Runs Characteristics of Streamflows

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Abstract

The theory of runs has been used for the analysis of hydrological data with regard to the wet and drought indicators, which are appropriately represented by the runs characteristics, viz. the wet and drought periods, the positive and negative run-sums. In this study, the effect of skewness and dependence of annual flows on these characteristics is first investigated, and a model for monthly sequence generation is proposed. Application to actual data indicates that the proposed model can reproduce the runs characteristics of historical monthly flows.

Introduction

The theory of runs has proven to be very useful in the analysis of wet and dry characteristics of hydrological data since Yevjevich (1967) attempted to use it in providing an objective approach to drought investigations. It can also be used for other purposes as pointed out by Salazar and Jevjevich (1975), and Phien (1982). Because of its usefulness, many investigations have been carried out in order to find out the runs characteristics for diverse processes. However, the results obtained so far are still very limited. As a continuing effort, the present study aims to (a) investigate the effect of skewness and dependence of stationary processes on the runs characteristics, and (b) propose a model for monthly streamflow generation which incorporates the preservation of the runs characteristics of historical data.

Definition of Runs Characteristics

Let $\{X_k, k=1, \ldots, n\}$ denote a sequence of random variables and x_o a given value. Let Y_k be defined as

$$Y_k = X_k - x_o \tag{1}$$

then Y_k is called a *surplus* if $Y_k > 0$ (i.e. $X_k > x_o$) and a *deficit* if $Y_k \leq 0$ (i.e. $X_k \leq x_o$). A *run* made up of m successive surpluses is called a *positive run of length m,* and a run of n successive deficits is called a *negative run of length n* (Fig. 1). For hydrological sequences, a positive run-length is referred to as a *wet period*, denoted by N_w and a negative run-length a dry or *drought period*, denoted by N_d. Corresponding to a wet period, there is a positive run-sum S which is the cumulative excess of $Y_k > 0$ for that duration (m):

$$S = \sum_{k=1}^{m} Y_k = \sum_{k=1}^{m} (X_k - x_o) \tag{2}$$

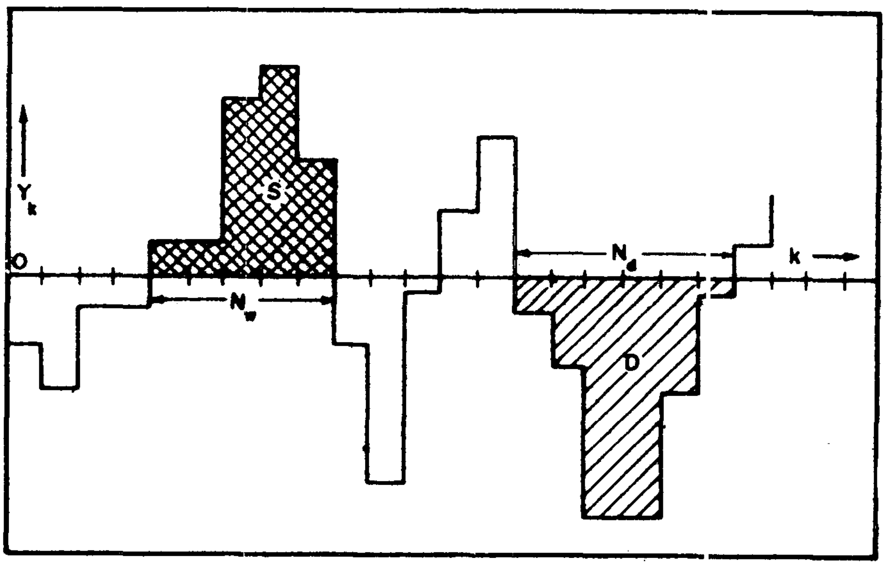

Figure 1
Definition sketch of runs characteristics

Likewise, corresponding to a drought period of length n, there is a negative run-sum D, which is the cumulative deficit of $Y_k \leq 0$ for that duration:

$$D = \sum_{k=1}^{n} Y_k \tag{3}$$

The positive run-sum S and the negative run-sum D represent the severity for wet and drought events, respectively (Dracup *et al.*, 1980). The stochastic nature of hydrological events induces that of the variables N_w, N_d, S and D. Thus it is appropriate to treat them in the framework of stochastic processes.

