The tank model in rainfall-runoff modelling

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Abstract

Having a simple structure, the Tank Model has been widely used in modelling the rainfall-runoff processes of many watersheds. In most cases, a daily input and output basis was employed as it was intended originally. In this study, both daily and monthly inputs and outputs were used, and an attempt is made to describe the important related factors and to classify the parameters involved into two types. The parameters of the first type can be obtained by means of an automatic calibration while those of the second type are adjusted by trial and error procedure. Application of the model to two watersheds in Thailand, one small and the other relatively large, shows that the Tank Model can simulate the discharge volume satisfactorily, but is not capable of simulating daily or monthly peaks. This finding holds for both cases, namely when input and output are daily and monthly values. Moreover, the contribution of evaporation is found to be less significant in producing monthly discharges.

Introduction

The relationship between rainfall and runoff is very important. This is due mainly to the fact that rainfall data are commonly used in flood forecasting and good forecasting methods may be obtained once the appropriate relationship (with relevant values of parameters estimated) is established. Moreover, since rainfall data are normally available for a longer period than runoff, this availability can be used for filling in missing values of runoff, or in extending (most frequently backward) runoff records. For these purposes, the above relationship is also very useful.

There has been a variety of models developed for the transfer of rainfall to runoff. One of the most important achievements in the early stage was the unit hydrograph theory, developed by Sherman (1932). By means of this theory, rainfall could be converted to runoff. After this, several researchers refined Sherman's work and introduced the concept of Instantaneous Unit Hydrograph (IUH) which in effect is the result of routing an instantaneous effective rainfall of unit quantity through the assumed linear catchment system. In order to correlate physical catchment characteristics with the parameters of the IUH, Nash (1960) found that it is necessary to specify a catchment model whose IUH could be found in terms of the model parameters. The resulting model was a cascade of identical linear reservoirs. For any catchment, the first two moments of the corresponding IUH can be found directly from rainfall and runoff data, and these are used to estimate the two parameters involved, namely the number of linear reservoirs and the storage constant of each.

Many other developments followed, resulting in many models in use in various parts of the world. Among them, the Streamflow Synthesis And Reservoir Regulation, (SSARR) Model (Rockwood, 1968), the Stanford Watershed Model IV (Crawford and Linsley, 1966), and the Tank Model (Sugawara, 1961) have been widely used in Southeast Asia. In this study, the Tank Model was selected for the analysis of the rainfall-runoff processes of the two watersheds in Thailand, one representing a small catchment area and the other with a relatively large area. The reason for this selection was that the Tank Model has the simplest structure and an automatic calibration procedure is available (Sugawara, 1979).

Description of the Tank Model

Simple Tank Model

For the two watersheds considered in this study, namely those for the Huai Sato River at Ban Sato Kaeng Kung and the Ping River at Ban Tha Sala, a Simple Tank Model for humid regions may be used. In this case, the model comprises four tanks laid vertically in series (Fig. 1a). The top tank has two side outlets corresponding to the conceptual structure of the *surface discharge*, and one bottom outlet representing the *infiltration*. The second and third tanks have two outlets each, while the fourth tank has only one outlet. Water in the second tank partly moves to the stream channel through the side outlet and this corresponds to the *interflow*. The bottom outlet of the second tank provides *percolation* to the

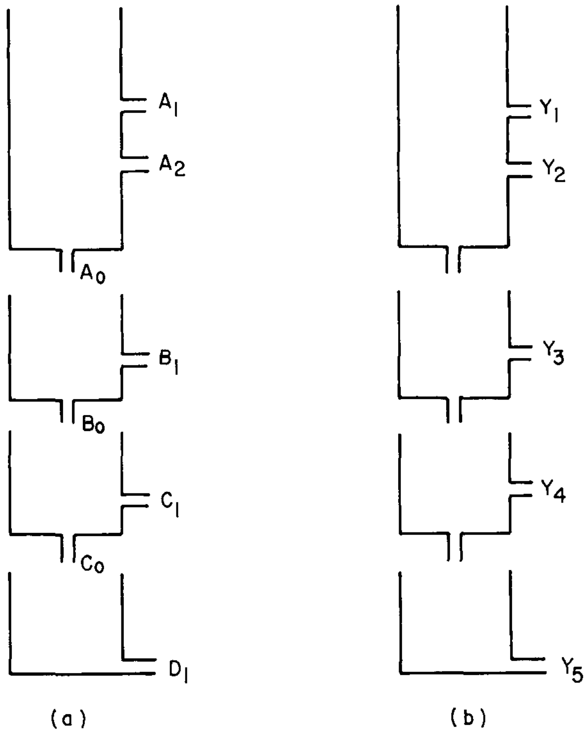


Figure 1
A Simple Tank for humid regions: (a) sketch of four tanks; (b) runoff components

third tank and the side outlets of the last two tanks provide the *base flow*.

The parameters of the Tank Model may be grouped into two types. The first type consists of the side and bottom outlet coefficients (A_0, A_1, A_2, B_0, B_1, C_0, C_1 and D_1) as shown in Fig. 1a. The parameters of the second type are the storage parameters. These are briefly described in the following and more details can be found in Sugawara *et al.* (1974, 1976).

- ***Top Tank:*** The parameters are commonly denoted by HA_1, HA_2, PS, SS, HS, XA and XS, where HA_1 (HA_2) is the storage between the lower (upper) side outlet and the maximum level of primary soil moisture; PS(SS) is the maximum storage of the primary (secondary) soil moisture; XA(XS) is the initial storage in the primary (secondary) soil moisture; and HS is the storage above the maximum storage level of the primary soil moisture.
- ***Second Tank:*** The parameters are HB and XB, where HB is the storage from the bottom to the side outlet; and XB is the initial storage above the bottom.
- ***Third Tank:*** For this tank, there are two parameters, denoted as HC and XC, where HC is the storage from the bottom to the side outlet; and XC is the initial storage above the bottom of the third tank.
- ***Fourth Tank:*** There is only one parameter, denoted as XD. This is the initial storage above the bottom.

