A correlation model for calculating groundwater level responses in the Sishen North Mine

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Abstract

Due to a lack of observation wells and geohydrological knowledge at greater distances from the centre of pumping in a mine where groundwater is abstracted on a large scale for dewatering purposes, it is not always possible to delineate the aquifer and to establish the spreading of the depression cone of dewatering. Consequently, it will be impossible to make a fairly good assessment of the groundwater supply.

A method has been developed to calculate the future drawdown of the groundwater level inside the mine, which reflects the depletion of the groundwater supply, by correlating the variable daily pumping figures and the measured groundwater level responses. The application of this computation method is illustrated by the dewatering of the Sishen North Mine.

Introduction

At Sishen (Fig. 1) extensive iron ore mines belonging to ISCOR have been developed.

The results from an environmental isotope study, as well as the different responses of the groundwater hydrographs to pumping, indicate that the North Mine area and the Sishen compartment act as separate hydrological systems with the dividing NE-SW trending dyke (Fig. 2) as a perfect aquiclude (Dziembowski and Verhagen, 1979).

The bounding dykes of the Sishen compartment were established geologically and by intensive geophysical investigations in most locations. The area affected by the dewatering operations in the South Mine has been delineated using an extended network of observation wells. Consequently, it was quite possible to estimate the groundwater storage of the Sishen compartment fairly accurately with the aid of the available geohydrological data (Dziembowski and Verhagen, 1979). In case of the North Mine area, however, both the delineation of the aquifer and the spreading of the depression cone of dewatering could not be established properly due to lack of sufficient geohydrological information. Hence, another method based on a correlation between daily pumping figures and the measured groundwater hydrograph was developed to assess the total groundwater volume which can be abstracted before the groundwater level has reached a critical level at a selected height above the base of the aquifer inside the North Mine.

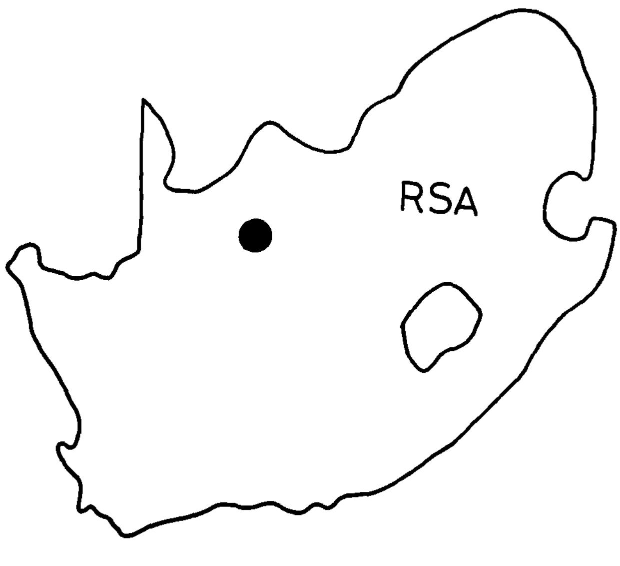

Figure 1
Location of the Sishen area in South Africa

Hydrogeology of the North Mine area

The North Mine area is defined as the area wherein the groundwater regime has been influenced by the dewatering of the North Mine. The generalized geological map (Fig. 2) and the generalized cross-section (Fig. 3) illustrate the hydrogeological framework of the North Mine. According to available borehole information the jointed and fissured rocks of the Gamagara and Asbesberg Formations and the chert breccia of the Ghaap Plateau Formation can be considered to form part of the same bedrock aquifer with the solid dolomite as a hydrological base. In the bedrock aquifer sub-artesian conditions prevail with the lava of the Ongeluk Formation and the red clay of the Kalahari Group as confining beds, except in the vicinity of the open cast workings where water table conditions are present because of dewatering of the water-bearing rocks.

The Kalahari Group sediments consist of basal clayey gravel lenses, a continuously thick layer of red clay, upper gravels, calcrete and sand on the surface. Gravity drainage of groundwater out of the upper Kalahari gravels into the bedrock aquifer is negligibly small with regard to the pumping rate because of the very low permeability of the heavy red clay.

Exploratory drilling approximately 7 km northwesterly from the North Mine during 1975 showed artesian groundwater in the Gamagara and Asbesberg Formations overlain by the impervious Ongeluk lava. The piezometric level has since dropped considerably (about 45 m) due to the dewatering operations in the North Mine, which may indicate that the N – S trending dyke does not act as a pefect subterranean barrier.

