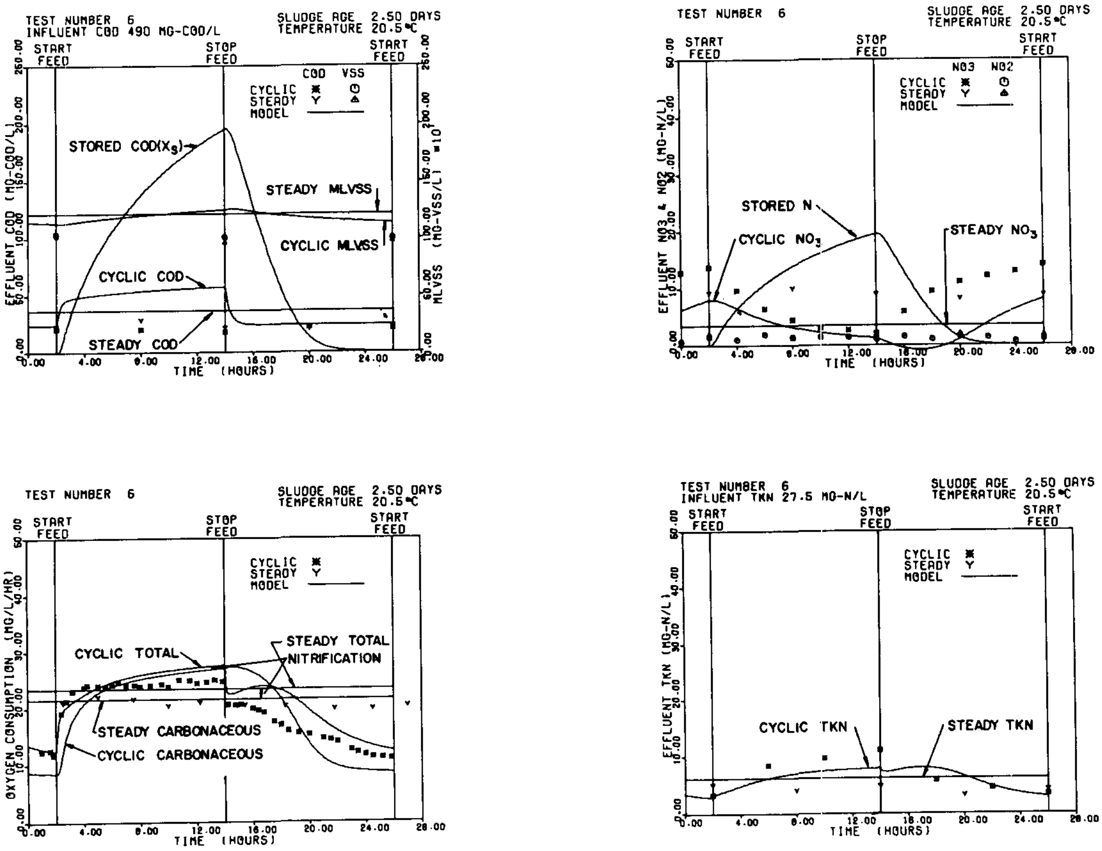

Figure 30
Comparison of 26 hour test number 6, 2,5 days sludge age units, and theoretical model.

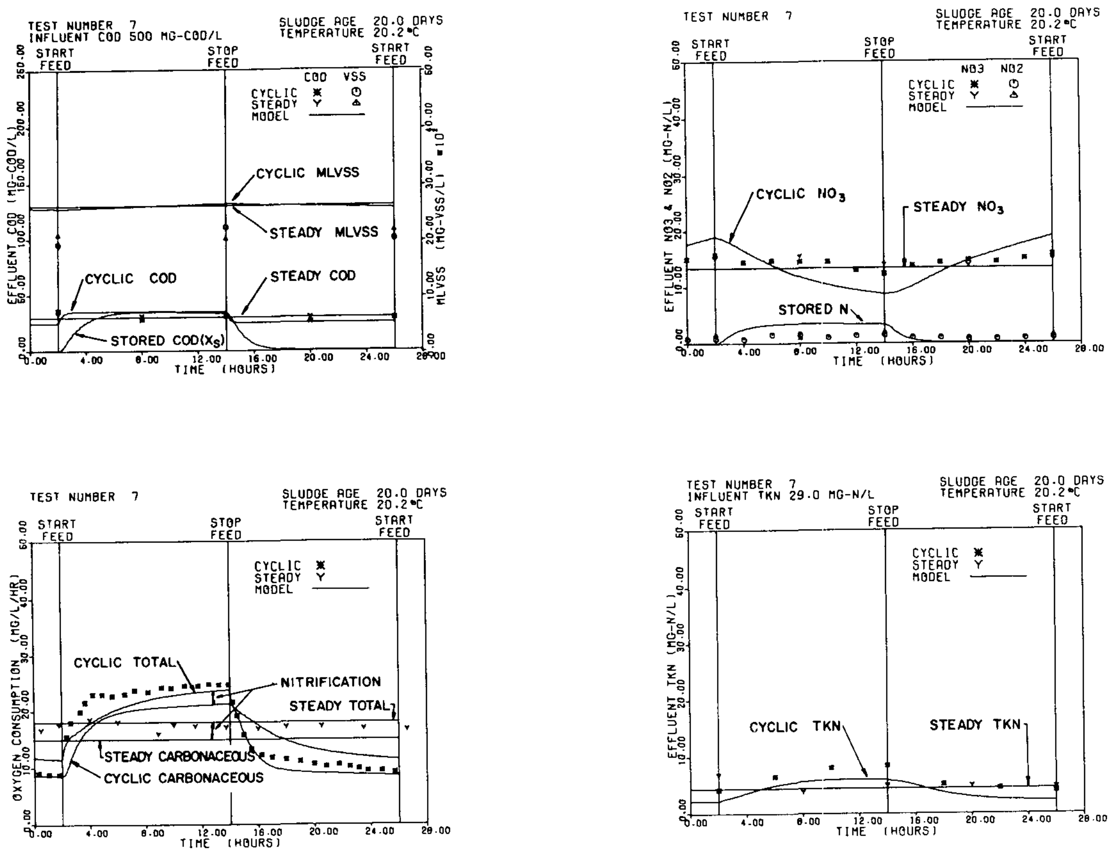

Figure 31
Comparison of 26 hour test number 7, 20 day sludge age units, and theoretical model.

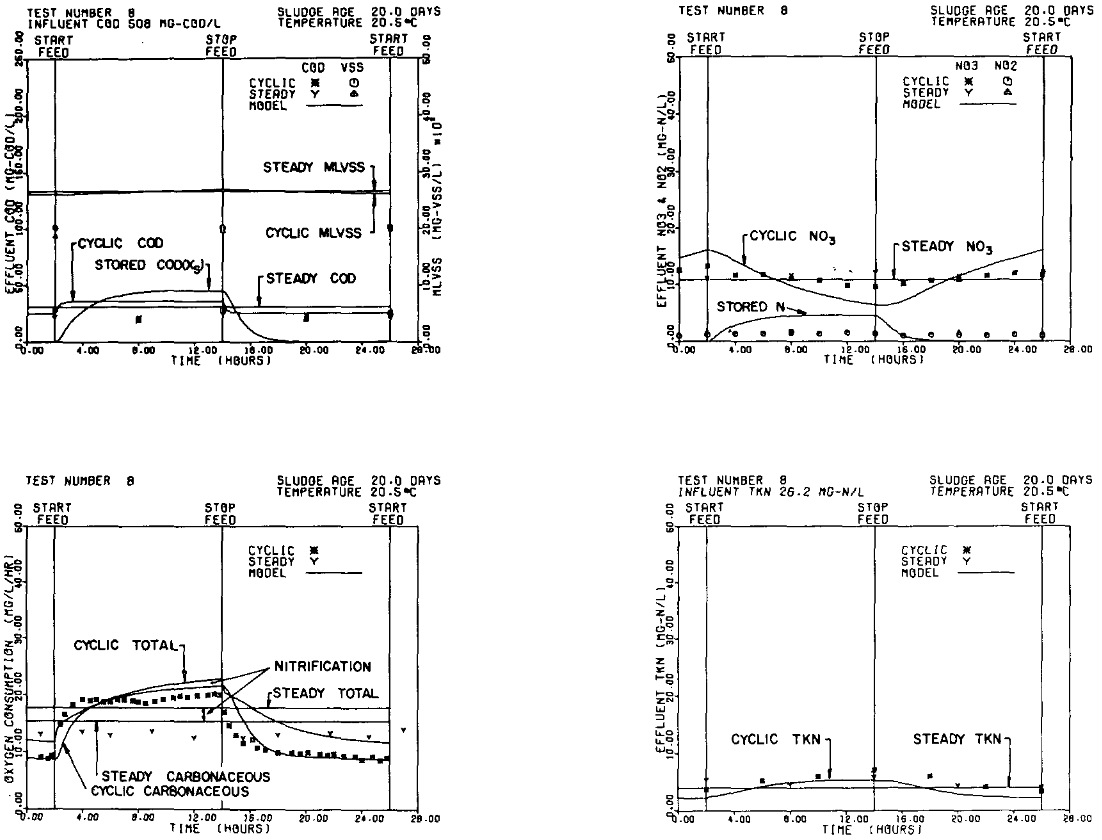

Figure 32
Comparison of 26 hour test number 8, 20 days sludge age units, and theoretical model.

Oxygen consumption rate was measured as follows: The dissolved oxygen concentration in the aeration tank was raised from the general operating concentration of about 2 to 3 mg/ℓ up to 6 to 7 mg/ℓ. The aeration was then stopped and the change in the dissolved oxygen concentration was recorded with time for about six minutes, while the aeration tank was still being fed and well stirred. The total oxygen consumption rate was given by the slope of line (usually linear) of the dissolved oxygen concentration versus time plot.

The COD and TKN tests on the influent were done on unfiltered samples. Occasionally the influent was tested for NO_3-N and NO_2-N concentrations which were always found to be less than 0,1 mg/ℓ.

Comparison of Experimental Data and the Theoretical Model

The mathematical model (considering the nett effect in the formulation of endogenous respiration) was programmed to the same input forcing waves as those imposed on the experimental units during each 26 hour test, i.e. steady state and cyclic state of nutrients and flow conditions. From these inputs, the computer model calculated both the steady state and dynamic steady state response taking due account of the sludge wastage sequence. A Calcomp subroutine plotted both the results of the process variables measured during the experimental 26 hour tests, and the results of the corresponding process variables as calculated by the mathematical model (Figs. 6 and 26 to 32).

By comparing the predicted values of the mathematical model with the experimental data, adaptive changes in the model itself and the kinetic constants were made until a best mean correspondence between predicted values and experimental data was obtained. In this fashion the model described above evolved.

