

# Design of Flocculation Systems from Batch Test Data

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## Abstract

Up to the present time, for the design of continuous completely mixed flocculation systems, it has been necessary to obtain experimental data by performing continuous type pilot plant trials. Because of the considerable expenditure involved in constructing the necessary apparatus and the lengthy testing procedures involved, design of flocculation systems (especially for relatively small plants) has often been based on rule of thumb procedures.

In this paper the theoretical principles and experimental procedures are set out by which it is possible to obtain design data for continuous completely mixed flocculation systems using batch test results only. The validity of the theoretical link between batch type (or plug flow) behaviour and continuous type performance, presented in the paper, is checked by comparing the data gained from continuous and batch test data.

The principles set out should prove of considerable value to design engineers faced with the problem of designing flocculation systems in which both performance and total costs require optimisation.

## Introduction

Flocculation, a water and wastewater treatment process, is a necessary adjunct to destabilization. After the addition of coagulant chemicals and the rapid mixing stage the destabilized stream passes to a flocculation system where destabilized particles are induced to make contact and gradually form settleable or filterable agglomerates.

For a given destabilized water, flocculation reactor geometry and agitation device, the principle parameters governing the effective degree of flocculation are the retention time and velocity gradients applied. These parameters influence the rate and extent of particle aggregation and the rate and extent of breakup of these aggregates.

Equations describing both the aggregation and breakup phases of the process have been developed (Argaman and Kaufman, 1968 and 1970). These equations contain certain 'constants' which must be evaluated for each particular water. At present the determination of these constants can only be carried out by performing continuous type pilot plant trials. Such experiments require relatively complicated and costly apparatus and lengthy testing procedures. If it were shown possible to determine the appropriate terms by a simple batch testing procedure, a powerful means would be available to design engineers for the design of continuous completely mixed flocculation systems. Without the incentive of a quick and inexpensive experimental procedure, designers through economic necessity (especially for relatively small plants) would tend to base their design on empirical and subjective assessments, a practice not unusual in the past.

To date, there has been no demonstration that batch test results can be linked to those of a continuous completely mixed system. O'Melia (1967), for example, limits batch test data to the design of plug flow reactors only.

In this paper, the theoretical link between batch type behaviour and continuous test performance is demonstrated and experimental data presented to test the validity of the connection.

## Flocculation Kinetics

After the addition of coagulants at the rapid mixing stage destabilized particles commence aggregation into larger units. This stage of the overall treatment process is termed flocculation. There are two phases in the flocculation process: The first is given the name perikinetic flocculation and arises from aggregation brought about by random Brownian motion causing particles to make contact. This phase is complete within seconds as there is a limiting floc size beyond which Brownian motion has little effect. The limiting floc size is of the order 0.1 to 1 micrometer.

Von Smoluchowski (1916) developed an expression for the rate of flocculation of a bimodal floc size distribution during perikinetic flocculation:

$$H_{ij} = 4\pi D_{ij} R_{ij} n_i n_j \quad (1)$$

where

$H_{ij}$  = the number of contacts per unit time between particles of radius  $R_i$  and  $R_j$   
 $D_{ij}$  = the mutual Brownian diffusion coefficient of particles  $i$  and  $j$  (approximately  $D_i + D_j$ )  
 $R_{ij}$  = radius of interaction of the two particles, i.e. the distance between centres of two particles forming a lasting contact i.e.  $R_{ij} = R_i + R_j$

Once perikinetic flocculation is complete the only way in which appreciable contact between particles can be promoted is to induce shear motion in the liquid. This is achieved by inducing velocity gradients where particles achieve mutual contact through motion of the surrounding liquid. This latter assumption is valid since the density of suspended particles in water and wastewater treatment is usually very close to that of water. The process of floc aggregation by this procedure is termed orthokinetic flocculation.

The greater the velocity gradients induced in the liquid the more particle contacts there will be within a given time. However, the greater the velocity gradients the smaller will be the ultimate floc size due to a continuous breakdown of the larger flocs. Hence, for a given velocity gradient there will be a limiting flocculation time beyond which flocs will not show further growth. The lower the velocity gradient the longer will be the time needed to reach the optimum floc size, but the larger will be the final floc size.

Von Smoluchowski (1917) formulated a mathematical model for the orthokinetic flocculation of colloidal particles under laminar shear flow conditions:

$$H_{ij} = \frac{4}{3} n_i n_j R_{ij}^3 \frac{dv}{dz} \quad (2)$$

where

$\frac{dv}{dz}$  = velocity gradient in laminar flow

The applicability of von Smoluchowski's orthokinetic flocculation equation to the design of flocculation systems is limited in that his theoretical treatment is based on laminar flow conditions. The more usual condition existing during flocculation is a turbulent regime.

Since the local velocity gradients are not known in turbulent mixing, Camp and Stein (1943) replaced the velocity gradient term  $dv/dz$  by a measurable average value termed the root mean square velocity gradient,  $G$ , which is defined by Camp (1955) as follows:

$$G = (W/\mu)^{1/2}$$

where

$W$  = mean value of the work of shear per unit volume per unit time at each point in the liquid

$\mu$  = absolute viscosity

Replacing  $dv/dz$  in Equation (2) by  $G$ , Camp and Stein's equation for orthokinetic flocculation is given by

$$H_{ij} = \frac{4}{3} n_i n_j R_{ij}^3 G \quad (3)$$

The principal objection to the use of the average velocity gradient,  $G$ , is that it does not describe the length scale over which local velocity gradients extend. That is, velocity gradients which extend for a given mixing length do not influence the flocculation of particles which are larger than this distance.