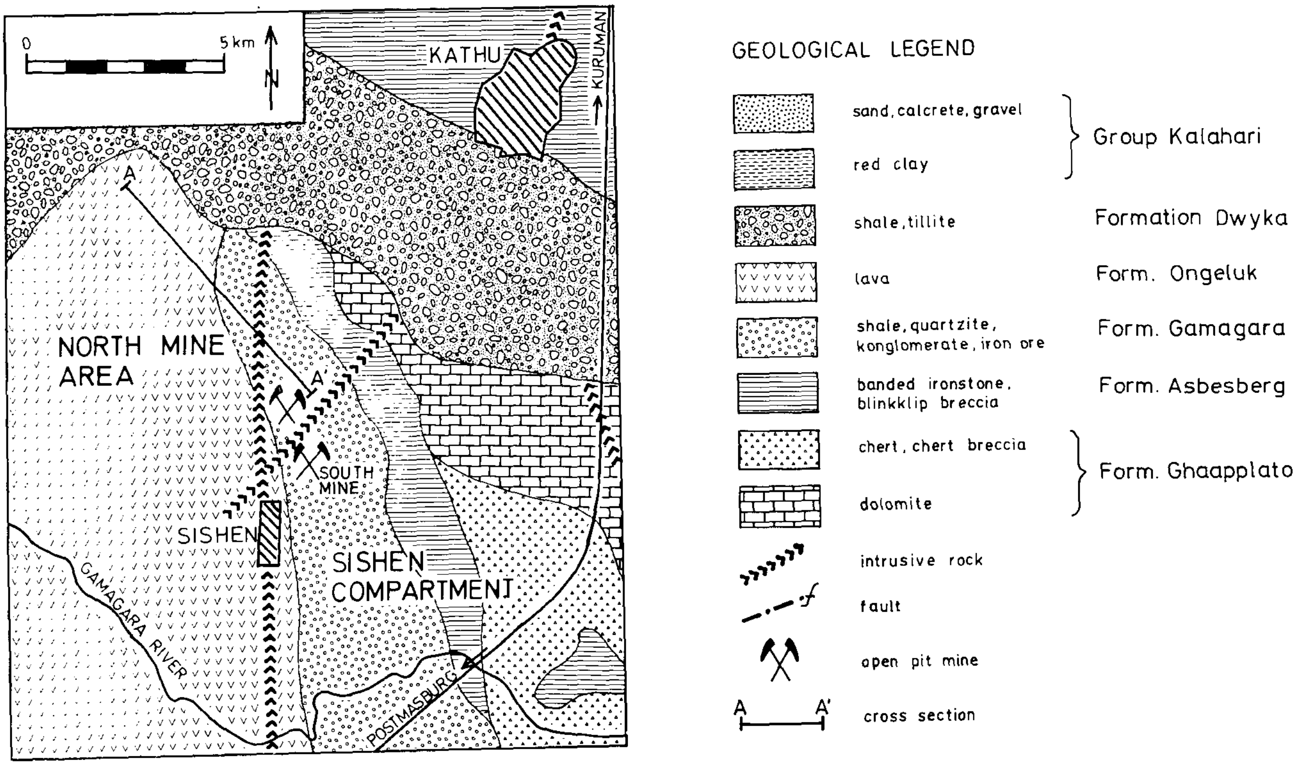

Figure 2
Map of the generalized pre-Kalahari geology of the Sishen area.
(For cross-section A – A' see Fig. 3)

