Comparison of empirical settling-velocity equations in flux theory for secondary settling tanks

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Abstract

Secondary settling tank design and operation based on graphical application of solids flux theory is greatly facilitated by functionally relating zone settling velocity to solids concentration. Two mathematical expressions linking the settling velocity and solids concentration are widely used, i.e. the semi-log ($v_s = v_o e^{-nx}$) and log-log ($v_s = v_o^1 x^{-n^1}$) models. Acceptance of one or the other of these formulations in the flux theory leads to significant differences in the interpretation of the behaviour of settling tanks. This paper investigates the implications of using the two expressions in the flux theory for interpreting secondary settling tank behaviour and describes the differences between the predicted behavioural patterns. Theoretically, the semi-log expression yields a more constant and intuitively satisfying flux theory for secondary settling tanks. Statistical evaluation of 159 sets of experimental data (from the writers and literature) indicates that the semi-log expression gives a better correlation between settling velocity and sludge concentration than the log-log expression.

Nomenclature

A	surface area of settling tank (m^2)
e	natural logarithm base
G_s	solids flux ($kg.m^{-2}.d^{-1}$)
G_{ap}	applied flux ($kg.m^{-2}.d^{-1}$)
G_L	limited flux ($kg.m^{-2}.d^{-1}$)
n, v_o, n^1, v_o^1	experimentally derived constants which depend on the physical characteristics of the sludge
Q_i	influent wastewater flow to process (m^3/d)
Q_r	underflow recycle rate (m^3/d)
s	underflow ratio $= Q_r/Q_i$
u_o	overflow rate $= Q_i/A$
u_u	underflow velocity $= Q_r/A$
v_s	settling velocity (m/d)
x	sludge concentration (kg MLSS/m^3)
x_c	critical sludge concentration (kg/m^3)
x_L	limiting sludge concentration (kg/m^3)
x_o	operation MLSS sludge concentration in biological reactor (kg/m^3)
x_r	MLSS concentration in underflow recycle (kg/m^3)

Introduction

The importance of flux theory in settling tank design and operation was first established by Coe and Clevenger (1916) and Kynch (1952). This early work was performed on unflocculated suspensions. Although activated sludge is a flocculant suspension, it has been shown that the mass-flux concept also can be applied to activated sludge (Dick, 1970; 1972).

Application of the mass flux concept to the settling tank behaviour has been particularly assisted by the graphical method of Yoshioka *et al.* (1957). This method was developed from multiple batch settling test (Coe and Clevenger, 1916) for the purpose of obtaining the settling velocity as a function of concentration, upon which the criteria for settling tank area for the *thickening function* (solids transfer to the bottom of a tank – specifically not compression) is based.

Despite developments in the graphical flux theory, the procedure remains rather tedious, as for each concentration the settling velocity versus sludge concentration curve needs to be constructed to estimate the settling velocity.

In order to overcome the difficulties associated with the graphical representation of the flux theory, considerable research has been undertaken to find an empirical relationship between the settling velocity and sludge concentration. This relationship allows direct analysis of settling tank behaviour. Many mathematical expressions have been proposed, but by far the two most widely accepted are those of Dick and Young (1972) and of Vesilind (1968). However, the respective functional relationships proposed by the two authors result in different predictive behavioural patterns of the settling tank.

This paper is concerned with study of the two mathematical expressions and with their comparison. The applicability of the flux theory to secondary settling tank design and operation lies outside its scope and would need to be considered separately.

Physical characteristics of flux curves based on the two models

Model 1 Vesilind (1968)

$$v_s = v_o e^{-nx} \tag{1a}$$

Model 2 Dick and Young (1972)

$$v_s = v_o^1 x^{-n^1} \tag{1b}$$

where v_s = settling velocity (m/d)
x = MLSS concentration (kg/m)
v_o, n, v_o^1, n^1 = constants describing settling characteristics of sludge

Equation (1a) can be represented by a straight line on a log v_s vs x plot (semi-log, see Fig. 1a) i.e.

$$\log v_s = \log v_o - nx/2{,}303 \tag{2a}$$

Equation (1b) can be represented by a straight line on a log v_s vs log x (log-log, see Fig. 1b) i.e.

$$\log v_s = \log v_o^1 - n^1 \log x \tag{2b}$$

The flux, G_s, is defined as the product of the settling velocity, v_s, and concentration, x, i.e.