Fair and Gemmell (1964) extend von Smoluchowski's Equation (2) to describe the rate of change in number concentration of flocs of a given size  $R_k$ :

$$\frac{dn_k}{dt} = \frac{2}{3} \left[ \sum_{i=1}^{k-1} \sum_{j=k-1}^{\infty} n_i n_j R_{ij}^3 - 2n_k \sum_{i=1}^{\infty} n_i R_{ik}^3 \right] \frac{dv}{dz} \quad (4)$$

The first term in the brackets describes the formation of particles of radius  $R_k$  from a range of particles i.e. from primary particles ( $i$  and  $j = 1$ ) to  $(k-1)$  fold particles, for example the joining of a  $(k-1)$  fold particle and a primary particle produces a  $k$  - fold particle with radius  $R_k$ . The second term in the brackets depicts the elimination of  $k$  - fold particles due to their collisions with all other particles i.e. with the full range of particles from  $i = 1$  (primary particles) to infinity.

In all the equations above floc breakup was not considered. Furthermore, all collisions were assumed to result in lasting contact.

Harris and Kaufman (1966) and Harris, Kaufman and Krone (1966) modified Equation (4) to include floc breakup and the possibility of non-lasting collisions. They considered the case of  $k = 1$  which describes the rate at which primary (1 - fold) particles are removed from the system. Their expression for the rate of change in primary particle concentration is as follows:

$$\frac{dn_1}{dt} = -\frac{\alpha a^3}{\pi} \delta \phi n_1 \frac{dv}{dz} \quad (5)$$

where

$\alpha$  = fraction of collisions which result in aggregation  
 $a$  = ratio of collision radius of a floc to its physical radius  
 $\phi$  = floc volume fraction  
 $\delta$  = size distribution function given by

$$\delta = \frac{\sum_{i=0}^{p-1} n_i (i^3 + 1)^3}{\sum_{i=0}^p i n_i}$$

The maximum floc size permitted is  $p$ -fold. Harris *et al* presented other distribution functions for various floc formation and breakup models.

Hudson (1965) adopted a different approach by assuming that in a continuous flow system a simplified bimodal floc size distribution is valid. In a system composed of only primary particles and large flocs, assuming size variations within each group are small compared to the difference between the groups, the removal of primary particles is hypothesised to occur primarily by their collisions with flocs. Accordingly Equation (4) is simplified to

$$\frac{dn_1}{dt} = -\frac{4}{3} n_1 n_F R_F^3 \frac{dv}{dz} \quad (6)$$

where

$R_F$  = radius of flocs

$n_F$  = floc number concentration

Flocculation as described by the above equations probably occurs whenever applied velocity gradients are linear over a distance of a particle diameter. In a turbulent regime, however, flocculation kinetics are better described by turbulent diffusion.

Argaman and Kaufman (1968 and 1970) proposed a diffusion model for orthokinetic flocculation analogous to that of von Smoluchowski for perikinetic flocculation. It is based on the hypothesis that particles suspended in a turbulent regime experience a random motion resembling gas molecules. In their analysis a simplified bimodal floc size distribution comprising primary particles and large flocs was assumed. This assumption was also made by Hudson (1965). Argaman and Kaufman experimentally verified the validity of the bimodal distribution by analysing floc size measurements. Their expression for the rate of collision of primary particles and flocs is given by:

$$H_{1F} = 4\pi K_S R_F^3 n_1 n_F \overline{u^2} \quad (7)$$

where

$K_S$  = proportionality coefficient expressing the effect of the turbulence energy spectrum on the effective diffusion coefficient

$R_F$  = radius of floc

$n_1, n_F$  = number concentration of primary particles and flocs respectively

$\overline{u^2}$  = mean square velocity fluctuation, which is related to the root mean square velocity gradient,  $G$ , and is a measure of the intensity of turbulence

Although Equation (7) was derived for orthokinetic flocculation using the concept of turbulent diffusion it is equivalent in form to the orthokinetic flocculation equation of von Smoluchowski using the concept of laminar flow velocity gradients. In fact all the equations derived for particle aggregation kinetics during orthokinetic flocculation are similar in form to Equation (2). The only differences lie in the constant terms and the definitions of velocity gradient.

It was recognised by Argaman and Kaufman that two opposing processes are responsible for the change in particle concentration during flocculation, that is, aggregation of primary particles and small flocs to form larger flocs (as described by Equation (7)) and the breakup of flocs into smaller fragments. The breakup mechanism assumed by Argaman and Kaufman is described by the rate of formation of primary particles by shearing from the floc surface, i.e.

$$\left[ \frac{dn_1}{dt} \right]_{\text{breakup}} = B R_F^2 \frac{n_F}{R_1^2} \overline{u^2} \quad (8)$$

where

$B$  = breakup constant

Parker, Kaufman and Jenkins (1970 and 1972) on reviewing alternative mechanisms for floc breakup (i.e. erosion of primary particles from the floc surface or floc deformation and subsequent splitting) concluded that the surface erosion concept used by Argaman and Kaufman was the more plausible. The rate of change in concentration of primary particles is thus given by combining Equations (7) and (8) i.e.

$$\frac{dn_1}{dt} = -4\pi\alpha K_S R_F^3 n_1 n_F \overline{u^2} + B \frac{R_F^2}{R_1^2} n_F \overline{u^2} \quad (9)$$

where

$\alpha$  = fraction of particle collisions resulting in lasting aggregation

Argaman and Kaufman applied Equation (9) to the case of a single completely mixed continuous flow tank reactor and, at steady state, obtained the following equation:

$$(n_0 - n_1) = (4\pi\alpha K_S R_F^3 n_1 n_F \overline{u^2} - B \frac{R_F^2}{R_1^2} n_F \overline{u^2}) T \quad (10)$$

Rearranging Equation (10)

$$\frac{n_0}{n_1} = \frac{1 + 4\pi\alpha K_S R_F^3 n_F \overline{u^2} T}{1 + \frac{B R_F n_F \overline{u^2} T}{n_0 R_1^2}} \quad (11)$$

where

$n_0$  = number concentration of primary particles at time  $T = 0$

$n$  = number concentration of primary particles at time  $T$

$n_0/n_1$  is referred to as the performance parameter.