The data from the 2,5 day sludge age units proved the most useful for developing the model as the process appears to be very sensitive at this sludge age to any input perturbation or changes in kinetic constants. This sensitivity was so great that it was possible to converge without undue trouble to the best estimate of the parameters and formulation of the process.

Once a satisfactory correlation between the predicted values and experimental data was obtained in the 2,5 day tests, the model was applied without modification to the 20 day units, to test the predictive power of the model at this sludge age. This method of testing the model was not as good as originally hoped for as the process response at 20 day sludge age is relatively insensitive and appreciable deviations from the parameter values giving a good fit at 2,5 days sludge age resulted in small changes in the response at 20 days sludge age.

Using sewage as influent, it was impossible to ensure close control of input COD and TKN every day. Inevitably daily changes were encountered and especially when a new consignment of sewage was received the constitution of the sewage changed. The response on a particular day is not only dependent on the input values of the COD and TKN at that day, but also on the history of the process extending back two or more days. This was particularly evident in units operating at short sludge ages. The influent COD and TKN measured in a particular 26 hour test constituted the input forcing wave for the mathematical model, which assumes that these values are repeated every day until the model converges to the dynamic steady state solution. Consequently, from this effect alone, the response of the experimental units would show discrepancies above and below experimental values.

Initially, the mathematical model included the equations for nitrification of the generally accepted nitrification theory, (Downing *et al*, 1964; Andrews and Paduska, 1974). The equations were based on the Monod relationship for nitrosomonas and nitrobacter organism synthesis utilizing nitrogen as substrate. However, after many attempts to fit the total oxygen consumption, effluent TKN and NO_3 curves of the model to experimental data, it was found that the formulation of the generally accepted nitrification theory was inadequate. Initially it was thought that the rate constants in the theory were incorrect. A range of values for the constants are reported, differing by an order of magnitude. (Andrews and Paduska, 1974; Stern, 1974; Lijklema, 1973). However, application of all of these reported values in various combinations led to no significant improvement between the experimental and theoretical values of the oxygen consumption rates. The clue to the behaviour was found after careful comparison of the TKN, NO_3 and O_2 utilization experimental and predicted curves; from the cyclic load tests on laboratory units it seemed that nitrification was not complete. It was eventually concluded that the influent TKN consisted of three fractions, an inert, a slowly biodegradable, and one immediately available for synthesis and nitrification. This hypothesis has support from the response of the oxygen demand rate at the termination of feed when a precipitous decrease in the rate resulted apparently from the termination of the supply of readily available ammonia for nitrification in the influent. The magnitude of this drop was indicative of the fraction of ammonia immediately available. As nitrification continued in the process during the non-feed period, at a rate greater than can be accounted for by the release of nitrogen through organism die off, it was concluded that a slowly biodegradable fraction existed. The major fraction of influent TKN appeared to be of a slowly biodegradable nature so that the rate of nitrification was in a large measure dependent upon the rate of conversion of slowly biodegradable nitrogen to immediately available ammonia for nitrification.

The most sensitive parameter to changes in the model or the kinetic constants was found to be the oxygen utilization rate. The yield factor, Y, with a value of 0,43 mg VSS/mg COD, which was obtained from steady state tests, always gave rise to an over-estimation of the oxygen utilization rate irrespective of adjustment in the other constants. The best fit for the oxygen utilization rate was obtained when Y = 0,49 mg VSS/mg COD was used in the model. It is difficult to pinpoint the reason for this higher value necessary for Y. It may be that the COD test over-estimates the oxygen consumption potential of the waste or that the value of b, (the nett endogenous respiration rate per day) and P, (the COD/VSS ratio) are not quite correct. (The stoichiometric value of P is 1,42 mg COD/mg VSS, but when tested experimentally was found to range between 1,40 and 1,50 for the activated sludge processes with a mean nearer 1,45 (Marais and Ekama, 1975). The net endogenous respiration rate per day was found to be 0,24 from tests on aerobic digestion of sludge (Marais and Ekama, 1975)). It should be noted that the values of Y, P and b are mutually interactive; for example, if Y is selected low, the value of P must increase to give approximately the same oxygen consumption response. Generally, it can be said that the product of Y and P should be about 0,67 and b about 0,24 to obtain a good correlation between experimental and theoretical oxygen consumption rates.

Conformity between the experimental and theoretical oxygen consumption rate was given precedence in developing and calibrating the model. Inevitably this, in some instances, led to lower correspondence between the other parameters. The theoretical parameters showing the largest deviation from their experimental counterparts are the effluent TKN and NO_3. For some yet unknown reason, the best fit for these parameters is obtained when no storage of COD is assumed, but then the peak oxygen utilization rate is grossly overpredicted (compare Fig. 6 and 33). The MLVSS also tends to be overpredicted at the longer sludge ages (Figs. 29, 31 and 32).

It is clear that improvements can still be made to the model, but this will depend on more information on the biological mechanisms in the process.

From this study it would seem that the Monod function mechanism should be reassessed. It would appear that the rate of metabolism is a Monod type function relating to the *stored* substrate concentration and not the substrate concentration surrounding the organism to the rate of synthesis. At steady state the Monod function appears to be adequate because the stored substrate concentration remains constant and in this fashion is effectively bypassed to link the substrate concentration surrounding the organism directly with the rate of organism synthesis. Under transient nutrient input conditions the stored nutrient concentration changes in a manner different from the nutrient concentration in the liquid and leads to a different rate of synthesis from that indicated by the nutrient concentration in the liquid.

The storage mechanism presented in this paper appears to give a reasonable account of the behaviour observed in the contact stabilization activated sludge process. In the "contact" phase mainly adsorption of substrate onto the organism occurs. High COD removal rates with a disproportionately low oxygen utilization rate are observed to occur in a very short contact period. (The model presented in this paper indicates very high adsorption rates.) The increase in VSS observed in the contact phase is mainly due therefore to the volatile solid equivalent of the substrate adsorbed (stored) onto the sludge mass. In the stabilization tank, the stored substrate is metabolized. This is indicated by the observation that very small changes in filtered COD take place, yet the oxygen utilization rates are high.

Plant Design

In design, an important factor is the maximum daily oxygen demand rate, in order to size the aerators and prevent unwanted anoxic conditions during peak oxygen demand periods. The variation in oxygen demand will be different between plants, depending on the cyclic variation of the COD and TKN load, so that it is not possible to predict the oxygen utilization behavioural pattern except in general terms.

At present these daily cyclic variations are estimated empirically, based on observations of large plant behaviour. Vosloo

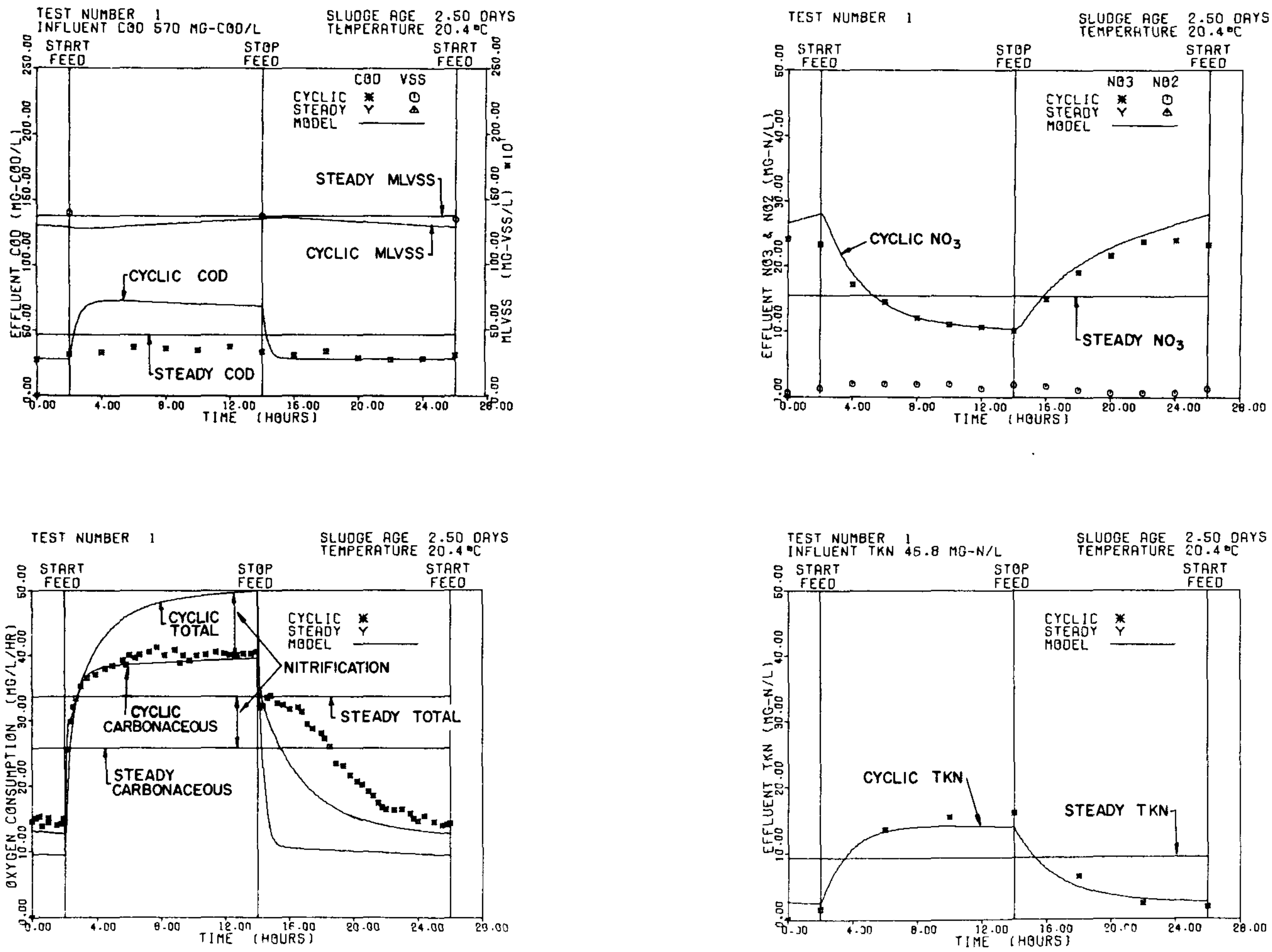

Figure 33

Comparison of experimental data of 26 hour test 1 and the theoretical model for 2,5 days sludge age assuming no storage of COD.

(1970) gave an account of such estimations. His estimations of the probable peak to average and minimum to average influent flow and BOD concentrations are given in Table 4. His conclusions, as regards the peak, average and minimum oxygen requirement per mg BOD used, based on a cyclic load, are summarized in Table 5, but these values specifically exclude the nitrification oxygen demand.