In this study, both annual and monthly streamflow sequences are considered. For annual streamflows, $\{X_k\}$ may be treated as a stationary process, for which, the *truncation level* x_o is set equal to a constant. For monthly streamflows, periodicities exist, and it is convenient to express X_k as follows:

$$X_k = Q_{i,j}, \; k = 12(i-1) + j \tag{4}$$

where j = 1, ..., 12 and i = 1, ..., L (the length of record in years, for example). The truncation level should vary with j, and it is convenient to define

$$x_o(j) = \alpha \bar{Q}_j \tag{5}$$

where α is a positive constant, and $\bar{Q}_j$ is the mean flow in month j, i.e.

$$\bar{Q}_j = \frac{1}{L} \sum_{i=1}^{L} Q_{i,j} \tag{6}$$

The sketch in Fig. 1 remains applicable for

$$Y_k = X_k - x_o(j) = Q_{i,j} - \alpha \bar{Q}_j \tag{7}$$

With a wide variety of applications of the theory of runs given in the aforementioned references, the statistical properties of the runs characteristics N_w, N_d, S and D are of important interest. In the following the annual sequence is first considered where the effect of skewness and dependence of X_k on the runs characteristics is investigated; and then the monthly sequence is treated in the context of streamflow generation schemes.

Annual Streamflow Sequences

Annual streamflows may be independent or dependent. For independent flows, analytical results have been made available quite extensively (Phien, 1981), but for dependent sequences, very few results have been obtained.

Independent Sequences

General Considerations

Independent annual (rainfall or runoff) sequences are commonly found to be fitted by the normal, lognormal or gamma distributions (Markovic, 1965). For independent sequences, the wet and dry periods can be obtained using the result of Feller (1957) as follows:

$$P(N_w = m) = qp^{m-1}, \; m = 1, 2, \ldots \tag{8}$$

$$P(N_d = n) = pq^{n-1}, \; n = 1, 2, \ldots \tag{9}$$

where

$$p = \int_{x_o}^{\infty} f(x)\,dx \text{ and } q = 1 - p = \int_{-\infty}^{x_o} f(x)\,dx \tag{10}$$

f(x) being the probability density function of the X_k (treated as identically and independently distributed random variables). Let denote the random variable representing all the X_k by X;

$$X_+ = \begin{cases} X \text{ if } X > x_o \\ 0, \text{ otherwise} \end{cases} \tag{11}$$

and

$$Y_+ = X_+ - X_o \tag{12}$$

then the expected value of the positive run-sum S can be obtained as:

$$E(S) = [E(X)_+) - x_o]/q = E(Y_+)/q \tag{13}$$

Explicit formulas for the first four moments of X_+ for the normal, lognormal and gamma distributions have been given by Phien (1982). Some of them will be used later when the effect of skewness is considered.
Similarly, define

$$X_- = \begin{cases} 0 \text{ if } X > x_o \\ X \text{ if } X \leq x_o \end{cases} \tag{14}$$

and

$$Y_- = X_- - x_o \tag{15}$$

then the expected value of the negative run-sum can be shown to be:

$$E(D) = [E(X_-) - x_o]/p = E(Y_-)/p \tag{16}$$

Important Remarks

When the truncation level x_o is the *median* of X, then p = q = ½ and eqs. 8 and 9 reduce to:

$$p(N_w = m) = (½)^m = P(N_d = m)$$

Thus the wet and drought periods have the same distribution.

When the truncation level x_o is equal to the *mean* of X, it will be shown that the expected values of S and D are of the same magnitude, regardless of the distribution of X.