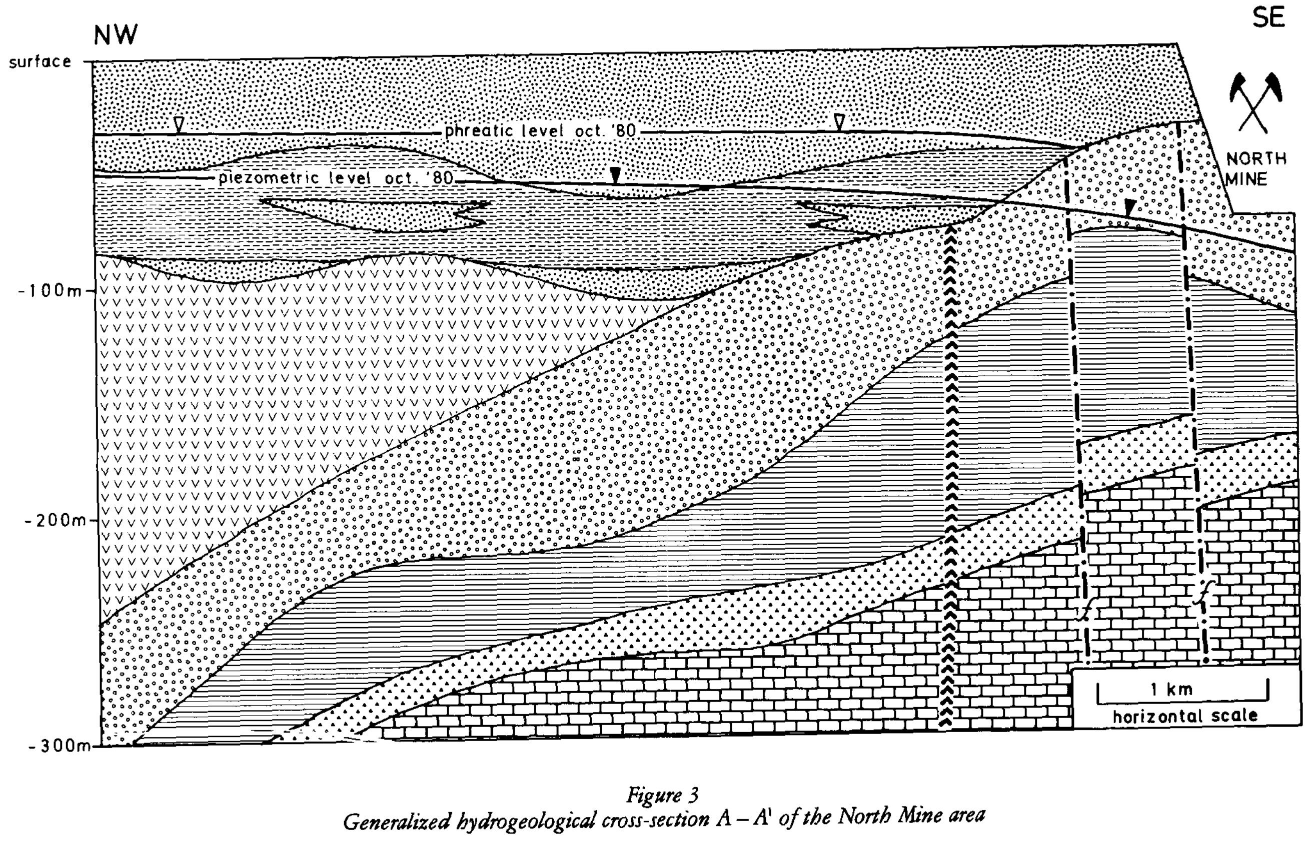

Figure 3
Generalized hydrogeological cross-section A – A' of the North Mine area

As was stated in the previous section, the NE – SW trending dyke is an impervious subterranean groundwater divide between the North Mine and the Sishen compartment, as is evidenced from the actual difference between the groundwater levels on both sides of the dyke (about 20 m).

The pre-Karoo glacier valley filled up with impervious shale and tillite of the Dwyka Formation (maximum thickness greater than 200 m) can be considered as being the northern and north-eastern boundary of the aquifer.

The water-bearing strata dip in a westerly direction, which is probably coupled with a progressive reduction of the permeability because of the increasing load of the overlying lava.

Development of the correlation model

The correlation between the variable, daily pumping quantities and the groundwater level in an observation well at the end of every day is based on the Theis equation (Theis, 1935):

$$s(t) = \frac{Q}{4\pi kD} W(u) \qquad (1)$$

where

$u(t) = \frac{r^2S}{4kDt}$

$s(t)$ = the drawdown in an observation well at a distance r from the pumped well (m)

Q = the constant well abstraction (m^3/day)

S = the storage coefficient (–)

kD = the transmissivity of the aquifer (m^2/day)

t = the time since pumping started (day)

$$W(u) = -0{,}5772 - \ln u + \Sigma_1(-1)^{n-1}\frac{u^n}{n.n!}$$

Eq.(1) represents an unsteady-state groundwater flow to a fully penetrating pumped well in a homogeneous, confined aquifer. A value for transmissivity and storage coefficient obtained by means of the Theis equation for the bedrock aquifer consisting of fissures and joints, will differ from the values of these hydraulic properties for any given small portion of the aquifer. However, the bedrock aquifer as a whole will behave as if it was homogeneous if the fissures and joints are relatively numerous and sufficiently evenly distributed (Venter, 1969).

The North Mine is dewatered by means of various pumped wells at rather long distances from each other (varying from a few hundred metres to more than 1 km) and not all of them penetrate to the bottom of the aquifer. However, for the sake of simplicity this cluster of wells will be considered as one hypothetical centre-well penetrating the whole aquifer.

As was stated earlier, confined conditions prevail except in the vicinity of the mine where water table conditions are present. With continuing dewatering, the area in which water table conditions occur will become progressively greater. Strictly speaking this feature of continuing change from sub-artesian to water table conditions should be included in the Theis equation.

Correlation models have been developed based on two variants of the Theis equation respectively, viz. the Cooper-Jacob equation (Cooper and Jacob, 1946) and a modified Theis equation. The model based on the modified Theis equation produced the best simulation of the groundwater level responses.