In order to develop a working equation incorporating experimentally measurable parameters, Argaman and Kaufman made a number of assumptions as follows:

1. The average floc size is closely related to the mean square fluctuating velocity as follows:

$$R_F = K_2 / \overline{u^2} \quad (12)$$

where

$K_2$  is a constant

They showed experimentally that for practical purposes this assumption is valid.

2. The mean square fluctuating velocity fluctuations can be estimated from the root mean square velocity gradient as follows

$$\overline{u^2} = K_p G \quad (13)$$

where

$K_p$  = performance parameter dependent on the type of stirring arrangement and referred to as 'stirrer performance coefficient'.

This assumption was supported by experimental observations using a hot-film anemometer for a range of  $G$  values from 30 to 240  $\text{sec}^{-1}$ . For a stake and stator stirrer arrangement they found  $K_p = 0,0458 \text{ cm}^2 \text{ sec}^{-1}$  and for a turbine stirrer  $K_p = 0,0233 \text{ cm}^2 \text{ sec}^{-1}$ .

The difference in  $K_p$  for the different stirrers supports the contention made earlier that  $G$  is a parameter whose influence is dependent on one particular stirrer arrangement and reactor configuration.

3. Assuming spherical particles, the total volume of flocs in a reactor is given by

$$\phi = \frac{4}{3} \pi n_f R_f^3 \quad (14)$$

where

$\phi$  = floc volume fraction

4. The aggregation - collision ratio,  $\alpha$ , is constant. They point out, however, that as the time of flocculation increases the flocs become more compact and of a more regular shape, and therefore, the probability of successful adhesion on collision decreases.

Substituting Equations (12), (13) and (14) into Equation (11) and describing a flocculation constant,  $K_F$ , as

$$K_F = 3 \alpha \phi \quad (15)$$

and a breakup constant,  $K_B$ , as

$$K_B = \frac{3}{4\pi} \frac{B \phi K_p^2}{n_0 R_1^2 K_1} \quad (16)$$

the flocculation equation becomes

$$\frac{n_0}{n_1} = \frac{1 + K_A G T}{1 + K_B G^2 T} \quad (17)$$

where

$$K_A = K_F K_S K_P$$

$K_B$  and  $K_A$  are constants for a single tank.

By making a further assumption that with a number of reactors,  $m$ , in series the constants  $K_A$  and  $K_B$  retain the same values, a general working equation for the  $i$ -th tank in the series is given by

$$\frac{n_{i-1}}{n_i} = \frac{1 + K_A G T / m}{1 + K_B \frac{n_0}{n_{i-1}} G^2 T / m} \quad (18)$$

The overall performance of an  $m$  tank series system is therefore given by

$$\frac{n_0}{n_m} = \frac{(1 + K_A G T / m)^m}{1 + K_B G^2 T / m \sum_{i=0}^{m-1} (1 + K_A G T / m)^i} \quad (19)$$

From experimental data they showed that the constants  $K_A$  and  $K_B$  do remain essentially unchanged through each reactor in series and that Equations (18) and (19), therefore, are valid for an in-series completely mixed system.

## Validity of Batch Test Data in predicting Flocculation Performance

In Equation (19), if  $K_A$  and  $K_B$  remain constant for all tanks in a series of completely mixed reactors then these constants should theoretically remain unchanged when the number of tanks per unit retention time is increased to infinity, that is, for plug flow or batch conditions.

Batch test results are governed by a relationship of the form given by Equation (9). Substituting Equations (12), (13) and (14) into Equation (9) and expressing  $K_A = K_F K_S K_P$  and  $K_B = \frac{3}{4\pi} (B \phi K_p^2) / (n_0 R_1^2 K_1)$  as before, the equivalent flocculation equation for a batch system is

$$\frac{dn_1}{dt} = -K_A n_1 G + K_B n_0 G^2 \quad (20)$$

Integration of Equation (20) yields

$$n_1 = \frac{K_B}{K_A} n_0 G + n_0 \left(1 - \frac{K_B}{K_A} G\right) e^{-K_A G T}$$

Rearranging

$$\frac{n_0}{n_1} = \left[ \frac{K_B}{K_A} G + \left(1 - \frac{K_B}{K_A} G\right) e^{-K_A G T} \right]^{-1} \quad (21)$$

If Equation (21) is valid and, further, the constants  $K_A$  and  $K_B$  have the same value for an equivalent completely mixed continuous system, then by conducting a series of batch flocculation tests for a range of  $G$  and  $T$ , the values of  $K_A$  and  $K_B$  can be determined and applied to the design of a continuous completely mixed system.

## Experimental Work

The objectives of the experimental investigations were as follows:

- (1) Test the validity of Equation (21);
- (2) Determine the constants  $K_A$  and  $K_B$  by a batch test procedure and
- (3) Compare the values of  $K_A$  and  $K_B$  determined from batch experiments with those for an equivalent continuous completely mixed system.

## Apparatus

The reactor used for the batch experiments is shown in Figure 1. A stake and stator stirring mechanism was used and  $G$  values determined by installing a torquemeter on the drive shaft. The relationship between the measured torque and  $G$  is given by

$$G = \left[ \frac{2\pi t N g}{60 V \mu} \right]^{1/2} \quad (22)$$

where

$t$  = torque measured at drive shaft (gm cm)

$N$  = speed of rotation of stirrer (rpm)

$V$  = volume of water in reactor (cm<sup>3</sup>)

$g$  = weight per unit mass on earth's surface (9,81 10<sup>-3</sup> N g<sup>-1</sup>)

$\mu$  = viscosity of water (10<sup>-7</sup> N s cm<sup>-2</sup> @ 20°C)

reactors using the equation developed by Camp (1955) i.e. for a stake and stator system  $G$  is given by

$$G = \sqrt{\frac{1,24 \cdot 10^5 \cdot C_D (1 - k)^3 S_s^3}{V \mu}} \Sigma A r_b^3 \quad \text{sec}^{-1} \quad (23)$$

where

$C_D$  = drag coefficient = 2,0 for flat blades

$S_s$  = speed of rotation of shaft (rev s<sup>-1</sup>)