Computation of Runoff Components

The relationship between the runoff through an outlet, denoted as Y, and the storage above the outlet, denoted as S, is in its simplest form as follows

$$Y = aS \tag{1}$$

where a is a constant. This equation is applied repeatedly to compute the amount of water flowing from one tank to another, or the runoff components shown in Fig. 1b. In the computation of runoff, the following considerations on the structure of soil moisture, the evapotranspiration and the mean areal rainfall are useful.

Structure of Soil Moisture Soil moisture is divided into two parts, primary and secondary. When primary soil moisture is filled up, the excess rain water goes gradually to the secondary soil moisture with a transfer velocity T_d (mm/day) given by:

$$T_d = \alpha_0 + \alpha_1(1 - XS/CS) \tag{2}$$

where α_0 and α_1 are two constants, and CS is the saturation capacity. If the primary soil moisture is not saturated and if there is free water in the lower tanks, water will rise by capillary action so as to fulfil the primary soil moisture with a velocity T_u (mm/day) given by

$$T_u = \beta_0 + \beta_1(1 - XP/CP) \tag{3}$$

where β_0 and β_1 are two constants; and XP and CP are the storage and saturation capacity of the primary soil moisture, respectively. In most cases, the following values may be used:

$$\alpha_0 = 0{,}5 \text{ mm/day},\ \alpha_1 = 1 \text{ mm/day},\ CS = 250 \text{ mm}$$
$$\beta_0 = \beta_1 = 3 \text{ mm/day},\ CP = 50 \text{ mm}.$$

Evapotranspiration To account for evapotranspiration, an amount, ET is substracted from the top tank according to the following rule:

$$ET = \begin{cases} 0{,}8E \text{ if } S_f \geq 0{,}8E \\ 0{,}75\,(0{,}8E - S_f) \text{ if } 0 < S_f < 0{,}8E \\ 0{,}6E \text{ if } S_f = 0 \end{cases} \tag{4}$$

where S_f denotes the free water storage (expressed in mm), and E is the class A pan evaporation.

Mean Areal Rainfall One of the problems concerning the estimation of runoff from rainfall data using a rainfall-runoff model is the mean areal rainfall which is frequently used as input into such model. Usually this is obtained as the weighted mean, computed from the equation:

$$P = \sum_{i=1}^{N} W_i P_i / \sum_{i=1}^{N} W_i \tag{5}$$

where P is the mean areal rainfall, P_i and W_i are respectively the rainfall and the corresponding weight at station i, and N is the number of rainfall stations employed.

Sugawara *et al.* (1974) indicated that both the Thiessen and Isohyetal methods are unreasonable, and suggested the use of eq. 5 with the following four weights: 1; 0,5; 0,25 and 0. At the start of the calibration of the Tank Model, all stations have the same weight equal to 1. After fairly good results are obtained for the computed runoff values, modifications of the weight will start. If rainfall data at a station are found to cause larger differences between computed and observed values of runoff, they should be less representative, i.e. should have less weight. Consequently, the weight is decreased from 1 to 0,5, or from 0,5 to 0,25 or even 0,25 to zero, where zero weight means that the rainfall station concerned is neglected.

Model Calibration

The automatic calibration provided by Sugawara (1979) was not based upon any standard optimization method. It was instead based upon a trial and error procedure which is ***carried out automatically by a computer***. The two criteria used in this calibration are the discharge volume and the shape of the hydrograph. Careful inspection reveals that it is applicable only to the parameters of the first type, i.e. the coefficients of the side and bottom outlets. For other parameters of the second type, trial and error procedure must be used. For this purpose, as well as for the provision of good initial values for the parameters of the first type, the following remark is useful: According to Sugawara (1979), the tank model consists of two types of tank (type 1 is

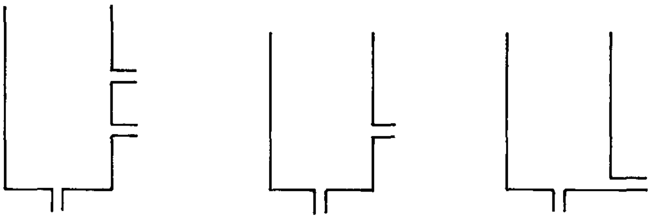

(a) (b) (c)

Figure 2
Two types of tank: (a) type 1 tank; (b) type 2 tank; (c) approximate linear tank of (a) and (b)

represented by the first tank, while type 2 is represented by the second or the third tank) as shown in Fig. 2. These may be approximated by a linear model by moving the side outlets or outlet to the bottom. This linear tank model is a first order lag system which depends on two parameters denoted as γ and δ. The ratio of runoff to rainfall is represented by $\delta/(\gamma+\delta)$ while $1/(\gamma+\delta)$ is the time constant.

- To change the shape of the hydrograph, $\gamma+\delta$ must be modified. In order to make the hydrograph steeper, $\gamma+\delta$ must be increased.
- To change the total volume of the hydrograph, the ratio $\delta/(\gamma+\delta)$ must be modified. For example, to make the total runoff volume larger without changing the shape of the hydrograph, δ must be increased while γ is decreased so that $\gamma+\delta$ is unchanged.

The adjustment of the parameters is carried out using the following guidelines:

- The parameters of the top tank are adjusted according to the shape and volume of the computed hydrograph in periods of high discharge resulting from high rainfall.
- The parameters of the second tank are adjusted by examining the hydrograph of the intermediate period that follows the peak discharge.
- The parameters of the last two tanks are adjusted by examining the part of the hydrograph which corresponds to the base flow.

