Calculating runoff from catchment physiography in South Africa

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Abstract

Multivariate statistical techniques and regression analysis are used to develop runoff equations for small catchments (< 100 km^2) in two environmentally different regions of South Africa, the winter rainfall region and the summer rainfall region east of the 800 mm isohyet. Fourteen physiographical and three rainfall variables are selected as independent variables while the dependent runoff variables are the mean annual runoff, mean annual flood and 10-year return period flood. The statistical procedures are described and results are presented and discussed. The catchments of the two regions are shown to be physiographically and hydromorphometrically distinct. Regression equations for these two regions are also notably different and despite inadequacies in the runoff data are considered to be highly satisfactory.

Introduction

Conventional methods of collecting data for hydrological studies are generally time consuming and costly. In recent decades hydrologists have turned increasingly to mathematical and statistical techniques in order to reduce to a minimum the potentially large number of variables which may be required for a particular investigation (Seyhan and Keet, 1981). Greater use is now being made of multivariate statistical techniques to study multidimensional hydrological problems and to develop uncorrelated components which are able to define the given multivariate population in a more simplified structure. Numerous multivariate statistical techniques exist along with a wide variety of solution procedures and similarity measures which may be used in these procedures.

Some of the most widely used multivariate techniques are factor analysis, principal components analysis, cluster analysis, canonical correlation analysis and discriminant analysis while examples of solution procedures are principal components, principal factors, centroid and maximum-likelihood. Similarity measures which are adopted in most analyses are square Euclidean distance, product-moment correlation coefficient, variance, matching coefficient and average distance. A description of these multivariate statistical techniques is given by Seyhan (1981a).

Runoff from a catchment is largely a function of catchment variables which operate on the primary input, rainfall. In attempting to develop empirical equations to calculate catchment runoff the first step is to identify and quantify those climatic and cathment variables which influence runoff. Since these variables are generally highly interrelated it is necessary to establish the underlying dimensions of interrelatedness.

The first thorough quantitative study of river systems in a catchment was undertaken by Horton (1945). Following Horton (1945), pioneering work in the field of quantitative geomorphology directed at the formulation of catchment variables was conducted by, *inter alia*, Miller (1953), Schumm (1956), Morisowa (1958) and Maxwell (1960). These early works showed that the catchment variables which are most significantly related to runoff are: climate, catchment area, mean catchment altitude above the outlet, mean distance to the outlet, channel slope, catchment lag, surface storage, vegetation and soil. However, none of these authors made any attempt to study the interdependency of the variables in their multiple regression analyses.

Research conducted by Hope (1979) on small catchments (<100 km^2) in Natal, South Africa indicated that readily measurable physiographical and climatic variables could be used to calculate selected runoff characteristics, such as the mean annual runoff, of these catchments. The purpose of this investigation is to extend the research initiated by Hope (1979) to include a larger sample of catchments, catchments from different environmental regions and to incorporate multivariate statistical techniques in the development of predictive equations.

Aims and Procedures

The mean annual runoff, mean annual flood and 10-year return period flood are runoff characteristics which are widely used in the design of water storage and control structures in small catchments. This study had, as its central aim, the objective of developing physiographically based regression equations to calculate these three runoff variables for catchments in two regions of South Africa. The two selected regions were the winter rainfall region of the South-West Cape and summer rainfall region east of the 800 mm isohyet (Figure 1). Background information to these two regions is given in Table 1.

In order to achieve the objective of this study three specific aims were defined, viz., to

- determine whether the catchments of the winter and summer rainfall regions differed in terms of the measured runoff, rainfall and physiographical variables (Q-mode analysis);
- establish the underlying dimensions and interrelationships of the catchment variables in the two regions (R-mode analysis); and to

