Kinematic study of effects of storm dynamics on runoff hydrographs

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Abstract

Hydrologists frequently employ simplifying assumptions when using so called "design" storms for estimating runoff. In particular, uniform rainfall rates are often assumed as well as a uniform spatial distribution and a stationary storm. In this paper attention is brought to the variability of real rainfall rate in time and space, and to the movement of storms. The kinematic equations are employed to simulate the effect of these variables on runoff from a simple catchment. Generalized graphs indicating the effect of each variable on the peak runoff are included as well as guides to the selection of design storms to account for the storm dynamics and distribution.

Introduction

It is common practice to design stormwater systems for uniform intensity, uniformly distributed, stationary storms. Lack of data often makes any other basis for design difficult. There is little information available on instantaneous precipitation rates, storm cell size and cell movement. Time average precipitation rate or total precipitation depth can be predicted from intensity-duration-frequency curves (e.g. Van Wyk and Midgley, 1966) or equations such as that of Bell (1969). The most common method of abstracting data from rainfall records is to select a duration and calculate the maximum storm precipitation in that period. The so defined storm may include times of low rainfall intensity immediately preceding and succeeding a more intense precipitation rate.

Such simplifications in data render runoff calculation simplistic. Even when employing numerical models it is simplest to use a uniform intensity hyetograph for every point on the catchment. Although time varying storms are sometimes used, the precipitation pattern is seldom related to the maximum possible runoff rate.

Warnings have been made against simplification in rainfall patterns. For example, James and Scheckenberger (1983) indicated that storm movement can affect the runoff hydrograph significantly. Eagleson (1978) has expounded on the spatial variability of storms and Huff (1967) studied the time variability of storms.

Although much research has been done on storm variability, relatively little has been published on the resulting effects on runoff hydrographs. Research appears rather to have concentrated on models of particular (monitored) storms over particular catchments. The design engineer or hydrologist does not have sufficient guidance as to what storm pattern to design for. Presumably certain rainfall sequences, spatial variations and storm movements will result in a higher rate of runoff than other rainfall patterns for a particular catchment. Apart from an indication of what storm pattern produces the worst flood, one needs an indication of what storm pattern could be expected for the design catchment. Such data should be available on a frequency basis in order to estimate the likelihood of the worst hyetograph shape, spatial storm distribution and movements occurring. Although isolated catchments are being studied by the Water Systems Research Programme at the University of the Witwatersrand, considerably more information is required for the country as a whole. Analysis and use of such data in different combinations would require many trials before the worst storm patterns would emerge. An alternative approach is a deterministic one. Before calculating runoff, the analyst determines the following in order to select the correct design storm:

- the storm duration (for small catchments this is usually equated to the time of concentration of the catchment. Exceptions are irregular shaped catchments, and when losses are not proportional to rainfall (Stephenson, 1982));
- variation in precipitation rate during the storm;
- spatial distribution of the storm; and the
- direction and speed of movement of the storm.

The above information could be employed in modelling the design storm using numerical models. Alternatively, for minor structures, simplistic methods such as the Rational method could be employed. Since data shortage often limits the accuracy of modelling, the latter, manual approach, is often sufficiently accurate. The guides presented below may assist both the modeller by providing information on which design storm would produce the highest runoff rate and the formula orientated solution by providing factors to account for storm variability.

Storm patterns

Variation in rainfall intensity during a storm

In order to understand the reasons for and extent of variability (spatial and temporal) of rainfall, it is useful to describe the physical process of cloud formation and precipitation. The majority of storms in the interior of South Africa are convective, that is, the clouds originate from rising air masses. On the other hand, storms of winter and all year rainfall regions e.g. 65% of Cape Town's annual events are from frontal activity and 70% of Pietersburg's "extreme" annual events are caused by tropical and mass movements. The size and shape of the rising air mass depends on the topography and the air masses will usually be of smaller scale than the air mass which has been brought by advection and which contains sufficient moisture for raindrops to precipitate. Mader (1979) concluded from radar observations of storms in South Africa that storm areas, duration and movement were related to mean 500 mbar winds, thermal instability and wind shear.

Most recorded hyetographs indicate that rainfall intensity is highest somewhere in the middle of the storm duration. Huff (1967) presented extensive data on rainfall rates for storms of varying intensity indicating a time distribution somewhat between convex upward and triangular. In order to create a hyetograph which could be used for simple design of interconnecting stormwater conduits, Keifer and Chu (1957) proposed an exponential distribution termed the Chicago storm. The position of the peak intensity could be varied and was observed to occur at about 0,375 of the storm duration from the start for Chicago (Fig. 1).