TABLE 4

INFLUENT SEWAGE VARIATIONS AS A FRACTION OF THE MEAN ACCORDING TO VOSLOO, 1970

Parameter	Minimum	Peak	Peak
BOD (mg BOD/ℓ)	0,50	1,0	1,5
Flow (ℓ/d)	0,67	1,0	2,0
Load (mg BOD/d)	0,33	1,0	3,0

TABLE 5

CARBONACEOUS OXYGEN DEMAND PER MG BOD USED (MG O/MG BOD USED) ACCORDING TO VOSLOO, 1970

R_s(d)	Mean*	Peak/Mean	Min./Mean
1,0	0,65	2,5	0,5
1,3	0,68	2,4	0,5
1,8	0,74	2,2	0,5
2,9	0,88	2,0	0,5
4,0	1,00	1,9	0,5
5,0	1,10	1,8	0,5
6,7	1,22	1,7	0,5
10,0	1,38	1,6	0,5
20,0	1,60	1,5	0,5

*To obtain approximate mg O/mg COD used, divide by 1,8.

There are two aspects to be considered when evaluating the peak design oxygen demand rates:

(1) The average cyclic COD load rate over the day varies for each particular plant. This cyclic variation depends on many factors – layout of sewerage system, i.e. whether the peak load from different parts of the town reaches the plant sequentially, or at approximately the same time, the intensity and type of industrialization and so on.

(2) There is a day to day random fluctuation in load about the average cyclic load which results in occasional extreme loading conditions. Correspondingly, the response of the plant will be of a two-fold character:
   (i) The average cyclic variation in load will lead to an average response in cyclic oxygen demand rates.
   (ii) The random fluctuation about the mean cyclic load will result in a concomitant random fluctuation of the oxygen demand rates about the mean cyclic oxygen demand rates.

The average cyclic load variations over the day, giving rise to the dynamic response of the plant, have been investigated in this paper. The random fluctuation however, has not been investigated.

The plant response to cyclic loading shows severe damping. However, it is difficult to assess the extent of damping of the random fluctuations of the feed and the random fluctuations generated in the reactor. Even when the feed is closely controlled to be a constant value, to achieve average cyclic load conditions, a random response is detectable.

TABLE 6

DESIGN PARAMETERS OF LABORATORY SCALE COMPLETELY MIXED ACTIVATED SLUDGE UNITS A AND B

Parameter	Unit A	Unit B
State	Cyclic	Cyclic
Sludge age (d)	21,5	10,7
Volume of reactor	12,0 ℓ	12,0 ℓ
Volume of feed per day	14,0 ℓ/d	17,6 ℓ/d
Mean influent COD (mg COD/ℓ)	600	596
Mean influent TKN (mg N/ℓ)	44,6	44,2
Mode of feed	square wave 12 hrs. on/ 12 hrs. off	square wave 12 hrs. on/ 12 hrs. off

For example, in the investigation two completely mixed activated sludge units were operated for a period of about three months. The design parameters of both units are listed in Table 6. A flow variation in the form of a square wave was imposed and an effort was made to feed the same mass of COD daily. All the process variables were monitored over the three month

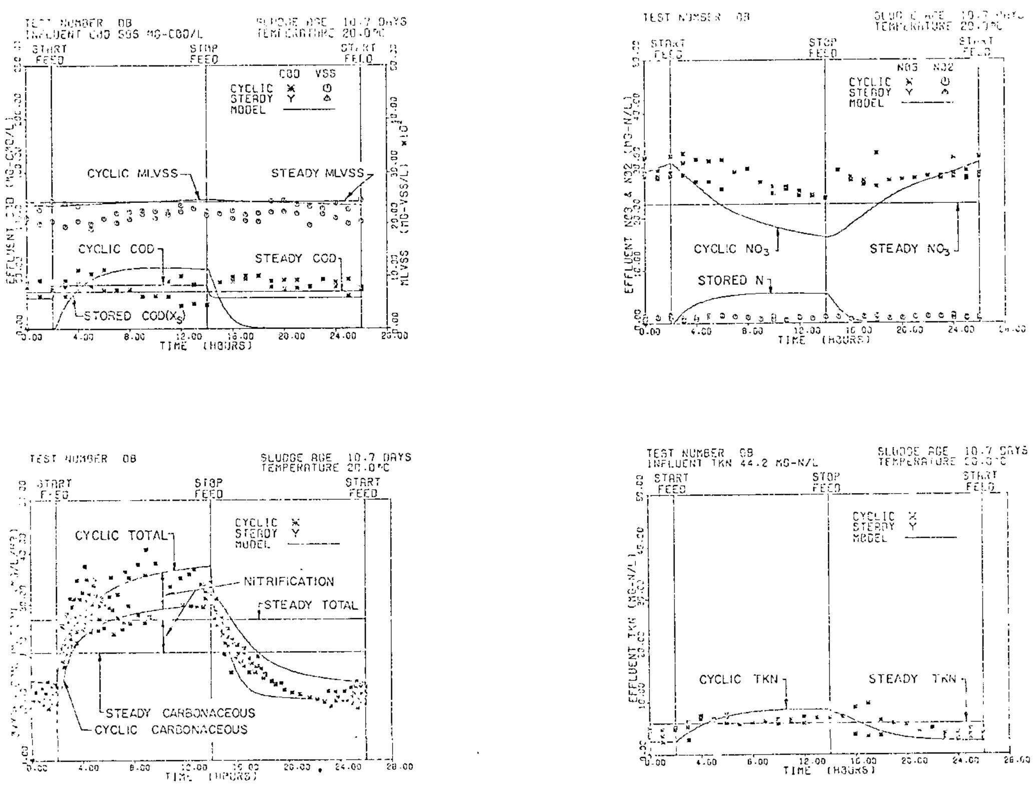

Figure 34
Experimental results showing the random fluctuation response at a sludge age of 10,7 days. Also shown is the average cyclic response of the mathematical model.

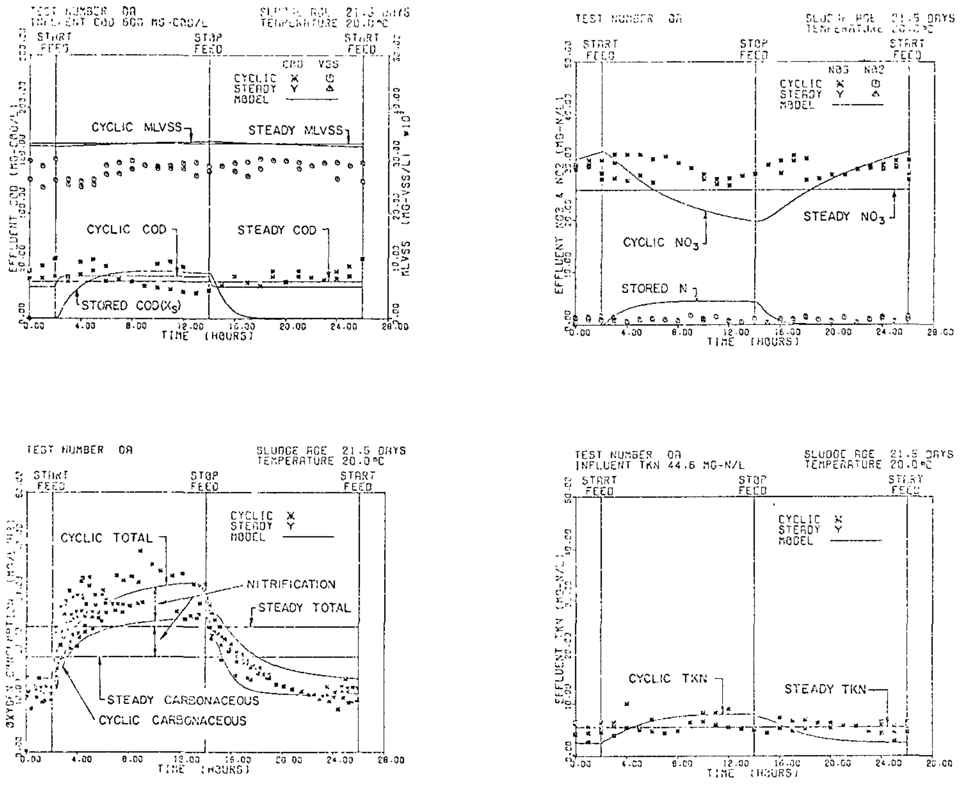

Figure 35
Experimental results showing the random fluctuation response at a sludge age of 21,5 days. Also shown is the average cyclic response of the mathematical model.

period paying special attention to the oxygen consumption rates. The experimental results for both laboratory units are given in Figs. (34) and (35). A rather large fluctuation in the oxygen consumption rates is observed. Incorporation of these random effects in design will best be effected from observations of full scale plant behaviour.

In order to give some guidance on the response of a plant under average cyclic load conditions, two load patterns were investigated using the mathematical model: a sinusoidal and a square wave influent COD and TKN load variation over the day.

1. Sinusoidal varying load

Figure 36 illustrates the sinusoidally varying influent COD load. Different amplitudes of the sine wave, a, (as a fraction of the average load) were imposed from a = 0 (steady state) to a = 1, in which the peak load is twice the average and the minimum load is zero.

In practice, the influent flow, COD and TKN vary independently. Such variations can be simulated in the computer model for a practical plant, but for general design purposes it makes the design chart very complicated: For an influent flow wave, Q_i, with an average of Q_{ave} (ℓ/d), and with a variation of peak to average of say, x, and an influent COD wave of S_{oi} with an average of S_{oave}, and with a variation of peak to average of say, y, the average of the COD mass flow wave, MS_{oave}, increases as x and y increase, i.e.

$$S_{oave} = \frac{1}{n} \sum_{i=1}^{n} S_{oi} \qquad (56)$$

$$Q_{ave} = \frac{1}{n} \sum_{i=1}^{n} Q_i \qquad (57)$$

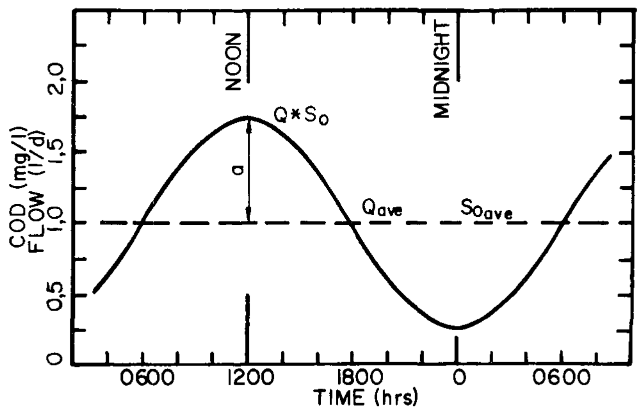

Figure 36
Sinusoidal COD mass load imposed on mathematical model for the construction of the design charts.

$$MS_{oave} = \frac{1}{n} \sum_{i=1}^{n} Q_i S_{oi} \qquad (58)$$

$S_{oave} * Q_{ave} < MS_{oave}$

Q_i and $S_{oi} > O$ and both vary

n = number of intervals considered per day.