It follows from eq. 13 that

$$E(S) = (pq)^{-1} \int_{x_o}^{\infty} (x - x_o)\, f(x)\,dx \tag{17}$$

By relating the probability density function of X_- to X in the same way as that of X_+ to X (Phien, 1982) one obtains:

$$E(D) = (pq)^{-1} \int_{-\infty}^{x_o} (x - x_o)\, f(x)\,dx \tag{18}$$

Thus

$$E(S) + E(D) = (pq)^{-1} \int_{-\infty}^{+\infty} (x-x_o) f(x)\, dx \tag{19}$$

Now if $x_o = E(x)$, the integral in eq. 19 vanishes, and consequently:

$$E(S) = -E(D) \tag{20}$$

This result would have been thought to be valid only for symmetrical distributions such as the normal distribution. But from the derivation, no assumption is made on f(x); in other words, eq. 20 holds true for any distribution.

Effect of Skewness

Foir the normal distribution,

$$f(x) = [\sigma(2\pi)^{1/2}]^{-1}\exp[-1/2\{(x-\mu)/\sigma\}^2], \ -\infty < x < \infty$$

where μ and σ are the mean and standard deviation, respectively. In this case, the expected value of S can be obtained using the result provided by Phien (1982) as:

$$E(S) = (\mu-x_o)/q + \sigma\exp(-t_o^2/2)/[pq(2\pi)^{1/2}] \tag{21}$$

where

$$t_o = (x_o-\mu)/\sigma \tag{22}$$

Similarly,

$$E(D) = (\mu-x_o)/p - \sigma\exp(-t_o^2/2)/[pq(2\pi)^{1/2}] \tag{23}$$

Note that in this case,

$$1-p = q = (2\pi)^{-1} \int_{-\infty}^{t_o} \exp(-t^2/2)dt = \Phi(t_o) \tag{24}$$

where Φ is the distribution function of standard normal variables. This function has been tabulated and can be approximately computed using the formulas given by Abramowitz and Stegun (1964).

To see the effect of the skewness on the expected values of S and D, the gamma distribution is employed, because of the availability of models for dependent gamma variables, as will be seen later.

In this case,

$$f(x) = [b^a\Gamma(a)]^{-1} x^{a-1} \exp(-x/b)\ , \ x > 0$$

where a and b are the shape and scale parameters, respectively and Γ is the gamma function defined by

$$\Gamma(a) = \int_0^{\infty} x^{a-1} e^{-x} dx$$

The expected values of S and D can be obtained as:

$$E(S) = (ab-x_o)/q + x_o^a \exp(-x_o/b)/[pq\, b^{a-1}\Gamma(a)] \tag{25}$$

and

$$E(D) = (ab-x_o)/p - x_o^a \exp(-x_o/b)/[pq\, b^{a-1}\Gamma(a)] \tag{26}$$

The computation of the expected values of S and D involves that of the incomplete gamma function, because

$$q = 1-p = [b^a\Gamma(a)]^{-1} \int_0^{x_o} x^{a-1} a^{-x-b} dx$$

$$= \frac{1}{\Gamma(a)} \int_0^{y_o} y^{a-1} e^{-y} dy \tag{27}$$

where $y_o = x_o/b$. Although the value of the integral in eq. 30 can be obtained from existing tables (Pearson, 1965), it is more convenient to compute it by using the recently developed algorithms of Gautschi (1979) and Lau (1980).

Having obtained the expressions for computing the expected values of the positive and negative run-sums, one can readily investigate the effect of the skewness on these expected values. In Table I, both the normal and gamma distributions have a constant mean equal to 5,0 m³/s, and varying standard deviation which is represented by the values of σ for the normal distribution. The parameters and skewness coefficient of the gamma distribution are computed using the following equation:

$$a = \mu^2/\sigma^2\ ,\ b = \sigma^2/\mu\ ,\ Cs = 2a^{-1/2} \tag{28}$$

TABLE 1
PARAMETERS AND SKEWNESS COEFFICIENT OF THE NORMAL AND GAMMA DISTRIBUTIONS EMPLOYED

Normal*	μ	5,00	5,00	5,00	5,00	5,00
	σ	2,10	2,20	2,30	2,40	2,50
	a	5,669	5,165	4,726	4,340	4,0
Gamma	b	0,882	0,968	1,058	1,152	1,250
	Cs	0,840	0,880	0,920	0,960	1,0

*Skewness = 0

Their values are also collected in Table 1. It is seen that with a given mean, the skewness coefficient of this distribution increases as the standard deviation increases. This means that when the fluctuation of observed data, for example, is high, their distribution becomes more skewed to the left.