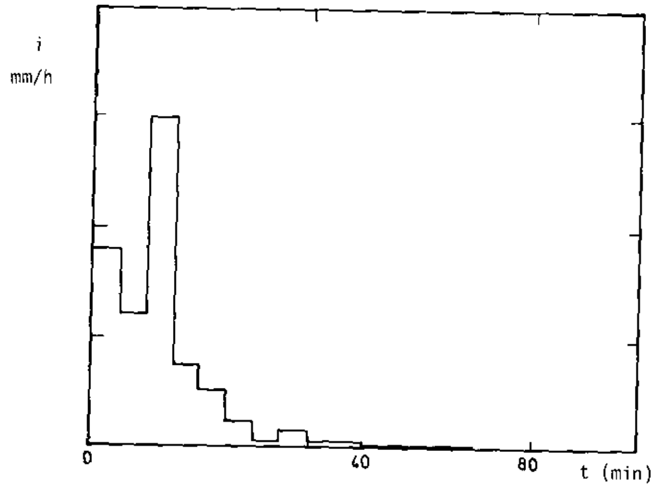

Figure 1
Hyetograph with peak near beginning.

Spatial distribution

The nature of storm cells within a potential rain area has been documented by many researchers e.g. Waymire and Gupta (1981). The persistence of storms observed in the Northern Hemisphere has not been found in South Africa however (Carte,

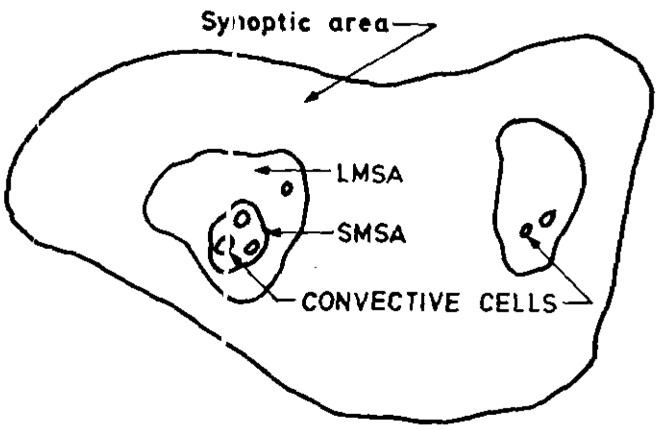

Figure 2
Typical areal distribution of a convective storm.

1979). The larger air mass within which storm cells occur is referred to as the synoptic area (see Fig. 2). The synoptic area can last for 1 to 3 days and the size is generally greater than 10^4 km^2. Within the synoptic area are large mesoscale areas (LMSA) of 10^3 to 10^4 km^2 which have a life of several hours. Sometimes small mesoscale areas (SMSA) of 10^2 to 10^3 km^2 can exist simultaneously. Within the mesoscale areas or sometimes on their own, convective cells, which are regions of cumulus convective precipitation, exist. These may have an areal extent of 10 to 30 km^2 and have an average life of several minutes to half an hour. These cells are of concern to the hydrologist involved in stormwater design. By comparing the storm cell size with the catchment size he can decide whether the cell scale is significant in influencing spatial distribution over the catchment. There may be overlapping cells which could result in greater intensity of precipitation than for single cells. Eagleson (1984) investigated the statistics of storm cell occurrences in a catchment and found the possibility of large storms can be computed assuming overlapping small storms.

The shape of the storm cell has significance for catchments larger than the cell. Scheckenberger (1984) indicates that the cells are elliptical which may be related to storm movement. The rain-

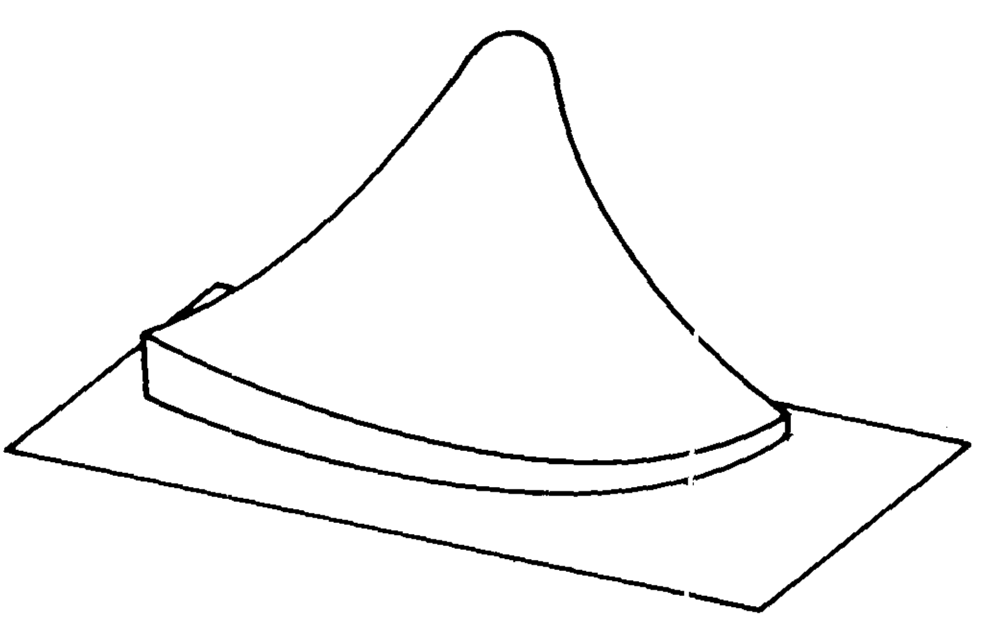

Figure 3
Illustration of spatial distribution of precipitation intensity.