The development of a suitable correlation model, based on the original Theis equation, for long-term pumping at variable rates is from an algebraic point of view complicated due to the series

$$\Sigma_1(-1)^{n-1}\frac{u^n}{n.n!} \quad \text{in } W(u).$$

The series is convergent for every value of u, therefore

$$\lim_{n\to\infty} \Sigma_1(-1)^{n-1}\frac{u^n}{n.n!} = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{u^n}{n.n!} \qquad (2)$$

has a finite value.

Differentiating Eq.(2) with respect to u yields

$$\frac{d}{du}\left[\sum_{n=1}^{\infty}(-1)^{n-1}\frac{u^n}{n.n!}\right] = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{u^{n-1}}{(n-1)!}\cdot\frac{1}{n} \qquad (3)$$

The expotential function $e^{-u/2}$ can be expanded according to Taylor into a convergent series for values of u around u = 0, as follows:

$$e^{-u/2} \simeq \sum_{n=1}^{\infty}(-1)^{n-1}\frac{u^{n-1}}{(n-1)!}\cdot\frac{1}{2^{n-1}} \qquad (4)$$

The first two terms of the series in Eqs.(3) and (4) are equal, so for values of u around u = 0 it follows that

$$\frac{d}{du}\left[\sum_{n=1}^{\infty}(-1)^{n-1}\frac{u^n}{n.n!}\right] \simeq e^{-u/2} \quad , \qquad (5)$$

which yields upon integration (boundary condition: for u = 0 the series in Eq.(2) equals zero)

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{u^n}{n.n!} \simeq 2 - 2e^{-u/2} \qquad (6)$$

Hence, it follows that for values of u near u = 0

$$W(u) \simeq 2 - 0{,}5772 - \ln u - 2e^{-u/2} \qquad (7)$$

Arbitrarily, Eq.(7) is applicable for $u<1$ (for $u=1$ the relative error of the approximated value of W(u) is about 4,5%). Consequently, the modified Theis equation then reads

$$s(t) \simeq \frac{Q}{4\pi kD}[2 - 0{,}5772 - \ln u - 2e^{-u/2}]\ ;\ u<1 \qquad (8)$$

The daily abstraction diagram (Fig. 4) illustrates the variability of the daily total abstraction in the North Mine. A constant pumping rate (ℓ/s) is assumed during one day.

The difference between the daily abstraction on two successive days $(\Delta Q)_i = Q_{i+1} - Q_i$; $(i = 1, \ldots, n-1)$ can be considered as

a. the abstraction of an additional imaginary well after the i-th day, in case $(\Delta Q)_i > 0$

or

b. the recharge of an additional imaginary source after the i-th day, in case $(\Delta Q)_i < 0$

Fig. 5 illustrates the pattern of abstraction and recharge.

After n days the drawdown caused by the pumping that started at t = 0 is according to Eq.(8)

$$s_1 = \frac{Q_1}{4\pi kD}\left[2 - 0{,}5772 - \ln\left(\frac{r^2S}{4kDn}\right) - 2e^{-\frac{r^2S}{8kDn}}\right]$$

After n days the rise of the groundwater level (i.e. negative drawdown) caused by the recharge that started at t = 1 (Fig. 4 and 5) is

$$s_2 = \frac{(Q_2 - Q_1)}{4\pi kD}\left[2 - 0{,}5772 - \ln\left(\frac{r^2S}{4kD(n-1)}\right) - 2e^{-\frac{r^2S}{8kD(n-1)}}\right]$$