$$G_s = xv_s = xv_o e^{-nx} \tag{3a}$$

$$G_s = xv_s = xv_o^1 x^{-n^1} \tag{3b}$$

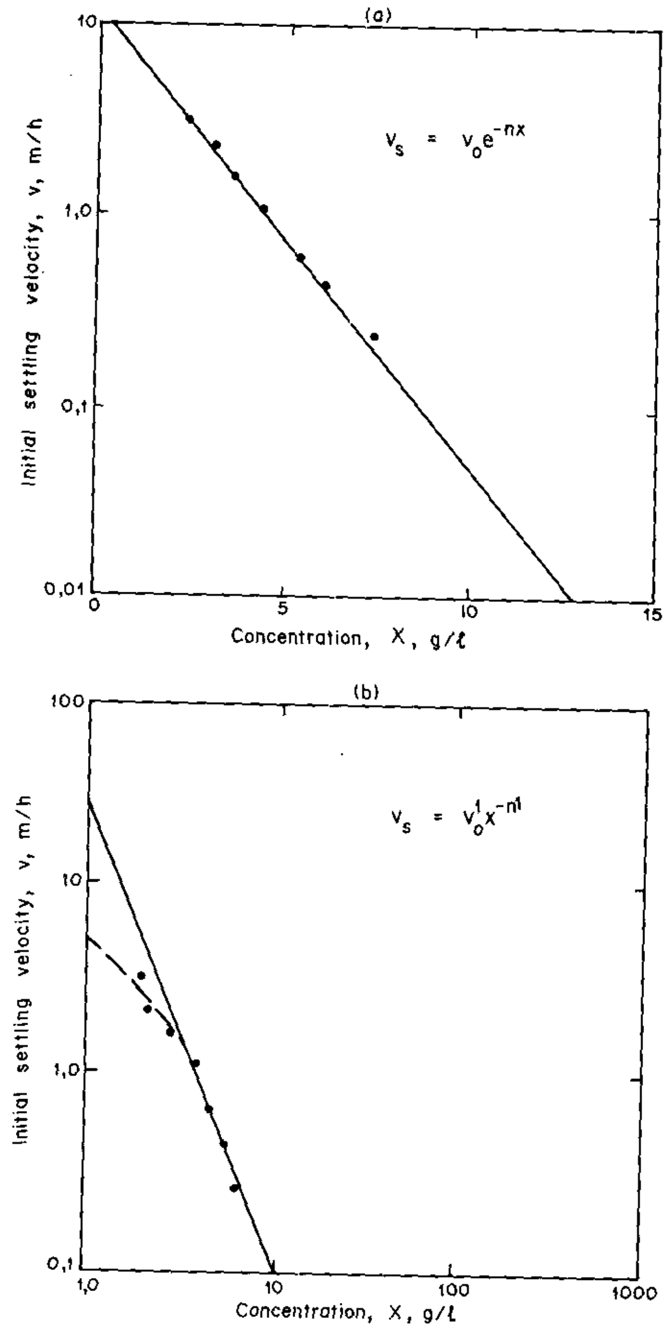

Figure 1
Settling velocity vs concentration (a) semi-log approach (b) log-log approach.

A diagrammatic representation of Equations (3a and b) is shown in Figure 2(a) and (b) respectively.

The slope of the flux curves is given by:

Model 1

$$dG_s/dx = v_o e^{-nx}(1 - nx) \tag{4a}$$

which gives turning point at x = + 1/n and a flux of v_o/ne. Differentiating Eq. (4a) and substituting 1/n for x show that the turning point of co-ordinates (1/n; v_o/ne) is a maximum. Setting the second derivative to zero shows that there is an inflection point in the flux curve at co-ordinates (2/n, $2v_o/ne^2$).

Model 2

$$dG_s/dx = v_o^1(1 - n^1)x^{-n^1} \tag{4b}$$

differentiating Eq. (4b) the nature of the turning point can be determined, i.e.

$$d^2G_s/dx^2 = -n^1 v_o^1 (1 - n)x^{-(1+n^1)} \tag{5b}$$

Equation (5b) is positive only when n^1 is greater than 1 with the result that a minimum is defined by $X \rightarrow \infty$. This is not contrary to expectation, because from observation, when x becomes large, v_s tends to zero. However, of greater importance is that when n > 1, as $x \rightarrow 0$, $G_s \rightarrow \infty$ which is clearly contrary to observation.

Graphical interpretation of settling tank loading state

Graphic application of the flux theory for interpreting the loading state of the settling tank can be obtained if it is assumed that the flux, G_s, – concentration, x, curve applies to the settling tank behaviour.

Using Yoshioka's graphic method, which was further developed by Keinath *et al.* (1976), the overflow rate $Q_i/A = u_o$ is superimposed on the flux-concentration curve (Fig. 3). The intersection of Q_i/A line with the mixed liquor concentration, x_o, defines the "state point". The line of a negative slope which passes through the state point represents the underflow rate, $Q_r/A = u_u$. The intersection of the underflow line with (i) the vertical axis, gives the applied flux $G_{ap} = x_o(Q_i + Q_r)/A$ and (*ii*) the horizontal axis, the underflow concentration, x_r (but only valid for underloaded cases).

An overloaded state of settling tank will develop if applied flux, G_{ap}, exceeds the maximum flux, G_L, that can be transmitted to the bottom of the tank, and thickened to the desired underflow concentration, x_r. As a result of $G_{ap} > G_L$, the sludge layer of limiting concentration, x_L, will build up until sludge spills over the weir. This type of failure is known as the thickening failure.