fall intensity is highest at the centre and decreases outwards. The intensity has been shown to decrease exponentially, radially outwards from the focus, in various localities as in Fig. 3 (Colyer, 1982; Wilson *et al.*, 1979). Generally the variability in intensity does not necessarily cause higher runoff intensities, but on small catchments near the centre of the cell the average precipitation can be higher than for a larger catchment, and as a rule, the rainfall depth increases the smaller the storm area (Natural Environment Research Council, 1975).

It should also be noted that local effects, e.g. built-up areas, can influence the amount of precipitation. Huff and Changnon (1972) indicated that local thermal effects over urban areas can add 15% to the storm precipitation on the leeward side of the town.

Storm movement

Clouds generally travel with the wind at their elevation. Rain-bearing wind speeds can vary widely with speeds between 2 and 60 km/h not uncommon in South Africa. As the rain falls it goes through lower speed wind movements so that the most significant speed is that of the clouds. The direction of lower winds can also differ from the general direction of movement of the upper strata. This may be the reason Changnon and Vogel (1981) observed slightly different directions for storm and cloud movements. Dixon (1977) analysed storm data from Nelspruit and other areas and indicated storm cells have a circulation in addition to a general forward movement.

Numerical solutions

The effects of storm dynamics and distribution can be studied numerically and the results for simple plane catchments are presented below. The kinematic equations are employed in the numerical scheme. Although these solutions are no substitute for detailed catchment modelling when there are sufficient data, they do indicate which variables are likely to be the most important in storm dynamics. It must be pointed out that the following studies are simplified to the extent of assuming constant speed storms with unvarying spatial distribution. True storms are considerably more complex as explained in the above references.

Kinematic equations

The one-dimensional kinematic equations are for a simple plane catchment (Brakensiek, 1967):

the continuity equation $\frac{\partial y}{\partial t} + \frac{\partial q}{\partial x} = i_e$ and

flow resistance $q = zy^m$

y is water depth on the plane, q is discharge rate per unit width of plane, i_e is excess rainfall rate, t is time, x is longitudinal distance down the plane, z is assumed a constant and m is a coefficient. Employing the Manning discharge equation in SI units $z = \sqrt{S_o}/n$ where S_o is the slope of the plane, n is the Manning roughness coefficient, and m is 5/3. (Fig. 4).

The number of variables can be reduced to facilitate solution by re-writing the equations in terms of the following dimensionless variables:

$$X = x/L$$
$$T = t/t_c$$
$$I = i_e/i_a$$
$$Q = q/i_aL$$

where L is the length of overland flow, i_a is the time and space averaged excess rainfall rate and t_c is the time to equilibrium, or time of concentration, for an average excess rainfall i_a. Subscript c refers to time of concentration, d to storm duration, a to time and space average and p to peak. Then the following expression for t_c can be derived:

$$t_c = (L/zi_e^{m-1})^{1/m}$$

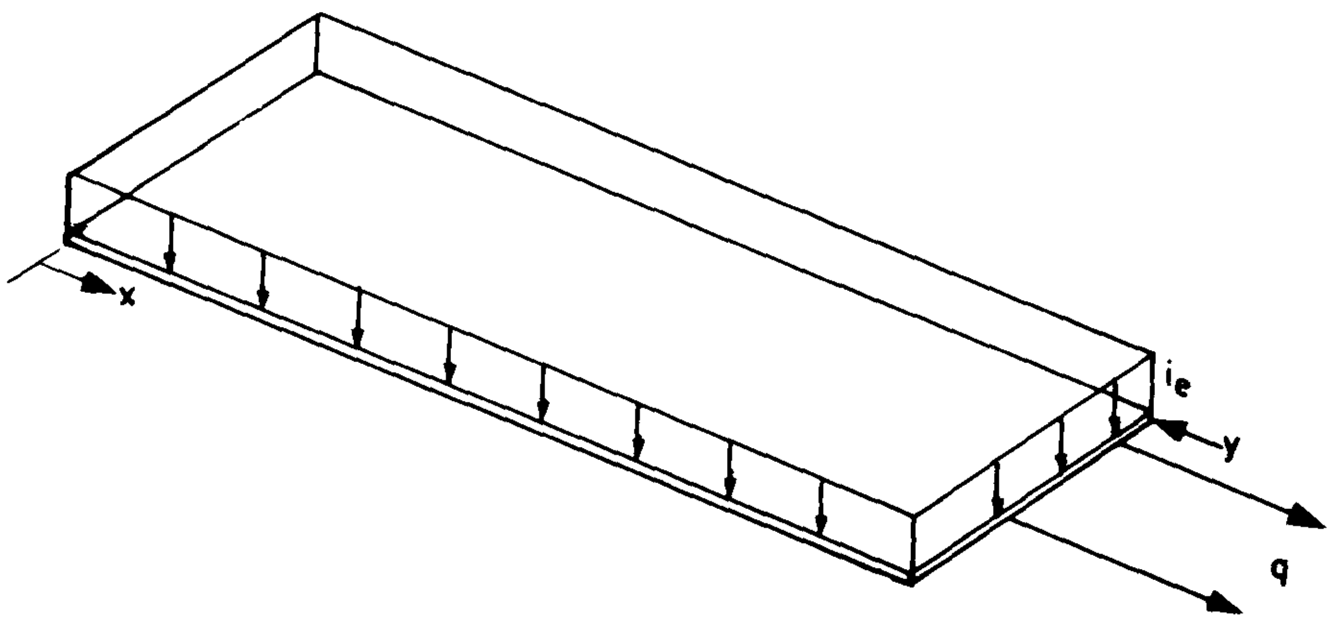

Figure 4
Plane rectangular catchment studied with storm.

In general the dimensionless variables are proportional to the dimensioned variables. Thus Q is the proportion of maximum flow at equilibrium. Substituting $y = (q/z)^{1/m}$ from the resistance equation and for X, T, I and Q from the equations for the dimensionless terms, the following equation replaces the continuity equation:

$$\frac{\partial Q}{\partial T} = 5/3\, Q^{2/5}\left(I - \frac{\partial Q}{\partial X}\right)$$

This single equation can be solved for Q in steps of T and X for various distributions of I.

Numerical scheme

Although it appears a simple matter to replace differentials by finite difference, there can be problems of convergence and speed of solution unless the correct numerical scheme is employed. The simplest finite difference schemes are explicit, employing values of Q at a previous T to estimate new values at the next time T. This method is not recommended as it is often unstable when discontinuities in rainfall intensity occur. Upstream differences are usually taken in such schemes, as downstream effects cannot be propagated upstream according to Huggins and Burney (1982). It is also necessary to limit the value of $\Delta T/\Delta X$ to ensure stability.

Woolhiser (1977) documented various numerical schemes including very accurate methods such as Lax-Wendroff's. Brakensiek (1967) suggested 3 schemes: four point, implicit and explicit. His second scheme (implicit) is adopted here as it is accurate and rapid for the examples chosen.

Employing the notation in the grid in Fig. 5:

$$\frac{\partial Q}{\partial x} = \frac{Q_1 - Q_2}{\Delta x}$$

$$\frac{\partial Q}{\partial T} = \frac{Q_1 + Q_2 - Q_3 - Q_4}{2\Delta T}$$

Since $\partial Q/\partial T$ is not sensitive to $Q^{2/5}$, (the power is less than one), $Q^{2/5}$ is approximated by $((Q_3 + Q_4)/2)^{2/5}$, i.e. an explicit form is employed here or else the resulting equations would be difficult to solve. The finite difference approximation to the differential equation is thus:

$$\frac{Q_1 + Q_2 - Q_3 - Q_4}{2\Delta T} = 5/3\left(\frac{Q_3 + Q_4}{2}\right)^{2/5}\left(I - \frac{Q_1 - Q_2}{\Delta X}\right)$$

solving for Q_1:

$$Q_1 = \frac{5/3\left(\frac{Q_3 + Q_4}{2}\right)^{2/5}\left(I + \frac{Q_2}{\Delta X}\right) + \frac{Q_3 + Q_4 - Q_2}{2\Delta T}}{\frac{1}{2\Delta T} + \frac{5/3}{\Delta X}\left(\frac{Q_3 + Q_4}{2}\right)^{2/5}}$$

Starting at the upstream end of the catchment where $Q_2 = 0$ and replacing Q_2 at the next point by Q_1 at the previous point, all the variables on the right hand side are known and one can solve for Q_1. The dimensionless time step used was 0,05. The difference for smaller time steps was found by trial to be unnoticeable.