Therefore, if the influent flow and COD vary independently, for variations in x and y about the means Q_{ave} and S_{oave}, different masses of COD are fed per day, the greater the values of x and y, the more mass of COD is fed.

The most important factor influencing the behaviour of the plant is the COD mass load per day. The magnitude of the flow has little significance: provided the cycle of the COD mass is specified, the concentration at which it comes into the reactor is not important. For this reason, in developing the design chart the flow is kept constant and the COD concentration varied sinusoidally so that the COD mass load per day thus also varies sinusoidally (Fig. 36).

An example of the response to sinusoidal COD mass input is shown in Fig. 37 (see p 45). In order to allow a comparison to be made, the same input data, i.e. influent mass of COD and N, and design parameters, i.e. volume and Q_{ave}, etc. as those for the 2,5 day sludge age units in Test 1 (Fig. 6) are used, except that the COD mass rate input wave is sinusoidal with an amplitude equal to 0,6 times the average. The flow is constant and sludge draw is over 24 hours. The steady state results and average results computed from the dynamic behaviour of both, are identical – as is to be expected. The peak total oxygen demand is 38 mg/ℓ/hr from the sine wave input, whereas it is 44 mg/ℓ/hr for the square wave input.

To develop a design chart, the total mass of COD input per day is taken to be constant. This value cannot be taken as unity as the effluent COD is not a function of the influent COD, and the model is only valid within realistic simulations. An influent volume of 15,0 ℓ at a COD concentration of 600 mg COD/ℓ (feeding a total mass of 9 000 mg COD/d) was selected to obtain the results of the mathematical model responses in all cases. (Fig. 38 is an example of one case, a = 0,8, R_s = 5,0 days). However, the results were generalized finally by dividing the mathematical model responses by the COD utilized (S_i–S).

The influent TKN is assumed to be 10 per cent of the equivalent VSS of the influent COD, i.e.

$$N_i = 0{,}1\ S_o/P \text{ (mg N/ℓ)} \qquad (59)$$

The amplitude of the influent sine wave is varied to give different values for the daily peak COD load. Amplitudes (as a fraction of the base value) considered are 0,40, 0,60, 0,80 and 1,0 to give maximum daily load values of 1,4, 1,6, 1,8 and 2,0 times the average, and the corresponding minimum daily load values of 0,60, 0,40, 0,20 and 0,0 times the average respectively.

The response to each of the four input sine waves is calculated for sludge ages 2,5, 5, 10, 15, 20 and 30 days. The results of the oxygen requirements are recorded in Tables 7 and 8. The results are also plotted in Figs. 39 and 40, these two figures constituting the design charts for sinusoidal response wave.

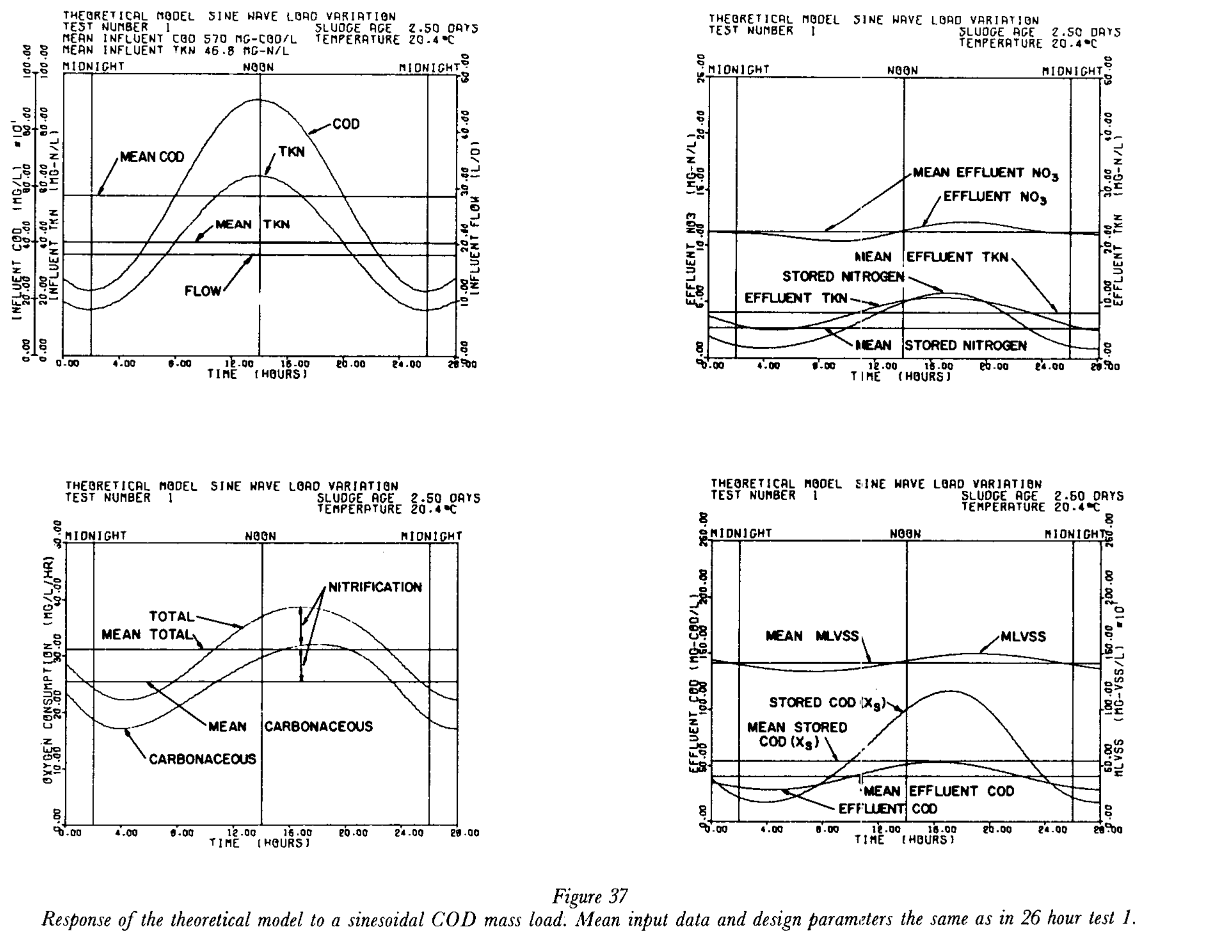

Figure 37

Response of the theoretical model to a sinesoidal COD mass load. Mean input data and design parameters the same as in 26 hour test 1.

TABLE 7

MILLIGRAM OXYGEN REQUIRED PER MG COD UTILIZED IN THE ACTIVATED SLUDGE PROCESS AT 20°C RECEIVING SINUSOIDAL COD MASS LOAD

Sludge Age (days)	Steady state		Dynamic State – maxima and minima oxygen demands								
	a = O			a = 0,40		a = 0,60		a = 0,80		a = 1,0	
R_s	C*	N*		C	N	C	N	C	N	C	N
2,5	0,497	0,116	max.	0,524	0,138	0,545	0,149	0,558	0,160	0,671	0,176
			min.	0,419	0,083	0,388	0,066	0,356	0,047	0,238	0,053
5,0	0,596	0,159	max.	0,686	0,163	0,728	0,168	0,767	0,176	0,809	0,187
			min.	0,497	0,151	0,446	0,145	0,393	0,137	0,338	0,135
10,0	0,692	0,196	max.	0,798	0,205	0,849	0,212	0,900	0,220	0,951	0,232
			min.	0,579	0,176	0,523	0,167	0,466	0,157	0,407	0,145
15,0	0,737	0,212	max.	0,848	0,227	0,902	0,234	0,956	0,243	1,009	0,254
			min.	0,623	0,189	0,565	0,177	0,507	0,166	0,448	0,153
20,0	0,763	0,222	max.	0,875	0,238	0,931	0,247	0,985	0,255	1,041	0,267
			min.	0,647	0,196	0,589	0,184	0,531	0,172	0,473	0,158
30,0	0,792	0,232	max.	0,906	0,251	0,963	0,261	1,081	0,270	1,073	0,287
			min.	0,675	0,205	0,617	0,192	0,558	0,179	0,499	0,162

*C = Carbonaceous oxygen demand per mg COD utilized.
*N = Nitrification oxygen demand per mg COD utilized.
C + N = Total oxygen demand per mg COD utilized.
Milligram oxygen demand/d/mg COD utilized for average mass of 1 mg COD/d with sinusoidal load variation = $1 + a \sin (2\pi x/24)$, $0 \leqslant a \leqslant 1$, $0 \leqslant x \leqslant 24$.

TABLE 8

MILLIGRAM OXYGEN REQUIRED PER MG COD UTILIZED IN THE ACTIVATED SLUDGE PROCESS AT 20°C RECEIVING SINUSOIDAL COD MASS LOAD

Sludge Age (days) R_s	Steady State a = 0 T*		Dynamic State – maxima and minima oxygen demands											
			a = 0,4			a = 0,6			a = 0,8			a = 1,0		
			T	max./mean	max./min.	T	max./mean	max./min.	T	max./mean	max./min.	T	max./mean	max./min.
2,5	0,613	max.	0,641	1,046	1,194	0,664	1,083	1,302	0,687	1,121	1,428	0,718	1,171	1,595
		min.	0,537			0,510			0,481			0,449		
5,0	0,755	max.	0,840	1,113	1,276	0,882	1,168	1,444	0,922	1,215	1,638	0,972	1,287	1,913
		min.	0,658			0,611			0,563			0,508		
10,0	0,888	max.	0,996	1,131	1,298	1,051	1,184	1,482	1,105	1,244	1,694	1,169	1,316	1,981
		min.	0,767			0,709			0,651			0,590		
15,0	0,949	max.	1,067	1,124	1,296	1,127	1,188	1,479	1,187	1,251	1,696	1,251	1,318	1,970
		min.	0,823			0,762			0,700			0,635		
20,0	0,985	max.	1,106	1,122	1,294	1,168	1,186	1,475	1,229	1,248	1,688	1,296	1,316	1,961
		min.	0,855			0,792			0,728			0,661		
30,0	1,024	max.	1,149	1,122	1,290	1,213	1,185	1,467	1,276	1,246	1,675	1,345	1,313	1,944
		min.	0,891			0,827			0,762			0,692		

T* = Total oxygen demand/mg COD utilized.
Milligram oxygen demand/d/mg COD utilized for average mass of 1 mg COD/d with sinusoidal load variation = $1 + a \sin (2\pi x/24)$, $0 \leqslant a \leqslant 1$, $0 \leqslant x \leqslant 24$.

