A Depth-Duration-Frequency Diagram for Point Rainfall in SWA-Namibia

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Abstract

From an analysis of available daily and autographic rainfall data an isohyetal map of SWA-Namibia has been compiled as well as a co-axial diagram from which, given the mean annual precipitation at a location, it is possible to estimate for a specified recurrence interval, the maximum likely precipitation for any duration between one-tenth of an hour and one day.

Introduction

Information on extreme rainfall is a basic requirement for the design of hydraulic structures, from roof gutters to dam spillways. The first report published by the Hydrological Research Unit (HRU) comprised a comprehensive study of storm rainfall in South Africa (Wiederhold, 1969). However, until now there has been no such study undertaken for SWA-Namibia. Designers in SWA-Namibia usually had to rely on doubtful extrapolations from the South African data.

This paper presents the methods adopted and the results obtained in an analysis of extreme one-day point rainfalls in SWA-Namibia. Extrapolation of the results to durations shorter than one day is also described. The outcome is a co-axial diagram and a map of mean annual rainfall that enables the designer to estimate, at any location in SWA-Namibia, point rainfalls for durations of 6 min to one day with return periods from 2 to 100 years.

The Data

Records from 572 daily-read rain gauges were transferred to computer cards in the Weather Office, Windhoek, by a firm of consulting engineers under directions from the HRU. Fig. 1 shows the distribution of the rainfall stations. Data from the seven official autographic recording stations in SWA-Namibia and two outside the territory (Maun and Upington), were used to disaggregate daily to short-duration rainfall extremes.

Earliest records date from about 1890 and the closing date of the data assembled is December 1976. In general, the records prior to about 1910 were merely monthly totals. An editing program was developed to check for errors such as incorrect number of days in the month, discrepancies between monthly totals listed and the sum of daily falls recorded for the month and various other inconsistencies in the records. After correction the data set, comprising about 120 000 cards, was copied onto magnetic tape. Table 1 lists the length-of-record statistics.

TABLE 1
NUMBERS OF STATIONS ACCORDING TO RECORD LENGTH

Records longer than	Number	% Total
2 years	572	100
10 years	548	96
20 years	362	63
30 years	198	35
40 years	96	17
50 years	45	8

Mean annual precipitation (MAP) was computed for each station with length of record greater than ten years and from the results the mean annual isohyetal map of SWA-Namibia was compiled as shown in Fig. 2.

Analyses of Maximum Daily Rainfalls

Maximum daily rainfalls (MDR) recorded in each hydrological year (October — September) were abstracted for each station. In the light of experience gained in analyses of similar data in South Africa (Midgley and Pitman, 1978) the Log-Gumbel distribution was adopted for frequency analyses of the MDR.

The Log-Gumbel cumulative density function (Gumbel, 1958) is given by:

$$f(x) = e^{-a + cy} \quad \ldots\ldots (1)$$

For the case in point $f(x)$ = MDR

$y = -\ln(-\ln(1-\frac{1}{T}))$

T = return period

$c = \sigma\sqrt{6}/\pi$

$a = \gamma c - \mu$

σ = standard deviation

μ = mean

γ = 0,57721 ..... (Euler's constant)

There are various ways of fitting a distribution to a set of data. In this study the method of moments was employed to