Utilizing the above guidances, Sugawara (1979) developed an automatic calibration of the Tank Model to make the computed discharge volume and computed hydrograph *close* to the observed discharge volume and observed hydrograph, respectively. His scheme is in fact applicable only to the parameters of the first type, as mentioned previously. The parameters of the second type are to be adjusted by trial and error, viz. a set of values are assigned to these parameters, then the discharge volume and hydrograph are computed. If they are not *close* to the observed discharge volume and observed hydrograph, a new set of values should be tried until they become close respectively to the observed ones. It was found from studies by Sugawara (1979 – See also Loria, 1980 and Thang, 1981) that a relative error of $\leq 25\%$ would be required for the computed values of a criterion (discharge volume or hydrograph) to be *close* to its observed values.

Selected Watersheds

Although the Tank Model is intended for the modelling of rainfall-runoff processes on a daily basis (i.e. daily rainfall and daily discharge were used as basic input and output, respectively), its applicability on a monthly basis (i.e., monthly rainfall and monthly discharge were used respectively as basic input and output) was also evaluated in the present research. In its applications, data in two watersheds in Thailand were used.

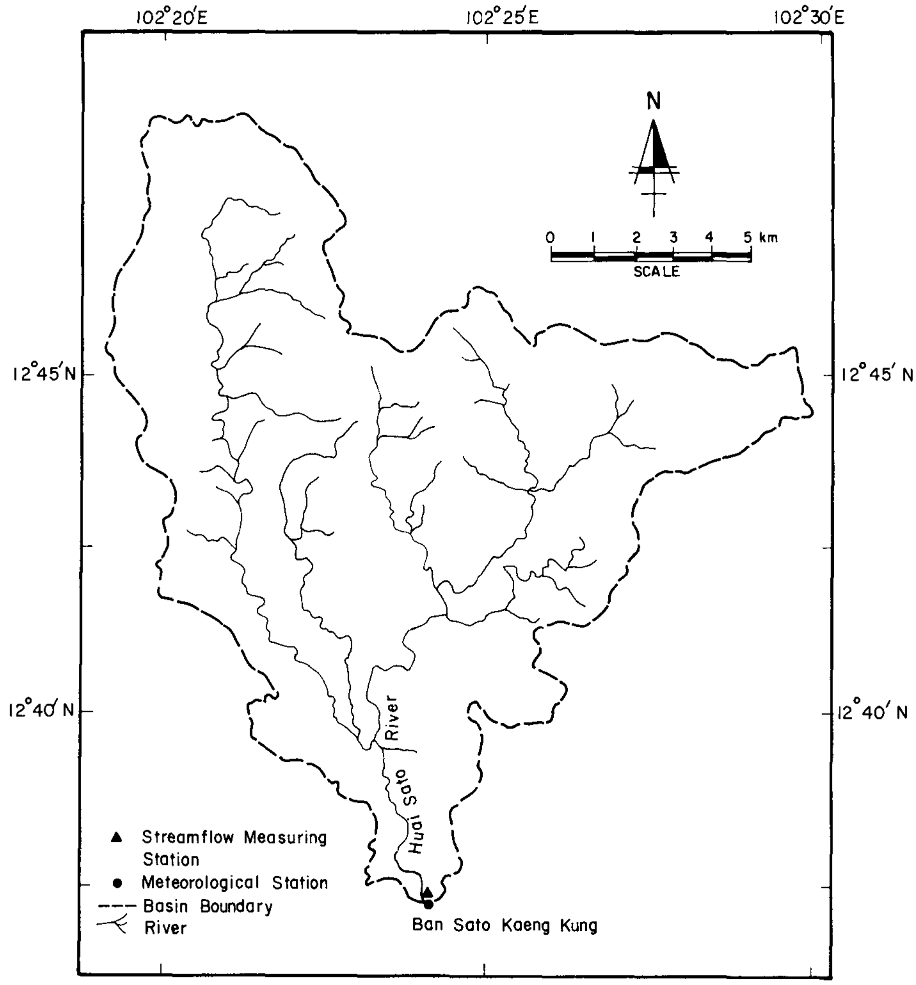


Figure 3
The watershed of the Huai Sato River at Ban Sato Keang Kung

Huai Sato Basin

The Huai Sato River has a small drainage area of 190 km² situated in Eastern Thailand. The streamflow gaging station for this river is located at latitude 12° 37,4' N and longitude 102° 24,3' E. It is on the right bank of the Huai Sato River (Fig. 3), in Ban (village) Sato Kaeng Kung, Amphoe (District) Khung, Chantaburi province. There is only one meteorological station in this headwater catchment. It is located at latitude 12° 37,1' N and longitude 102° 24,2' E. In this study, eight years of records (1971-1978) were used. Data in the first five years (1971-1975) were employed in the calibration of the model, while those in the last three years (1976-1978) were used for model verification.

Ping River Basin

The Ping River basin is located in the western area of Northern Thailand. Of the four tributaries of the Chao Phraya River (the greatest waterway of Thailand), the Ping River is the largest. The valley floors are at elevations from 150 m to 380 m above the mean sea level. The sub-basin under study is the area above Ban Tha Sala gauging station (station P.19A, according to the code system provided by the Irrigation Department of Thailand) with a drainage area of 14 023 km², which consists of steep land with some limestone crags and red-brown earth. Mountains are rugged and peaks range from 1 500 m to almost 2 600 m. After combining discharge from its tributary (Mae Chaem), the Ping River flows into the Bhumibol Dam which is about 50 km downstream of Ban Tha Sala. There are 12 rainfall stations located in the sub-basin as shown in Fig. 4 (with codes adopted from the Irrigation Department of Thailand). Data from April 1973 to March 1978 were employed in this research. Data in the first three water years (April 1973 – March 1976) were used for model calibration, and those in the remaining years (April 1976 – March 1978) were used for model verification.