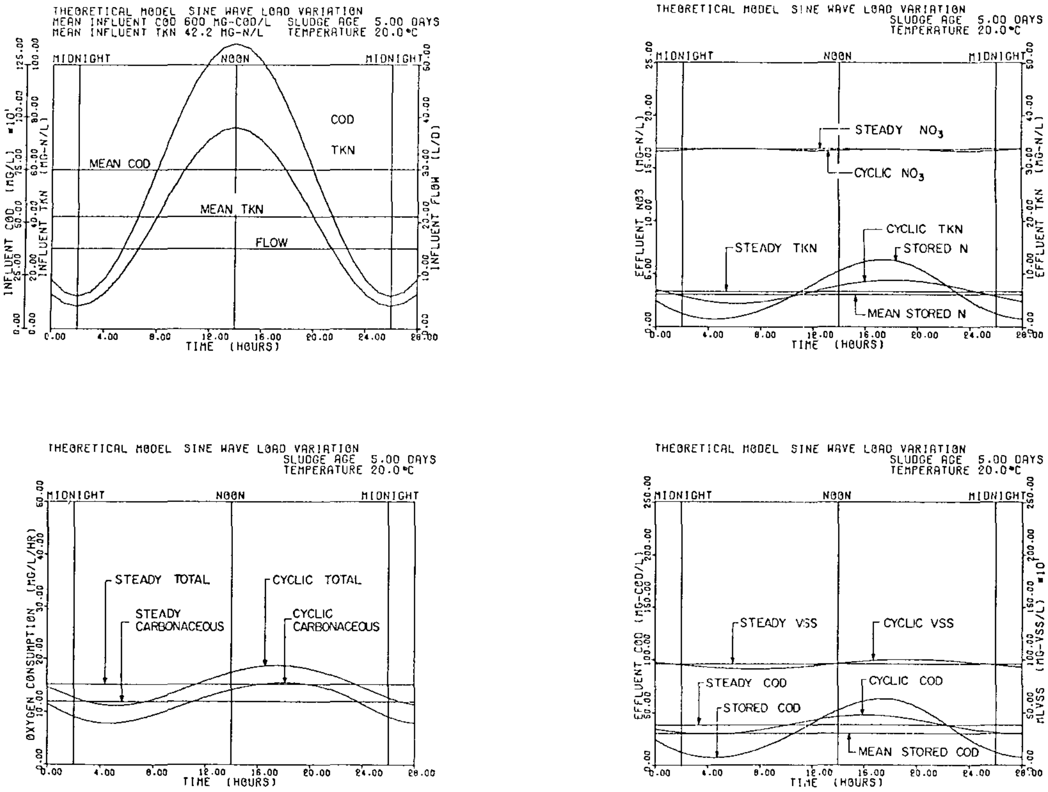

Figure 38
Example showing the response of the mathematical model under sine wave input of amplitude 0,8 times the average at a sludge age of 5 days.

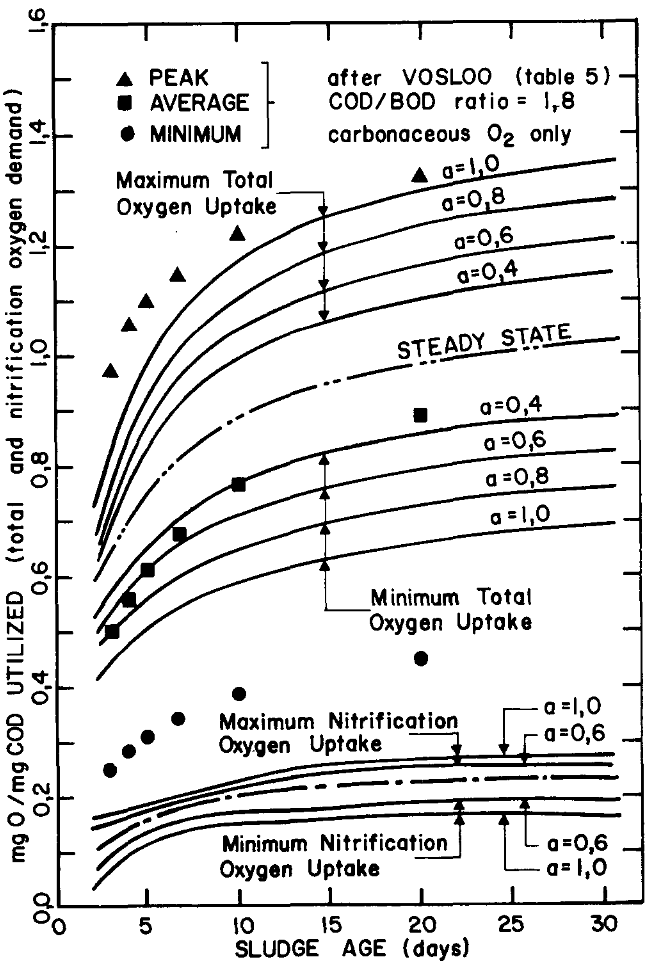

Figure 39
Design chart for total oxygen demand per mg COD utilized per day as predicted by the mathematical model under sine wave input load.

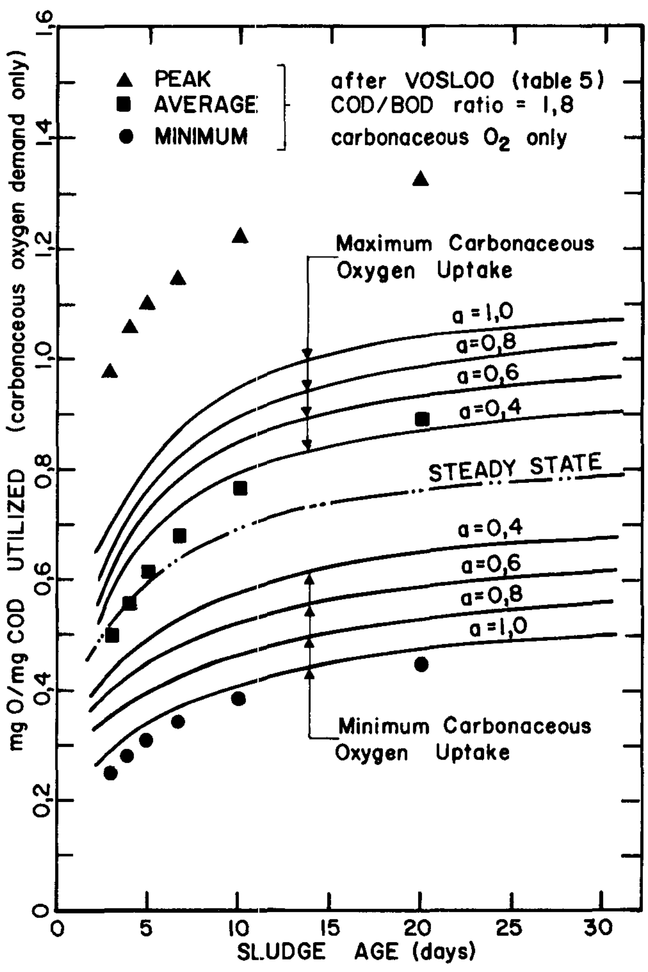

Figure 41
Comparison of peak, average and minimum carbonaceous oxygen demands estimated by Vosloo with those calculated by the mathematical model.

A comparison of the empirical carbonaceous oxygen requirements of Vosloo (1970) and the corresponding carbonaceous oxygen requirements predicted by the computer model under sine wave simulation, is given in Fig. 41.

2. Square wave load variation

Figure 42 illustrates the different loading conditions imposed on the mathematical model in the investigation of the response to square wave load variations. In all cases the same mass of COD was fed per day (9 000 mg COD/d), the variation being the length of the feed period which ranges from 6 to 14 hours, the corresponding non-feed periods ranging from 18 to 10 hours (Fig. 42).

The development of the oxygen demand design chart (Fig. 43) was done under the same conditions as those for the sine wave oxygen requirement design charts.

The response to each of the five input square waves was calculated for sludge ages 2,5, 5, 10, 15, 20 and 30 days. The results of the oxygen requirements are recorded in Tables 9 and 10. The results are also plotted in Fig. 40 (sludge masses) and Fig. 43 (oxygen requirements), these figures constituting the design charts for square wave response.

An example of the response to square wave input of the particular case where the sludge age is 20 days and length of the feed period is 8 hours is given in Fig. 44.

In using the charts the following features should be noted:

(i) Nitrification is included.

(ii) Sludge wastage is continuous over the whole day. For design purposes the period of sludge wasting has negligible effect.

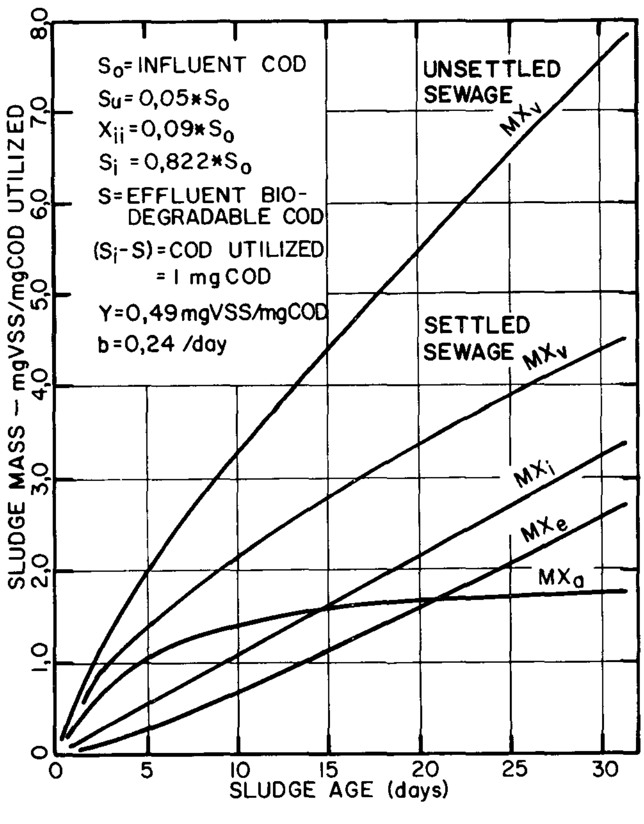

Figure 40
Design chart for estimating sludge accumulation in the aeration reactor for any COD input wave.

(iii) The design charts (Figs. 39, 40 and 43) give the total peak and minimum oxygen demand rate (mg 0/day) per average mass of COD *utilized* per day (mg COD/day). The design charts can therefore be used for both settled and unsettled municipal sewage, i.e. in the construction of the design charts $M(S_i–S)$ is taken as 1 mg COD utilized/day (Eq. (67)). Also given is the maximum and minimum oxygen utilization for nitrification.