Under conditions of high underflow rates, a sludge layer of limiting concentration, x_L, does not necessarily form. This occurs when the underflow rate is so high that a minimum total flux (i.e. the sum of the gravity and bulk flux) does not arise. With the semi-log expression, this occurs when the underflow rate (u_u) is greater than the slope of the flux curve at the inflection point, i.e. (v_o/e^2) (Fig. 2), and with log-log model this occurs when the underflow rate (u_u) is greater than the slope of the flux curve at the critical sludge concentration x_c (Figs. 2 and 4). For these high underflows, a solids handling capacity criterion needs to be met to avoid failure of the settling tank. The solids handling criterion is met when the applied flux (G_{ap}) is equal to, or less than total combined gravity and bulk flux at the feed concentration. This leads to the conclusion that the overflow Q_i/A must be less than the settling velocity of sludge v_s, at feed concentration, x_o, i.e.

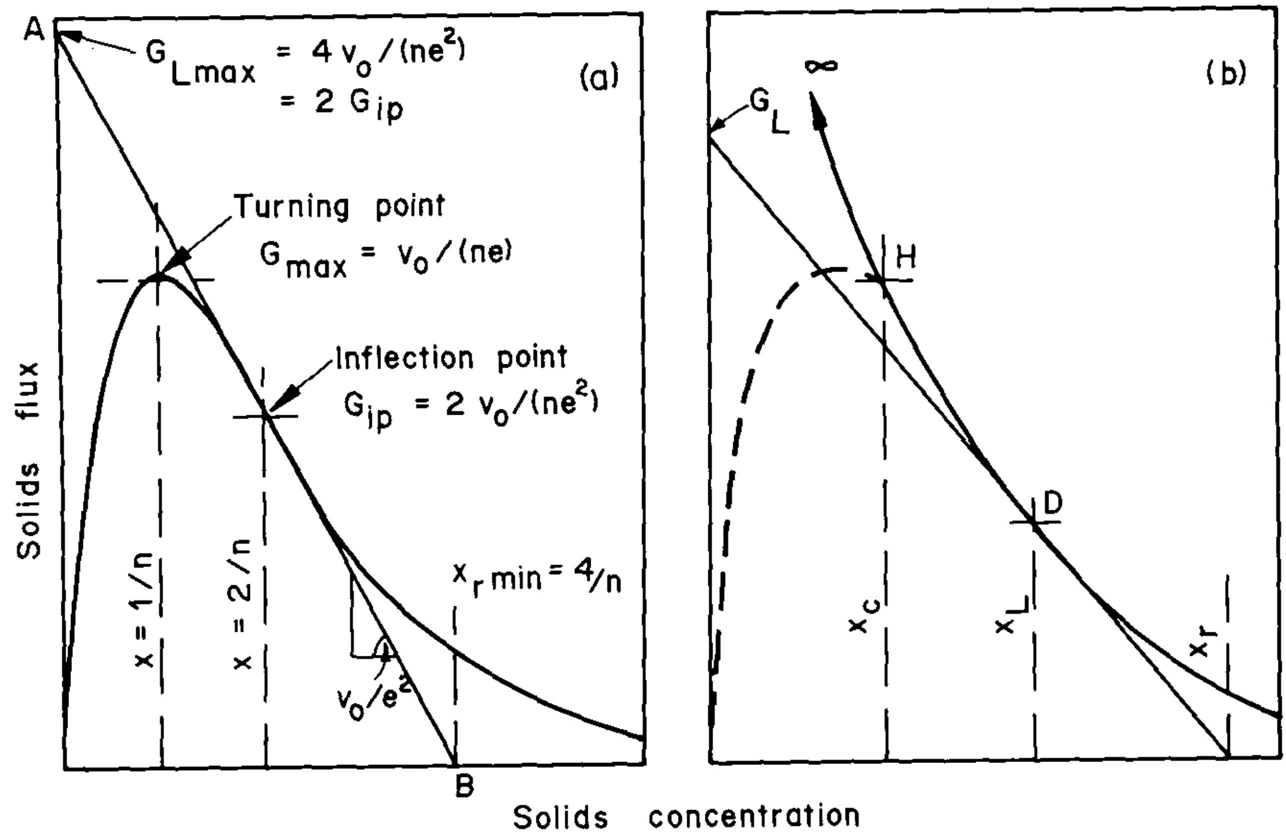

Figure 2
Mathematical properties of the flux curve based on the settling velocity-sludge concentration equations.
(a) $v_s = v_o \exp(-nx)$ (b) $v_s = v_o^l x^{-n^l}$