The expected values of S and D are computed at various values of the truncation level x_o, expressed as

$$x_o = \alpha\mu \tag{29}$$

where α is a constant, varying from 0,8 to 1,2 in the present work. Results of this computation are collected in Table 2. Inspection of these tabulated values reveals the following:

- As α (hence x_o) increases, the expected value of S decreases and that of D increases. This is obvious from the definition of S and D.
- When α = 0,8 – 1,2, the expected values of S and D increase with the standard deviation for the normal distribution, or with the skewness coefficient for the gamma distribution.
- For $\alpha \le 1$, with the *same mean and standard deviation*, the expected values of S and D for the normal distribution (Cs = 0) are greater than those for the gamma distribution (Cs > 0). The reverse is true for $\alpha \ge 1$.

TABLE 2
EXPECTED VALUES OF POSITIVE AND NEGATIVE RUN-SUMS

α	0,8		0,9		1,0	1,1		1,2	
	(I)	(II)	(I)	(II)	(I) & (II)	(I)	(II)	(I)	(II)
	(1)6,61	1,44	4,61	2,54	3,35	2,54	4,61	1,99	6,61
	(2)6,69	2,13	4,76	2,69	3,51	2,69	4,76	2,13	6,69
Normal	(3)6,78	2,27	4,90	2,84	3,67	2,84	4,90	2,27	6,78
	(4)6,87	2,41	5,05	2,99	3,83	2,99	3,05	2,41	6,87
	(5)6,98	2,55	5,20	3,15	3,99	3,15	5,20	2,55	6,98
	(1)5,97	1,61	4,32	2,30	3,34	2,73	4,91	2,33	7,30
	(2)6,02	1,72	4,45	2,44	3,50	2,89	5,07	2,48	7,41
Gamma	(3)6,08	1,83	4,59	2,58	3,66	3,06	5,23	2,64	7,53
	(4)6,16	1,94	4,72	2,72	3,82	3,22	5,39	2,80	7,65
	(5)6,23	2,06	4,86	2,86	3,98	3,38	5,56	2,97	7,78

Notes *(I) and (II): Values of E(S) and E(-D), respectively.*
(1) – (5): for σ = 2,10; 2,20; 2,30; 2,40 and 2,50, respectively.

Dependent Sequences

General Considerations

Analytical results are available only for the first order Markov model of normal variables. Let r denote the conditional probability:

$$r = P(X_k > x_o \mid X_{k-1} > x_o) \quad (30)$$

then the distribution of the wet period was given by Sen (1976) as:

$$P(N_w = m) = (1\text{-}r)\, r^{m-1}, \; m = 1, 2, \ldots \quad (31)$$

Although Sen (1976) obtained this result for normal variables, it can be shown easily by following the same reasoning as that used by Sen, that eq. 31 remains valid for any (first-order) Markov process. Moreover, the same procedure can also be used to show that the distribution of the drought period for any Markov process is given by

$$P(N_d = n) = qs^{n-1}, \; n = 1, 2, \ldots \quad (32)$$

where

$$s = P(X_k \leq x_o \mid X_{k-1} \leq x_o) \quad (33)$$

It follows from eqs. 31 and 32 that

$$E(N_w) = 1/(1\text{-}r) \quad (34)$$

and

$$E(N_d) = 1/(1\text{-}s) \quad (35)$$

The reason for referring to normal variables may be due to the popularity of models for them. Now several models for non-normal variables have been proposed (see Phatarfod, 1976; Gaver and Lewis, 1980; Lawrance and Lewis, 1980), and thus the validity of eqs. 31, 32, 34 and 35 should be noted.