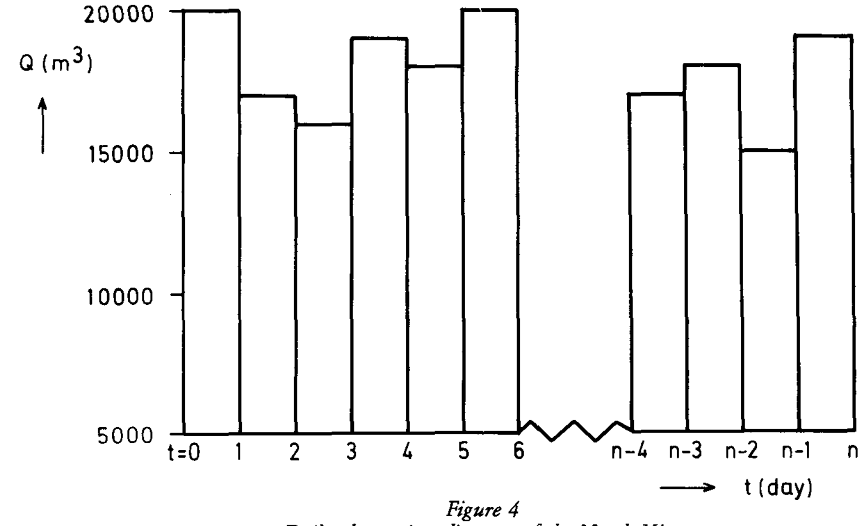

Figure 4
Daily abstraction diagram of the North Mine

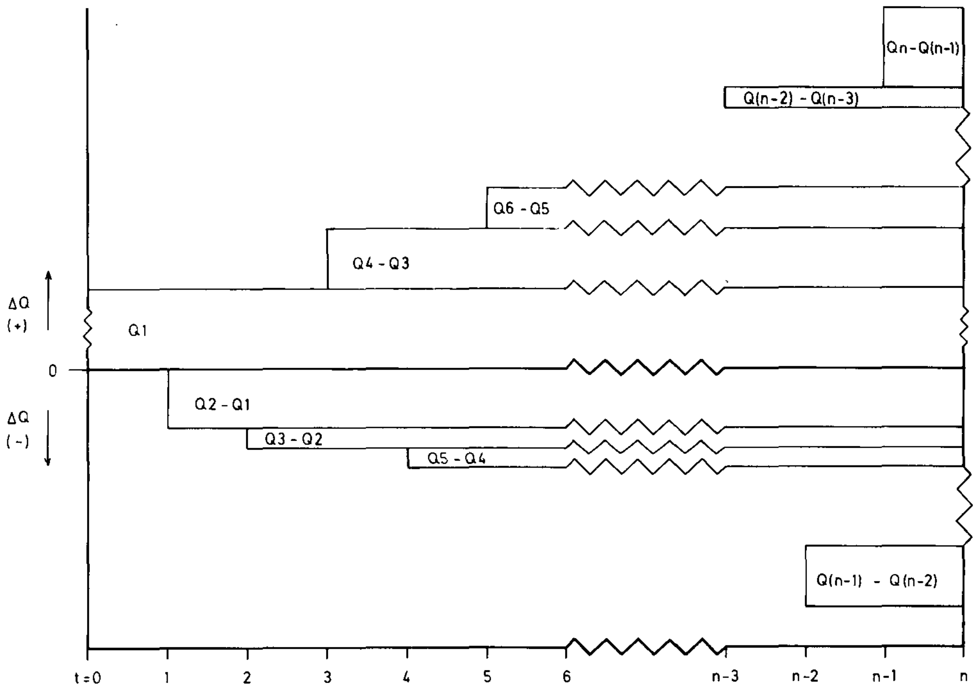

Figure 5
Pattern of abstraction and recharge; derived from Fig. 4

The remaining partial drawdowns can be expressed in a similar way. Assuming the principle of superposition is applicable, the real drawdown at the end of the n-th day is the sum of the drawdowns caused by the pumping of each hypothetical well and the recharge of each hypothetical source. After some mathematical operations the following equation can be derived

$$h_{t=n} \simeq \alpha + \beta_1 Q_n + \beta_2 \sum_{i=1}^{n-1} Q_i \left[\ln\left(\frac{n-i+1}{n-1}\right) - 2e^{\beta_3/(n-i+1)} + 2e^{\beta_3/(n-i)} \right] \quad \text{(model A)}$$

where

$h_{t=n}$ = the elevation of the groundwater level at the end of the n-th day (m + mean sea level)

Q_i = the abstraction on the i-th day (m^3/day)

n = either the number of days since pumping started, or the number of days from the beginning of the calibration period; $n \geq 2$

α, β_1, β_2 and β_3 are model parameters.

In the North Mine the aquifer is limited by the NE – SW trending dyke. Stallman (as quoted by Kruseman and De Ridder,

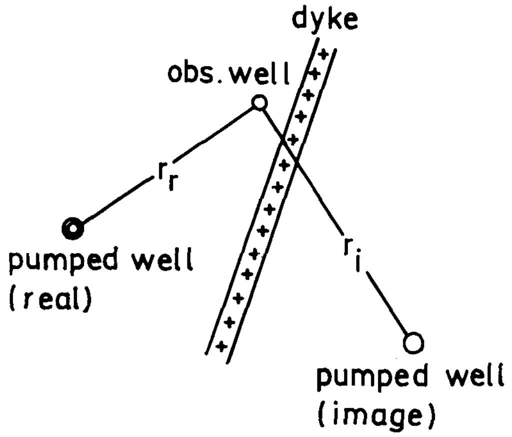

Figure 6
Simulation of an impervious dyke by an image well

1976) adjusted the basic Theis equation to the condition where the aquifer is crossed by one straight, fully penetrating barrier boundary within the zone influenced by the pumping, as follows

$$s(t) = \frac{Q}{4\pi kD}\left[W(u) + W(\gamma^2 u)\right] \qquad (9)$$

where (Fig. 6)

