Peak Runoff From Small Areas — A Kinematic Approach

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Abstract

The kinematic approximation to the equations of flow facilitates analysis of overland flow. When used in conjunction with a head loss equation such as that of Manning and Strickler, the equations can be solved for specific cases. Here the equations are solved together with an empirical equation for the rainfall intensity-duration relationship. Using two assumptions for losses, generalized equations are developed for time of concentration for overland flow. The peak runoff rate is thus derived and the results compared with those derived by conventional methods. Regional parameters are presented for South African conditions enabling concentration times and peak runoffs for various locations and risks to be read off charts. The effect of urbanization and canalization on the peak runoff is discussed.

Introduction

Considerable work has been done in South Africa on rainfall patterns for small areas, in particular that by the Hydrological Research Unit of the University of the Witwatersrand (Midgley and Pitman, 1978), Schulze (1978), Schwartz and Culligan (1976), Van Heerden (1978) and Van Wyk and Midgley (1966). The studies centre on rainfall data collected and analyzed by the Weather Bureau (1974). The rainfall data for various stations in the Republic have been analyzed to yield depth of precipitation versus storm duration and return period. The form of the results varies with the analysis and the mathematical distribution selected but essentially the intensity of precipitation decreases with storm duration for any selected recurrence interval or return period. It is this fact which results in non-linearity between catchment area and peak runoff rate. Thus the time of concentration, or time to peak, for any catchment is a function of the catchment size amongst other things. So it is apparent that the larger the catchment, other factors remaining constant, the longer will be the storm duration resulting in maximum runoff, even though intensity of rainfall is bound to decline the longer the storm duration.

The popular method of assessing peak runoff from small catchments (less than 15 m²) is the so-called Rational Method. The time of concentration is estimated from an empirical equation and the storm with duration equal to the concentration time is selected as the design storm. The intensity of precipitation corresponding to that duration and return period is then multiplied by an empirical coefficient (C) to yield a runoff rate per unit area of catchment.

The technique is an oversimplification of the true runoff process which depends on many factors other than those indicated above, i.e.

The time-intensity pattern of the storm and its movement in space.

- The form and magnitude of losses, i.e. infiltration, dis-(ii) persion, storage, etc.
- The concentration time of a catchment depends on its characteristics, such as slope, shape, roughness and per-
- The concentration time is also dependent on the rate of runoff. The time to peak shortens as the intensity of the storm increases.
- The degree of urbanization and the alterations made to the catchment characteristics by man affect the runoff process. These include paving of permeable areas, thereby increasing the volume of runoff and reducing frictional resistance, and so increasing the speed of runoff. Canalization and construction of storm-water drains also accelerates the runoff process. The resulting shorter concentration time corresponds to more intense storms and higher runoff rates.
- Construction of storage in the catchment could attenuate the peak runoff rate and reduce total volume of runoff.
- The longer the storm duration the greater the losses but the lower the final rate of loss.
- (viii) Antecedent moisture conditions affect infiltration losses.

A hydrodynamic analysis of the runoff process would overcome the shortcomings of the Rational Method, but would be complicated and expensive. Many modern computer simulation program packages e.g. SWMM (Metcalf and Eddy, 1971) were developed for use in such circumstances. The use of these programs is recommended for major projects to confirm a preliminary design and to facilitate stormwater management.

An approximation to the rigorous St. Venant hydrodynamic equations may, however, be made for most cases of overland flow. The simplified equations are termed the kinematic equations. The resulting simplifications enable the runoff process to be analyzed mathematically. The techniques developed by Woolhiser and Liggett (1967) and others, enable analytical solutions, which can readily be generalized, to be achieved. This article employs these techniques to determine concentration times for various catchment surfaces. Dimensionless charts are produced to facilitate design. Concentration times, peak runoffs and the effect of canalization may be studied with the assistance of the charts. By rendering the results dimensionless, they may be applied globally, although the specific rainfall figures apply only to South African conditions. It is necessary to assume a rectangular catchment and uniform storm distribution. Numerical techniques must be employed for non-uniform and time-varying storms (Stephenson, 1980).

An attraction of the kinematic approach is that all the variables are physically measurable. No empirical factors are required. The slope, roughness and length of catchment are all measureable, although an approximate equation for friction gradient is employed. The infiltration and initial losses are still difficult to assess, but the U.S. Soil Conservation Service (1972) has given guidelines.