Remarks

For Markov processes, the expected values of the wet and drought periods are equal when the truncation level is equal to the *median* of the X_k treated as identically distributed variables. This is obvious because it is always true that

$$s = 1\text{-}p(1\text{-}r)/q \quad (36)$$

When $p = q$, eq. 36 reduces to $s = r$, and in view of eqs. 34 and 35, one obtains

$$E(N_w) = E(N_d)$$

As for the case of independent sequences, the expected values of S and D have the same magnitude if x_o is equal to the *mean* of the X_k. This result may be reached as follows. From the general formula given by Sen (1977):

$$E(S) = E(Y_+)E(N_w)$$

and from eqs. 12 and 34: one obtains

$$E(S) = [E(X_+)\text{-}x_o]\,(1\text{-}r) \quad (37)$$

Similarly, the following expression can also be reached:

$$E(D) = [E(X_{-2})\text{-}x_o]/(1\text{-}s) \quad (38)$$

The dependence structure of X_k has no effect on the expected value of X_k, and hence of X_+ so that one can still write:

$$E(X_+)\text{-}x_o = (1/p) \int_{x_o}^{\infty} (x\text{-}x_o).f(x)dx$$

Likewise,

$$E(X_-)-x_o = (1/q) \int_{-\infty}^{x_o} (x-x_o)\, f(x)dx$$

Thus:

$$E(S) + E(D) = \frac{1}{p(1-r)} \int_{x_o}^{\infty} (x-x_o)f(x)dx + \frac{1}{q(1-s)} \int_{-\infty}^{x_o} (x-x_o)f(x)dx$$

In view of eq. 36, this can be rewritten as:

$$E(S) + E(D) = \frac{1}{p(1-r)} \int_{-\infty}^{\infty} (x-x_o)f(x)dx \qquad (39)$$

Now, if $x_o = \mu$ the integral in eq. 39 vanishes, giving:

$$E(S) = -E(D) \qquad (40)$$

Effect of Dependence

The effect of the dependence structure of the $\{X_k\}$ on the runs characteristics has been considered by Sen (1976, 1977) for Markov normal sequences. In this study, that effect is investigated for non-normal processes. Although several models have been proposed for dependent gamma sequences as summarized by Phatarfod (1976), their practical use remains to be limited chiefly because of the difficulty involved in their generation scheme. More recently, Gaver and Lewis (1980) have devised a simple way for generating Markov sequences from the exponential distribution (treated herein as a special case of the gamma distribution). It is as follows:

For $O \leq \varrho < 1$, the Markov sequence of X_k is formed by taking:

$$X_k = \varrho X_{k-1} + \begin{cases} O \text{ wp } \varrho \\ E_k \text{ wp } 1-\varrho \end{cases} \qquad (41)$$

where the E_k, $k = 1, 2, \ldots$ are independent exponential variables with parameter $\lambda > 0$, and wp stands for "with probability". The model of eq. 41 can be suitably referred to as the first order exponential autoregressive model denoted by EAR(1), because the serial correlation of lag h satisfies the equation:

$$\varrho_h = \varrho^h \qquad h = 0, 1, 2, \ldots$$

It can be verified that X_k has an exponential distribution with parameter λ, i.e.:

$$f(x) = \frac{1}{\lambda} e^{-x/\lambda}, \; x > 0$$

This distribution is just a special case of the gamma distribution where $a = 1$ and $b = \lambda$. In this case, one can easily show that:

$$E(X_+) = \lambda + x_o \exp(-x_o/\lambda)/p$$

But since

$$p = \int_{x_o}^{\infty} f(x)dx = \exp(-x_o/\lambda),$$

one obtains:

$$E(X_+) = \lambda + x_o$$

Thus

$$E(S) = \lambda/(1-r) \qquad (42)$$

Similarly, the expected value of X_- is

$$E(X_-) = \lambda - x_o(p/q)$$

and

$$E(D) = \frac{\lambda - x_o \; x_o(p/q)}{1-s} = (\lambda - x_o/q)/(1-s)$$

which can be rewritten as:

$$E(D) = \frac{q\lambda - x_o}{p(1-r)} \qquad (43)$$

For example, if x_o is equal to the mean of the X_k, i.e. $x_o = \lambda$, then

$$E(S) + E(D) = \frac{\lambda}{1-r} + \frac{q\lambda - \lambda}{p(1-r)} = 0$$

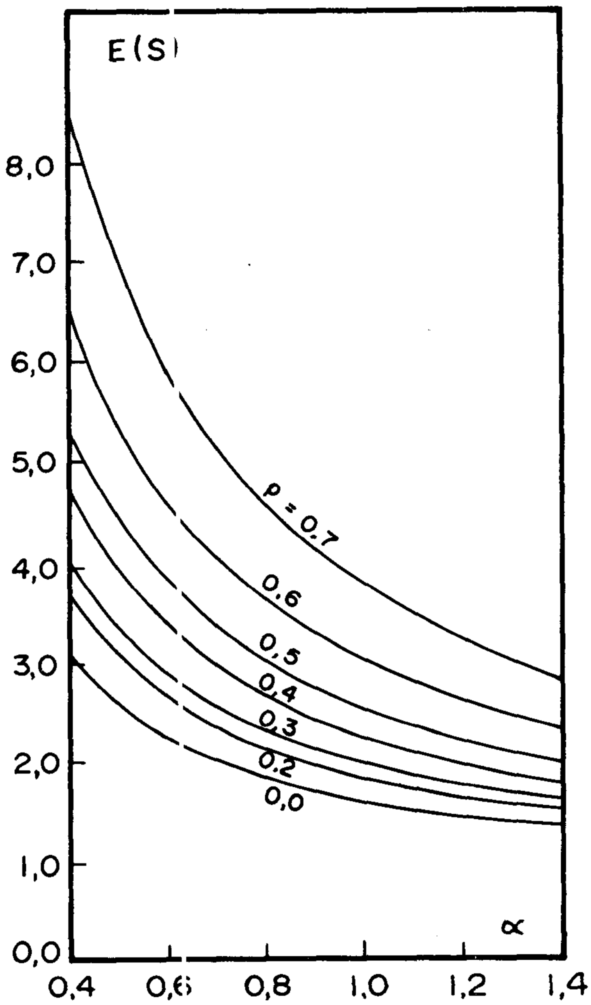

Figure 2
Expected value of positive run-sum of dependent exponential variables

which is been proven to hold true for any sequence in eq. 40.

The expressions for the expected values of the runs characteristics are quite simple, yet their computation is not simple because of the involvement of r and s.

In order to see the effect of the dependence structure of flows on the runs characteristics, a Monte Carlo simulation has been carried out instead of an analytical derivation. The results for the expected value of the positive run sum are shown in Fig. 2 for the case $\lambda = 1$. It is seen that E(S) is an increasing function of ϱ, and a decreasing function of α (or x_o). Thus the dependence of annual flows increases the volume of cumulative surplus water.

Monthly Streamflow Generation

In this part of the study, a model is proposed for the generation of monthly streamflows incorporating the preservation of the historical runs characteristics. For a general consideration of monthly streamflow models see Phien and Ruksasilp (1981).

First of all, the Method of Fragments (Svanidze, 1969, 1980) has been found to be capable of reproducing the mean, standard deviation, skewness coefficient, and correlation of monthly streamflow sequences. The definition of *segments* in this method indicates that the whole structure of monthly historical data is not interfered, hence its capability of preservation of the historical record in terms of whatever the statistics employed is expected. However, the Method of Fragments requires the use of a *suitable model* for producing annual flows which are to be disaggregated into monthly values by selecting the appropriate segment (Srikanthan and McMahon, 1980). To generate annual flows, the first-order autoregressive AR(1) model is extensively used. The flow in year h + 1 is expressed as:

$$q_{h+1} - \mu = \varrho(q_h - \mu) + \sigma(1-\varrho^2)^{1/2}\, t_{h+1} \qquad (44)$$

where μ, σ are respectively the mean and standard deviation of annual flows, ϱ is the correlation of annual flows, and t_{h+1} is a random variable of zero mean, unit variance, and skewness coefficient γ_t given by

$$\gamma_t = \gamma(1-\varrho^3)(1-\varrho^2)^{-3/2} \qquad (45)$$

γ being the skewness coefficient of annual flows.