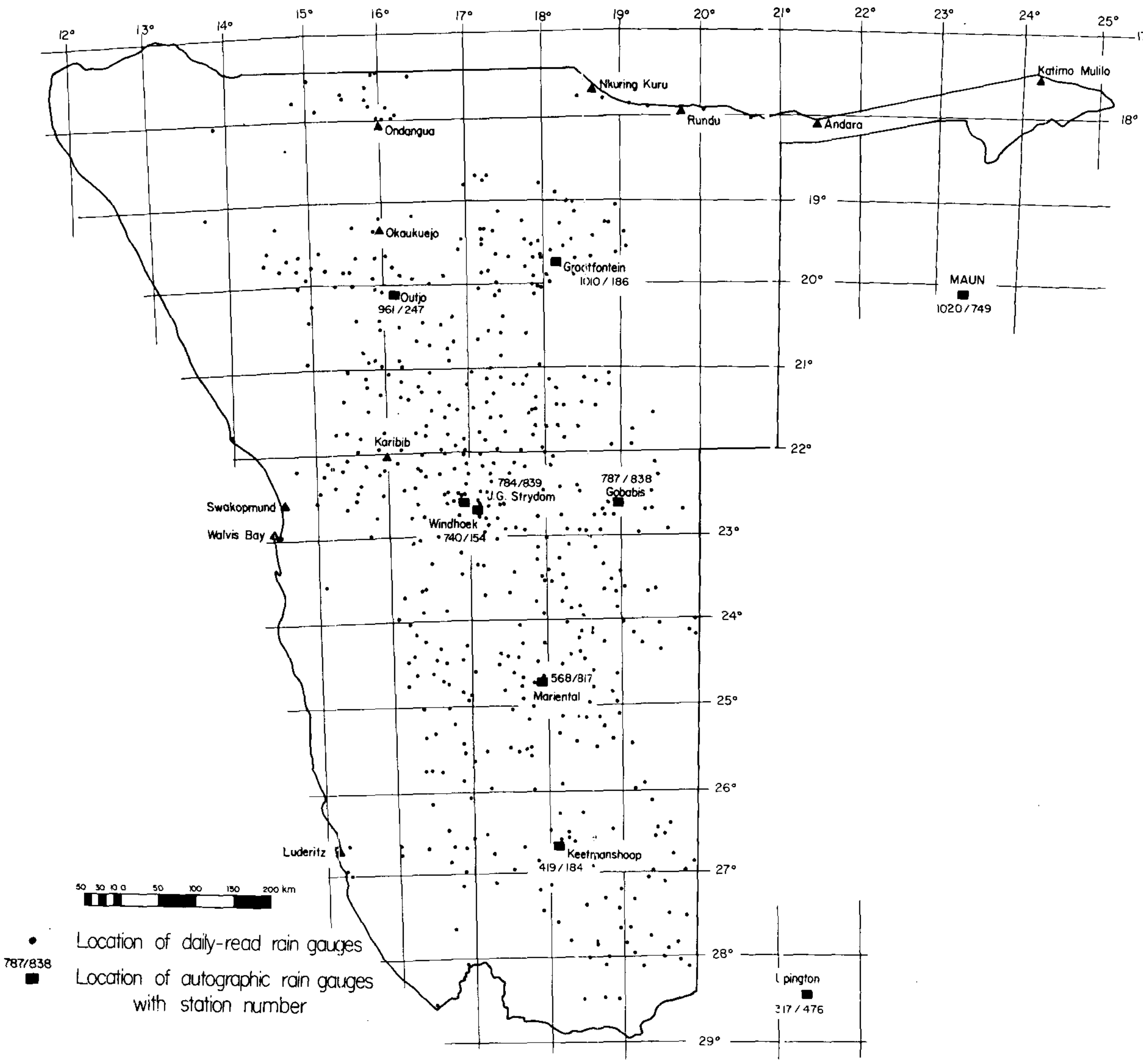

Figure 1
SWA/Namibia: Distribution of rainfall stations

derive μ and σ and, in turn, c and a. This method has the advantage of not relying on the somewhat arbitrary selection of a plotting position formula.

The chi-square goodness-of-fit test was performed on the Log-Gumbel distribution as fitted to the 20 longest records of MDR. In all cases the distribution was accepted at the 5% level of confidence.

After fitting the Log-Gumbel distribution to all records of MDR the influence of mean annual precipitation (MAP) on extreme rainfall events was sought. Means (μ) and standard deviations (σ) of the logarithmically transformed MDR records were plotted against MAP, as shown on Figs. 3 and 4. Data were group-weighted (i.e. stations were given weights proportional to length of record) according to "sections" (30 min squares) to minimize sampling error and to reduce the number of points to be plotted.