Specific cases

Time varying storms

One of the most frequently used simplifying assumptions, but a dangerous assumption, in many rainfall-runoff models is that of constant precipitation rate throughout the storm duration. The temporal variation of precipitation intensity for storms over Illinois was documented by Huff (1967) whose findings are often extrapolated to other regions. He suggested identifying the quartile of maximum precipitation and further employing probabilities of the rain occurring sooner or later than the median. Huff (1967) plotted his results as mass rainfall curves so it is not easy to discern the shape of the hyetographs unless his curves are differentiated with respect to time. In general they are found to be convex upwards. Apart from Keifer and Chu's (1957) synthetic hyetograph, evidence points to convex up hyetographs. The assumption of a triangular hyetograph is thus extreme as a real storm would tend to be less 'peaky' than a triangular one. The general triangular-shaped rainfall rate versus time relationship depicted in Fig 6 is therefore studied. The time of the peak is varied between the start of the storm ($T_p = 0$) and the end ($T_p = 1$).

Simple models of hyetographs assume a single peak in rainfall intensity. Storms with multiple major peaks can be synthesized from overlapping compound storms. It is a single peak-storm which is considered here and the time of the peak intensity permitted to vary.

It was found by Sutherland (1983) that design storms for flood estimation generally peak in intensity in the first half of the storm. This is an alleviating factor in peak runoff, as indicated in Fig. 7. That is a plot of hydrographs from the simple catchment depicted in Fig. 4 with various hyetographs imposed, i.e. a rectangular hyetograph and triangular hyetographs with various peak times were employed. The ordinate in Fig. 7 is the discharge rate expressed as a fraction of the mean excess precipitation rate, and the abscissa is time as a fraction of the time of concentration for a uniform storm with precipitation rate equal to the mean rate over the storm for each of the triangular hyetographs.

It will be observed from Fig. 7 that if the storm intensity peaks in the first part of its duration ($T_p<0,5$) the peak runoff is less than that for a uniform storm of the same average intensity. This holds for peaks up to 80% of the duration after the commencement of rain. Only for the peak at the end of the storm (e.g. $T_p = 1,0$) does the peak runoff exceed that for a uniform intensity storm. Then the peak runoff is approximately 10% greater than for a uniform storm of the same duration.

If the storm duration is not equal to the time of concentration for a uniform storm however, the peak can be higher. Fig. 8 is for a storm of constant volume peaking at its termination ($T_p = 1$) and for durations represented by $T_d = 0,4$ to 1,2. It appears that the time to equilibrium for this type of hyetograph is the same as for a uniform storm. These hydrographs are for storms of equal volume so that the shorter duration storms are of a higher intensity than longer duration storms.

It should be recalled that all other hydrographs plotted are for a specified excess rate of precipitation. That is, if the hyetograph is uniform so are the abstractions. In practice, losses will be higher at the beginning of a storm, resulting in a late peak in excess rain even for a uniform precipitation rate. This has the same effect as a storm peaking in the latter part as it increased the peak runoff. The effect is compounded as a storm which peaks near the end will occur on a relatively saturated catchment so a greater proportion of the higher rate of rain will appear as runoff