However the annual values generated by the AR(1) model often underestimate the longest drought of historical data, as pointed out by Mandelbrot and Wallis (1968). To overcome this shortcoming, an additional correlation is built when needed by using the scheme of Jackson (1975), where annual flows are classified into those in wet and drought periods. For a selected truncation level, the following transition matrix is obtained:

$$P = \begin{bmatrix} 1-c & c \\ d & 1-d \end{bmatrix}$$

where

c = P (flow in wet state | flow in drought state), and
d = P (flow in drought state | flow in wet state).
This mechanism introduces a correlation coefficient ϱ_o given by Jackson (1975) as

$$\varrho_o = Acd\,(1-c-d)(\mu_1-\mu_2)^2/(c+d) \qquad (46)$$

where μ_1 and σ_1 (or μ_2 and σ_2) are respectively the mean and standard deviation of flows in drought (or wet) state, and

$$A = [d\sigma_1^2 + d(\mu-\mu_1)^2 + c\sigma_2^2 + c(\mu-\mu_2)^2]^{-1} \qquad (47)$$

where μ is the overall mean of annual flows, i.e.

$$\mu = (d\mu_1 + c\mu_2)/(c+d) \qquad (48)$$

Assume that the flow q_h in the hth year is of type i (i = 1 for flows in drought state, i = 2 for flows in wet state), and that the state transition mechanism determines that the next flow q_{h+1} is of type j (j = 1, 2), then one computes q_{h+1} by the equation

$$q_{h+1} = \mu_j + \sigma_j\, \varrho(q_h-\mu_i)/\sigma_i + \sigma_j(1-\varrho^2)^{1/2}\, t_{t+1} \qquad (49)$$

With this mechanism, the correlation coefficient between successive flows becomes (Jackson, 1975);

$$\varrho' = \varrho_o + \varrho A[(1-c)d\sigma_1^2 + 2cd\sigma_1\sigma_2 + (1-d)c\sigma_2^2] \qquad (50)$$

The random component t_{h+1} in eq. 49 was assumed to be a standard normal variable by Jackson. However, to make the procedure more general, it can have a skewness coefficient γ_t different from zero. It is then very easy to show that

$$\gamma_t = (\gamma_j - \gamma_i\varrho^3)(1-\varrho^2)^{-3/2} \qquad (51)$$

when q_h is in state i and q_{h+1} is in state j (i,j = 1,2).

Instead of using Jackson's procedure for classification of flows into the drought and wet states, in this study, the truncation level equal to the mean μ was employed. The procedure for generating annual flows is then summarized as follows:

- Estimate c and d by the maximum likelihood method, using the equations:

  $$\hat{c} = f_{12}/(f_{11}+f_{12})\ ,\ \hat{d} = f_{21}/(f_{21}+f_{22})$$

  where f_{ij} is the number of transitions from state i to state j in the historical data (i, j = 1, 2).

- Compute ϱ' according to eq. 50. If $\varrho' \leq \varrho$, annual values are generated according to the AR(1) model in eq. 44. If $\varrho' > \varrho$, they are generated according to eq. 49.

- Annual values are then disaggregated into monthly values using the corresponding segment, according to the suggestion of Srikanthan and McMahon (1980). When a negative flow occurs in the generated annual sequence, it is neglected.

To evaluate the reproduction of the runs characteristics of historical monthly streamflows by the proposed model, several sets of actual data were employed. They are shown in Table 3. Even though they were run at different truncation levels expressed by α in eq. 5, only the results for $\alpha = 1,0$ are collected in Tables 4 and 5 (to save space) for the means and standard deviations of the runs characteristics, respectively. These simulated results are obtained as the average values of the respective statistics from 50 generated sequences each having the same length as the historical data. Inspection of Tables 4 and 5 leads to the following observations:

- The wet and drought periods, on average, last for less than 6 months in all cases. Their standard deviations (Table 5) are larger than or nearly equal to their means (Table 4), indicating that these periods fluctuate very much.

- Both the means and standard deviations of the wet and drought periods are reproduced very satisfactorily by the proposed method. The historical and generated values are almost the same in all cases.

- Both the mean and standard deviation of the positive run sum are also reproduced. The differences between historical and generated values are negligible.