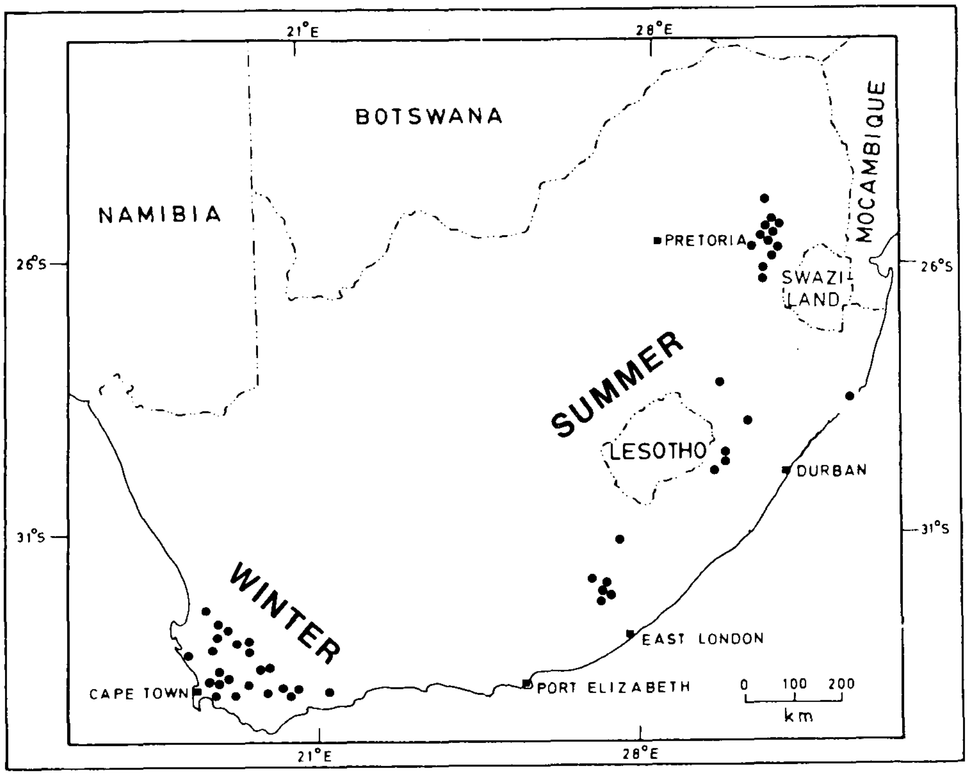


Figure 1
Study catchments in the summer and winter rainfall regions of South Africa

- screen and select independent physiographical and rainfall variables to be included in regression equations for calculating the selected runoff characteristics.

TABLE 1
BACKGROUND INFORMATION TO THE WINTER RAINFALL REGION AND SUMMER RAINFALL REGION

Variable	Rainfall Region	
	Winter	Summer
Mean Annual Rainfall (mm)	300 - 1000	600 - 1000
Mean Daily Temperature (°C)	15,0 - 17,5	12,5 - 22,5
Free Water Evaporative Losses (mm/yr)	<1250 - 1600	<1250 - 1750
Major Natural Vegetation	Mediterranean	Temperate Forest, Sub-Tropical Coastal Forest, Temperate Grasses
Geology	Sandstone, quartzite, unconsolidated superficial deposits	Shale, mudstone, sandstone, limestone, tillite, granite

Statistical Procedures

The selection of suitable multivariate statistical techniques for analysing a given body of data is not determined by set rules. Generally, such a selection is based on the past experiece of the investigator who should take cognisance of the sampling procedures applied, measurement errors and the statistical assumptions involved as well as the statistical and mathemetical structure of the selected multivariate techniques.

Factor analysis is the most widely used multivariate statistical method in hydrology. The primary objective of this technique is to represent a given group of variables in terms of several factors (Hotelling, 1933; Kaiser, 1958; Harman, 1968; and Seyhan, 1981a). In factor analysis an attempt is made to reduce the original group of variables to a smaller number of factors which will account for the observed variance in the given data. Various types of factor analysis exist and are described by Seyhan (1981a). These methods differ in terms of the format of the input data matrix, the procedure for extracting the initial factors, the type of rotation applied to established factors and the method of computing factor scores (Seyhan, 1981a).

In order to determine whether the selected catchments for this study could be classified into two significantly distinct groups (Q-mode analysis), linear discriminant analysis was selected as the

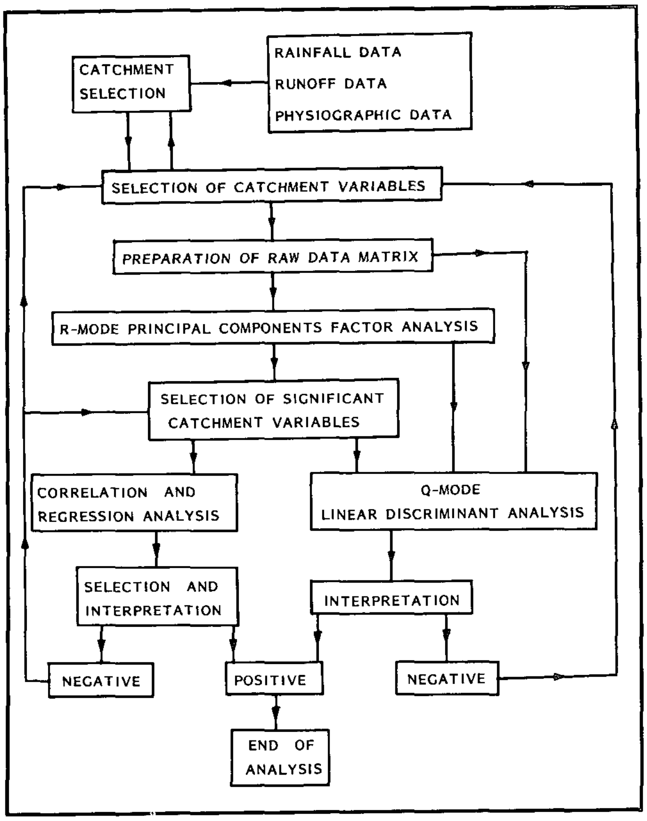


Figure 2
Procedural steps for R-mode and Q-mode analyses

TABLE 2
SELECTED DEPENDENT AND INDEPENDENT VARIABLES

Runoff variables:
MAR = mean annual discharge (x10^6 m^3 s^{-1})
MAF = mean annual flood (m^3 s^{-1})
MA10 = 10-year return period flood (m^3 s^{-1})