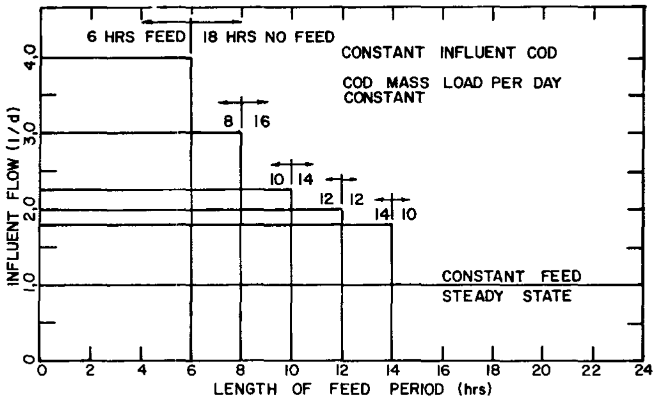

Figure 42
Square wave load variations imposed on the mathematical model for the construction of the design charts for square wave load patterns.

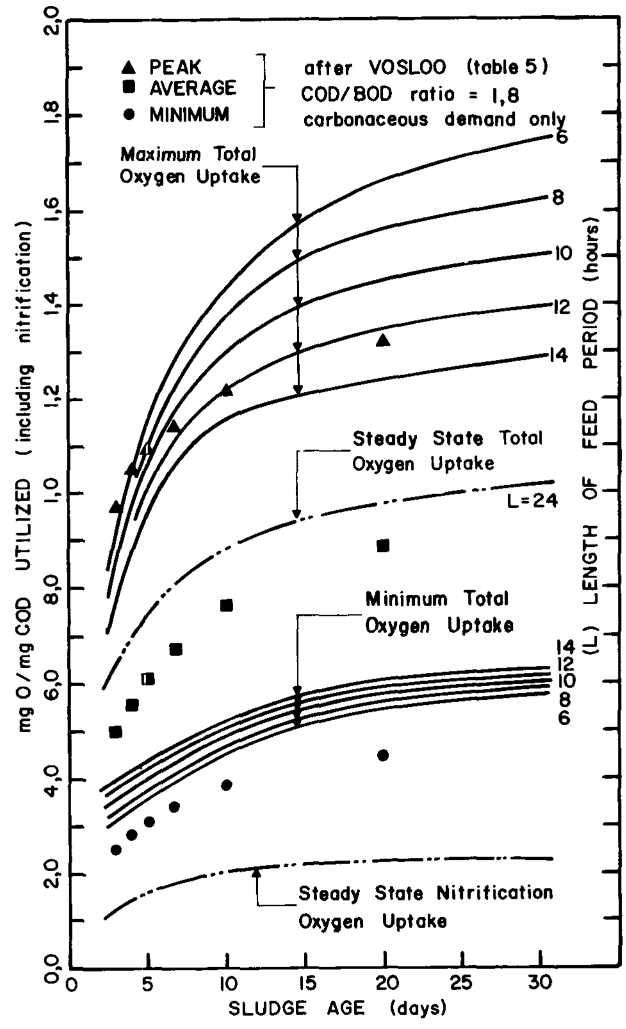

Figure 43
Design chart for total oxygen demand per mg COD utilized per day as predicted by the mathematical model under square wave input load.

TABLE 9

MILLIGRAM OXYGEN REQUIRED PER MG COD UTILIZED IN THE ACTIVATED SLUDGE PROCESS AT 20°C RECEIVING SQUARE WAVE COD MASS LOAD

Sludge Age (days) R_s	Steady State $\ell^* = 24$		Dynamic State – maxima and minima oxygen demands											
				$\ell = 6$		$\ell = 8$		$\ell = 10$		$\ell = 12$		$\ell = 14$	
	C*	N*		C	N	C	N	C	N	C	N	C	N
2,5	0,493	0,117	max.	0,553	0,410	0,577	0,345	0,588	0,308	0,594	0,284	0,588	0,266
			min.	0,182	0,000	0,195	0,000	0,209	0,000	0,226	0,000	0,246	0,000
5,0	0,602	0,159	max.	0,912	0,404	0,920	0,337	0,903	0,301	0,864	0,282	0,813	0,275
			min.	0,250	0,000	0,260	0,000	0,260	0,000	0,276	0,007	0,283	0,033
10,0	0,696	0,196	max.	1,212	0,415	1,177	0,346	1,103	0,307	1,014	0,284	0,930	0,275
			min.	0,338	0,000	0,348	0,000	0,355	0,022	0,362	0,061	0,308	0,086
15,0	0,741	0,212	max.	1,335	0,426	1,275	0,356	1,171	0,316	1,065	0,292	0,977	0,280
			min.	0,380	0,000	0,389	0,000	0,397	0,048	0,404	0,087	0,411	0,109
20,0	0,770	0,222	max.	1,411	0,433	1,326	0,362	1,206	0,322	1,093	0,298	1,003	0,285
			min.	0,404	0,000	0,414	0,006	0,422	0,063	0,429	0,101	0,436	0,122
30,0	0,790	0,232	max.	1,560	0,441	1,375	0,370	1,241	0,329	1,122	0,305	1,031	0,291
			min.	0,432	0,000	0,442	0,021	0,450	0,081	0,457	0,117	0,463	0,135

C* = Carbonaceous oxygen demand.
N* = Nitrification oxygen demand.
ℓ^* = Length of feed period of the square wave (Fig. 42).

TABLE 10

MILLIGRAM OXYGEN REQUIRED PER MG COD UTILIZED IN ACTIVATED SLUDGE PROCESS AT 20°C RECEIVING SQUARE WAVE COD MASS LOAD

Sludge Age (days) R_s	Steady State $\ell^* = 24$ T		Dynamic State – maxima and minima oxygen demands														
			$\ell = 6$			$\ell = 8$			$\ell = 10$			$\ell = 12$			$\ell = 14$		
			T	max./mean	max./min.	T	max./mean	max./min.	T	max./mean	max./min.	T	max./mean	max./min.	T	max./mean	max./min.
2,5	0,610	max.	0,847	1,389	2,805	0,791	1,297	2,456	0,759	1,244	2,213	0,737	1,208	2,019	0,716	1,174	1,836
		min.	0,302			0,322			0,343			0,365			0,390		
5,0	0,761	max.	1,152	1,514	3,182	1,105	1,452	2,915	1,066	1,401	2,692	1,024	1,346	2,473	0,962	1,264	2,201
		min.	0,362			0,379			0,396			0,414			0,437		
10,0	0,892	max.	1,446	1,621	3,171	1,381	1,548	2,926	1,307	1,465	2,695	1,226	1,374	2,452	1,148	1,287	2,212
		min.	0,456			0,472			0,485			0,500			0,519		
15,0	0,953	max.	1,582	1,660	3,114	1,498	1,572	2,870	1,402	1,471	2,621	1,304	1,368	2,375	1,217	1,277	2,150
		min.	0,508			0,522			0,535			0,549			0,566		
20,0	0,992	max.	1,658	1,671	3,076	1,561	1,574	2,823	1,452	1,464	2,565	1,347	1,358	2,326	1,255	1,265	2,109
		min.	0,539			0,553			1,505			0,579			0,595		
30,0	1,022	max.	1,741	1,704	3,033	1,628	1,593	2,769	1,505	1,473	2,504	1,392	1,362	2,267	1,297	1,269	2,062
		min.	0,574			0,588			0,606			0,614			0,629		

T* = Total oxygen demand per mg COD utilized per day.
ℓ^* = Length of feed period of square wave (Fig. 42).

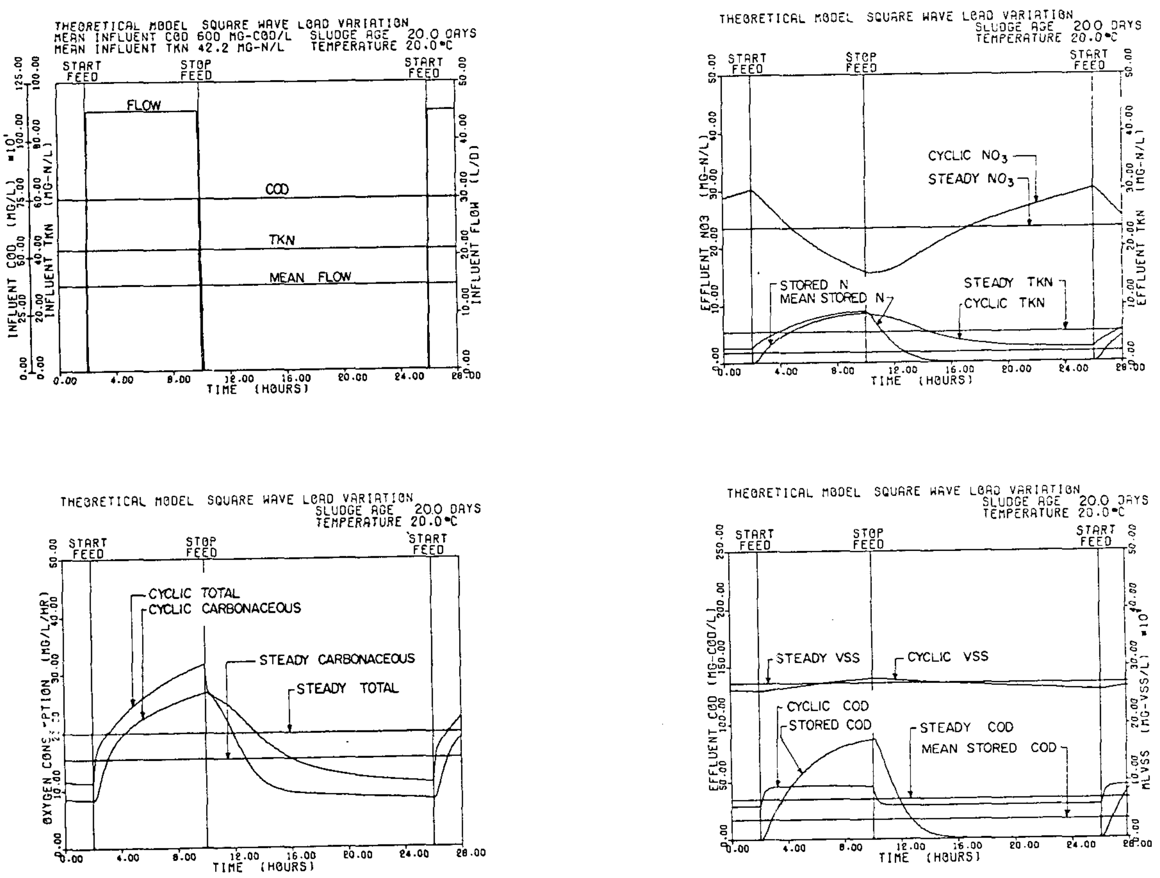

Figure 44

Example showing the response of the mathematical model under square wave input load of an 8 hour feed period at a sludge age of 20 days.