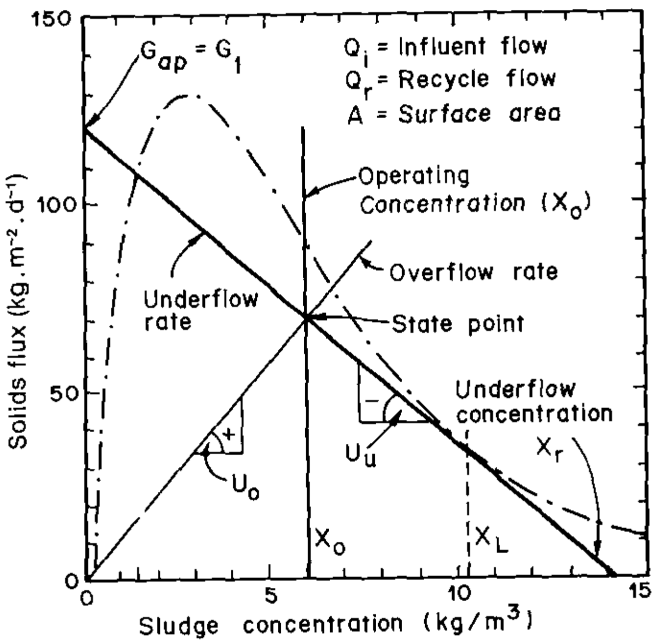

Figure 3
The gravity flux curve onto which is superimposed the settling tank behaviour; the state point defines the loading state of the tank and the underflow rate (u_u), overflow rate (u_o) and operating solids concentration (x_o) lines intersect at the state point.

G_{ap} < total gravity and bulk flux at feed concentration
$\therefore x_o (Q_i + Q_r)/A < (v_s \text{ (at } x_o) + u_u) x_o$
$\therefore Q_i/A < v_s$ at x_o

A third criterion, i.e. the clarification criterion also needs to be met to avoid failure of settling tanks. However, the clarification criterion is important only at severe changes in hydraulic load and then only for a short period of time, because, if the hydraulic load persists, the thickening or solids handling criterion will in time become the governing one (Laquidara and Keinath, 1983). Consequently in this paper, only the thickening and solids handling capacity criterion will be considered.

In terms of the graphical interpretation of settling flux theory, failure will occur in its thickening or solids handling function, depending on the state point and overflow and underflow lines on the flux-concentration curve, i.e. the following:

1. When the state point is within the envelope of the flux curve, the solids handling criterion is met and thickening governs the behaviour of the settling tank. When the underflow line

   - cuts the flux curve at one point only, safe operating conditions prevail;
   - cuts the flux curve at one point only, and is tangential

to the flux curve, critically loaded conditions prevail; and

- cuts the flux curve at 3 points, overloaded conditions prevail.

2. When the state point is on the gravity flux curve, critical conditions with respect to the solids handling criterion prevail. Under these conditions, thickening will also be critical if the underflow rate line is tangential to the flux curve on the state point; any underflow rate less than this will cause thickening failure.

3. When the state point is outside the envelope of the flux curve, solids handling failure conditions prevail (and thickening also, because conditions 1(i) and 1(ii) above cannot be met).

4. When the slope of the underflow line is too steep to make a tangent to the flux curve, but the state point is within the envelope of the flux curve, the thickening criterion no longer needs to be met and solids handling only governs the design.

Effects of mathematical differences on graphical analysis

Model 1 (Semi-log)

As illustrated in Fig. 2 (a) the curvature of the flux curve predicted by semi-log model allows an inflection point to be determined at co-ordinates ($x = 2/n$; $G = 2v_o/ne^2$).

From an inspection of the flux curve, no tangential underflow line can be constructed on the flux curve with a slope greater than that of the flux curve at the inflection point, and therefore, slope of the line A – B in Fig. 2 (a) represents the upper limit of underflow rate, for which the thickening function needs to be met (Ekama *et al.*, 1984). The intercepts of the tangent at the inflection point with the vertical and horizontal axes, give respectively, the maximum limiting flux (G_L max) and minimum underflow concentration (x_r min) for the thickening requirement to be met.

Model 2 (log-log)

As illustrated in Fig. 2 (b) the curvature of the flux curve does not allow an inflection point to be determined. Therefore, for the log-log model the upper limit for underflow rate must be subjectively assessed from a point by an arbitrary adjustment of the $G_s - x$ curve at the lower limit of sludge concentration (Fig. 4 (a) and (b)).

The line shown in Fig. 4 (a), deviates from the straight line conditions at point H, which corresponds with the critical concentration, x_c, which can be established by experimental observation and is a settling characteristic for a particular sludge (Fig. 1 (b)). The point H in Fig. 4 (b) describes the inflection point of flux curve, i.e. the tangent point, with the lowest x_L value.