Physiographical variables:
A = Area of catchment (km^2)
S85 = mean slope of main channel by 85-10 slope factor (m km^{-1})
D = drainage density (km^{-1})
P = drainage perimeter (km)
W = width of watershed (km)
LB = main channel length (km)
HM = maximum basin relief (m)
T = topographic factor (km^3 m^{-1})
RH = Schumm's relief ratio (m km^{-1})
RHP = Melton's relief ratio (m km^{-1})
RF = form factor (–)
RC = circularity ratio (–)
RE = elongation ratio (–)
C = compactness ratio (–)

Rainfall variables:
MAP = mean annual precipitation (m)
I2,33 = intensity of 2,33-years 24 h duration rainfall (mm d^{-1})
I10 = intensity of 10-years 24 h duration rainfall (mm d^{-1})

required statistical procedure. Discriminant analysis provides a numerical criterion for classifying the cases (catchments) into two or more statistically distinguishable groups (categories or classes). Discriminant analysis may be regarded as a special type of factor analysis that extracts orthogonal factors to show the differences among several groups. Mathematically, discriminant analysis can be viewed as the development of discriminant functions (of the discriminant axes) that best separate the multivariate samples (Nie, Hull, Jenkins, Steinbrenner and Bent, 1975).

Identification of the independent underlying dimensions in the physiographical and hydromorphometrical data of the selected catchments (R-mode analysis) was made using principal components factor analysis. The selection of this statistical technique was based on the findings of Seyhan and Keet (1981) who found it to be an appropriate technique for R-mode analysis of data from catchments in Italy. Principal components factor analysis was applied to the correlation matrix of the input data (using Pearson's product-moment correlation coefficients) with variance orthogonal rotation criterion (Seyhan, 1981a).

The procedural steps by which the R-mode and Q-mode analyses were conducted is presented in Figure 2. As illustrated in Figure 2, allowance had to be made for positive or negative interpretation of the results. In the case of negative or meaningless interpretation the statistical selection of significant catchment variables could have been repeated with different selection criteria. Alternatively, additional catchment variables may have had to be selected and the analysis repeated.

Study Catchments and Data

Forty-eight catchments were selected for this study, 25 from the summer rainfall region and 23 from the winter rainfall region. Catchments selection was based on a number of criteria, viz.:

- Only catchments with areas in the range 4 to 100 km^2 were considered. These limits ensured that all the required physiographical information could be derived from 1:50 000 scale maps and hence standardise the source of this information.
- A minimum of 10 years of corresponding rainfall and runoff data were required for each cathment.
- No substantial hydraulic structures or diversions were within the catchments.
- The catchments were not underlain by complex geological features such as large scale faulting or karstification.

Most of the potentially suitable catchments for this study were rejected on the basis of the quality or duration of the runoff record.

Numerous variables in a catchment may influence, directly or indirectly, the transfer of rainfall to runoff. Some of these variables, such as catchment area, determine the potential volume of runoff while others, such as land-use or soils, modify the output. Attempts to relate long term runoff characteristics to catchment and climatic variables by authors such as Seyhan (1976), Hope (1979), Seyhan (1981b) and Seyhan and Keet (1981) have revealed that in most areas physiographical variables have the best association with the runoff variables. The selection of independent variables for this study was based on these earlier findings and were of two types, namely, rainfall variables and physiographical variables. The dependent variables were the mean annual runoff, mean annual flood and the 10-year return period flood. The independent and dependent variables tested in this study are given in Table 2.