$\gamma = r_i/r_r$

r_r = the distance between the observation well and the real pumped well (m)

r_i = the distance between the observation well and the image pumped well

If the observation well is close to the dyke and the pumping centre is situated at a relatively large distance from the dyke, then γ will be approximately unity. From Eq.(9) it follows that the mathematical expression of model A does not change essentially. It appeared that model A did not produce realistic groundwater level responses after the calibration period (Fig. 7). This is comprehensible if one realises that model A is based on assumed sub-artesian conditions, while in the mine water table conditions exist. Therefore, an additional head loss is introduced which is increasing progressively because of continuous dewatering of the bedrock aquifer. This feature is illustrated in Fig. 7. The graph of the groundwater level in an observation well in the mine is running down steeper than the graph of the piezometric level in an observation well at 7 km north-west of the North Mine.

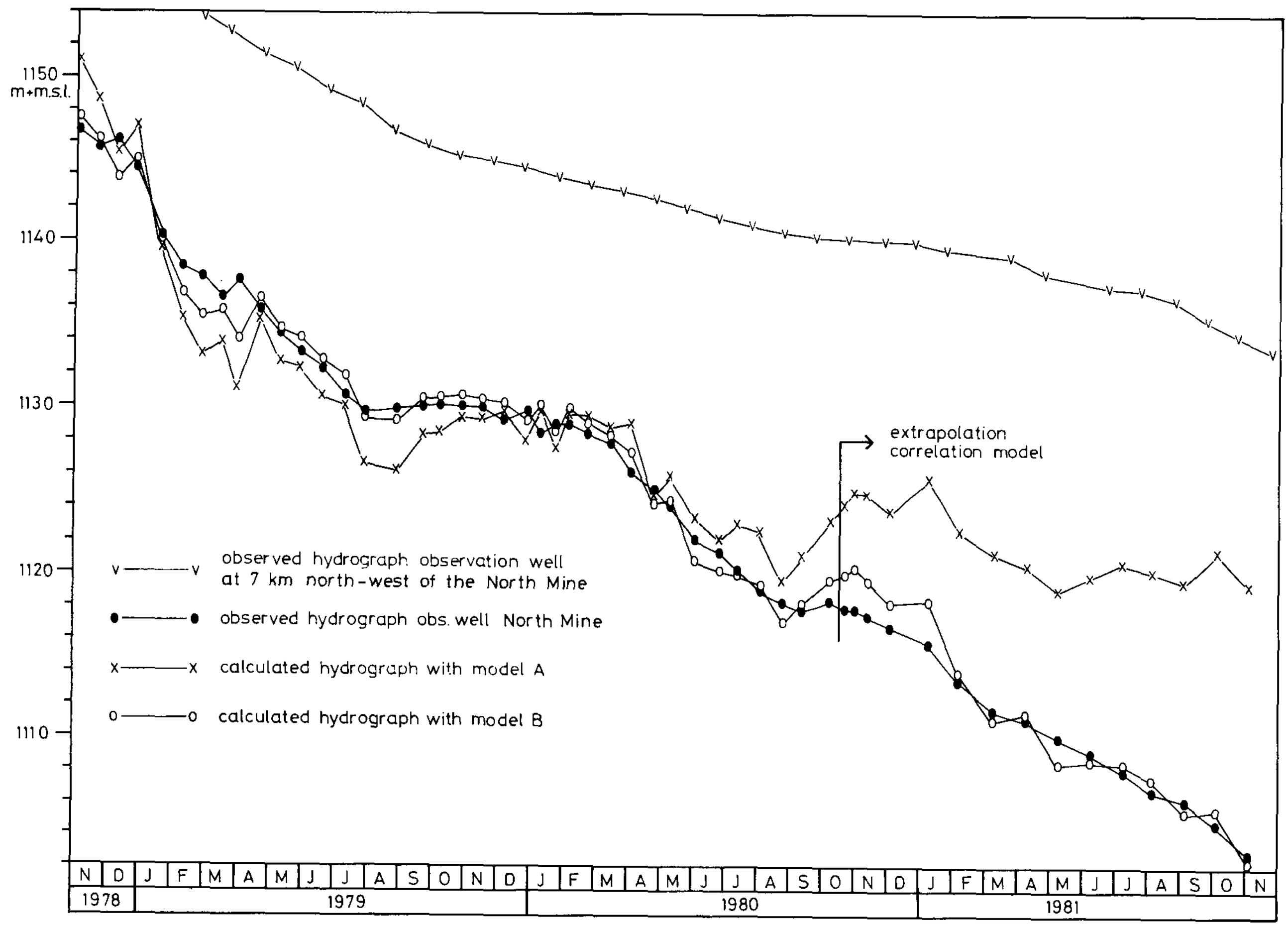

Figure 7
Observed and calculated groundwater level responses in the North Mine area