Comparison of response data

The design chart values for the sludge in the reactor was found to be independent of the cyclic input COD variations. This bears out the fact that the *mass* of sludge generated is *solely* a function of the *mass* of COD fed per day and the sludge age and is independent of hydraulic retention time (Marais, 1973) i.e. flow variations.

For the sine wave loading the peak oxygen consumption rate occurs approximately 3 hours after the peak COD mass flow (Fig. 37). Similarly, the minimum oxygen consumption rate occurs about 3 hours after the minimum COD mass flow. The variation of the oxygen consumption rate resembles a sine curve (Fig. 37), the amplitude being about 33 per cent of the COD mass flow amplitude.

This suggests a simple design rule to determine the peak oxygen demand for cyclic load conditions: Calculate the average oxygen demand for the carbonaceous and nitrification load according to the steady state equations. Estimate the peak/average COD load for the day. Increase the steady state total oxygen demand by one third of the amplitude (peak-average) of the COD load.

The peaks for nitrification and carbonaceous oxygen demand occur at nearly the same time (Figs 37 and 38). For this reason it is possible to assume that the peak total oxygen demand is the sum of the peak carbonaceous and peak nitrification oxygen demands. For sludge ages greater than 10 days, the assumption that the carbonaceous and nitrification oxygen demands

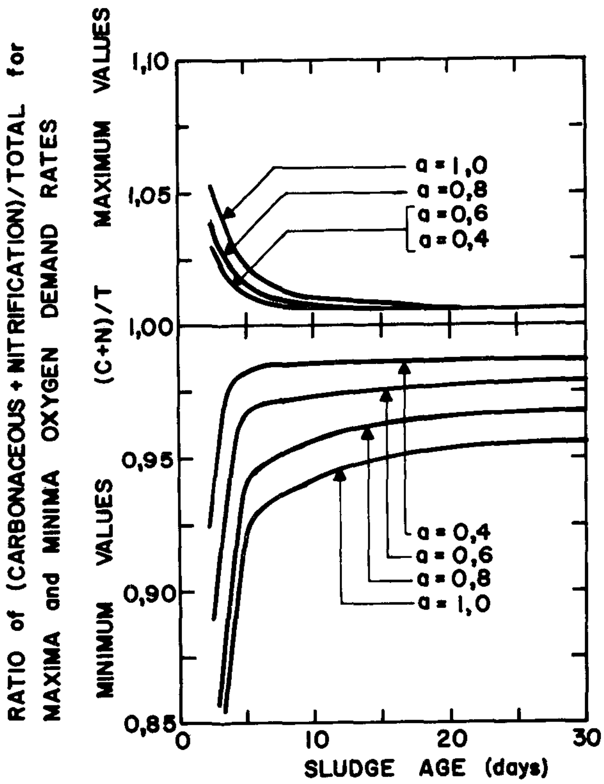

Figure 45

Ratio of the sum of carbonaceous and nitrification oxygen demand rates to total oxygen demand rates for maxima and minima values under sine wave input loading.

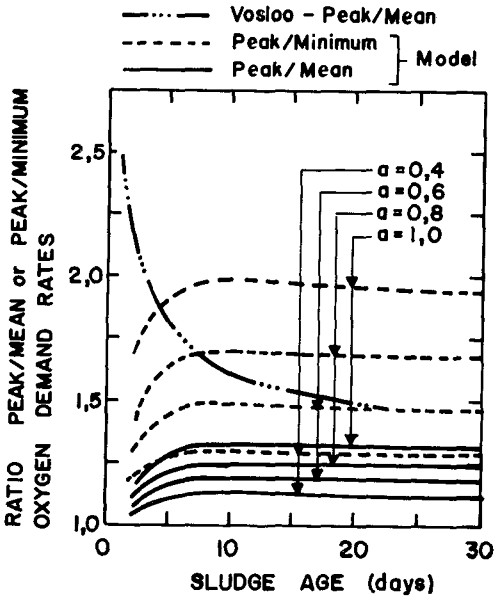

Figure 46
Ratios of peak to mean and peak to minimum total oxygen demand rates for sine wave input loading. Vosloo's values also given.

occur simultaneously, over-estimates the total oxygen demand by not more than 1,5 per cent. (Compare Tables 7 and 8) (Fig. 45).

An overview of the total oxygen requirement data as predicted by the mathematical model under sine wave loading conditions is given in Table 8. For each different case of sludge age and input COD sine wave amplitude, the ratio of maximum/mean and maximum/minimum total oxygen requirement is calculated. These values are plotted in Fig. 46. Also plotted are the values of the ratio maximum/mean *carbonaceous* oxygen requirements suggested by Vosloo (1970). His values for the ratio maximum/minimum carbonaceous oxygen requirement are double the value of the maximum/mean ratios. (He suggests a ratio of minimum mean carbonaceous oxygen requirement of 0,5 for all cases – Table 5.)

For the square wave loading the maximum carbonaceous and minimum nitrification oxygen requirement occurs approximately simultaneously at about two hours after feed termination at 2,5 days sludge age, decreasing with increasing sludge age to about half an hour at 30 days sludge age.

The minimum carbonaceous oxygen requirement occurs just prior to feed commencement with the maximum nitrification oxygen requirement occurring at about half an hour after feed commencement.

The maximum total oxygen requirement occurs at feed termination while the minimum total oxygen requirement occurs just prior to feed commencement.

With the square wave input COD load variation, it would be incorrect to assume that the maximum total oxygen requirement is the sum of the maximum carbonaceous and maximum nitrification oxygen requirement, i.e. that the two occur simultaneously. This assumption can lead to overestimation of the total oxygen requirements by 20% at the shorter sludge ages. (Compare Tables 9 and 10. Fig. 47.)

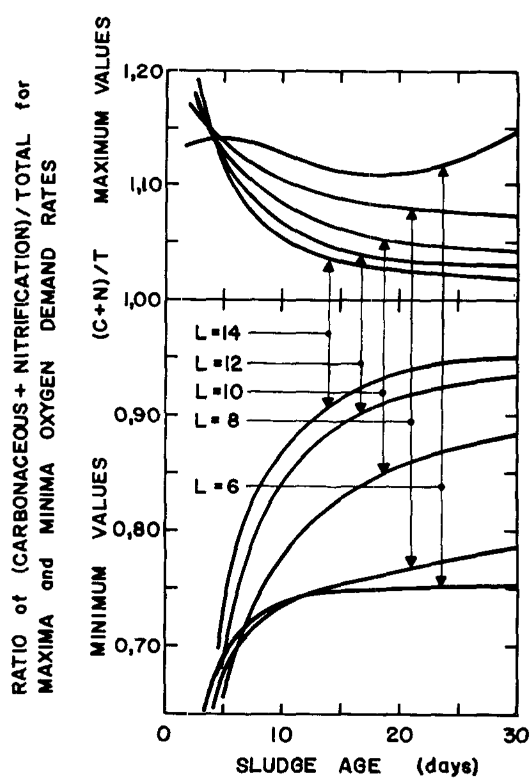

Figure 47
Ratio of the sum of carbonaceous and nitrification oxygen demand rates to total oxygen demand rates for maxima and minima values under square wave input loading.

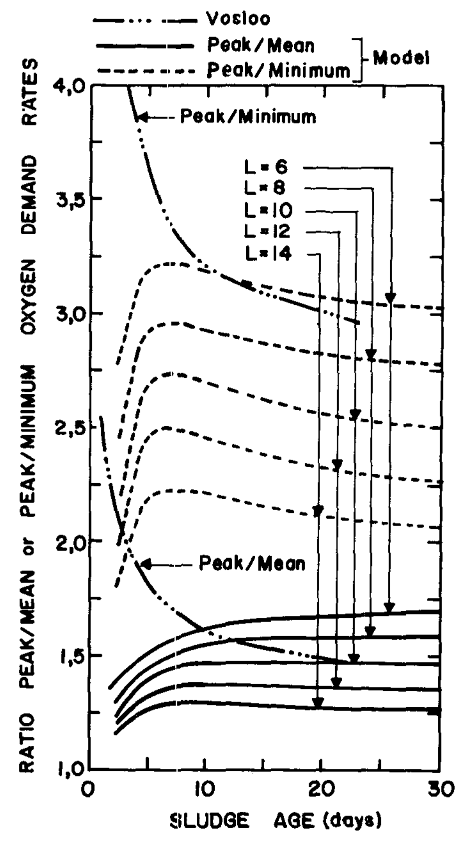

Figure 48
Ratios of peak to mean and peak to minimum total oxygen demand rates for square wave inpute loading. Vosloo's values also given.

An overview of the total oxygen requirements data as predicted by the mathematical model under square wave loading conditions is presented in Table 10. The ratios maximum/-mean and maximum/minimum total oxygen requirement are calculated and plotted in Fig. 48. Also plotted are the values suggested by Vosloo for carbonaceous oxygen requirement ratios maximum/mean and maximum/minimum under general cyclic loading conditions (Table 5).

The peak and minimum values of the oxygen requirements predicted by the mathematical model result only from the daily average cyclic loading variation. No attempt has been made to model the day to day fluctuations of this daily average cyclic loading. To include the extreme cases of loading due to these random fluctuations about the daily average cyclic load, it is suggested that a "factor of safety" be applied to the peak oxygen requirements as calculated here.

Design Procedure

From a plot giving daily cyclic variations of COD and flow, the COD mass flow can be calculated with Eq. (58). With the average flow calculated from Eq. (57) the effective average influent COD can be found:

$$S_{oave} = MS_{oave}/Q_{ave} \quad (\text{mg COD}/\ell) \tag{60}$$

1. Unsettled municipal sewage

The mass of COD *utilized* (requiring oxygen) in unsettled municipal sewage is calculated as follows:

Let S_{oave} = average total influent COD (mg COD/ℓ). The unbiodegradable COD, S_u, and the inert solid COD, S_{xii}, in the influent are first calculated:

$$S_{ui} = f_u S_o \quad (\text{mg COD}/\ell) \quad f_u = 0{,}05 \tag{61}$$

$$X_{ii} = f_i S_o \quad (\text{mg VSS}/\ell) \quad f_i = 0{,}09 \tag{62}$$

therefore

$$S_{xii} = P\, X_{ii} \quad (\text{mg COD}/\ell) \tag{63}$$

Therefore the biodegradable COD in the influent, S_i, is given by:

$$S_i = S_o(1 - f_u - Pf_i) \quad (\text{mg COD}/\ell) \tag{64}$$

The biodegradable COD in the effluent is calculated from the steady state equations:

$$S = (1/R_s + b)/(YK) \tag{65}$$

where

b = endogenous respiration rate (/day)

R_s = Sludge age (/day)

Y = Yield factor 0,49 (mg VSS/mg COD)

K = rate constant 0,07 (ℓ/mg VSS/d)

$$S_u = S_{ui} \quad (\text{mg COD}/\ell) \tag{66}$$

Hence the COD utilized by the organisms (COD degradation requiring oxygen is given by $(S_i - S)$. The average COD mass load utilised per day, $M(S_i - S)$, is given by:

$$M(S_i - S) = \sum_{t=1}^{t} Q_t(S_{it} - S) \Big/ \sum_{t=1}^{t} Q_t$$

$$= Q_{ave}\,(S_i - S)_{ave} \tag{67}$$

The total effluent COD, S_t, is given by:

$$S_t = S + S_u \tag{68}$$

The total mass of solids generated in the reactor is the sum of the active, endogenous and inert masses.

$$MX_v = MX_a + MX_e + MX_i \tag{69}$$

2. Settled municipal sewage

The design procedure is identical, except that for municipal settled sewage the inert volatile solids fraction in the influent, f_i, is zero. All Eqs. (61 to 69) are applicable if zero is substituted for f_i. Equation (64) reduces to:

$$S_i = S_o(1 - f_u) \tag{70}$$

Design Example

A completely mixed activated sludge plant is to be designed to treat an average expected flow of 10 Mℓ/d. The sludge age is set at 20 days. The anticipated peak COD mass flow is 1,8 times the average COD mass flow and varies sinusiodally.