Results

Station weights for rainfall data

For the Huai Sato River, there is only one rainfall station available in the drainage area. Determination of the station weight is trivial. For the case of the Ping River, station weights are needed. The suggestions by Sugawara *et al.* (1974) were followed, but instead of using their procedure which is time consuming and sometimes difficult to apply, the correlation between historical rainfall and discharge was used. The result for a monthly basis is shown in Table 1. However, after several test runs, it was found that when all stations had the same weights (i.e. the arithmetic mean was used), better results were obtained for the discharge

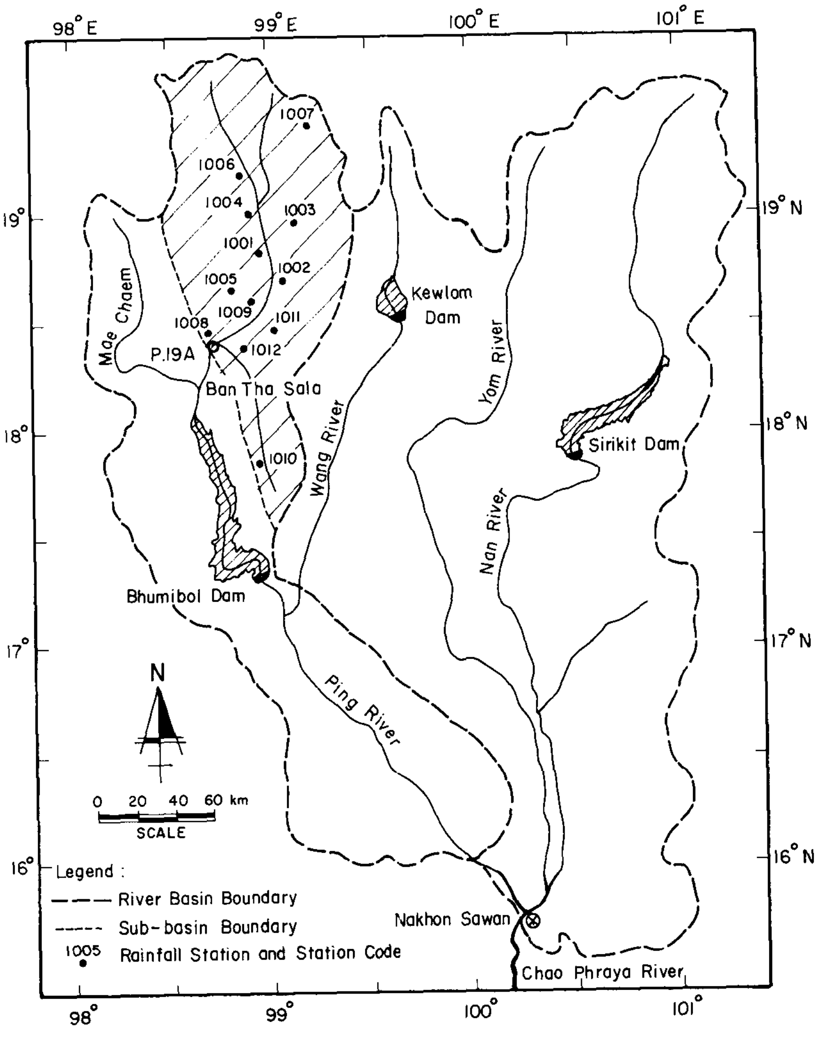


Figure 4
The watershed of the Ping River at Ban Tha Sala

TABLE 1
CORRELATION COEFFICIENTS BETWEEN MONTHLY RAINFALL AND MONTHLY DISCHARGE AT P.19A AND STATION WEIGHTS

Rainfall Station	Correlation Coefficient	Station Weight
1001	0,683	1
1002	0,600	¼
1003	0,635	½
1004	0,600	½
1005	0,795	1
1006	0,597	¼
1007	0,533	0
1008	0,672	1
1009	0,588	0
1010	0,550	0
1011	0,618	½
1012	0,581	0

TABLE 2
SELECTED VALUES OF TYPE 1 PARAMETERS

River		A_1	A_2	A_0	B_1	B_0	C_1	C_0	D_1
Huai Sato	(1)	0,035	0,035	0,050	0,040	0,050	0,005	0,005	0,0002
	(2)	0,220	0,220	0,250	0,040	0,050	0,005	0,005	0,0002
Ping	(1)	0,045	0,045	0,140	0,057	0,115	0,005	0,005	0,0002
	(2)	0,100	0,100	0,140	0,057	0,115	0,005	0,005	0,0002

Notes: (1) For daily basis
(2) For monthly basis

TABLE 3
SELECTED VALUES OF TYPE 2 PARAMETERS (mm)

River	HA_1	HA_2	HB	HC	HS	PS	SS	XS	XA	XB	XC	XD
Huai Sato	30	35	10	10	150	50	250	5	5	5	5	50
Ping	40	45	6,5	6,5	150	50	250	100	30	6,5	6,5	6,5

volume and hydrograph – the two criteria employed by Sugawara (1979). Consequently, this procedure of assigning equal weights to all rainfall stations was adopted in the present work.

Values of parameters

Following the combined approach consisting of trial and error procedure and automatic calibration, the parameters of the Tank Model were obtained and shown in Tables 2 and 3 for Type 1 and Type 2, respectively.

Comparison between computed and observed discharges

Many criteria have been used in the evaluation of rainfall-runoff models. In this study, both the annual discharge and the hydrograph were used as in most studies of Sugawara (1961, etc.). For the annual discharge (which is the mean of daily or monthly discharges in a selected year), the results obtained are shown in Tables 4 and 5 for the Huai Sato River and Ping River, respectively.

For the Huai Sato River, the computed annual discharge was less than the observed discharge in many years (indicated by a negative sign for the relative error in Table 4) when a daily basis was employed, during both the valibration and verification stages. However, the Tank Model seemed to overestimate the annual discharge when a monthly basis was concerned. The model performed slightly better for this basis as well.