$k S_s$  = speed of rotation of water (rev s<sup>-1</sup>)

$V$  = volume of water in tank (m<sup>3</sup>)

$A$  = cross sectional area of each blade in the plane perpendicular to direction of motion (m<sup>2</sup>)

$r_b$  = distance of centre of each blade from shaft (m)

$\mu$  = absolute viscosity of water (= 10<sup>-3</sup> N s m<sup>-2</sup> @ 20°C)

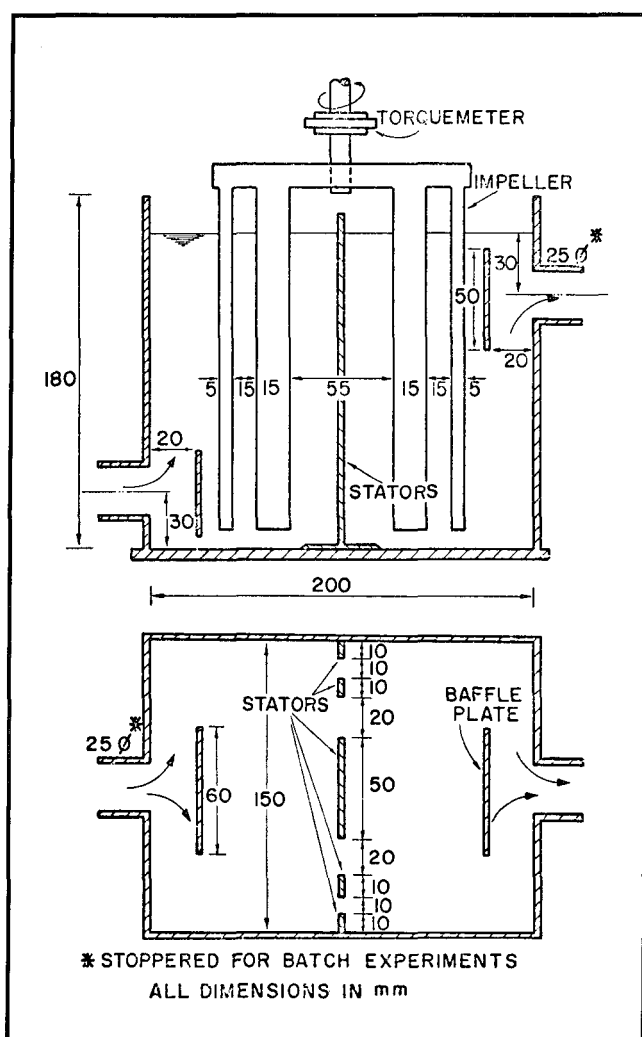


Figure 1

Details of reactor vessel, stirrers and stators used for both batch and continuous experiments.

Figure 2 shows the relationship between impeller speed and  $G$  for the reactor and impeller used. The results obtained demonstrate the validity of determining the value of  $G$  in such

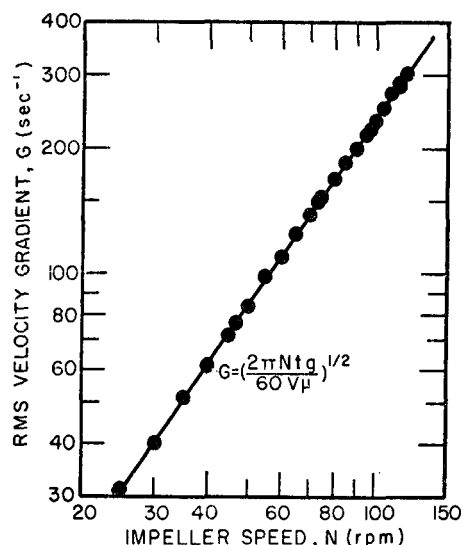


Figure 2

Root mean square velocity gradient,  $G$ , as a function of impeller rotational speed,  $N$ .

The rotational speed of the stirrer was varied using a thyristor speed control fitted to an electric motor (ℓ) – see Fig. 3.

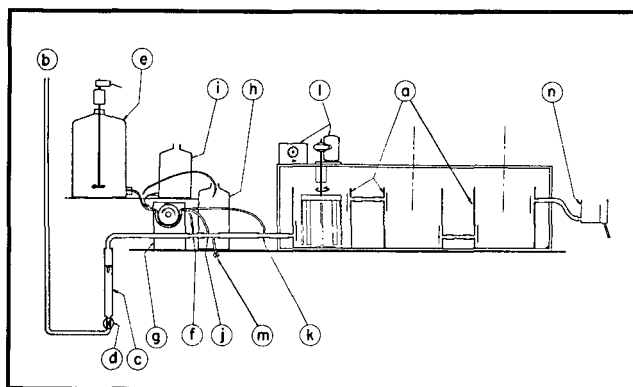


Figure 3

Schematic layout of apparatus for continuous experiments.

For the present study  $k$  was found to have a constant value of 0,24 for the full range of rotational speeds applied.