With recognition of the apparent discontinuity at a MAP of about 90 mm the relationship between μ and MAP in Fig. 3 was expressed by the following two equations:

$$\mu = -1{,}26 + 0{,}98 \ln \mathrm{MAP} \quad (\mathrm{MAP} < 86 \text{ mm}) \qquad (2)$$

$$\mu = 0{,}93 + 0{,}49 \ln \mathrm{MAP} \quad (\mathrm{MAP} > 86 \text{ mm}) \qquad (3)$$

As is clear from Fig. 4 the scatter of plotted standard deviations of MDR in areas of MAP below 50 mm (*viz.* the coastal strip) is considerable. Moreover, when the plotted points lying above the fitted curve (eqn. 4) were distinguished on a map from those lying below the curve, no obvious regional groupings could be identified. Equation 4, therefore, can be used with reasonable confidence only in areas where MAP exceeds 50 mm.

$$\sigma = 0{,}47\ e^{0.00435\,\mathrm{MAP}} + 0{,}30 \quad (\mathrm{MAP} > 50 \text{ mm}) \qquad (4)$$

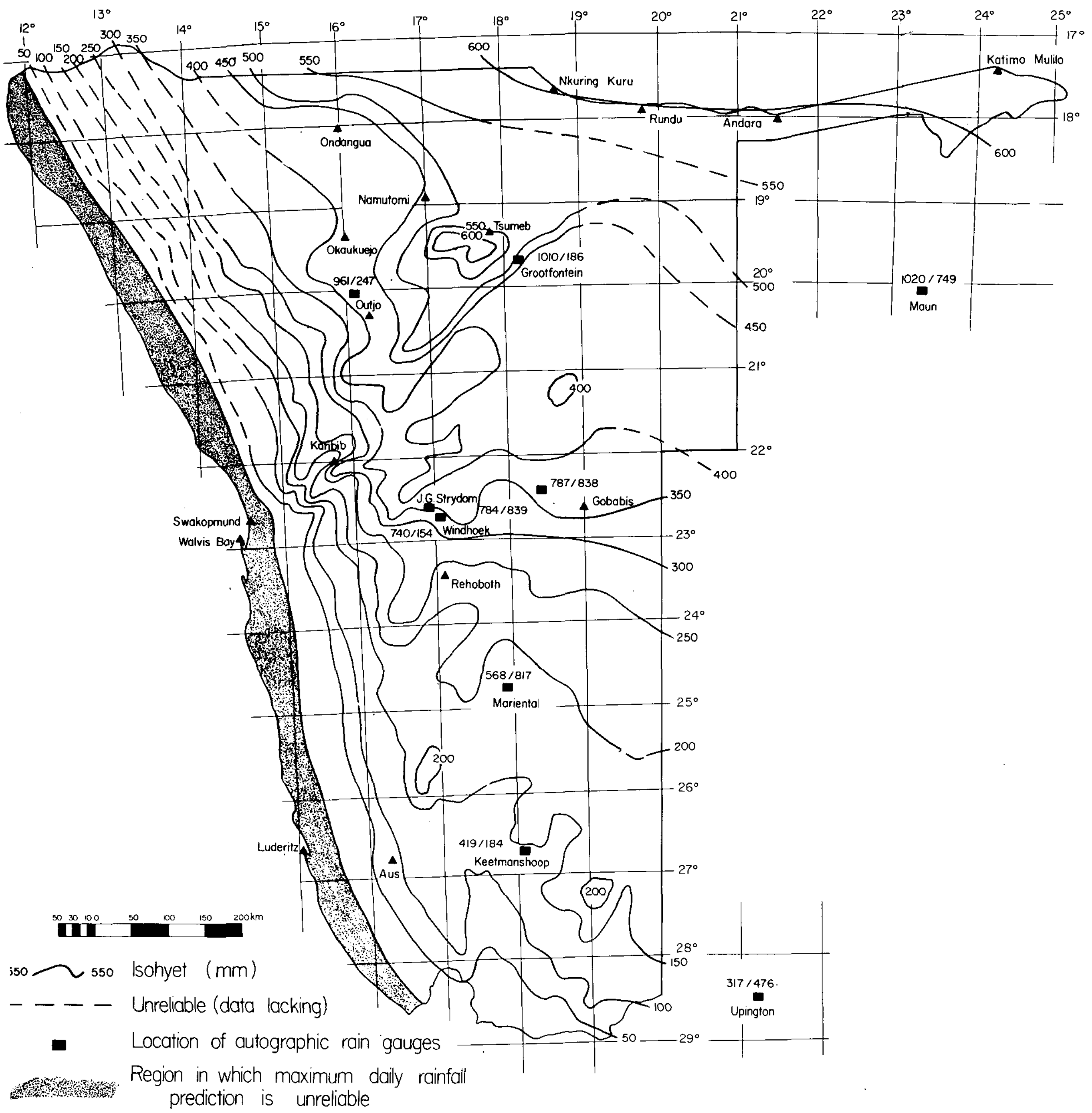

Figure 2
SWA/Namibia: Isohyetal map

With the exception of the coastal strip, once the MAP of a place is known (from Fig. 2) the one-day extreme rainfall of given probability or return period can be calculated from equations 1, 2, 3 and 4.