For the Ping River, when a daily basis was employed, the Tank Model underestimated the annual discharge throughout the entire period of five years considered in this study, both for the calibration and verification stages. But for a monthly basis, the computed values of annual discharges are very close to the observed ones, as can clearly be seen from Table 5.

TABLE 4
COMPARISON BETWEEN OBSERVED AND COMPUTED DISCHARGES FOR THE HUAI SATO RIVER

Basis	Year	Annual Discharge (m³/s)		
		Observed	Computed	Relative error[*] (%)
Daily[+]	**Calibration**			
	1971	10,0	9,4	-6,1
	1972	10,2	10,6	+3,9
	1973	10,3	12,2	18,5
	1974	12,4	11,5	-7,2
	1975	14,9	13,4	-10,2
	Verification			
	1976	11,0	10,4	-5,5
	1977	10,9	7,3	-10,9
	1978	13,6	15,1	11,1
Monthly[+]	**Calibration**			
	1971-1975	11,6	12,1	4,5
	Verification			
	1976-1978	11,9	12,9	8,4

[*] relative error = 100% x (Computed-Observed)/Observed

[+] Daily (monthly) basis: both input and output data are given day by day (month by month)

TABLE 5
COMPARISON BETWEEN OBSERVED AND COMPUTED DISCHARGES FOR THE PING RIVER

Basis	Water Year	Annual Discharge (m³/s)		
		Observed	Computed	Relative Error[*] (%)
Daily[+]	**Calibration**			
	Apr. 1973-Mar. 1974	188,7	143,0	-24,2
	Apr. 1974-Mar. 1975	116,5	101,5	-12,9
	Apr. 1975-Mar. 1976	186,8	145,1	-22,3
	Verification			
	Apr. 1976-Mar. 1977	86,6	67,2	-22,3
	Apr. 1977-Mar. 1978	102,2	101,4	-0,8
Monthly[+]	**Calibration**			
	Apr. 1973-Mar. 1976	164,0	166,8	1,7
	Verification			
	Apr. 1976-Mar. 1978	94,4	94,0	-0,4

(*) and (+): same as in Table 4

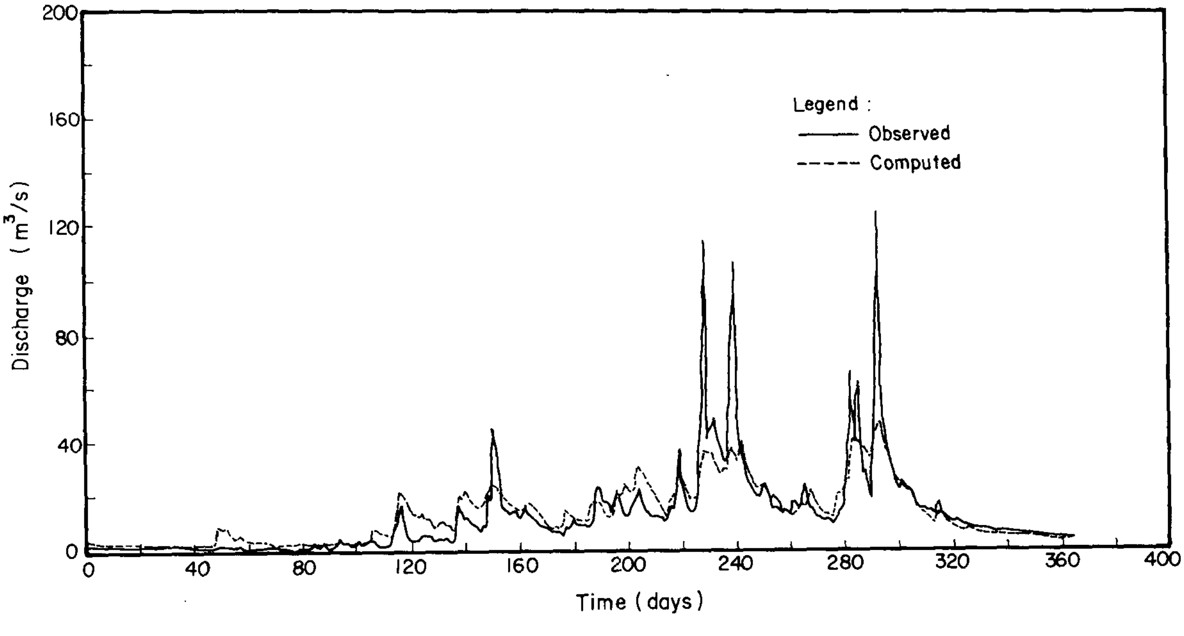


Figure 5
Hydrographs of computed and observed daily discharges for the Huai Sato River in 1974 (Calibration)

Comparison between computed and observed hydrographs

The hydrographs of computed and observed daily discharges were plotted each year during both the calibration and verification stages, while those for the monthly basis were separated into two corresponding to the two stages.

For the Huai Sato River, it was found that the peaks of daily discharge were underestimated by the Tank Model. Typical cases are illustrated in Fig. 5 for the year 1974 (calibration) and Fig. 6 for the year 1977 (verification). For monthly data, the peaks of computed discharges were lower than the observed peaks in most years during the calibration stage (Fig. 7) and the computed peaks were always lower in all the three years of the verification period (Fig. 8).

For the Ping River, the same situation was observed for daily discharges. Typical results are shown in Fig. 9 for the water year April 1975 – March 1976 (calibration), and Fig. 10 for the water year April 1976 – March 1977 (verification). However, for monthly data, the observed peaks were found to be higher than the computed peaks throughout the five years employed (Fig. 11, a and b).