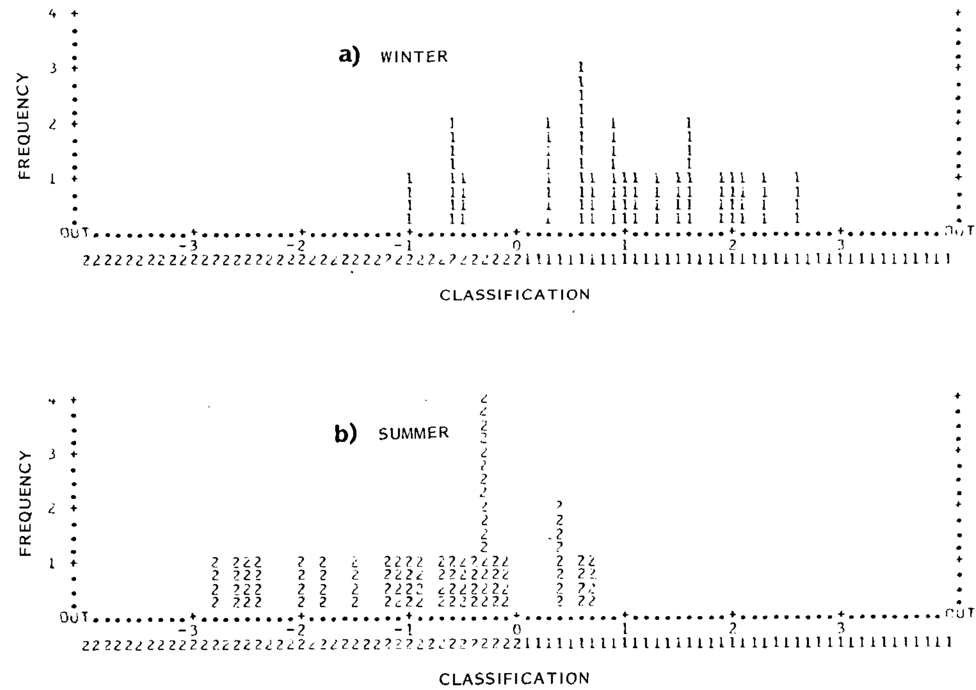
TABLE 3
MEAN (x̄) AND STANDARD DEVIATION (s) VALUES OF RUNOFF, PHYSIOGRAPHICAL AND RAINFALL VARIABLES FOR CATCHMENTS IN THE WINTER RAINFALL REGION (N = 23) AND SUMMER RAINFALL REGION (N = 25)

Variable	Winter Rainfall Region x̄	Winter Rainfall Region s	Summer Rainfall Region x̄	Summer Rainfall Region s
MAR	23,306	36,854	11,625	12,306
MAF	30,493	59,229	14,464	22,246
MA10	58,029	92,514	51,982	96,978
A	29,114	17,583	45,026	31,628
S85	86,795	60,485	43,412	40,187
P	2,248	0,805	2,073	0,663
D	26,032	10,766	29,778	13,779
W	2,856	1,088	3,426	1,251
LB	9,951	4,004	12,035	6,331
HM	1041,755	417,935	664,912	390,615
T	1,458	0,971	2,528	1,782
RH	102,377	55,371	52,966	36,191
RHP	43,199	21,374	25,966	18,705
RF	0,319	0,140	0,348	0,188
RC	0,565	0,174	0,602	0,126
RE	0,622	0,143	0,645	0,161
C	1,403	0,336	1,309	0,138
MAP	0,976	0,563	0,915	0,195
I2,33	60,217	22,711	73,720	13,183
I10	94,000	36,692	115,040	27,388

Procedures for calculating the physiographical variables are described by *inter alia* Seyhan (1976), Seyhan (1977) and Seyhan and Keet (1981). Mean annual precipitation values were obtained from the standard 1:250 000 rainfall maps of South Africa published by the Government Printer while the 2,33-years, 24 h duration rainfall and 10-years, 24 h duration rainfall were obtained from maps published by Schulze (1982). Runoff variables were calculated from data obtained from the Directorate of Water Affairs, Department of Environment Affairs, Pretoria. The mean and standard deviation values for each of the rainfall, physiographical and runoff variables for the selected catchments in the two regions are given in Table 3. Analyses were conducted using linear data and then repeated using logarithmically transformed data.

Results and Discussion

In all analyses where linear data were used the results were markedly poorer than the results obtained using logarithmic data. This observation was in keeping with the findings reported by Seyhan and Keet (1981) and Seyhan and Keet (1982). Only the results of analyses conducted using logarithmic data are presented.



RAINFALL REGION	NO. OF CATCHMENTS	PREDICTED GROUP MEMBERSHIP WINTER	PREDICTED GROUP MEMBERSHIP SUMMER
WINTER	23	19(82,6%)	4(17,4%)
SUMMER	25	4(16,0%)	21(89,0%)

PERCENT CLASSIFIED CORRECTLY = 85,8

Figure 3
Classification histogram based on discriminant functions for the (a) winter and (b) summer rainfall regions using physiographical data