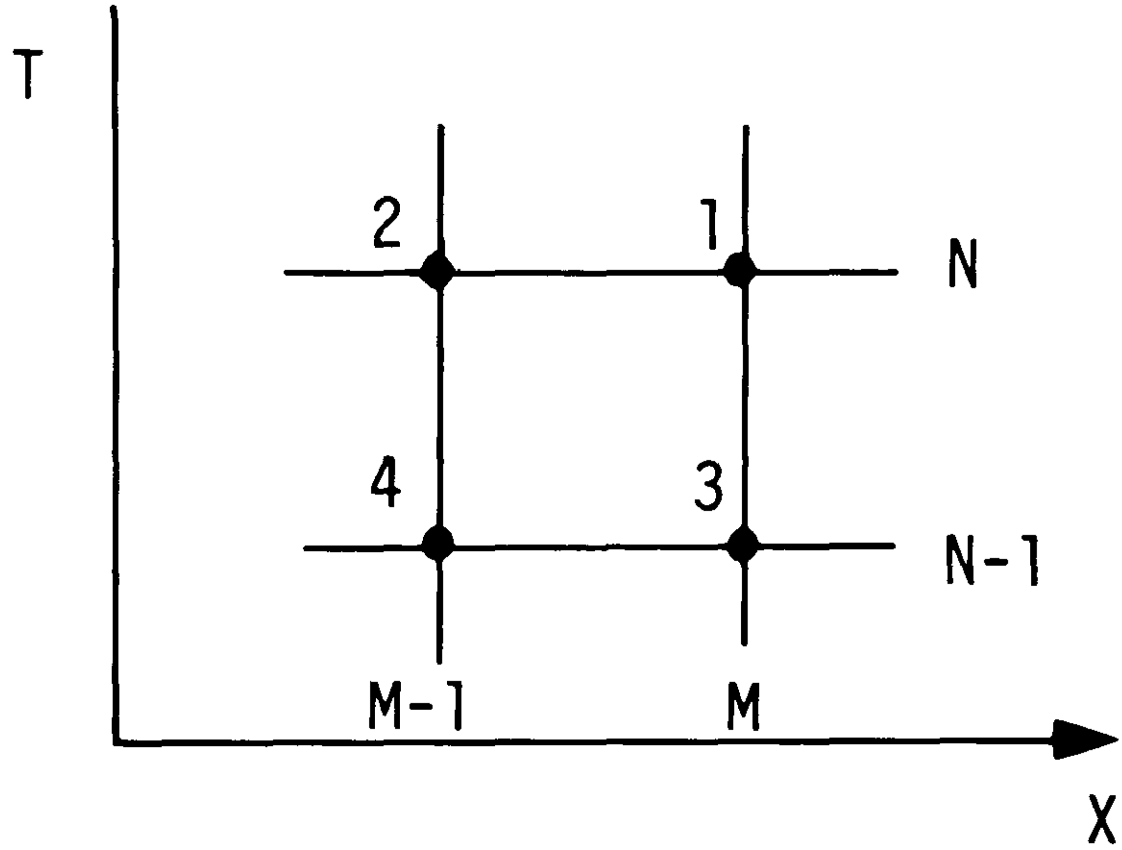

Figure 5
X-T grid employed in numerical solution.

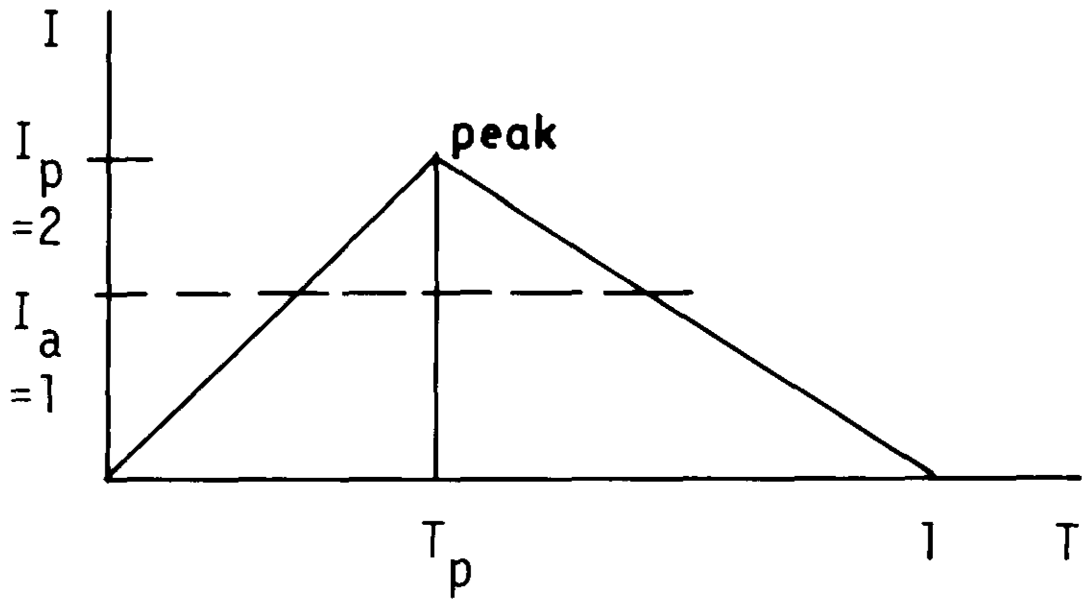

Figure 6
Temporally varying storm.

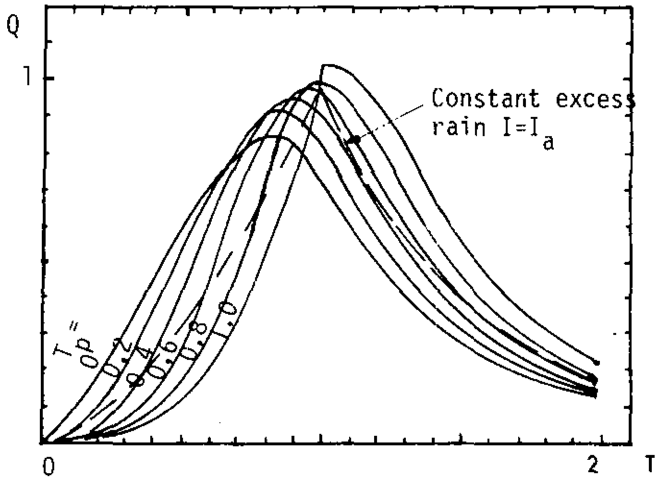

Figure 7
Simulated dimensionless hydrographs caused by storms with time varying rainfall intensities (Fig. 6) but the same total precipitation.

near the end. This tends to make the excess rain versus time graph concave upwards if the hyetograph was a straight-lined triangle. This effect is not modelled here but all the effects result in a higher peak than for a uniform input. Sheckenberger (1984) in fact indicates peaks up to 30% greater than for uniform storms due to the sum of these effects.