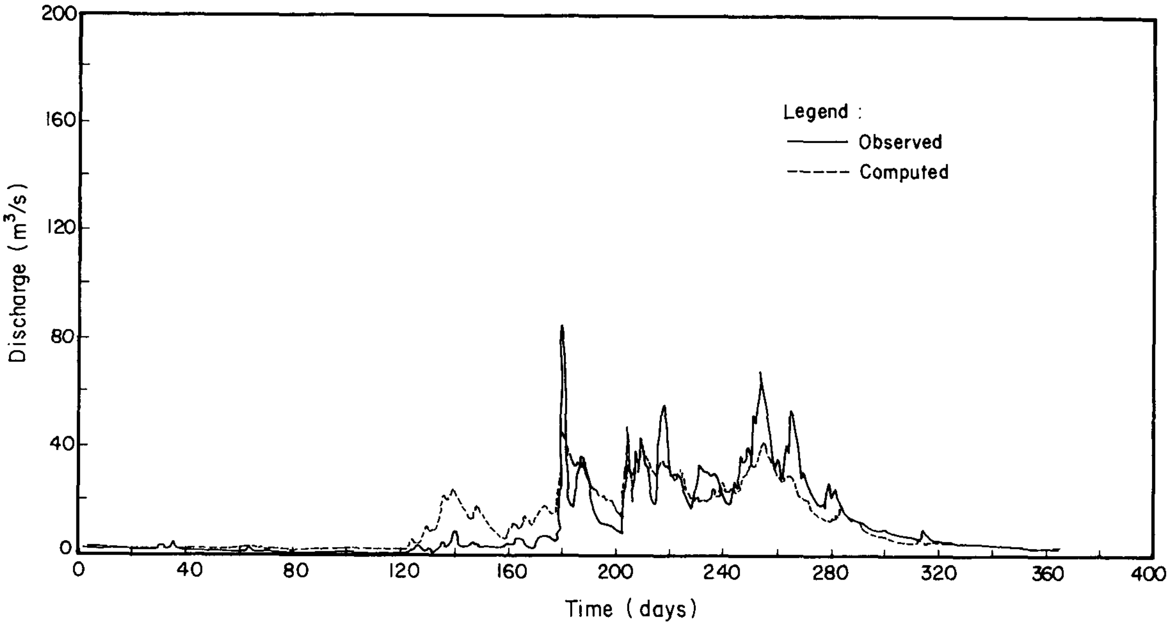


Figure 6
Hydrographs of computed and observed daily discharges for the Huai Sato River in 1977 (Verification)

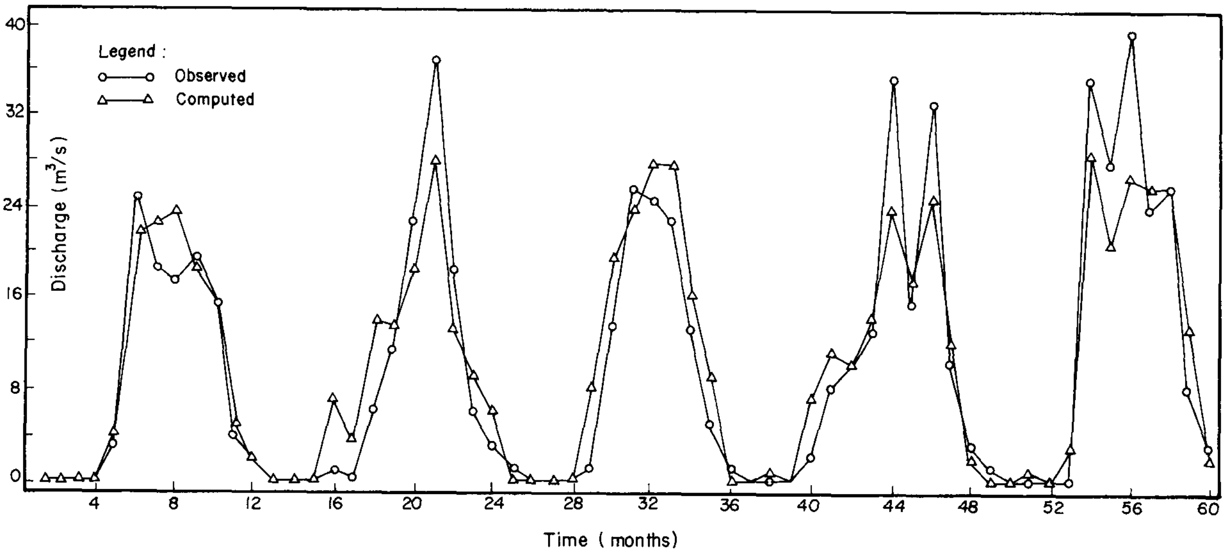


Figure 7
Hydrographs of computed and observed monthly discharges for the Huai Sato River (Calibration: 1971-1975)