Q-mode linear discriminant analysis

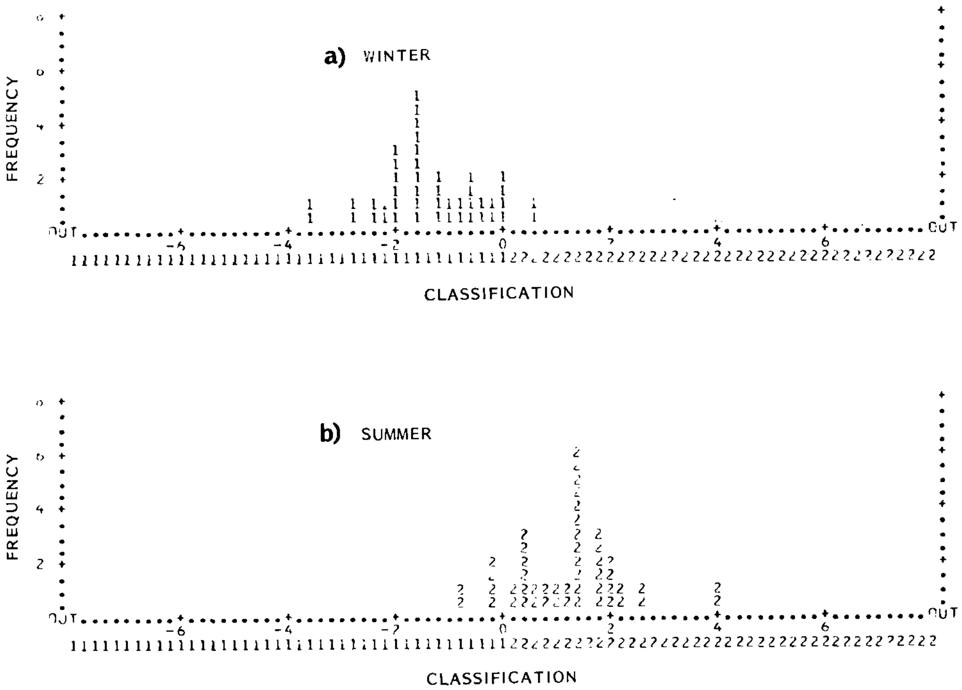
Linear discriminant analysis was used to determine whether the catchments of the winter rainfall region and summer rainfall region could be considered as statistically different in terms of physiography and hydromorphometry. Fourteen physiographical variables (Table 2) were used for the first analysis and the results of the discriminant analysis are given in Figure 3.

The histograms presented in Figure 3 illustrate the number of catchments classified into either group 1 (winter rainfall region) or group 2 (summer rainfall region) according to the discriminant function developed for catchments from these two regions using 14 physiographical variables (Table 2). The histogram presented in Figure 3a indicates that the discriminant function for the winter rainfall region predicted correctly the group membership of 82,6% of the catchments in this region. The proportion of catchments from the summer rainfall region which were classified correctly was 89,0% (Figure 3b). The minimal overlap in the histograms of Figure 3 reveals that the catchments in the two regions are, on the basis of physiography, two distinct groups.

The inclusion of rainfall and runoff variables (Table 2) in the discriminant analysis also resulted in a distinct hydromorphometrical classification of the catchments into the winter and summer rainfall regions (Figure 4). Eighty seven percent of the catchments of the winter rainfall region and 88% of the catchments in the summer rainfall region were classified correctly by the discriminant functions for these two regions. The results from both these analyses thus substantiated the hypothesis that the underlying physiographical and hydromorphometrical dimensions of catchments in the two regions were different and that they should be considered independently in developing runoff equations.

R-mode principal components factor analysis

The R-mode principal components factor analysis (Seyhan, 1981a) was adopted to analyse the interrelationships between the selected independent and dependent variables. Analyses were conducted using linear data and then repeated using data with a logarithmic transformation. For each of these raw data matrices a symmetrical correlation matrix of Pearson's product-moment correlation coefficients was developed. Factor identification was based on this correlation matrix, these factors explaining the variance of the input data. The degree of association between an individual variable and other variables contained in a factor is given by the factor loadings with the sign indicating a direct (+) or inverse (−) relationship. These factor loadings may be taken as having a meaning similar to that of a correlation coefficient in a correlation matrix. According to Seyhan (1981a) it is difficult to interpret a factor matrix which contains all the factor loadings. Thus, an arbitrary factor loading is selected and only variables



<table>
<tr><th rowspan="2">RAINFALL REGION</th><th rowspan="2">NO. OF CATCHMENTS</th><th colspan="2">PREDICTED GROUP MEMBERSHIP</th></tr>
<tr><th>WINTER</th><th>SUMMER</th></tr>
<tr><td>WINTER</td><td>23</td><td>20(87,0%)</td><td>3(13,0%)</td></tr>
<tr><td>SUMMER</td><td>25</td><td>3(12,0%)</td><td>22(88,0%)</td></tr>
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PERCENT CLASSIFIED CORRECTLY = 87,5

Figure 4
Classification histogram based on discriminant functions for the (a) winter and (b) summer rainfall regions using hydromorphometrical data

TABLE 4
SIMPLIFIED ROTATED FACTOR MATRIX OF PHYSIOGRAPHICAL DATA (LOGARITHMIC) FROM CATCHMENTS IN THE WINTER RAINFALL REGION
($N = 23$, factor loadings $\geq$ 0,70 and $\leq$ –0,70 retained)