Extrapolation to short durations

Nine autographic recording rainfall stations located as shown on Fig. 1 provided data (Midgley and Pitman, 1978) from which relationships between MDR and maximum rainfalls of duration shorter than one day could be established. The ratio, R, of the D-hour value of maximum rainfall for a given return period to the one-day value (MDR) of the same return period can be approximated by an equation of the following type (Institute of Hydrology, 1975):

$$R = D \cdot R_o/(1 + BD)^n \qquad (5)$$

in which R_o, B and n are constants.

Analysis of the maxima for 0,25; 0,5 and 1 h durations indicated that the ratio R was not dependent on return period. Furthermore, no obvious regional differences were apparent in the data. It was therefore possible to derive for the constants in equation 5 unique values which would be applicable anywhere in SWA-Namibia. With the constants evaluated for SWA-Namibia the equation reads:

$$R = D \cdot 2{,}09/(1 + 2{,}88D)^{0{,}92} \qquad (6)$$

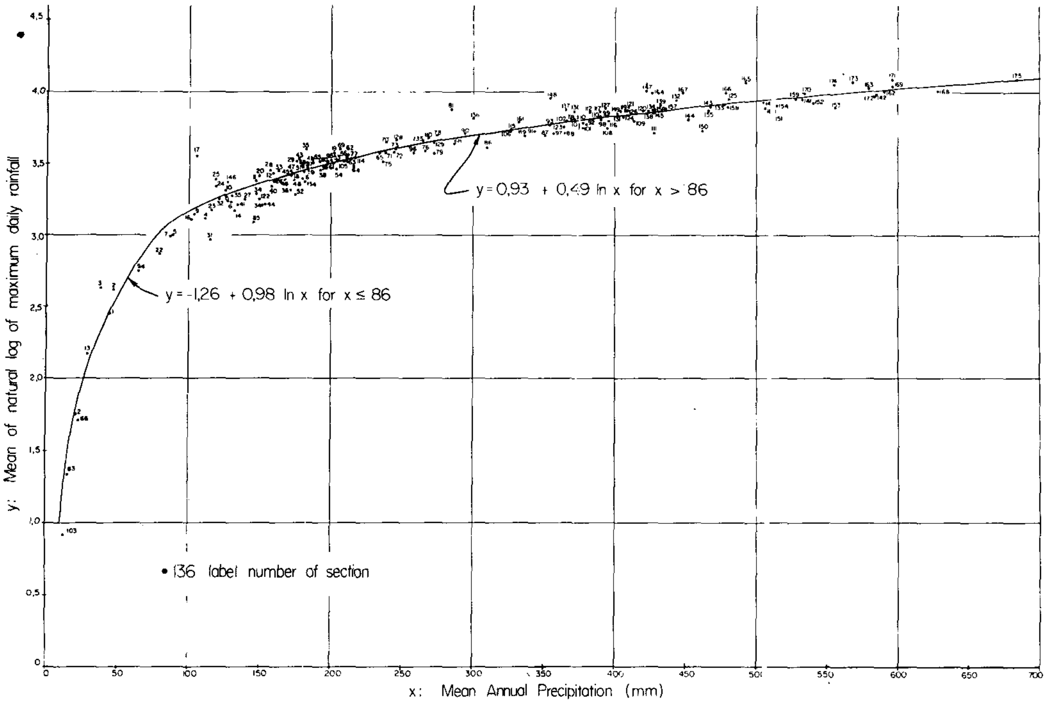

Figure 3
Relationship between MAP and mean of natural log of maximum daily rainfall for sections