Spatial variations

It appears that areal distribution of the storm is less effective than temporal distribution in influencing peak runoff rate. Fig. 10 represents the simulated runoff from a 2-dimensional plane subjected to various distributions of a steady excess rain. The storm duration was made infinite in case the time to equilibrium exceeded the storm duration. The spatial (or longitudinal in this case) distribution was assumed triangular, the peak varying from the top to the bottom of the catchment as in Fig. 9.

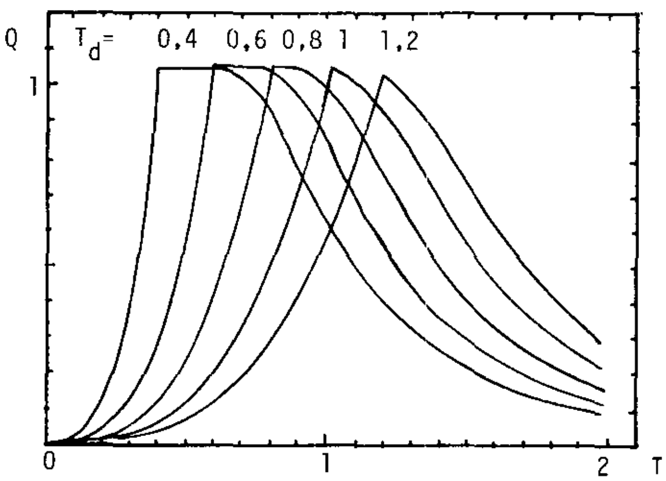

Figure 8
Simulated dimensionless hydrographs caused by late peaking storms of constant volume and varying duration.

The same example would apply to a uniform intensity storm over a wedge-shaped catchment, the catchment width increasing linearly to X_p and then decreasing linearly towards the outlet where X = 1.

Fig. 10 depicts the resulting simulated hydrographs which indicate that the runoff never exceeds that for a rectangular spatial distribution of rainfall.

The resulting dimensionless time to equilibrium is nearly unity for all cases, implying the same time of concentration holds for uneven distribution as for uniform distribution of rain. There is therefore not a chance of a shorter duration storm with a higher intensity contributing to a greater peak than the uniform storm (unless the intensity-duration curve is abnormally steep) since the time to equilibrium is not reduced relative to a uniform storm.

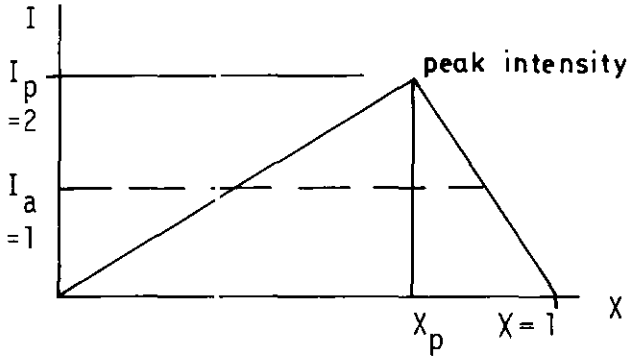

Figure 9
Catchment with longitudinally varying storm.

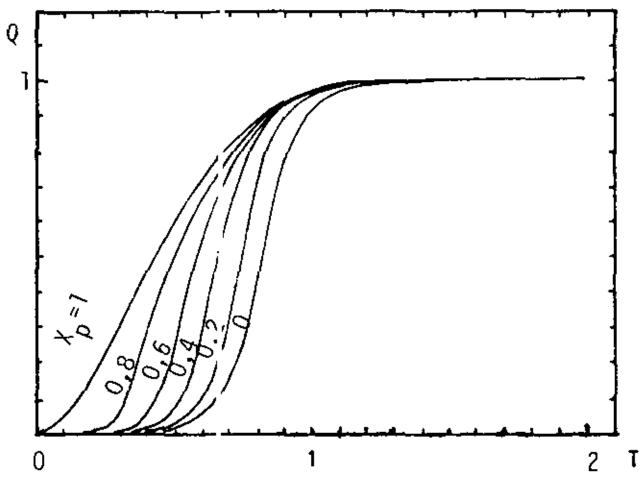

Figure 10
Simulated dimensionless hydrographs caused by steady semi-infinite storms of varying distribution down catchment (Fig. 9).