Effects of the mathematical differences on design and operating charts

Model 1 (Semi-log)

For thickening, the limiting flux G_L defines the area required for the settling tank, i.e. the applied flux must be equal to or less than the limiting flux.

$$x_o (Q_i + Q_r)/A \leqslant G_L \tag{5a}$$

The mass balance over the settling tank under steady-state conditions, with no solids loss in the overflow (i.e. safe operational condition) is:

$$(Q_i + Q_r)\, x_o = Q_r x_r \tag{6a}$$

Defining, s, as the underflow recycle ratio, i.e.

$$s = Q_r / Q_i \tag{7a}$$

and substituting Eq. (6a) yields

$$x_r = (1 + s)\, x_o / s \tag{8a}$$

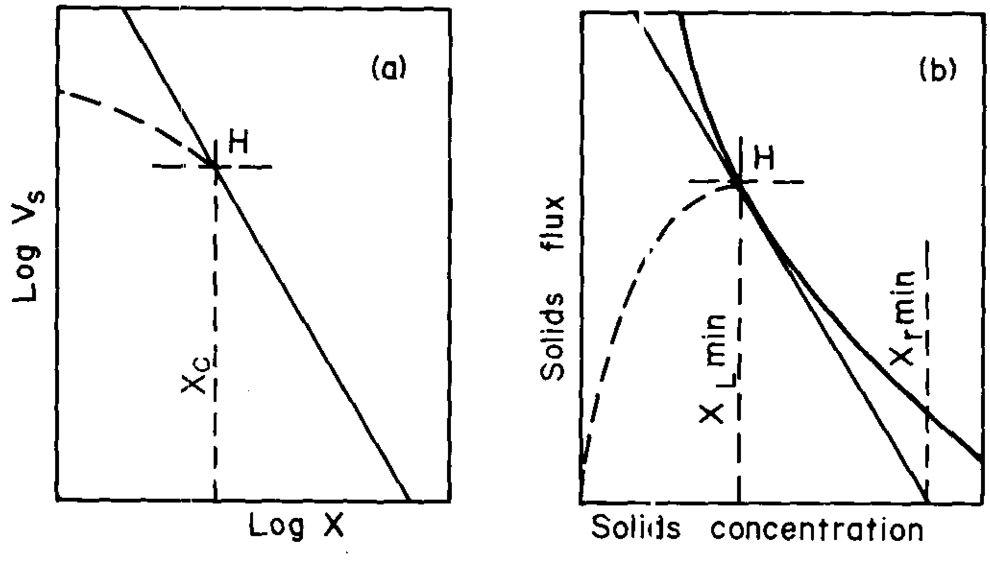

Figure 4
Adjustment of the flux curve based on log-log model.

The maximum overflow rate that ensures that all the solids reach the settler bottom, from Eqs. (5a, 6a and 8a) is

$$Q_i/A = G_L\{x_o(1+s)\} = G_L(x_r s) \quad (9a)$$

The equation of the underflow line with point of tangency to the flux curve is

$$G_s = G_L(1 - x/x_r) \quad (10a)$$

and its slope is

$$dG/d_x = -G_L/x_r \quad (11a)$$

At the point of tangency $x = x_L$, both the fluxes, the slopes of the flux curve and the underflow tangent are equal, i.e. Eq. (3a) = Eq. (10a) and Eq. (4a) = Eq. (11a). From these equations, taking x_r as known, solving for x_L and ignoring the impractical solution, yields

$$x_L = (x_r/2)\{1 + \sqrt{1 - 4(nx_r)}\} \quad (12a)$$

Equating Eqs (4a) and (11a), substituting Eq. (12a) for x_L and solving for G_L yields

$$G_L = v_o x_r \frac{(1+\alpha)}{(1-\alpha)} \exp\{-nx_r(1+\alpha)/2\} \quad (13a)$$

where $\alpha = \sqrt{1 - 4/(nx_r)}$

No solution for thickening function of a settler is possible for $x_r < 4/n$. Substituting Eq. (8a) for x_r into Eq. (13a) and substituting Eq. (13a) into Eq. (9a) yields

$$\frac{Q_i}{A} = \frac{v_o(1+\alpha)}{s(1-\alpha)} \exp\{-n(1+s)x_o(1+\alpha)/2s)\} \quad (14a)$$

where

$$\alpha = \sqrt{1 - 4s/\{n(1+s)x_o\}} \quad (15a)$$

Equation (14a) relates the overflow rate $Q_i/A = u_o$, to the recycle ratio s, for a selected value of sludge concentration, x_o, in the reactor, and the settling characteristics of the sludge expressed by v_o and n values to meet the thickening criterion. However, this is only possible if $x_r > 4/n$, i.e. for G_L values less than G_L max.

From Eq. (15a) if $\alpha = o$, $x_o = 4_s/(1+s)n$.

Hence from Eq. (15a) if $\alpha = o$

$$Q_i/A = v_o/e^2 s \quad (16a)$$

This equation is represented by a hyperbolic type curve which defines the limiting conditions imposed by the underflow rate line tangential to the flux curve, giving G_L max and x_r min. If Q_i/A is less than v_o/e^2s, then thickening and solids handling criteria need to be met simultaneously. When Q_i/A is greater than v_o/e^2s, the thickening criterion need not be met, only the solids handling capacity.