The apparatus used for the continuous tests is shown in Fig. 3. It comprised three reactors (a) identical to that shown in Fig. 1 for the batch experiments. Municipal tap water was fed from a constant head tank (b) above the apparatus and the flow monitored with a rotameter (c) and controlled with a needle valve (d). A kaolin clay suspension (e) was fed in-line (f) downstream of the rotameter by means of a multi-tube peristaltic pump (g). The rate of delivery of the pump was adjusted to correspond to the flow rate set in any particular experiment. By fitting additional tubes to the pump rollers, the same peristaltic pump served to feed the alum solution (h) and sodium hydroxide (i) to the suspension. Sodium hydroxide was fed in-line (j) upstream of the point of addition of alum (k). A sampling point (m) was located just upstream of the point of coagulant addition to monitor initial turbidity. Water level in the reactors was maintained constant by means of an overflow weir (n). The alum solution was added through a plug-flow type rapid mixer (k) shown in Fig. 4 designed to obtain optimal destabilization (at the optimum coagulant dosage and pH) according to the principles set down by Vrale and Jorden (1971). They propose a plug flow type mixer when the destabilization mechanism is principally adsorption. To ensure the required head loss through the mixer (estimated from Vrale and Jorden's results as approximately 40 mm) a plate was installed at the point of enlargement in the rapid mixer and coarse grained sand packed for a sufficient distance downstream of the plate. The length of packed sand was varied to attain the required head loss at the different flow rates applied. To verify that the head loss was achieved manometer tubes were fitted upstream and downstream of the mixer. The rapid mix unit was arranged to be as near to the first reactor in the series as possible. This ensured that orthokinetic flocculation was principally due to velocity gradients applied in the reactors and not to velocity gradients in the pipe.

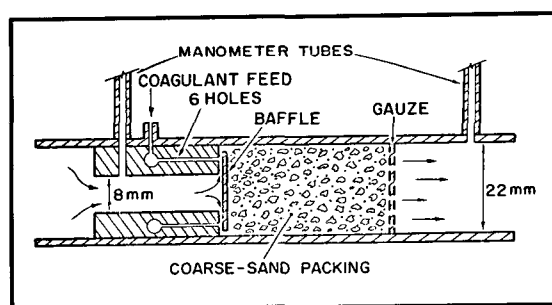


Figure 4  
In-line rapid mix unit for coagulant addition.

### Experimental procedure

In the batch experiments, the reactor was filled to the 5 ℓ mark with the kaolin clay suspension. The suspension was always maintained at a constant turbidity of 40 JTU measured with a Hach model 2100A turbidimeter. The optimum alum dosage of 4,5 mg ℓ<sup>-1</sup> as Al<sup>3+</sup> and pH of 7,5 was determined previously by means of the standard jar test following the principles set down by Stumm and Morgan (1962) – See Appendix I. The optimum dosage and pH were used throughout for both the batch experiments and continuous experiments. A feed alum concentration of 0,5 g ℓ<sup>-1</sup> as Al<sup>3+</sup> was used throughout to maintain

the same dilution effect. The importance of coagulant feed concentration has been pointed out by Griffith and Williams (1972) and Kawamura (1973).

The procedure for the batch experiments was as follows: The stirrer was set to the maximum rotational speed (300 rpm) giving a  $G$  value of approximately 1 000 sec<sup>-1</sup>. By immersing a pH probe in the reactor contents, the pH was adjusted to the optimal value of 7,5. The correct quantity of coagulant stock solution was then added (4,5 mg ℓ<sup>-1</sup> as Al<sup>3+</sup>) as near to the impeller as possible and, simultaneously, sodium hydroxide added to maintain the pH at all times at the optimum value of 7,5. After a 15 sec rapid mix period the impeller speed was reduced to that giving the  $G$  value set for the experiment and so maintained for the period set for the experiment. After the set period the impeller was stopped and the flocculated suspension allowed to settle for a constant 30 min period. A supernatant sample was then taken at a constant depth of 25 mm from the surface using a 100 ml pipette. The bottom of the pipette was bent through 90° at a distance of 15 mm from the tip to ensure that the supernatant sample was always withdrawn horizontally at the set distance of 25 mm from the surface. The sample was transferred to the turbidimeter tubes and supernatant turbidity determined.

The procedure for the continuous experiments was as follows: The impeller rotational speeds, made equal in all the tanks, were set at the beginning of the experiment. Having set the flow rate, stock clay, sodium hydroxide and alum dosage rates, the three tank in-series system was allowed to reach steady state. The time to attain steady state was taken as twice the hydraulic retention time set for the system. At the end of this period all flows were stopped and the impellers switched off. As in the batch experiments, the flocculated suspension in each tank was allowed to settle for 30 min and a supernatant sample taken from each reactor. Supernatant turbidity was then determined for each reactor using the turbidimeter.

### Results and Discussion

In this study it was assumed that the number concentration of primary particles,  $n$ , is proportional to supernatant turbidity,  $N$ . Equations (19) and (21) may then be rewritten as

$$\frac{N_o}{N_m} = \frac{(1 + K_A GT/m)^m}{1 + K_B G^2 T/m \sum_{i=0}^{m-1} (1 + K_A GT/m)^i} \quad (24)$$

for a continuous in-series completely mixed system, and

$$\frac{N_o}{N} = \left[ \frac{K_B}{K_A} G + \left( 1 - \frac{K_B}{K_A} G \right) e^{-K_A GT} \right]^{-1} \quad (25)$$

for a batch reaction,

where

$N_o$  = initial turbidity at time  $T = 0$

$N_m$  = supernatant turbidity of  $m$ -th reactor with total retention time in  $m$  reactors,  $T$ .

$N$  = supernatant turbidity of batch reactor after flocculation period,  $T$ .

Expressing  $N_0/N$  as a measure of the flocculation performance, Fig. 5 shows the performance obtained with the batch experiments for  $G$  values ranging from  $40 \text{ sec}^{-1}$  to  $222 \text{ sec}^{-1}$  and flocculation periods,  $T$ , ranging from 2 min to 40 min. Equation (25) is fitted to the data with values for  $K_A$  of  $2,5 \cdot 10^{-4}$  and  $K_B$  of  $4,5 \cdot 10^{-7} \text{ sec}$ .