Charts presented here are for uniform infiltration or an initial surface storage. A combination of these will approximate to a diminishing loss with time i.e. a decay in loss rate.

Equations of Motion

The one-dimensional continuity equation may be written as follows:

$$\frac{\partial y}{\partial r} + \frac{\partial q}{\partial x} = i - f = i_e \text{ say,}$$
 (1)

where y is the flow depth, t is time, q is the discharge per unit width in the x-direction, i is the precipitation rate and f the infiltration rate. It is assumed that the flow plane or channel is reasonably wide $(y \le b)$, the bottom slope is moderately small so that $\theta \approx \sin \theta \approx \tan \theta$, and the velocity distribution coefficient is unity (Eagleson, 1970). The equation applies equally to flow in a wide channel or over a uniform flood plane. The momentum equation for similar conditions is

$$\mathbf{v} \frac{\partial \mathbf{v}}{\partial \mathbf{x}} + \frac{\partial \mathbf{v}}{\partial \mathbf{t}} + \mathbf{g} \frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \mathbf{g} (\mathbf{S}_o - \mathbf{S}_f) - (\mathbf{i} - \mathbf{f}) \mathbf{v} / \mathbf{y}$$
 (2)

where v is the flow velocity, g is gravitational acceleration, S_o is the bed slope and S_f the energy gradient. Equation (1) may be simplified further for cases of overland flow. Inflow, change in depth and inertia terms are negligible in comparison with bottom slope and the equation may be approximated by the equation

$$S_o = S_f \tag{3}$$

This form of the momentum equation with the inertia terms neglected and the continuity equation (1) comprise the kinematic equations. Any non-uniformity will manifest itself only in the continuity equation.

The friction gradient may be calculated with the aid of the Manning equation which in SI units is

$$\mathbf{v} = -\frac{1}{n} \mathbf{y}^{\frac{2}{3}} S_f^{\frac{1}{2}} \tag{4}$$

or

$$\mathbf{q} = \frac{1}{n} y^{\frac{5}{3}} S_f^{\frac{1}{2}} \tag{5}$$

For turbulent flow the roughness factor n is a function of the absolute roughness k, and in dimensionless form it may be expressed as

$$n = k^{\frac{1}{6}}/7, 7\sqrt{g} \tag{6}$$

Therefore

$$q = \frac{7.7 \sqrt{Sg}}{k^{\frac{1}{6}}} y^{\frac{5}{3}} = 7.7 y \sqrt{Sgy} (y/k)^{\frac{1}{6}}$$
 (7)

This equation is termed the Manning-Strickler equation. It is dimensionally hornogeneous and does not suffer the drawback of the Manning equation, the form of which is dependent on the units employed.

The kinematic equations can be employed to derive an equation for the concentration time of a catchment. When rain falls at a uniform rate over a plane catchment, the depth at any point will gradually increase until an equilibrium is reached. The depth will also increase down the catchment, being a maximum at the mouth or discharge end, where the equilibrium flow will be a maximum, namely (i-f)L per unit width where L is the catchment length.

Consider an element of water which starts at the top end of the catchment at the beginning of a storm. As it flows towards the mouth the elepth increases due to the net rain, so that the depth of the element at any time t is

$$y = i_e t \tag{8}$$

where i, is the excess rainfall rate i-f. By the time the element from the furthest point has reached the mouth the system will be in equilibrium. Then the discharge rate at the mouth will equal the net rainfal rate, or

$$q = i_e L (9)$$

where L is the catchment length and q is the discharge per unit width of catchment. Now from (7)

$$y = (qk^{\frac{1}{6}}/7, 7\sqrt{Sg})^{\frac{3}{5}}$$
 (10)

At equilibrium,
$$y = (i_e L k^{\frac{1}{6}} / 7, 7 \sqrt{Sg})^{\frac{3}{5}}$$
 (11)

So from (8)
$$t_c = (Lk^{\frac{1}{6}}/7, 7\sqrt{Sg} i_c^{\frac{2}{3}})^{\frac{5}{5}}$$
 (12)

A more rigorous proof of (12) is given by Overton and Meadows (1976).

t_e is referred to as the concentration time of the catchment. It will be observed that it is a function of the catchment characteristics as well as the rate of excess rain i_e. It is therefore necessary to solve for time of concentration as a function of excess rainfall rate, which in turn is a function of storm duration for any locality and return period. The rain is assumed uniform in time and space, but the losses may not be. Initial storage may absorb some of the rain and infiltration may vary with time and antecedent conditions. Horton (1939) proposed an exponentially decreasing infiltration. Various infiltration models have been proposed, and Overton and Meadows (1976) used the U.S. Soil Conservation Service uniform loss relationships for an analytical solution for runoff from catchments in Tennessee.