By plotting Q_i/A versus s (from Eqs. 5, 14a and 16a) for the different values of x_o, a steady-state design and operating chart for the settling tank can be obtained (Fig. 6). The hyperbolic type curve in the diagram described by Eq. (16a) distinguishes between the domains in which thickening is a governing criterion (area between axes and hyperbola) and the domain in which the solids handling criterion has to be met (area above the hyperbola).

The thickening criterion is given by Eq. (15a) which for different x_o values is a family of curves from the hyperbola to the origin. The solids handling criterion is given by Eq. (5) and being independent of s, is a family of horizontal lines. In the region below the hyperbola, the thickening and solids handling criteria have to be met simultaneously. In some instances in this region, the allowable overflow rate for thickening is higher than for the solids handling capacity to be satisfied, and therefore solids handling will become the governing criterion for the settling tank (see Q_i/A in area between s_1' and s_2' in Fig. 6). This arises when the state point is below the flux curve (see point B in Fig. 5) and two tangential lines can be drawn i.e. line s_1, tangential to the flux curve at point H, and line s_2, tangential to the flux curve at point D. The s_1 line represents the lower and s_2 line the higher limit of underflow rate allowable for Q_i/A, defined by state point B. The maximum overflow rate at x_o concentration is given when the state point lies on the flux curve (see point A). In this case only one underflow line, s_c, tangential to the flux curve at point A, can be drawn. The maximum overflow line, which passes through point A and the two tangential lines s_1 and s_2, in the accompanying diagram correspond with point A′, lines s_1' and s_2' in Fig. 6 respectively.

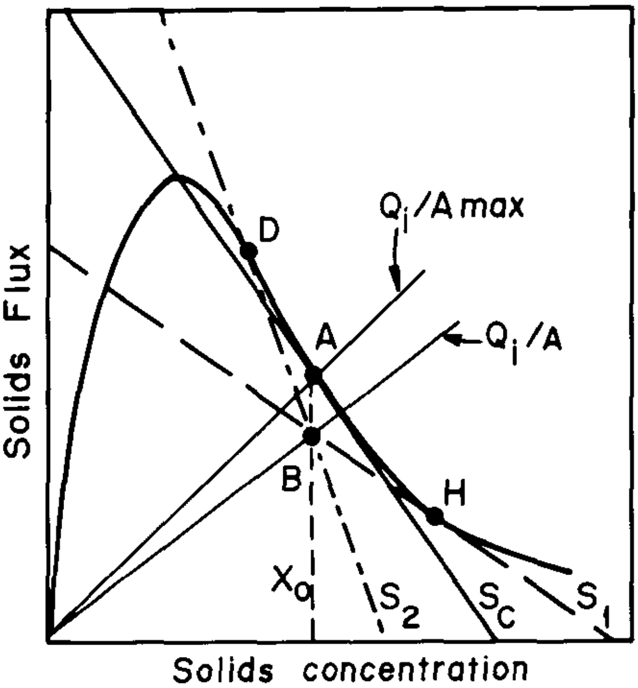

Figure 5
Graphical analysis of recycle ratio limitation.

Model 2 (log-log)

The thickening criterion for settling tank design is obtained by drawing a straight line from the underflow sludge concentration, x_r, tangential to the flux curve. The tangent point D (see Fig. 2b) defines the limiting concentration, x_L. The intersection with the vertical axis defines the limiting flux G_L. At point D, the flux of the tangent Eq. (10a) equals the flux of the flux curve Eq. (3b), and the slope on the tangent Eq. (11a), equals the slope of the