Moving storms

Fig. 12 represents simulated hydrographs from a storm with a constant precipitation rate and spatially uniform travelling down the catchment. The longitudinal extent of the storm cell is the same as the length of the catchment since in general smaller area storms are reputed to be more intense than larger cells. (Fig. 11) C is X/T_c or the speed divided by the rate of concentration. For slow storms ($C \leq 1$) the dimensionless hydrograph peak is unity while for faster storms the peak is less. The faster storms do not

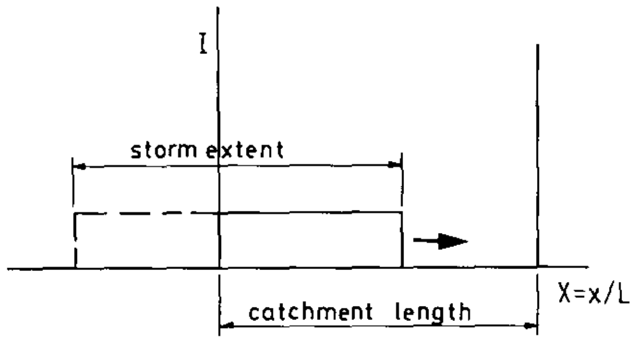

Figure 11
Catchment with a storm moving down it.

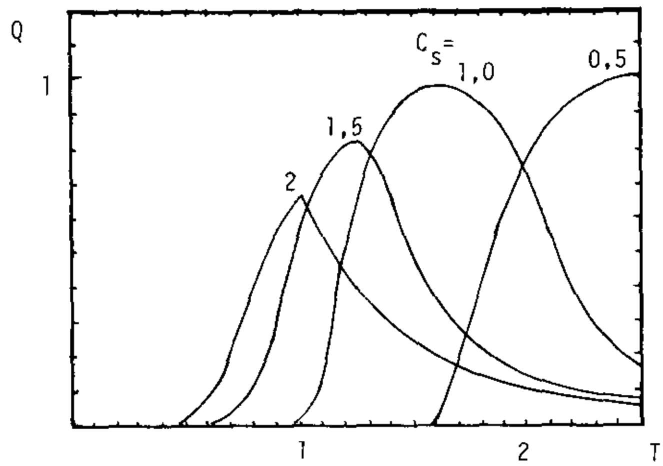

Figure 12
Simulated dimensionless hydrographs caused by unit steady uniform storms moving down catchment at different speeds (See Fig. 11).

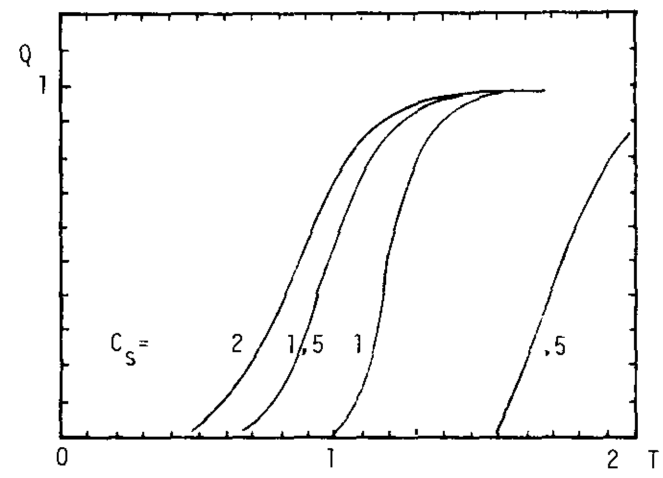

Figure 13
Simulated dimensionless hydrographs caused by steady uniform semi-infinite storms moving down catchment at different speeds.

fall on the catchment long enough to reach equilibrium.

Fig. 13 indicates there is also no increased peak for storms of semi-infinite longitudinal extent (never ending once they enter the catchment). All peaks converge on unity and there is no peak greater than unity. Thus movement does not appear to result in a hydrograph peak greater than for a stationary storm.

For storms of limited extent travelling up the catchment, the peak flow was observed to be less than for a stationary storm and the faster the speed of travel of the storm the smaller the peak runoff.

Conclusions

It has been demonstrated using numerical solutions to the kinematic equations for simple catchments that non-uniformity in rainfall intensity can affect peak runoff rates.

Temporal variation in excess precipitation rate can increase runoff rate above that for a steady rate of rain. Since storms usually peak sometime after commencing and time diminishing abstractions tend to cause a later peak in excess rainfall rate, the assumption of steady rainfall can be dangerous as peak runoff is underestimated.

Uneven spatial distribution of a storm does not directly contribute to higher peak runoff unless it results in a shorter duration storm being the design storm.

Storm movement reduces the peak flow unless the movement is down-catchment, when this model shows no change in peak runoff rate. A smaller, more intense storm than the one to equilibrium for the catchment may however result in a higher peak runoff rate.

The model did not account for time varying losses (abstractions and infiltration). Although the results are in generalized form (applicable to plane rectangular catchments of any size, slope or roughness), they may not be universally applicable as peak runoffs can occur for storms of limited lateral extent. The resulting theoretical hydrographs presented herein have not been verified.

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