In the following, two simplistic loss models are employed in deriving analytically the concentration times and runoff for rectangular catchments. One model assumes all the losses to occur at the beginning of the storm, as would occur for catchment storage. The other model assumes a uniform rate of loss for the entire storm duration. Combinations of the two types of loss may be interpolated between the two extremes, which are plotted on accompanying charts.

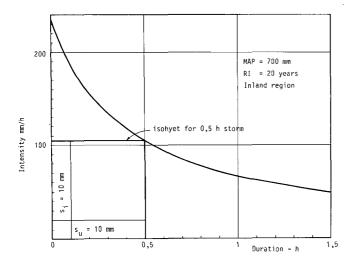


Figure 1 Rainfall intensity - duration relationship

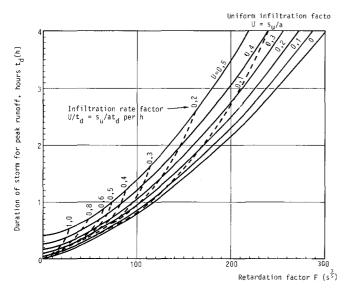


Figure 2 Design storm duration for uniform losses, b = 0.4 h

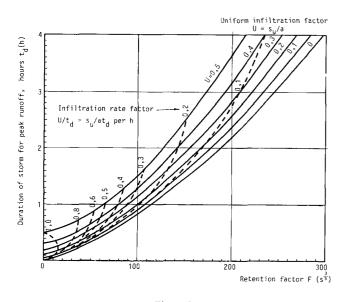


Figure 3 Design storm duration for uniform losses, b = 0.5 h

Storm Intensity — Duration Relationships and Solution for Time of Concentration

For any particular locality and recurrence interval, there is a statistical relationship between storm duration and intensity. Analysis of storms through the world (Overton and Meadows, 1976) and in South Africa in particular, indicate that the storm intensity may be predicted with reasonable accuracy with an equation of the form

$$i = \frac{a}{b + t_c} \tag{13}$$

where t_d is the storm duration, and a is a function of the locality and return period.

The peak runoff is found to occur for a storm duration equal to the concentration time of the basin. With this in mind, it is possible to solve (12) and (13) for various cases. The total depth of loss is s, in the same units as ti, where i is the rainfall rate and t is time.

Case I — Uniform loss rate

For a uniform loss rate over the entire storm duration, s_u/t_d ,

$$i_e = i - s_u/t_d \tag{14}$$

Substituting i from (13) into (14) and (14) into (12) yields

$$t_{c} = \frac{(Lk^{\frac{1}{6}}/7, 7\sqrt{Sg})^{\frac{3}{5}}}{(\frac{a}{b+t_{c}} - \frac{S_{u}}{t_{c}})^{\frac{2}{5}}}$$
(15)

$$= \frac{F}{\left(\frac{1}{b+t_c} - \frac{U}{t_c}\right)^{\frac{2}{5}}}$$
 (16)

where
$$F = (Lk^{\frac{1}{6}}/7, 7\sqrt{Sg} \frac{2^{\frac{3}{3}}}{3^{\frac{3}{5}}}$$
 (17)

is the catchment retardation factor and

$$U = s_u/a \tag{18}$$

U is defined as the infiltration factor. Eq. 16 cannot be solved explicitly for t_c so the equation was solved for F as a function of $t_{\mbox{\tiny c}}$ and U for various values of b. The results are summarized in Figs. 2 and 3 from which the concentration time may be read knowing the catchment characteristics, namely length L, absolute roughness k, slope S, storm characteristic α and uniform infiltration loss su. Unless the storm duration is known, it may be difficult to assess s_u . In many cases the infiltration rate, (s_u/t_d) is known instead of the total volume lost, and the dashed lines on Figs. 2 and 3 may therefore be of more use in estimating concentration times.