1. Unsettled municipal sewage

Assume the mean total influent COD, S_{oave} = 600 mg/ℓ. The unbiodegradable COD in the influent is given by Eq. (61):

S_{ui} = 0,05*600 = 30 (mg COD/ℓ)

The solid inert material in the influent is given by Eq. (62):

X_{ii} = 0,09*600 = 54 (mg VSS/ℓ)

The solid inert material equivalent COD is given by Eq. (63):

S_{xii} = 1,42*54 = 76,7 (mg COD/ℓ)

The biodegradable COD in the influent is given by Eq. (64):

S_i = 600 – 30 – 76,7 = 493,3 (mg COD/ℓ)

The biodegradable and unbiodegradable COD in the effluent is given by Eqs. (65) and (66):

S = (1/20 + 0,24)/(0,49*0,07) = 8,5 (mg COD/ℓ)

S_u = 30

$(S_i - S)$ = 493,3 – 8,5 = 484,8 (mg COD/ℓ)

The total effluent COD is given by Eq. (68):

S_t = 8,5 + 30 = 38,5 (mg COD/ℓ)

The *mass* of COD utilized per day is given by Eq. (67):

$M(S_i - S) = 10{*}10^6{*}484{,}8 = 4{,}85{*}10^9$ (mg COD/d)

From Fig. 37, the mass of oxygen consumed in utilizing 1 mg COD at 20 days sludge age, for a peak organic load (including nitrification for an influent TKN (given by Eq. (59)), is 1,23 mg O per mg COD utilized.

Therefore, assuming a factor of safety of 1,3 the peak maximum oxygen uptake rate expected (including nitrification) is:

$$dO/dt\ (max) = 1{,}3*1{,}23*4{,}85*10^9 \quad (mg\ O/d)$$
$$= 7{,}75*10^9 \quad (mg\ O/d)$$
$$= 322{,}1 \quad (kg\ O/hr)$$

The mass of sludge generated in the reactor is found from Fig. 38:

$X_a = 1{,}71*4{,}85*10^9 = 8{,}28*10^9$ (mg VSS/d)
$X_e = 1{,}60*4{,}85*10^9 = 7{,}76*10^9$ (mg VSS/d)
$X_i = 2{,}18*4{,}85*10^9 = 10{,}57*10^9$ (mg VSS/d)
$X_v = 5{,}50*4{,}85*10^9 = 26{,}66*10^9$ (mg VSS/d)

Assume the concentration of sludge has been specified at 3 000 mg VSS/ℓ, then the volume of the reactor, V, is MX_v/X_v.

$$V = \frac{26660*10^6}{3000} = 8{,}89\ M\ell$$

The hydraulic retention time, R, is V/Q.

$$R = \frac{8{,}89}{10} * 24 = 21{,}3 \text{ hours}$$

Peak oxygen utilization rate is:

$$\frac{322{,}1*10^6}{8{,}89*10^6} = 35{,}8 \quad (mg\ O/\ell/hr)$$

2. Settled municipal sewage

Assume the expected average total settled influent COD is 600 mg COD/ℓ.

$S_u = 0{,}05*600 = 30$ (mg COD/ℓ)

$X_{ii} = 0$

$S_i = 570$ (mg COD/ℓ)

$S = 8{,}5$ (mg COD/ℓ)

$S_t = 38{,}5$ (mg COD/ℓ)

$(S_i - S) = 561{,}5$ (mg COD/ℓ)

$M(S_i - S) = 10*10^6*561{,}5$ (mg COD/ℓ)

$= 5{,}62*10^9$ (mg COD/d)

Factor of safety = 1,3

$\therefore dO/dt\ (max) = 1{,}3*1{,}23*5{,}62*10^9$ (mg O/d)

$= 374{,}1$ (kg O/hr)

$X_a = 1{,}71*5{,}62*10^9 = 9{,}60*10^9$ (mg VSS/d)

$X_e = 1{,}60*5{,}62*10^9 = 8{,}98*10^9$ (mg VSS/d)

$X_i = 0$ (mg VSS/d)

$X_v = 3{,}31*5{,}62*10^9 = 18{,}58*10^9$ (mg VSS/d)

$$V = \frac{18{,}58*10^9}{3000} = 6{,}19\ M\ell$$

The hydraulic retention time, R, is V/Q.

$$R = \frac{6{,}195}{10} *24 = 14{,}9 \text{ hours.}$$

Peak oxygen utilization rate is:

$$\frac{374{,}1*10^6}{6{,}19*10^6} = 60{,}5 \text{ mg O}/\ell/\text{hr.}$$

APPENDIX

List of Symbols

Symbol	Description	Units	Value
b	endogenous respiration rate per day		0,24
b′	endogenous respiration rate per day		0,54
f_i	fraction of inert volatile solids in influent		0,09
f_u	fraction of unbiodegradable soluble COD in the influent		0,05
f	fraction of cell mass which is unbiodegradable		0,20
f′	fraction of cell mass which is unbiodegradable		0,09
f_r	maximum ratio of stored solids to active volatile solids		1,0
f_{nu}	fraction of unbiodegradable TKN in the influent		0,02
f_{sd}	fraction of available biodegradable TKN (N_{ai}) as slowly biodegradable		0,80
f_n	fraction of sludge as nitrogen	mg N/mg VSS	0,10
f_{ns}	fraction of stored solids (X_s) which is stored nitrogen	mg N/mg VSS	0,05
f_{sns}	fraction of stored nitrogen as slowly biodegradable		1,00
f_{ens}	fraction of nitrogen gained from endogenous respiration as slowly biodegradable	mg N/mg VSS	0,80
f_{ncs}	fraction of nitrogen required for cell synthesis as slowly biodegradable	mg N/mg VSS	0,00
K	rate constant for substrate entering into storage	ℓ/mg VSS/d	0,12
K′	rate constant for substrate entering into storage	ℓ/mg VSS/d	0,20
K_s*	saturation coefficient for substrate entering into storage	mg COD/ℓ	150,0
K_t*	substrate transfer coefficient	/d	3,0
K_r	rate of degradation of slowly degradable TKN	ℓ/mg VASS/d	0,01
K_{ml}	maximum rate coefficient for COD entering into storage	mg COD/mg VSS/d	12,0
K_{sl}	saturation coefficient for COD entering into storage	mg COD/ℓ	150,0

Symbols marked ′ have revised values to model endogenous respiration as in Approach (ii).
***After Andrews & Busby (1973).**

Symbol	Description	Units	Value
K_{m2} K'_{m2}	maximum rate coefficient for COD released from storage for cell synthesis	mg COD/ mg VSS/d	3,0 3,75
K_{s2} K'_{s2}	saturation coefficient for COD released from storage for cell synthesis	mg COD/ℓ	100,0 100,0
M	prefix indicating masses, not concentrations, are considered	mg	
N	general parameter for TKN concentration	mg N/ℓ	
N_i	total influent TKN	,,	
N_{ui}	unbiodegradable TKN in influent	,,	
N_{xii}	inert TKN bound in the inert volatile solids in the influent	,,	
N_{ai}	biodegradable TKN in the influent	,,	
N_{si}	biodegradable TKN in the influent which is slowly biodegradable	,,	
N_{bi}	biodegradable TKN in the influent which is immediately available	,,	
N_s	slowly biodegradable TKN in the effluent	,,	
N_u	unbiodegradable TKN in effluent	,,	
N_t	total effluent TKN	,,	
N_{h3}	ammonia which is instantaneously converted to nitrates	,,	
N_{o3i}	nitrates concentration in influent	,,	
N_{o3}	Nitrates concentration in effluent	,,	
O	general parameter for oxygen consumption	"mg O/ℓ"	
O_e	oxygen consumption for endogenous respiration	,,	
O_c	oxygen consumption for carbonaceous material oxidation	,,	
O_n	oxygen consumption for nitrification	,,	
O_t	total oxygen consumption		
P	COD/VSS ratio	mg COD/mg VSS	
Q	influent flow	ℓ/d	
Q_i	influent flow wave	ℓ/d	
Q_{ave}	mean of influent flow wave	ℓ/d	
q	waste flow	ℓ/d	
R	hydraulic retention time	d,hrs	
R_s	sludge age $= \frac{\text{mass of solids in system}}{\text{mass of solids wasted/d}}$ $= VX_v/qX_v = V/q$	d	
S	general parameter for substrate concentration – filtered biodegradable COD in the effluent	mg COD/ℓ	
S_u	filtered unbiodegradable COD in the effluent	,,	
S_t	total effluent COD	,,	
S_o	total influent COD	,,	
S_{ui}	unbiodegradable COD in the influent	,,	
S_{xii}	equivalent COD of inert volatile solids in influent	,,	
S_i	biodegradable COD in the influent	,,	
S_s	stored COD	,,	
S_x	equivalent COD of active volatile solids synthesized from stored COD	mg COD/ℓ	
S_{oi}	total COD influent wave	,,	
S_{oave}	mean of total COD influent wave	,,	
t	general parameter for time	d	
Δt	length of integration time interval	d	
V	volume of reactor	ℓ	
X	general parameter for sludge solids concentration	mg VSS/ℓ	
X_a	active volatile solids	,,	
X_e	endogenous residue volatile solids	,,	
X_i	inert volatile solids	,,	
X_v	total volatile solids	,,	
X_s	stored COD as volatile solids	,,	
X_{ii}	inert volatile solids in the influent	,,	
X_{a2}	active volatile solids lost due to endogenous respiration	,,	
x	maximum variation (as a fraction of the average) in Q_i	,,	
Y	organism yield factor	mg VSS/mg COD	0,49
y	maximum variation (as a fraction of the average) in S_{oi}		

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