The problem concerning an unsteady-state flow to a well in a phreatic aquifer, while the aquifer is dewatered considerably, cannot be solved analytically (Van der Molen, 1977). The following approximate solution is often used in practice

$$s^*(t) = s(t) - \frac{s^2(t)}{s\bar{D}} \simeq \frac{Q}{4\pi kD} W(u) \qquad (10)$$

where

$s^*(t)$ = the corrected drawdown for phreatic conditions (m)

$\bar{D}$ = the average saturated thickness of the aquifer before pumping started.

It can readily be seen that the derivation of a correlation model based on Eq.(10) will be cumbersome.

The term $s^2(t)/2\bar{D}$ in Eq.(10) can be regarded as being the additional head loss caused by decreasing of the saturated thickness of the aquifer in the vicinity of the pumping centre. From a practical point of view this additional head loss is assumed to be proportional to the time raised to a certain power and the weighted sum of the previous daily abstraction quantities. Consequently, the final mathematical expression for $h_{t=n}$ can be written as

$$h_{t=n} \simeq \alpha + \beta_1 Q_n + (\beta_2 + \beta_4 n^{\beta_5}) \sum_{i=1}^{n-1} Q_i \left[\ln\left(\frac{n-i+1}{n-i}\right) - 2e^{\beta_3/(n-i+1)} + 2e^{\beta_3/(n-i)} \right] \qquad \text{(model B)}$$

Calculation of the model parameters

One needs a sophisticated computer program for optimizing the parameter values in model B. However, a practical computational procedure was developed which is adaptable to a programmable calculator.

The parameters α, β_1, β_2 and β_4 can be solved using the multiple linear regression method. The other parameter values can be determined by 'trial and error'. As the application of the 'trial and error' method of parameter determination using a calculator will take a lot of time, a more practical procedure for calculating values of the parameters β_3 and β_5 was developed.

A value of the parameter $\beta_3 = -r^2S/8kD$ can be obtained with the aid of the Cooper-Jacob method for step-type pumping (Cooper and Jacob, 1946). According to Cooper and Jacob the Theis equation (Eq.(1)) can be simplified for small values of u ($u<0{,}01$) to

$$s(t) = \frac{Q}{4\pi kD}\left(-0{,}5772 - \ln\frac{r^2S}{4kDt}\right)$$

After rewriting and changing into decimal logarithms this equation reduces to

$$s(t) = \frac{2{,}30Q}{4\pi kD} \log\frac{2{,}25kDt}{r^2S} \qquad (11)$$

In case of step-type pumping Eq.(11) changes into

$$\frac{s_{t=n}}{Q_n} = \frac{2{,}30}{4\pi kD} \log\frac{2{,}25kD\bar{t}}{r^2S} \qquad (12)$$

where

$\frac{s_{t=n}}{Q_n}$ = the specific drawndown (day/m²) after n days with Q_n (m³/day) as the abstraction on the n-th day

$\log\bar{t}$ = the weighted logarithmic mean time (day)

$$= \frac{\sum_{i=1}^{n-1} \Delta Q_i \log(n-i+1)}{\sum_{i=1}^{n} \Delta Q_i}\ ; \ \Delta Q_i = Q_i - Q_{i-1}$$

The weighted logarithmic mean time represents the time at which the observed drawdown would have occurred if the pumping had been done at a constant daily abstraction equal to the actual abstraction on the n-th day.

After the linear relation between the specific drawdown and the weighted logarithmic mean time is established for a relatively short period, values of the transmissivity and the storage coefficient can be calculated.

It should be noted that in case of a cluster of pumped wells the position of the hypothetical centre well should be determined in order to know the distance between the observation well and the hypothetical centre well. This position can be fixed approximately by computing the weighted average of the x- and y-coordinates of the pumped well positions with an average figure of the pumping rate as weight.

Lumping of pumped wells will lead to an over-estimation of hydraulic characteristics, because the hypothetical centre well is pumped at a rate corresponding to the daily total abstraction. Hence, a relative, physical meaning must be attached to these values. The value of parameter β_5 in model B can be determined using the linear regression method applied to the equation

$$\log\left[\left(\frac{s^2_{t=n}}{2\bar{D}}\right) / \sum_{i=1}^{n-1} Q_i \left[\ln\left(\frac{n-i+1}{n-i}\right) - 2e^{\beta_3/(n-i+1)} + 2e^{\beta_3/(n-i)}\right]\right] \simeq \log\beta_4 + \beta_5 \log n \qquad (13)$$

Results

From mid – 1977 the North Mine was dewatered on a large scale, but continuous records of daily pumping figures and daily groundwater level readings are available as from November 1978. The models are calibrated for the period between November 1978 and October 1980.