The maximum storm runoff rate may be evaluated from the equation

$$\mathbf{i}_{ep} = \frac{\mathbf{a}}{\mathbf{b} + \mathbf{t}_d} - \frac{\mathbf{s}_u}{\mathbf{t}_d} \tag{19}$$

which is plotted in Figs. 6 and 7 in dimensionless terms

with $t_d = t_c$

evaluated from Figs. 2 and 3. Subscript e refers to excess rain and p to that corresponding to peak runoff rate. It will be observed that the maximum rate of runoff per unit area does not occur for small smooth basins, except for no losses. For real losses represented by U there is some basin configuration represented by F which results in a higher rate of runoff per unit area. This is because for any U the rate of loss reduces with increasing F and hence increasing t_c, and this effect predominates over the lower storm intensity. On the other hand for short storms, the rate of loss would have to be high to produce a certain U, hence the rate of runoff is affected.

Case II - Initial loss

If all the storm input is initially absorbed or taken up in filling depression storage, runoff will not commence till the storage is full. If the storage or loss volume is s_i per unit area, then the time till runoff commences is

$$t_i = s_i/i \tag{20}$$

For peak runoff,

$$t_{d} - t_{i} = t_{c} = \frac{(Lk^{\frac{1}{6}}/7, 7\sqrt{Sg})^{\frac{3}{5}}}{(\frac{a}{b+t_{d}})^{\frac{2}{5}}}$$
(21)

Therefore
$$t_d = F(b + t_d)^{\frac{2}{5}} + I(b + t_d)$$
 (22)

where I is the initial retention factor.

This equation was solved for F for various t_a and I and the results are plotted in Figs. 4 and 5 for various values of b. It will be noted that the resulting storm durations for peak runoff are invariably higher for initial losses than for uniform losses.

It sill be observed from Figs. 6 and 7 however, that the peak runoff rate is higher for initial loss than for uniform loss. For no loss both theories yield identical results as would be expected while for increasing losses the results diverge. The peak runoff per unit area for case II, however, occurs for the smallest, smoothest and steepest catchment.

For losses comprising a combination of initial storage and uniform infiltration, Figs. 2 to 7 may be interpolated, taking note that each line for a particular loss function is drawn assuming the other type of loss is zero.

Parameters for South African Conditions

Rainfall intensity — duration — frequency relationships for South Africa

Various hydrologists have proposed rainfall intensity — duration relationships of the form

$$i = \frac{a}{(b + t)^c} \tag{23}$$

where a, b and c are constants for any particular station and return period. Overton and Meadows (1976) found c values of approximately unity, whereas the Hydrological Research Unit (Midgley and Pitman, 1978) indicate values of the order of 0,8. The sensitivity to c is low however, and a value of unity was found by the author to produce results within acceptable accuracy for South African conditions. A value of 1 was similarly

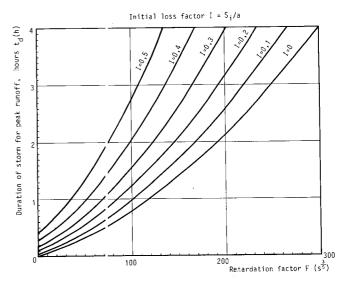


Figure 4

Design sterm duration for initial losses, b = 0.4 h

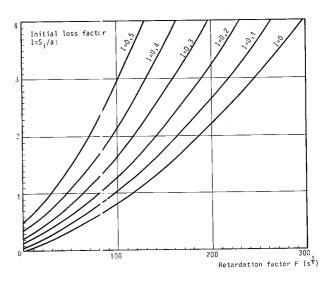


Figure 5
Design storm duration for initial losses, b = 0.5 h

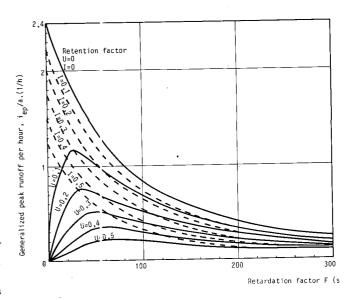


Figure 6
General peak runoff, b = 0.4 h

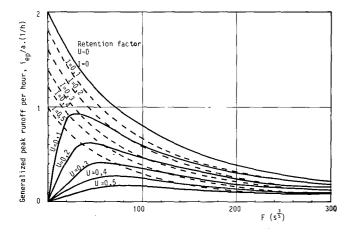


Figure 7 General peak runoff, b = 0.5 h

found by Overton (1976) to apply in Tennessee. Thus the intensity i in mm/h is given by

$$i = \frac{a}{b + t_d} \tag{24}$$

where t_d is the storm duration and b is a constant (both in hours). The value of b was found to be approximately 0,4 h for inland and 0,5 h for coastal regions in South Africa. Using these figures, values of a in millimetres may be calculated from the following equations