Variables	Factors			h_i^2(%)
	I	II	III	
A		0,982		96,83
S85			0,748	80,43
D				23,27
P		0,836		91,79
W		0,817		96,55
LB		0,775		95,34
HM			0,832	90,80
T				59,90
RH			0,753	72,87
RHP			0,882	89,45
RF	– 0,971			95,22
RC	– 0,875			82,66
RE	– 0,971			95,22
C	0,715			54,58
Eigenvalue	5,30	3,57	2,28	
Explained Variance (Cumulative %)	37,85	63,35	79,65	

TABLE 5
SIMPLIFIED ROTATED FACTOR MATRIX OF HYDROMORPHOMETRICAL DATA (LOGARITHMIC) FROM CATCHMENTS IN THE WINTER RAINFALL REGION
($N = 23$, factor loadings $\geq$ 0,65 and $\leq$ –0,65 retained)

Variables	Factors					h_i^2(%)
	I	II	II	IV	V	
MAR					0,656	72,83
MAF					0,854	95,97
MA10					0,838	96,00
A				0,949		99,35
S85			0,920			89,11
D					0,819	72,86
P				0,821		98,56
W				0,798		99,11
LB				0,733		97,67
HM			– 0,771			95,00
T						93,80
RH			– 0,771			80,58
RHP			– 0,949			91,17
RF	– 0,976					96,60
RC	– 0,890					89,99
RE	– 0,976					96,62
C	0,888					89,90
MAP		0,779				69,53
I2,33		0,914				95,95
I10		0,907				89,37
Eigenvalue	7,00	4,59	3,74	1,71	1,06	
Explained Variance (Cumulative %)	35,01	57,98	76,66	85,19	90,50	

TABLE 6
SIMPLIFIED ROTATED FACTOR MATRIX OF PHYSIOGRAPHICAL DATA (LOGARITHMIC) FROM CATCHMENTS IN THE SUMMER RAINFALL REGION
($N = 25$, factor loadings $\geq$ 0,70 and $\leq$ –0,70 retained)

Variables	Factors			h_i^2(%)
	I	II	III	
A	0,899			98,10
S85		0,890		84,69
D		0,763		70,71
P	0,870			99,85
W	0,951			99,60
LB	0,771			96,77
HM		0,787		93,35
T				95,22
RH		0,868		88,52
RHP		0,876		93,37
RF			0,972	96,20
RC			0,752	78,82
RE			0,971	96,20
C			– 0,750	78,75
Eigenvalue	6,90	4,48	1,32	
Explained Variance (Cumulative %)	49,27	81,26	90,72	

TABLE 7
SIMPLIFIED ROTATED FACTOR MATRIX OF HYDROMORPHOMETRICAL DATA (LOGARITHMIC) FROM CATCHMENTS IN THE SUMMER RAINFALL REGION
($N = 25$, factor loadings $\geq$ 0,63 and $\leq$ –0,63 retained)

Variables	Factors				h_i^2(%)
	I	II	II	IV	
MAR	0,706				89,01
MAF	0,636				79,76
MA10	0,638				80,26
A	0,832				91,35
S85		0,846			76,92
D		0,766			71,86
P	0,833				93,65
W		– 0,689			70,01
LB	0,954				96,50
HM		0,741			93,56
T	0,782				94,17
RH		0,841			85,28
RHP		0,958			92,87
RF	– 0,886				86,94
RC	– 0,910				82,91
RE	– 0,882				87,15
C	0,909				89,90
MAP				0,770	68,61
I2,33			0,943		92,55
I10			0,888		86,38
Eigenvalue	8,85	5,15	1,66	1,36	
Explained Variance (Cumulative %)	44,25	69,95	78,26	85,08	

having loadings greater than or less than this value are considered. The selection of this critical factor loading is, to a large extent, based on the experience of the researcher. However, in most analyses, there is a sharp transition between variables having high factor loadings and those having low loadings which assists in the selection of the threshold value.

The results of each analysis include the eigenvalue (explained variance) for each factor as well as the cumulative percentage of total explained variance. The communality, h_i^2, gives the percentage of variance of the *i* th variable in the matrix of *m* factors.

The simplified rotated matrix for the physiographical data of cathments in the winter rainfall region is given in Table 4. This analysis reveals that there are three major physiographical dimensions which may be identified for catchments of this region. These dimensions are catchment shape (RF, RC, RE and C), size (A, P, W and LB) and relief (S85, HM, RH and RHP). Only factors with eigenvalues greater than one were included in Table 4, an arbitrary decision recommended by Seyhan (1981a). This practice was adopted to simplify the results since, theoretically, the total number of factors is equal to the number of variables.

Although drainage density (D) was not reflected as a factor in Table 4, there was evidence to suggest that D also constituted an independent factor. The three factors given in Table 4 explain a mere 23,2% of the variance of D (communality) with the rest of the variance being explained by the remaining factors.