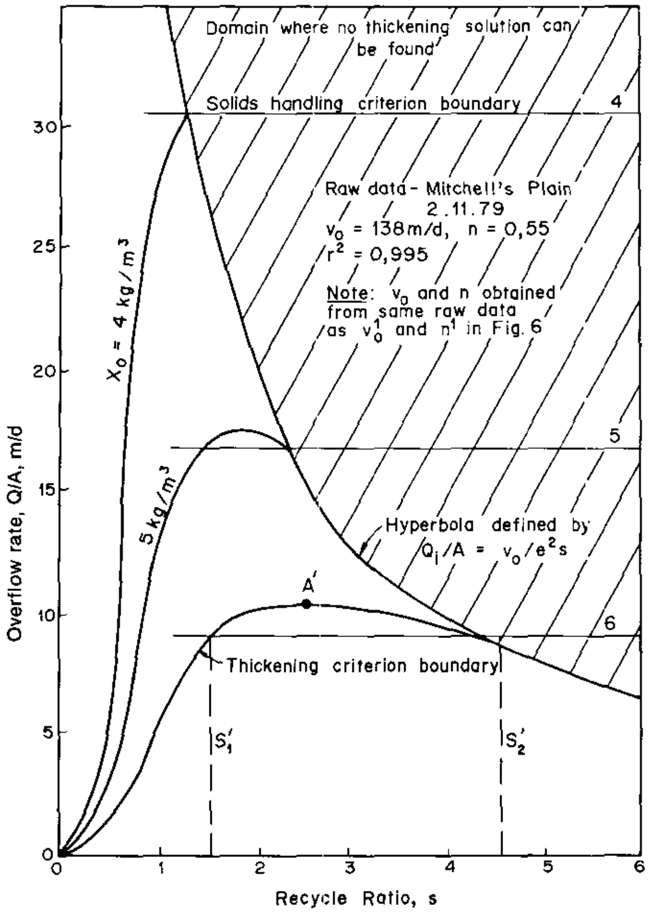

Figure 6
Diagram for settling tank design and operation for aerobic activated sludge from Mitchell's Plain sewage disposal works (1979-11-02). Semi-log approach.

flux curve (4b). From these equations, taking x_r as known and solving for x_L yields

$$x_L = \{(n^1 - 1)/n^1\}x_r \tag{6b}$$

Equating Eqs. (4b) and (11a), substituting Eq. (6b) for x_L and solving for G_L yields

$$G_L = v_o^1(n^1 - 1)\{n^1/(n^1 - 1)\}^{n^1} x_r^{1-n^1} \tag{7b}$$

Substituting Eq. (8a) for x_r into Eq. (7b) and substituting Eq. (7b) into Eq. (9a), the overflow rate is given by

$$Q_i/A = (v_o^1/s)(n^1 - 1)\{(n^1 s/\{x_o^1(n^1 - 1)(s + 1)\}\}^{n^1} \tag{8b}$$

The thickening criterion is expressed by Eq. (8b).

Differentiating Eq. (8b) with respect to s and setting to zero yields $s = n^1 - 1$.

Differentiating again and setting $s = n^1 - 1$, yields

$$d^2(Q_i/A)/ds^2 = -(n^1 - 1)^{(n^1-2)}/n^{1(n^1+1)} \tag{9b}$$

which has a negative solution provided $n^1>1$. (For biological sludges generally $n^1 > 1$). Hence, the turning point at $s = n^1 - 1$ is a maximum.

Substituting $s = n^1 - 1$ into Eq. (8b) yields expression for solids handling criterion, i.e.

$$u_o \leqslant Q_i/A = v_o^1 x_o^{-n^1} \tag{10b}$$

Equation (10b) indicates that the maximum overflow rate for thickening is obtained when the underflow recycle ratio $s = n^1 - 1$.

Based on Eq. (8b), a diagram for design and operation is contructed (Fig. 7). This diagram, similar to the diagram based on the semi-log model, relates the recycle ratio, s, the mixed liquor sludge concentration in the reactor x_o, and the settling characteristics, v_o^1, and n^1, to the overflow rate, Q_i/A.

Comparison of the two models

A comparison between the semi-log model and the log-log model for the settlement of biological sludges is undertaken in three parts:

- the comparison of the physical characteristics of the models;
- a comparison of a settling tank design based on each of the two models; and
- correlation coefficients of experimental data.

Comparison of the physical characteristics of flux curves based on the two models

The most important difference between the semi-log and log-log

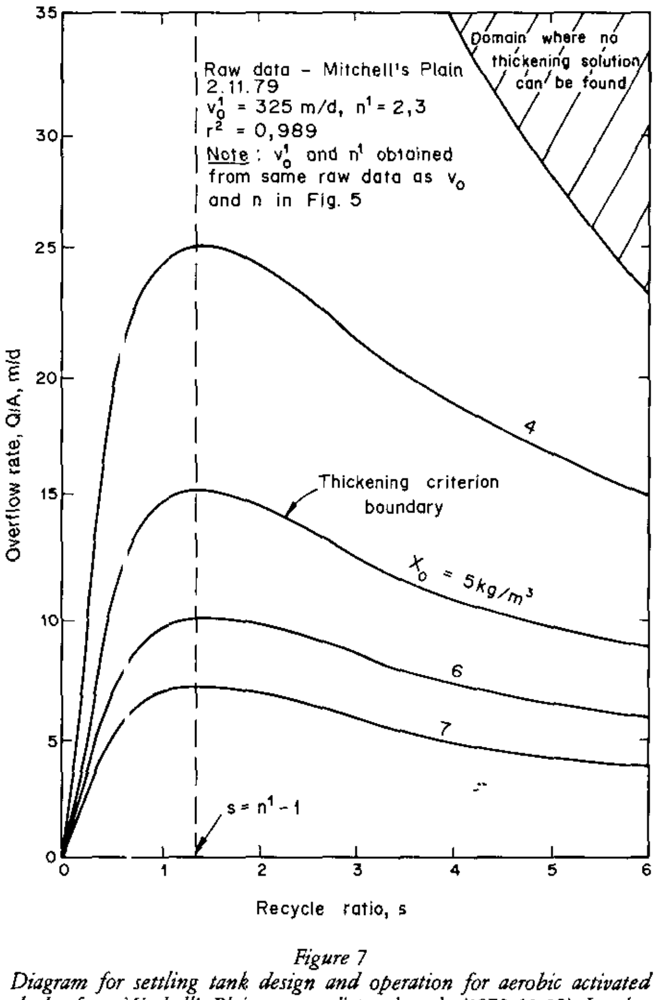

Figure 7
Diagram for settling tank design and operation for aerobic activated sludge from Mitchell's Plain sewage disposal works (1979-11-02). Log-log approach.