For inland regions
$$a = 74 (0.325 + 0.001 35 \times MAP)N$$
 (25)

For coastal regions
$$a = 51 (0.325 + 0.001 35 \times MAP)N$$
 (26)

MAP is the mean annual precipitation in mm, and may be obtained from the accompanying isohyetal map, Fig. 8. N is



Figure 8 Isohyetal map for South Africa

a factor for return period and values from the Report HRU 2/78 (Midgley and Pitman, 1978) are listed below:

100 10 20 50 Return period, years 0,47 0,64 0,81 1,00 1,30 1,60

The above relationships apply to storms with durations of between 0,1 and 2 h.

Surface losses

The losses to be deducted from precipitation include interception on vegetation and roofs, evapotranspiration, depression storage and infiltration. The remaining losses may be divided into initial retention and a time-dependent infiltration.

The loss function is really a function of many variables, including antecedent moisture conditions and ground cover. Infiltration is time-dependent and an exponential decay curve was proposed by Horton (1939), Holton (1961), Overton and Meadows (1976) and others. The infiltration typically reduces from an initial rate of about 50 mm/h down to 10 mm/h over a period of about an hour. The rates, especially the terminal loss rate, will be higher for coarse sands than for clays.

The time-decaying loss rate could be approximated by an initial loss plus a uniform loss over the duration of the storm. Values of initial and uniform losses used in the United States are tabulated in Table 1. The mean uniform loss rates are averages for storms of 30 minute duration, and the initial losses include the initial 10 minute rapid infiltration or saturation amount.

In the case of ploughed lands, and other especially absorptive surfaces an additional initial loss of up to 10 mm or more may be included. Allowance must also be made for reduced losses from covered areas (paved or roofed). The values should be used with caution for South African conditions until more appropriate data are available.

TABLE 1

SURFAC	E LOSSES	
Initia	Uniform infil- tration rate (mm/h)	
Surface	Infiltration	

	Retent		Infiltration	
Paved	up to	1	_	0
Clay	٠,,	5	20	2 - 5
Loam	,,	5	30	5 - 15
Sandy soil	"	5	40	15 - 25
Dense vegetation	**	12	_	

TABLE 2 ABSOLUTE ROUGHNESS, k (mm)

Concrete lined storm drains	0,5
	1
Concrete paving	5
Gravel	20
Lawn, turf	
Weeds	50
Ploughed land	150
Boulders and rubble	500
Dense vegetation	1 000 +

Roughness

The form of drag equation preferred (7) is dimensionally homogeneous. By including the absolute roughness as a variable, it loses the empiricism of the Manning equation. In fact the drag effect is very insensitive to the roughness, as it is raised to the power of 1/6. Thus any inaccuracy in selecting k is masked by the equation. It is preferable to overestimate k as the drag equation tends to predict too rapid flow concentration unless this is done. This is due to the tortuosity of the flow path over rough surfaces. In fact the original form of the Manning equation and the Strickler approximation for the roughness were never intended for overland flow where the depth of flow is comparable with the roughness and Reynolds numbers are of the order of 1 000. Table 2 may be used as a guide for surface roughness k.

The length of drainage path and slope influence the concentration time more than the roughness. Runoff follows a circuitous path over natural land and the ground slope along the flow path is therefore flatter than the net slope. A similar lag occurs with runofi from roofs (2 to 5 min lag). Allowance should be made for these effects in establishing the retardation factor F.

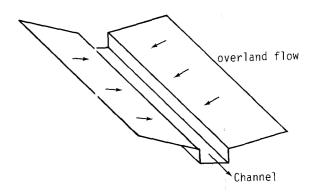


Figure 9 Rectanguar catchment with central collection channel

Example

The example ill strates the use of the design charts for determining design storm duration, peak runoff and the effect of canalization. The use of consistent units in the equations should be noted:

Calculate the peak 20-year runoff from a catchment before and after the installation of a drainage channel. The catchment is 500 m wide and 2 000 m long with a uniform slope of 1/500, and an effective absolute roughness of 10 mm before development. A 3 10 wide channel with an effective roughness of 1 mm will be constructed. The depression storage over the catchment may be taken as 10 mm and the infiltration as 20 mm for the soil conditions present. Neglect overland flow time for the purposes of the example.