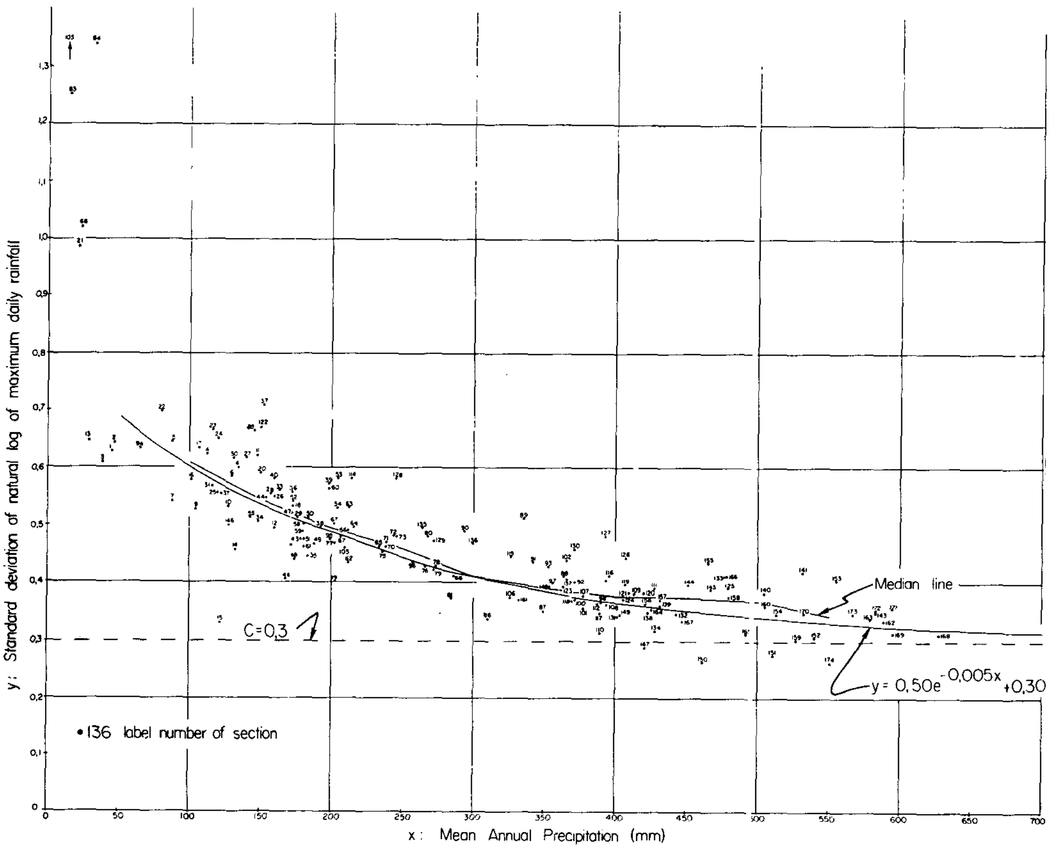

Figure 4
Relationship between MAP and standard deviation of natural log of maximum daily rainfall for sections

TABLE 2
RELATIONSHIP BETWEEN DURATION AND RATIO OF THE D-HOUR TO THE ONE-DAY MAXIMA OF LIKE RECURRENCE INTERVAL

D(h)	R
0,1	0,17
0,25	0,32
0,5	0,46
1	0,60
2	0,72
4	0,82
8	0,90
12	0,94
18	0,98
one day	1,00

Table 2 lists values of the ratio R for discrete durations D (hours).

Construction of the co-axial diagram

To facilitate solution of the equations 1, 2, 3 and 4, which can be used to compute the MDR of given return period at a place of known MAP, a co-axial diagram, Fig. 5, has been prepared.

At 15 stations (comprising only 2,6% of all stations) MDR's higher than the computed 100-year value were recorded and these have been plotted in the first quadrant of the co-axial diagram to provide some guidance to the designer as to the position of the probable maximum envelope in SWA-Namibia.

The radial lines in the second quadrant of Fig. 5 are based on the ratios in Table 2.

When the MDR's 2-, 5-, 10-, 20-, 50-, and 100-year return period for stations with records longer than 50 years were plotted in the third quadrant of the co-axial diagram against corresponding values derived from the model, it was found that, except for coastal strip, there was reasonable agreement; only three stations had values exceeding the model values by more than 30%.