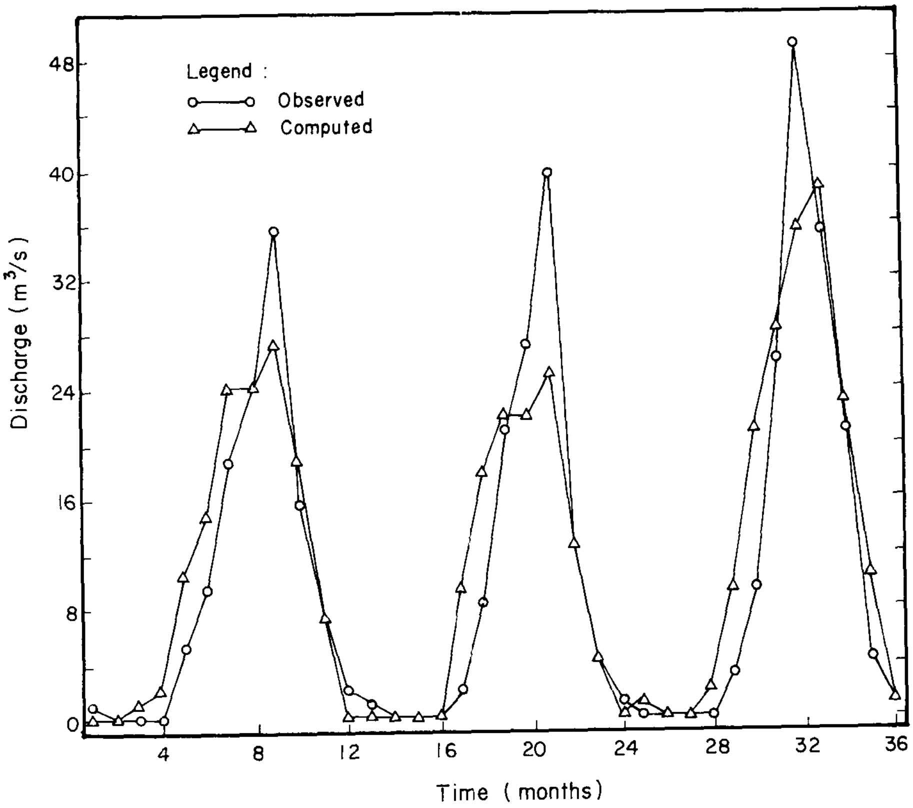


Figure 8
Hydrographs of computed and observed monthly discharges for the Huai Sato River (Verification: 1976-1978)

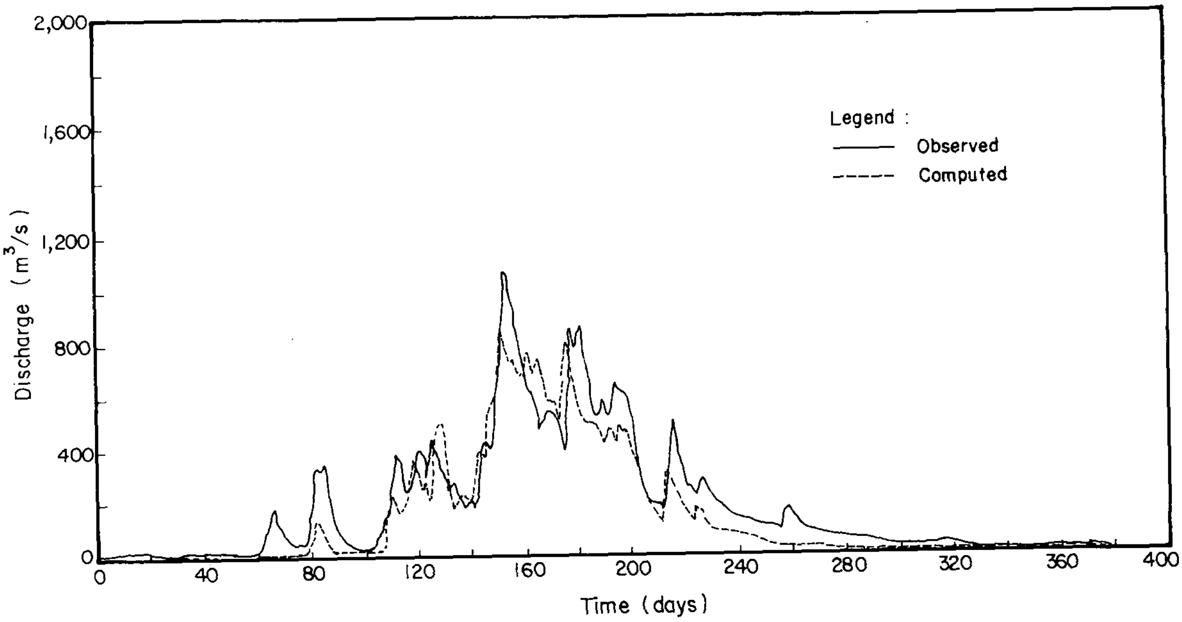


Figure 9
Hydrographs of computed and observed daily discharges for the Ping River during the water year 1975-1978 (Calibration)