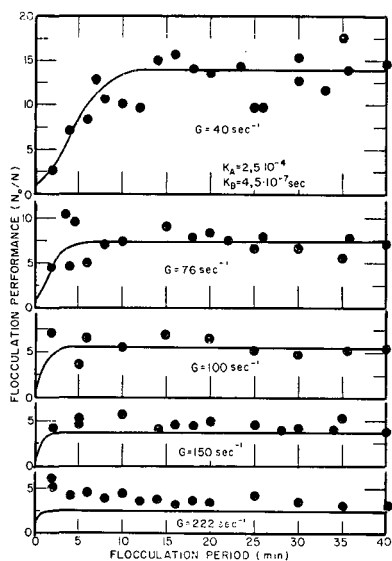


Figure 5

Flocculation performance,  $N_0/N$ , achieved for a range of  $G$  values and flocculation periods,  $T$ , in batch experiments using reactor shown in Fig. 1. Data fits theoretical curves when  $K_A = 2,5 \cdot 10^{-4}$  and  $K_B = 4,5 \cdot 10^{-7} \text{ sec}$ .

Using these values of  $K_A$  and  $K_B$  the experimental flocculation performance is closely correlated where, for  $G$  values of 40, 76 and  $100 \text{ sec}^{-1}$  respectively, the values of  $N_0/N$  achieved were 14,0, 7,5 and 5,7 respectively. It is seen however, that the experimental flocculation performance is underestimated at  $G$  values greater than  $100 \text{ sec}^{-1}$ . For example, at retention times greater than 2 min the experimental flocculation performances achieved at  $G$  values of  $150 \text{ sec}^{-1}$  and  $222 \text{ sec}^{-1}$  were  $N_0/N = 4,3$  and  $3,5$  respectively, whereas theoretically, values of 3,7 and 2,5 respectively are predicted. It is probable that this over-prediction is due to an inadequacy in the breakup equation (see later).

Figures 6 and 7 show the results from the continuous completely mixed experiments for  $G$  values ranging from  $22,8 \text{ sec}^{-1}$  to  $228 \text{ sec}^{-1}$ , total retention times ranging from 1 to 80 minutes and the number of tanks in series ranging from a single tank ( $m = 1$ ) to three tanks ( $m = 3$ ). It is seen that Eq. (24) fits the data closely when values for  $K_A$  and  $K_B$  of  $2,5 \cdot 10^{-4}$  and  $4,5 \cdot 10^{-7}$  respectively, obtained from the batch experiments, are used.

As in the batch test data the actual flocculation performances achieved at higher  $G$  values is under predicted when determined theoretically. The theoretical under prediction at high  $G$  values, also observed by Argaman and Kaufman (1968 and 1970), is probably due to an inadequacy in the break-up equation giving rise to an over-prediction of floc erosion at high  $G$  values. However, under normal circumstances, it is unlikely that the  $G$  values assigned to flocculation systems in practice will exceed approximately  $100 \text{ sec}^{-1}$  so the error evident in the above results is probably of little practical significance. In any case, the actual value of flocculation performance achieved at any  $G$  value is well predicted by the batch experiments.

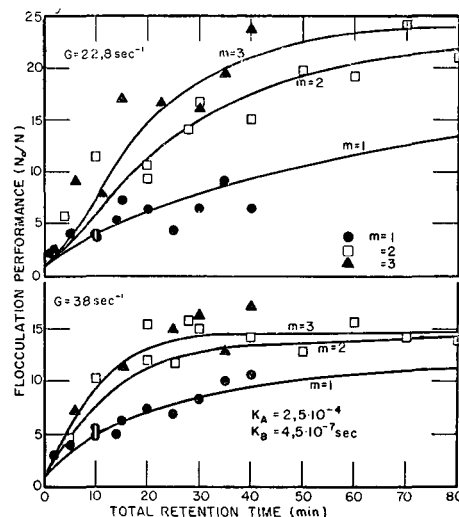


Figure 6

Flocculation performance,  $N_0/N$ , achieved for  $G = 22,8$  and  $38 \text{ sec}^{-1}$  respectively and retention times in continuous completely mixed series reactors up to 80 min. For  $m = 1, 2$  and 3 reactors in series data fits theoretical curves when values for  $K_A = 2,5 \cdot 10^{-4}$  and  $K_B = 4,5 \cdot 10^{-7} \text{ sec}$ , determined from Fig. 5, are used.

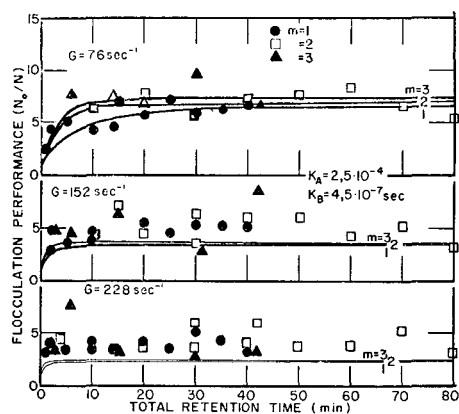


Figure 7

As for Fig. 6 but  $G = 76, 152$  and  $228 \text{ sec}^{-1}$  respectively. For  $G = 152$  and  $228 \text{ sec}^{-1}$  theoretical values underestimate actual performance achieved.

The close fit obtained when applying the values of  $K_A$  and  $K_B$  obtained from batch experiments to continuous completely mixed data, verifies the validity of the assumptions and the relationship between the performance of batch and completely mixed reactors as expressed by Eqs. (24) and (25).

The values of  $K_A$  and  $K_B$  determined in the present study are approximately 4,5 times those found by Argaman and Kaufman. The reason for the higher performance evident in the present investigation is difficult to identify since the methods of sampling and assessing flocculation performance are essentially the same in both cases. Most probably the higher results are due to differences in the kaolin suspensions used in the two studies. These differences illustrate the desirability of carrying out such tests before designing a flocculation system for a particular water. The convenience and validity of the batch test procedure presented in this paper should prove to be sufficient incentive to fulfil this requirement.

It is evident, from Figs. 6 and 7, that the advantage gained in using more than one tank in series diminishes as the velocity gradient increases. At  $G$  values greater than approximately  $150 \text{ sec}^{-1}$  in each tank, the difference in performance between a single reactor and a three reactor system is indiscernible.