The 20 year storm factor a for the station is 90 mm and the time factor t is 0,5 h.

For the or ginal catchment the retardation factor

$$F = \left\{ \frac{2 (00 \times 0.01^{\frac{1}{6}})}{7.7 \sqrt{9.8/500} (0.09^{\frac{2}{3}})} \right\}^{0.6} = 151 s^{\frac{3}{5}}$$

Uniform retention factor for total loss $U = s_u/a = 30/90 = 0.333$

Initial retention factor for total loss $I = s_i/a = 30/90 = 0.333$

It will be observed that the total loss is taken in each case. This is because each line in Figs. 2 to 7 is drawn assuming the other loss is zero.

Interpolating between these lines in Figs. 3 and 5 (one third of the way from the U = 0,33 to the I = 0,33 line), storm duration = 2,3 h and from Fig. 7, $i_{ep}/a = 0,27/h$. The peak excess runoff is therefore $i_{ep} = 0,27 \times 90 = 24,3 \text{ mm/h}$. The peak rate of runoff is $24,3 \times 500 \times 2 000/3 600 \times 1 000 = 6,75 \text{ m}^3/s$. The value of C in the rational formula is

$$\frac{i_{ep}}{i_{ep} + s/at_d} = \frac{24.3 \text{ mm/h}}{24.3 + 30/2.3} = 0.65$$

It should be noted that owing to the non-linearity of the equations, when uniform and initial losses are interpolated, the storm precipitation calculated from the storm duration from Fig. 7 and the equation $i = a/(b + t_d)$ does not necessarily quite equal the total of excess runoff and losses.

With the construction of the channel $a' = (500/3) \times 0.09 = 15$

F becomes
$$\left\{ \frac{2\ 000\ x\ 0.001^{\frac{1}{6}}}{7.7\sqrt{9.8/500}\ 15^{\frac{2}{3}}} \right\}^{0.6} = 15.5$$

The peak runoff thus becomes $0,45 \times 90 \times 500 \times 2000/3600 \times 1000 = 11,25 \, \text{m}^3/\text{s}$ and storm duration = $0,33 \, \text{h}$. It is very inaccurate to interpolate between values of U and I on Fig. 7 for low F though. In fact U would be less for the smaller duration (even negligible in this case) so I becomes 10/90 = 0,11 and runoff would become $1,46 \times 90 \times 500 \times 2000/3600 \times 1000 = 36,5 \, \text{m}^3/\text{s}$.

Effect of Canalization

It is evident from (15) and (21) that the rate of overland flow per unit width affects the concentration time. The higher the runoff rate, and consequently the greater the depth of flow, the faster will be the time of concentration. It follows that if the overland flow is concentrated in a channel down the catchment, the concentration time will similarly be reduced. This is assuming the overland flow time to the channel is negligible. The consequence of this is that the effective storm duration is reduced so that the intensity of precipitation is increased and the peak discharge rate is magnified.

The effect on peak discharge rate of a rectangular cross section channel of uniform width down a long rectangular plan catchment may be studied from previous results. Fig. 9 illustrates the catchment considered. Neglect side effects of the channel (i.e. $y \le w$). Then the catchment storm factor a should be multiplied by W/w where W is the catchment width and w is the channel width, to obtain an effective a'. The catchment retention factor s/a remains the same since it represents a proportional loss (both loss s per unit area and a increase in the proportion W/w). It will be observed from Figs. 2 and 3 that concentration time reduces as F is reduced due to the higher a', and from Figs. 6 and 7, peak runoff increases in most cases, due to smaller F.

Conclusions regarding the effect of urbanization

The effect of urbanization has the following effects on the runoff process:

- Losses are reduced due to paved and covered areas replacing soil and vegetation.
- (ii) Infiltration is reduced and losses tend to be more in the way of initial losses (storage or retention). Initial losses do not reduce excess runoff as much as the same loss spread over the duration of the storm.
- (iii) Effective catchment storm factor is increased by canalization, with the result that concentration times are reduced. Peak runoff is thereby increased for any location and recurrence interval.
- (iv) Roughness is reduced so that concentration is faster. This has the same effect as eliminating temporary surface storage.

Acknowledgements

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