The spatial distribution of pumped wells is changing in course of time. This factor could influence the correlation to a certain degree, but for the sake of simplicity this matter is ignored. Another practical problem is the regular destruction of observation wells because of continuing mining operations, so that long-term records of one observation well are scarce. For the period concerned two observation wells were selected with relatively long records of the groundwater level overlapping each other to a sufficient extent. Correlating the simultaneous water level readings of both observation wells the hydrograph of one observation well could be reconstructed for the whole period.

The root mean square deviation E between the computed groundwater levels and the observed ones, reading in mathematical terms

$$E = \sqrt{\frac{\sum_{i=1}^{n} (h^{obs}_{t=n} - h^{calc}_{t=n})^2}{n}}\ ,$$

is used here as a criterion for the quality of the mathematical expression of $h_{t=n}$.

The multiple linear regression technique applied to the correlation models A and B resulted in 0,947 and 0,986 as the respective, multiple correlation coefficients, and 2,51 m and 1,13 m as the respective values of the root mean square deviation. These values are fairly good taking into account that the groundwater level dropped nearly 30 m during the calibration period.

The hydrographs resulting from the optimized models A and B are drawn in Fig. 7. The calculated parameters of model B are listed below

γ = 1147,71, β_1 = 3,778 x 10^{-4}, β_2 = − 2,662 x 10^{-4}, β_3 = − 1,15, β_4 = − 1,830 x 10^{-9}, β_5 = 1,68

For the period between October 1980 and November 1981 ydrograph of the groundwater level in an observation well in lorth Mine was reconstructed as a response to the actual daily ping quantities.

The extrapolated hydrograph produced by model A sticks to ctuating level and consequently does not show any agree- with the observed hydrograph. The explanation is that this re is inherent in Theis's assumption of an extensive, confin- uifer. As long as the aquifer is effectively infinite the rate of ne decreases continuously as the area influenced by the ping expands, and will become negligibly small in the long which can be considered to be a pseudo-equilibrium situa- This mechanism is reflected by the convergent series in the el. However, referring to the outlined hydrogeological ework of the North Mine area in a previous section, the l extent of the confined bedrock aquifer is limited and e a pseudo-equilibrium is not likely to be established at the l pumping rates. Apart from periodical flattening due to antial lowering of the pumping rate, the hydrograph of the metric level at 7 km from the North Mine does not show a nuously decreasing rate of decline (Fig. 7).

The simulated hydrograph from October 1980 till mber 1981 produced by model B agrees well with the ved one. This model can be used to calculate the future ndwater level responses for several pumping rates. In- ting a particular daily abstraction rate over a period between tain date and the time on which the groundwater level has ed a defined critical level gives the total groundwater me which can be abstracted.

ussion

rrelation model purely based on the Theis equation does not tion for a limited, confined aquifer which is dewatered on a scale. To cope with the continuing change from sub-artesian ater table conditions an additional time-dependent term in nodel was needed. The developed correlation model B has so roved to be a fairly good mathematical tool for calculating uture groundwater level responses in the Sishen North Mine.

The bedrock aquifer is supposed to be homogeneous. How- there is some evidence that the lower part of the aquifer t breccia) is more permeable, so one can expect a rapid in- e in the rate of decline as soon as this section of the aquifer is tered. The correlation is based on daily pumping figures about 10 000 to 20 000 m³/day. It is uncertain to which ee the optimized model B will be satisfactory in the case of rkably higher or lower abstraction rates.

Although the area influenced by pumping has reached some boundaries already, the effect of the intersection by the expanding cone of depression of further boundaries is not involved in the model. The boundaries which are not yet reached by the depression cone are considered to be situated at a great distance from the centre of pumping and the effect of these boundary conditions will not reduce the computational accuracy seriously.

Model B is based on the assumption that the rate of decline increases progressively in course of time at a constant daily abstraction rate. Consequently, this model will fail should the groundwater level tend to stabilize. In other words, model B is not adaptable for recharge conditions.

Referring to the above-mentioned matters, one needs long term records (at least 2 years) of reliable groundwater level readings and pumping figures to develop a reliable correlation model for predicting the future trend of the groundwater level in an area with continuous, large-scale dewatering operations.

Although the applicability of model B is limited to a specific point in the North Mine for which the model has been calibrated, the model can be of great use for the planning of dewatering operations.

Acknowledgements

This paper is published with the permission of the Department of Environment Affairs and ISCOR. The author thanks Mr J R Vegter, who suggested the idea to develop a correlation model.

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