### Geometric and scale-up effects

It is very important to note that the geometry of the flocculation reactor(s) influence(s) significantly the flocculation performance obtained. The experiments described above were all carried out with reactors as shown in Fig. 1. To determine whether reliable results could be obtained using standard jar test apparatus, experiments of a similar nature to those described above for the batch tests were performed using 600 ml glass beakers. Using a stake and stator stirring arrangement as before, equivalent results to those described above were obtained. Figure 8 shows that the flocculation performance obtained in a circular reactor is far superior to that obtained in a square reactor – for the same corresponding values of  $G$ ,  $T$  and  $N_0$ . This is probably due to the presence of 'dead' pockets in square reactors which would not be present in circular reactors. It is recommended, therefore, that for the design of a flocculation system using reactors which are square in plan, experiments should be conducted using laboratory scale *square* reactors. Normal jar test (beaker) apparatus will probably over estimate the performance.

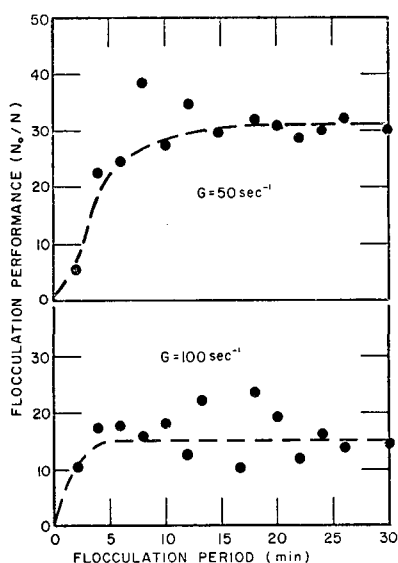


Figure 8  
Flocculation performance  $N_0/N$ , achieved for  $G = 50$  and  $100 \text{ sec}^{-1}$ , respectively and flocculation periods up to 30 min, in batch jar test (beaker) apparatus. Results far in excess of those achieved using square batch reactor (c.f. Fig. 5) shown in Fig. 1.

The comments above lend weight to the observation of Argaman and Kaufman that the  $G$  value in itself is not an adequate design parameter. Design values for  $G$  and  $T$ , whether obtained from laboratory batch experiments or pilot plant trials are valid only for one particular type of reactor and stirring mechanism.

A factor of crucial importance is the applicability of bench scale results to the design of full scale units, that is, whether

such results can be applied directly or whether scaling-up factors are required. Argaman and Kaufman found good correlation between results obtained from laboratory scale continuous type experiments and those obtained from a full scale plant treating the same type of water. They concluded that laboratory scale continuous experiments may be successfully extrapolated to full scale design. The good correlation between batch test results and continuous experiments demonstrated in the present study indicates that batch data may also be directly applied to full scale design provided the geometry and stirring mechanisms of the batch and full scale reactors are the same.

### Interpretation of batch test data for design

This study has demonstrated the validity of designing a continuous type completely mixed flocculation system using data from batch experiments. Based on the principles set down above a suggested design procedure is presented below in the form of an example.

It is required to design a flocculation system for a particular water. From jar test experiments it has been found that the optimum values for coagulant dosage and pH, using aluminium sulphate, are  $4.5 \text{ mg l}^{-1}$  as  $\text{Al}^{3+}$  at a pH of 7.5. Using a square reactor with dimensions and stirrer arrangement as shown in Fig. 1, the relationship between impeller rotational speed and the root mean square velocity gradient,  $G$ , is as shown in Fig. 2. From batch test data using this reactor, the flocculation constants are determined ( $K_A = 2.5 \cdot 10^{-4}$  and  $K_B = 4.5 \cdot 10^{-7} \text{ sec}$ ) and curves constructed describing the performance of an in-series completely mixed flocculation system for a range of  $G$  values using Eq. (24) – see Figs. 6 and 7.

It is seen that with the lower  $G$  values a significant reduction in total flocculator volume is effected by compartmentalization. For example, at a  $G$  value of  $40 \text{ sec}^{-1}$  and a performance parameter  $N_0/N_m = 10$ , a single completely mixed tank requires a retention time of 53.5 min, two tanks in series a total retention time of 15 min and three tanks, 12 min. The retention time required in a plug flow system is 6 min. It is probable that beyond three compartments, the savings in cost by having a smaller total volume is offset by the cost of multiple stirrer mechanisms, etc.

In the present example, the turbidity of the raw water is 40 JTU. It is desired that the final water, after destabilization, flocculation and sedimentation has an effluent turbidity of not more than 2 JTU. The overall performance parameter  $N_0/N_m$  is therefore 20. Using Eq. (24) it is evident that beyond a  $G$  value of approximately  $25 \text{ sec}^{-1}$ , the desired performance cannot be achieved because of the increasing influence of break-up at higher velocity gradients.

To achieve the desired flocculation performance of  $N_0/N_m = 20$ , the minimum total retention time is determined using a trial and error procedure. Assigning equal velocity gradients to each tank and assuming a three tank system, Eq. (24) gives a minimum total retention time of 31 min when the velocity gradient in each tank is  $20 \text{ sec}^{-1}$ . That is, the minimum retention time in each tank, for a three tank system is  $31/3 = 10.3$  minutes.



In practice, it has been found beneficial to provide a tapered velocity gradient through an in-series flocculation system. By providing a high velocity gradient in the first chamber and subsequently lower velocity gradients in the second and third compartments, etc. flocs with denser, more robust characteristics will be produced. Furthermore, the total retention time for the series is reduced. The calculation proceeds as follows:

Design and construction will be more convenient if the three tanks in series are of the same size, even though the  $G$  values change from tank to tank. The lowest total retention time, determined by trial and error and using Eq. (18), is obtained by assigning  $G$  values of  $55 \text{ sec}^{-1}$ ,  $25 \text{ sec}^{-1}$  and  $15 \text{ sec}^{-1}$  to the first, second and third tanks respectively. The optimum retention time in each tank, therefore, is 8 min with the flocculation performance of tank one,  $N_0/N_1 = 4.6$ ; for tank two,  $N_1/N_2 = 2.47$  and for tank three,  $N_2/N_3 = 1.81$ .  $N_0/N_3$  is given by the product, i.e.  $N_0/N_3 = 20.57$ .