In the fourth quadrant there is a small scale reproduction of Fig. 2: the mean annual isohyetal map.

The coastal strip

Mean annual precipitation along the west coast is everywhere well below 50 mm and is so highly erratic that it was not possible to derive a relationship between MAP and frequency distribution of MDR.

Although values derived for some short-record stations along the coast were found to agree well with those given by the model, the long-record stations yielded values very much higher than would be given by the coaxial diagram.

This lack of agreement among stations in the coastal strip could be ascribed to sampling error; with rainfall so erratic it would probably require records several hundred years in length to establish with any degree of confidence both MAP's and frequency distributions of MDR.

Typical stations along the coast can experience MDRs of *several times* higher than MAP (e.g. Stn. 735/11 — max. MDR = 49 mm, MAP = 15 mm); in such cases the concept of a mean annual precipitation is perhaps meaningless.

Comparison with South African design data

In order to compare the results of this study with those obtained by spatial extrapolation of South African data, Table 3 has been compiled. The figures clearly illustrate that use of design graphs based largely on South African data (autographic records only) can lead to inaccurate estimates for SWA-Namibia. Except for very few cases, the extreme rainfalls based on South African data tend to underestimate quite significantly. This is especially so for the combination of MAP less than 400 mm and return period greater than 10 years. On the other hand, the agreement is very close for locations with MAP of around 600 mm.

The discrepancy between results for the arid areas is no doubt due to the adoption in this study of a standard deviation of ln MDR, which increases with decreasing MAP (see Fig. 4), whereas in the previous study a blanket value of standard deviation was adopted. It would appear, therefore, that a similar analysis of MDR in the arid areas of South Africa might lead to improved estimates of extreme point rainfalls for such areas.

TABLE 3
COMPARISON BETWEEN RESULTS OBTAINED FROM THIS STUDY AND THOSE DERIVED BY EXTRAPOLATION OF SOUTH AFRICAN DATA

MAP (mm)	Duration (h)	Study*	Point rainfalls (mm) for return periods (years) 2	5	10	20	50	100
50	1	A	5	12	18	27	46	68
		B	9	13	16	19	26	33
	24	A	9	19	29	45	77	114
		B	16	22	28	35	45	56
100	1	A	12	22	31	43	65	87
		B	11	16	19	24	31	40
	24	A	21	36	52	72	109	145
		B	20	28	35	43	56	69
200	1	A	18	29	40	52	73	94
		B	15	20	25	31	39	51
	24	A	31	48	66	86	121	156
		B	25	35	44	54	70	86
400	1	A	26	36	46	56	77	96
		B	20	29	36	45	57	73
	24	A	44	60	76	94	128	160
		B	37	50	63	78	101	125
600	1	A	32	43	52	64	82	99
		B	27	38	48	60	75	98
	24	A	54	72	87	106	136	165
		B	50	67	85	105	136	167

*A: as obtained from Fig. 5 (this paper)
B: as obtained from Fig. 4 (HRU Report no. 2/78)