By including the rainfall and runoff variables in the factor analysis for the winter rainfall region, the same independent physiographical dimensions are identified as in the preceding analysis viz. shape, length and relief (Table 5). Two additional factors are included in Table 5, these being the dimensions of rainfall (MAP, I2,33 and I10) and runoff (MAR, MAF, MA10 and D). Together, these five factors explain most of the variance contained in the correlation matrix (90,5%). Since drainage density is included in factor V (Table 5), it may be concluded that this is the most important independent variable associated with runoff in these catchments.

The factor analysis of physiographical data from catchments in the summer rainfall region isolates the same three principal dimensions as reported for the winter rainfall region (Table 6). However, in comparing the results of Table 4 with those of Table 6, two major differences are notable. First, drainage density (D) is correlated directly with the relief factor (II) in the summer rainfall region and secondly the order of the three factors for this region differs to the order in the winter rainfall region. This latter finding indicates that the relative importance of these factors in explaining the variance in the correlation matrix differs in these two regions. This is also evident in comparing the cumulative percentages of explained variance for each factor in the two regions. Furthermore, the three factors identified explain more variance in the physiographical data of the summer rainfall region (90,72%) than they do for the corresponding data of the winter rainfall region (79,64%).

Factor analysis of the hydromorphometrical variables (physiographical, rainfall and runoff) from catchments in the summer rainfall region (Table 7) resulted in a different classification to that for catchments in the winter rainfall region (Table 5). Factor I in Table 7 reflects the intercorrelation of runoff (MAR, MAF and MA10), catchment size (A, LB and P), shape (RF, RC RE and C) and the topographical factor (T). Factor II defines the association of catchment relief (S85, HM, RH and RHP), width (W) and drainage density (D). The independent dimensions of rainfall intensity (I2,33 and I10) and mean annual precipitation (MAP) are revealed in factors III and IV respectively. The four factors identified for the hydromorphometrical data of catchments in the summer rainfall region are, however, not as clearly definable as those described for the winter rainfall region.

Having identified the major physiographical and hydromorphometrical dimensions of the catchments in the two selected regions, attention is now turned to developing runoff equations for these two regions.

Regression analysis

The results of the Q-mode analysis indicated that the catchments of the winter rainfall region could be considered to be physiographically and hydromorphometrically distinct from the catchments of the summer rainfall region. Furthermore, the major hydromorphometrical factors established using factor analysis were also found to be different for these two regions. In view of these findings separate regression analyses were conducted for the winter and summer rainfall regions.

A fundamental requirement of regression analysis is that the independent variables should be completely independent. This constraint was satisfied by referring to the factors identified in the R-mode analysis and selecting a single independent variable from each factor. Using correlation analysis, multiple linear regression and step-wise multiple linear regression, a suite of equations was developed to calculate the mean annual runoff (MAR), mean annual flood (MAF) and 10-year return period flood (MA10) for catchments in the two regions. The standard error of estimate (S_y) and multiple correlation coefficient (r) for each equation was determined. The significance of the regression coefficients ($\beta_{1-\gamma}$) was determined using Student's t-test while the significance of the multiple correlation coefficient was calculated using an F-test (Snedecor and Cochran, 1972).

The significant runoff equations for the winter rainfall region are given in Table 8. The most significant equations for this region are for the mean annual flood (MAF) and 10-year return period flood (MA10). These peak discharges are principally a function of drainage density, catchment shape and rainfall intensity (Table 8). The multiple correlation coefficients for these peak discharge equations are all in excess of 99% significance and the significance of the regression coefficients is greater than 95%. However, the standard error of estimate values range from 34,1% to 54,5%.

The selected regression equations for the summer rainfall region are notably different to those of the winter rainfall region (Table 9). While the equations for calculating mean annual runoff are less significant than equations 6 and 7 in Table 9 for calculating peak discharges, the peak discharge equations have markedly higher standard errors of estimate than mean annual runoff equations 2 and 3 in Table 9. In this region MAR is expressed as a function of catchment shape, area, relief and mean annual precipitation and the more significant peak discharge equations express the dependent variable as a function of catchment area alone.

In developing equations to calculate runoff from catchment physiographical and rainfall variables, the results of this study have revealed differences in the variables controlling the runoff of catchments in the winter and summer rainfall regions of South Africa. This finding may be attributable to a number of factors. Firstly, the two regions have distinctly different climatic and geological characteristics (Table 1). Secondly, the catchments selected for the winter rainfall region were spread over a considerably smaller area than those selected for the summer rainfall region. Thus, greater geological and climatological homogeneity existed for catchments of the winter rainfall region. Finally, the

TABLE 8
REGRESSION EQUATIONS AND ASSOCIATED STATISTICS FOR CATCHMENTS OF THE WINTER RAINFALL REGION (N = 23)