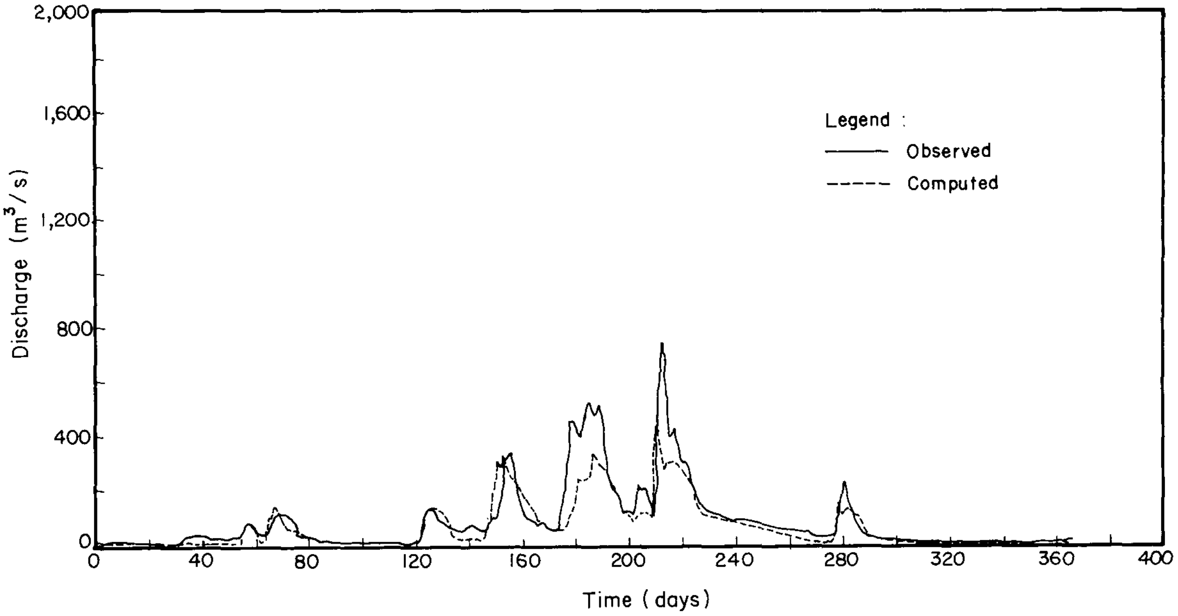
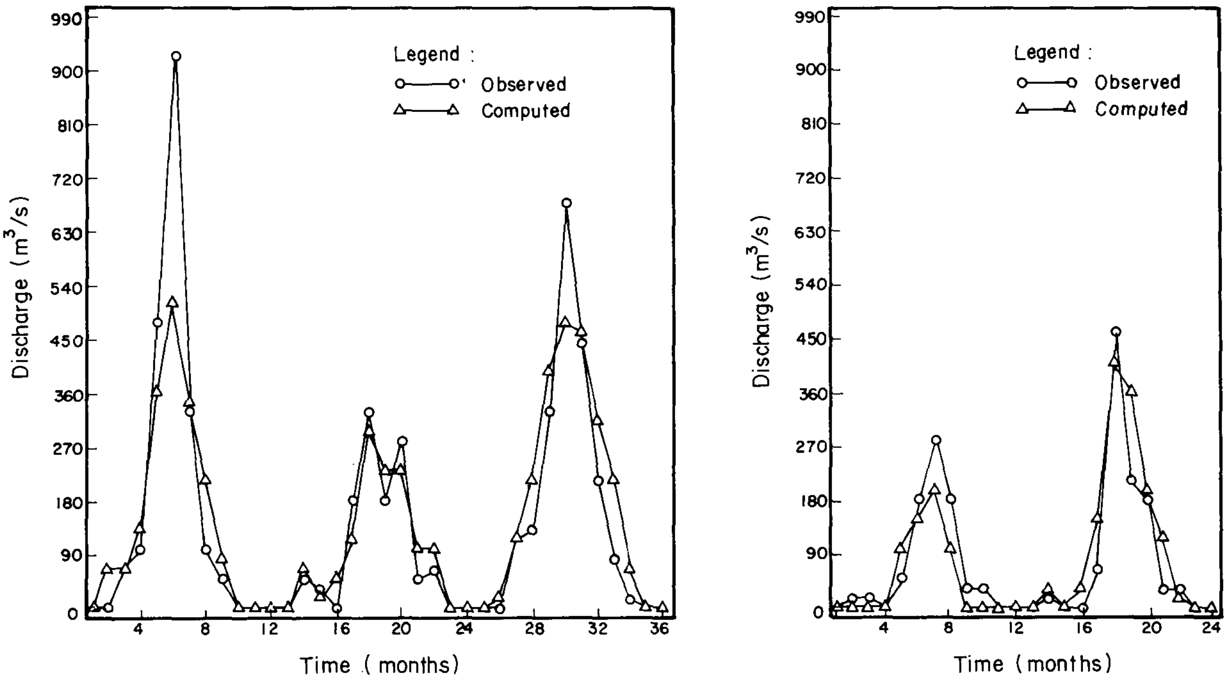


Figure 10
Hydrographs of computed and observed daily discharges for the Ping River during the water year 1976-1977 (Verification)



(a)

(b)

Figure 11
Hydrographs of computed and observed monthly discharges for the Ping River, (a) Calibration: April 1973 – March 1976; (b) Verification: April 1976 – March 1978

Discussion

With respect to the *annual discharges*, the results obtained for the Huai Sato River may be said to be very good, for the daily or monthly basis. Throughout the period under consideration, the relative error was found to be quite small. For the Ping River, the relative error was relatively high for many years when a daily basis was concerned. However, all the errors are less than 25%, and the results obtained may be said to be accpetable. It should be noted that actual daily evaporation data were not available for this case and average daily values recorded at the Bhumibol Dam were used in the analysis. This may account for some part of the errors produced by the Tank Model.

For monthly data, the Tank Model produced very good results for both rivers. The relative errors were very small for the calibration as well as for the verification stage.

From the *hydrographs*, except for the fact that the peaks of daily discharges were underestimated by the Tank Model, the results obtained may be said to be quite satisfactory if one compares those obtained in this study with those reported in the literature on rainfall-runoff modelling using the SSARR Model and the Stanford Watershed Model IV.

As mentioned earlier, the Tank Model was intended for the modelling of the rainfall-runoff process on a daily basis. However, from the results obtained in this study, it is clear that it may be used on a monthly basis as well. In this case, from several test runs for the Huai Sato River, it was found that use of actual values of monthly evaporation is not necessary. Instead, a simple average value obtained from the eight years of records may be used for each month, and the results obtained remained fairly good. This explains to some extent the fact that even with the use of average monthly values recorded at the Bhumibol Dam, the computed monthly discharges were very close to those of the observed data for the Ping River. However, more case studies should be carried out in order to give a definite conclusion on this observation.

As mentioned above, peak discharges are underestimated by the Tank Model. When the computed peak discharges were increased to be of the same range as the observed data, the annual discharges and hydrographs ceased to be close to this historical ones. These results together with those obtained by Thang (1981) indicated that the Tank Model might not be suitable for flood forecasting purposes, if the two criteria used by Sugawara (1979) were adopted.

In summary, the Tank Model can be used quite satisfactorily to simulate discharge volumes with both daily and monthly inputs. However, in terms of the hydrograph, it shows a fairly poor visual one-to-one fit in most high flow periods. Since a monthly basis is commonly used in the planning and design of water resources projects, the Tank Model would be very useful for the extension of streamflow records. Having calibrated its parameters using the available data for rainfall, runoff and evaporation (perhaps only rough estimates for evaporation), the model can be used to transfer rainfall to runoff when data on the former are available for a longer period. However, due to the fact that the peaks of computed daily discharges were lower than those observed, the Tank Model would not be good for flood forecasting.

Acknowledgements

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