- Although the historical and generated values are not very close for the negative run sum (mean and standard deviation), the reproduction of this characteristic may still be considered acceptable, because the relative error is less than 10% for the mean in *all cases*, and for the standard deviation in *most cases*.

TABLE 3
LIST OF STATIONS WHERE DATA WERE EMPLOYED

Station	River	Location	Length and period of record
S1	Mekong	Vientiane, Laos	59 (1914 – 1972)
S2	Mekong	Pamong Dam, Laos	62 (1913 – 1974)
S3	Mekong	Stung Treng, Cambodia	35 (1934 – 1968)
S4	Mekong	Mukdahan, Thailand	44 (1925 – 1968)
S5	Ping	Nawarat Bridge, Thailand	54 (1921 – 1974)
S6	Loire	Montejean, France	102 (1863 – 1964)
S7	Penobscot	West Enfield, U.S.A.	62 (1962 – 1964)
S8	Colorado	Lees Ferry, U.S.A.	54 (1911 – 1964)

TABLE 4
MEANS OF RUNS CHARACTERISTICS

Station	Wet Period (months)		Drought Period (months)		Positive Run Sum (m^3month/s)		Negative Run Sum (m^3month/s)	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
S1	4,36	4,36	5,03	5,03	3 887,5	3 887,4	3 939,3	4 365,6
S2	4,20	4,17	4,82	4,82	3 995,8	3 995,7	3 947,6	3 834,8
S3	3,38	3,38	3,23	3,23	7 825,2	7 825,1	7 702,9	8 206,5
S4	4,26	4,29	4,32	4,41	5 904,8	5 956,2	5 809,5	5 512,0
S5	3,08	3,08	4,54	4,54	84,9	84,8	84,9	87,9
S6	3,07	3,07	4,68	4,68	1 439,8	1 437,2	1 439,7	1 461,1
S7	2,59	2,60	4,11	4,11	350,9	350,9	350,9	372,9
S8	3,20	3,20	3,97	3,97	637,9	637,9	645,0	652,0

Note (1) from historical data.
(2) from 50 generated sequences.

TABLE 5
STANDARD DEVIATIONS OF RUNS CHARACTERISTICS

Station	Wet Period		Drought Period		Positive Run Sum		Negative Run Sum	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
S1	5,28	5,28	5,34	5,34	6 000,4	6 000,4	6 627,3	6 691,6
S2	5,18	5,18	5,19	5,19	6 265,2	6 265,3	6 773,6	6 911,6
S3	3,51	3,47	3,30	3,30	10 510,4	10 510,4	11 597,5	9 206,5
S4	4,57	4,59	4,63	4,41	8 387,3	8 342,5	9 022,0	8 150,4
S5	3,91	3,91	5,53	5,53	159,2	158,1	129,7	145,4
S6	3,01	3,01	4,72	4,72	2 029,2	2 029,1	1 960,9	2 084,9
S7	2,49	2,49	3,85	3,85	460,1	460,2	446,8	412,4
S8	3,12	3,12	5,51	5,51	1 075,1	1 075,1	1 492,9	1 536,9

Notes (1) from historical data.
(2) from 50 generated sequences.

From the last three observations the proposed method can be said to be capable of reproducing the runs characteristics of historical monthly streamflows.

Summary and Conclusions

The present study comprises two parts. In the first part, the effect of skewness and dependence of annual flows on the runs characteristics was investigated, and a model for monthly streamflow generation was proposed in the remaining part.

It was shown analytically that the expected values of the positive and negative run sums have the same magnitude when the truncation level is equal to the mean of the stationary (annual) process involved, regardless of the underlying distribution of its elements.

Application of the proposed model to actual data indicates that it can satisfactorily reproduce the runs characteristics of historical monthly streamflows. This, together with the fact that the method of fragments – which constitutes the essential part of the proposed method – is capable of reproducing all statistics of monthly streamflows as claimed by many researchers (see Svanidze, 1980 and references therein) suggests that the proposed method should be considered as a suitable scheme for the generation of monthly streamflows.

Acknowledgements

The authors are grateful to the Division of Computer Applications, Asian Institute of Technology for its support which made the present study possible.

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