	Regression Equation	r	Sy	Significance (%)	
				β	r
1.	$\mathrm{MAR} = (5{,}395 \times 10^{-4})(A)^{1{,}916}(S85)^{0{,}788}$	0,633	0,687	95,00	99,00
2.	$\mathrm{MAF} = (1{,}049 \times 10^{-4})(D)^{1{,}799}(I2{,}33)^{2{,}500}$	0,787	0,474	99,50	99,50
3.	$\mathrm{MAF} = (1{,}260 \times 10^{-4})(D)^{1{,}791}(RE)^{-2{,}003}(I2{,}33)^{2{,}781}$	0,839	0,429	97,50	99,50
4.	$\mathrm{MAF} = 2{,}358\,(D)^{2{,}150}(MAP)^{0{,}976}$	0,705	0,545	95,00	99,50
5.	$\mathrm{MA10} = (25{,}745 \times 10^{-4})(D)^{1{,}546}(I10)^{1{,}808}$	0,796	0,374	97,50	99,50
6.	$\mathrm{MA10} = (6{,}440 \times 10^{-4})(D)^{1{,}555}(RE)^{-1{,}527}(I10)^{1{,}945}$	0,843	0,341	97,50	99,50
7.	$\mathrm{MA10} = 1{,}104\,(D)^{1{,}817}(A)^{0{,}579}$	0,718	0,430	95,00	99,50

r = multiple correlation coefficient; Sy = standard error of estimate; β = regression coefficient

TABLE 9
REGRESSION EQUATIONS AND ASSOCIATED STATISTICS FOR CATCHMENTS OF THE SUMMER RAINFALL REGION (N = 25)

	Regression Equation	r	Sy	Significance (%)	
				β	r
1.	$\mathrm{MAR} = 0{,}112\,(RE)^{-3{,}801}(MAP)^{1{,}617}$	0,716	0,420	97,50	99,50
2.	$\mathrm{MAR} = 0{,}269\,(RE)^{-2{,}651}(MAP)^{1{,}668}(A)^{0{,}561}$	0,782	0,383	97,50	99,50
3.	$\mathrm{MAR} = 0{,}024\,(RE)^{-1{,}998}(MAP)^{1{,}279}(A)^{0{,}845}(S85)^{0{,}502}$	0,831	0,351	96,50	99,50
4.	$\mathrm{MAF} = 0{,}091\,(A)^{1{,}194}(MAP)^{1{,}137}$	0,707	0,489	90,00	99,50
5.	$\mathrm{MAF} = 0{,}093\,(LB)^{1{,}779}(MAP)^{1{,}207}$	0,714	0,484	90,00	99,50
6.	$\mathrm{MAF} = 0{,}093\,(A)^{1{,}371}$	0,681	0,496	99,95	99,50
7.	$\mathrm{MA10} = 0{,}120\,(A)^{1{,}371}$	0,692	0,563	99,95	99,50
8.	$\mathrm{MA10} = 0{,}118\,(A)^{1{,}395}(MAP)^{1{,}40}$	0,713	0,560	85,00	99,50
9.	$\mathrm{MA10} = 0{,}127\,(LB)^{2{,}060}(MAP)^{1{,}218}$	0,713	0,559	85,00	99,50

r = multiple correlation coefficient; Sy = standard error of estimate; β = regression coefficient

data obtained for catchments in the summer rainfall region were less reliable than those of the winter rainfall region, particularly for peak discharges. This latter point may account for the poor equations which were developed to calculate peak discharges in the summer rainfall region.

In both regions equations which did not include rainfall variables were not notably less significant than those equations which included these variables (Table 8 and Table 9). This finding suggests that physiographical variables are more important than rainfall variables in explaining the variance in runoff variables. The factor analysis also indicated that physiographical variables accounted for most of the variance in the correlation matrix.

Conclusions

The results of the three related analyses have been presented and discussed, namely, Q-mode analysis, R-mode analysis and regression analysis. From these analyses a number of conclusions may be drawn and summarised as follows:

- The catchments of the winter and summer rainfall regions are physiographically and hydromorphometrically distinct.
- In both regions the major physiographical dimensions are catchment shape, relief, length and drainage density.
- Physiographical variables appear to be more important than rainfall variables in explaining the variance in the runoff

variables. This observation is true for both the winter and summer rainfall regions and may be taken to indicate that the physiography of the catchments reflects, amongst other things, the climate which acted on them.

- Selected equations developed to calculate peak discharges (MAF and MA10) in the winter rainfall region are more significant than the equations developed to calculate the mean annual discharge (MAR). The reverse is true in the summer rainfall region.

While this study was conducted using a limited number of catchments and in some cases poor quality runoff data, the results obtained are considered to be highly satisfactory. The analyses have shown that multivariate techniques may be used beneficially to regionalise catchments and to assist in developing runoff equations. Finally, it may be concluded that the development of equations using physiographical and rainfall data could be a viable undertaking for other regions in South Africa.

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