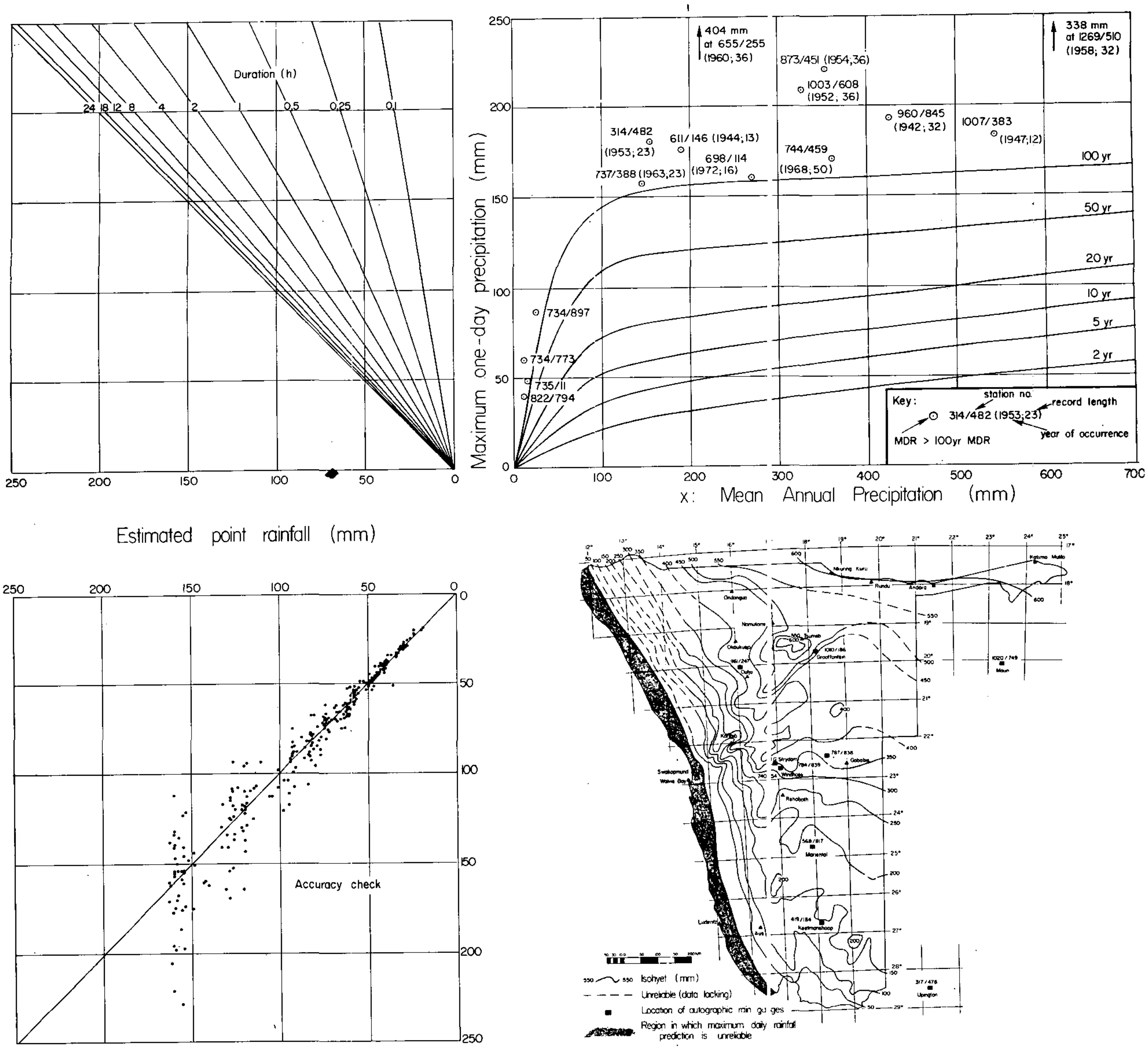

Figure 5
Depth-Duration-Frequency diagram for point rainfall

Conclusion

From an analysis of available daily rainfall data an isohyetal map of SWA-Namibia has been compiled (Fig. 2) as well as a co-axial diagram (Fig. 5) from which, given the mean annual precipitation at a location, it is possible to estimate, for given recurrence interval, the maximum likely one-day precipitation. The co-axial diagram was extended to cover storm durations down to one-tenth of an hour by making use of relationships between short-duration and one-day rainfalls as determined from data at nine autographic gauges in the study area.

Comparison with a recent study of South African data revealed that extreme rainfalls in SWA-Namibia — and possibly all arid areas in Southern Africa — could be much higher than indicated by earlier work.

References

GUMBEL, E.J. (1958) *Statistics of Extremes,* Columbia University Press, New York.

INSTITUTE OF HYDROLOGY (1975) *Flood studies Report:* Vol II Meteorological Studies. Natural Environment Research Council, London.

MIDGLEY, D.C. & PITMAN, W.V. (1978) A depth-duration-frequency diagram for point rainfall in Southern Africa *Hydrological Research Unit Report* 2/78, University of the Witwatersrand Johannesburg.

WIEDERHOLD, J.F.A. (1969) Design storm determination in South Africa *Hydrological Research Unit Report* 1/69, University of the Witwatersrand, Johannesburg.