It is seen that by providing a tapered velocity gradient profile in the above example, with  $G$  values in first, second and third compartments 55, 25 and  $15 \text{ sec}^{-1}$  respectively, the total retention time is 24 min. Applying a uniform velocity gradient of  $20 \text{ sec}^{-1}$  throughout, the total retention time is 31 minutes.

For square tanks, the final design dimensions, configuration of the stirrers and their rotational speeds may be determined following the procedure of Camp (1955) where  $G$  is given by Eq. (23), i.e.

$$G = \sqrt{\frac{1,24 \cdot 10^5 \cdot C_D \cdot (1-k)^3 \cdot S_3^2}{V \mu}} \cdot \Sigma A r_0^3 \quad \text{sec}^{-1}$$

## Summary and Conclusions

This study has demonstrated the validity of designing a continuous type completely mixed flocculation system using data gained from batch experiments. The equation for a continuous completely mixed system is as presented by Argaman and Kaufman (1968 and 1970), i.e.

$$\frac{N_0}{N_m} = \frac{(1 + K_A G T / m)^m}{1 + K_B G^2 T / m \cdot \sum_{i=0}^{m-1} (1 + K_A G T / m)^i} \quad (24)$$

Constants  $K_A$  and  $K_B$  are determined from batch test results using the equation developed herein, i.e.

$$\frac{N_0}{N} = \left[ \frac{K_B}{K_A} G + \left(1 - \frac{K_B}{K_A} G\right) e^{-K_A G T} \right]^{-1} \quad (25)$$

Normal jar test (beaker) apparatus should not be used to determine the flocculation characteristics of a particular water since flocculation performance will be over predicted. For the design of flocculation tanks square in plan, a laboratory scale *square* batch reactor should be used. By constructing the batch reactor with the same dimensions and stirrer-stator configuration as used in this study, the relationship between impeller rotational speed and  $G$  as shown in Fig. 2 may be used.

## Appendix I – Standard Jar Test

A 500 ml sample is placed on a magnetic stirrer and a high rate of mixing applied. A pH electrode is immersed in the sample at the beaker edge and the sample brought to the test pH by addition of NaOH or HCl. Alum is then added as near to the stirring magnet as possible. At the same time as the alum is added a solution of NaOH, of approximately the same normality as the alum, is added so that the pH does not change at any time. That is, the pH always remains at the constant value set for that test. After 30 secs. rapid mixing the sample is transferred to a standard jar test flocculation apparatus and slowly stirred – at the same rate for each experiment – for 15 minutes. The sample is then allowed to settle for 30 minutes and a supernatant sample, withdrawn at a depth of 20 mm, analysed for turbidity. The above procedure is followed for a range of alum dosages and pH values and a series of curves plotted, supernatant turbidity vs dosage, each curve at a particular pH. By this means the optimum dosage is identified. By plotting the pH-turbidity curve for the optimal alum dosage, the optimal pH is found.

## Appendix II – Notation

The following symbols are used in this paper:

- $a$  = ratio of collision radius of a floc to its physical radius;
- $A$  = cross sectional area of each stirrer blade in plane perpendicular to direction of motion;
- $B$  = constant (see Eq. 8);
- $C_D$  = drag coefficient of stirrer blades;
- $D_{ij}$  = mutual Brownian diffusion coefficient of particles  $i$  and  $j$ ;
- $g$  = weight per unit mass on earth's surface;
- $G$  = root mean square velocity gradient;
- $H_{ij}$  = the number of contacts per unit time between particles of Radius  $R_i$  and  $R_j$ ;
- $H_{IF}$  = the number of contacts per unit time between primary particles and flocs;
- $k$  = ratio between speed of rotation of water and stirrer;
- $K_2$  = constant (see Eq. 12);
- $K_B$  = break-up constant;
- $K_F$  = flocculation constant;
- $K_P$  = stirrer performance coefficient;
- $K_S$  = proportionality coefficient (see Eq. 7);
- $m$  = number of reactors of in-series flocculation system;
- $n_i$  = number concentration of particles of radius  $R_i$ ;
- $n_j$  = number concentration of particles of radius  $R_j$ ;
- $n_k$  = number concentration of particles of radius  $R_k$ ;
- $n_1$  = number concentration of primary particles;
- $n_F$  = number concentration of flocs;
- $N$  = speed of rotation of stirrer;
- $R_i$  = radius of  $i$ -particle;
- $R_j$  = radius of  $j$ -particle;
- $R_{ij}$  = radius of interaction of  $i$  and  $j$ -particles;
- $R_k$  = radius of  $k$ -particle;

- $R_i$  = radius of primary particles;  
 $R_F$  = radius of flocs;  
 $r_b$  = distance of centre of each stirrer blade from shaft;  
 $S_s$  = speed of rotation of stirrer shaft;  
 $t$  = torque measured at drive shaft of stirrer;  
 $T$  = retention time in flocculation reactor;  
 $\overline{\mu^2}$  = mean square fluctuating velocity in a turbulent regime;  
 $V$  = liquid volume in reactor;  
 $W$  = mean value of work of shear per unit volume per unit time at each point in the liquid;  
 $\alpha$  = fraction of particle collisions which result in aggregation;  
 $\delta$  = size distribution function (see Eq. 6);  
 $\phi$  = floc volume fraction; and;  
 $\mu$  = absolute viscosity.

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Messrs J. Ball and D.J. McLean, past undergraduate students in the Department of Civil Engineering, University of Cape Town, who, as part of their final year thesis assignments constructed the apparatus and conducted the preliminary experimental work.

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