mathematical models is that in the semi-log application, the mathematical prediction of the slope of the curve covers the full range of concentrations from zero to infinity, whereas the log-log model applies only when the concentration value is greater than the critical concentration (x_c). This critical concentration is obtained by observation, and defines the point commencing from which the experimental settling velocity deviates from linearity on the log v_s vs log x plot (Fig. 4). Thus, in the log-log application of the model, the inflection point must be established empirically on the flux curve, whereas, in the semi-log model, the inflection point forms an integral part of the flux curve and allows for the feasible solution for thickening to be readily separated from non-valid solutions.

Comparison of settling tank design criteria

The two mathematical models show considerable differences when used to establish settling tank design criteria. In the semi-log model (Fig. 6), the overflow rate (and recycle ratio) beyond which thickening solutions cannot be found is coincident (or very close to) with the overflow rate found from the solids handling criterion. This led to the conclusion that above a certain recycle ratio, the thickening criterion needs not to be met, as the overflow rate is governed by the solids handling criterion. This criterion states that the overflow rate remains constant irrespective of the recycle ratio. In contrast, in the log-log model, the overflow rate (and recycle ratio) beyond which thickening solutions cannot be found is *not* coincident with the overflow rate found from the solids handling criterion (Fig. 7). This results in the need for the thickening criterion to be met for all practical recycle ratios and consequently the solids handling criterion is not the governing one over the entire recycle ratio range in the log-log model.

In the semi-log model the thickening criterion needs to be met up to a certain recycle ratio. No constraints are imposed on the recycle ratios in the domain where the solids handling criterion has to be met, as Eq. (9b) which defines this criterion, is independent of s (see a family of horizontal lines in Fig. 6). In practice, however, too high rates of sludge recirculation could result in turbulence which would cause deterioration in sludge settleability. This problem lies outside the scope of the flux theory and would need to be considered separately. The design solutions based on the semi-log model are explicit, and the instances where thickening is not important are well defined.

In the log-log model, the maximum overflow rate allowable is obtained when underflow recycle ratio is equal to $n^1 - 1$. Furthermore, for a fixed Q_i/A, a decrease or increase in recycle ratio from $s = n^1 - 1$ may result in an overloaded condition with respect to thickening. Depending on the magnitude of the changes, failure caused by decrease in recycle ratio is understandable, and is the same as predicted by the semi-log model. However, failure due to increase in recycle ratio predicted by the log-log model cannot be reconciled with the interpretation of the semi-log model, ignoring the practical turbulence factor which neither model recognizes.

Correlation coefficients of experimental data

In order to obtain experimental data for the evaluation of the two equations, 84 sets of batch settling test were undertaken (Tables 1 and 2). The samples were obtained from plants with very different settling characteristics, i.e. anaerobic-biological, aerobic-biological and chemical-biological processes (Smollen, 1981).

Batch settling tests were conducted in standardized stirred settling column, according to the procedure developed by White (1975). For each set of batch settling tests, the velocity concentration data were obtained at 4 to 16 concentrations in the range 0,5 to 17,0 g/ℓ, and these values were plotted in accordance with both the semi-log and log-log models.

Additional 75 sets of data were obtained from other sources. Table 3 contains data from Pitman (1980), Table 4 data from Tuntoolavest *et al.* (1980), Table 5 data from Rachwal *et al.* (1982) and Table 6 data from Pitman (1984).

Each set of data was analysed statistically by performing a linear regression analysis to give constant – v_o, n, v_o^1, n^1 and correlation coefficients, r^2. The superiority of the one approach over the other was assessed by comparing (r^2) values. Out of 74 sets of settling data obtained from tests carried out by the authors, 63 gave higher correlation coefficients, r^2, for the semi-log model

TABLE 1
DATA FROM SMOLLEN (1981)

Data set no.	Number of data	Concentration range gMLSS/ℓ	Exponential equation			Logarithmic equation		
			v_0	n	r^2	v_0^1	n^1	r^2
1	6	2,5 – 15,1	2,16	0,32	0,943	13,99	2,34	0,961
2	6	1,5 – 6,4	4,83	0,56	0,994	5,74	1,87	0,950
3	6	1,7 – 9,4	2,73	0,73	0,978	5,93	2,36	0,982
4	7	1,4 – 6,8	3,90	0,61	0,994	4,60	2,07	0,963
5	8	1,4 – 11,0	3,63	0,47	0,979	5,68	1,99	0,834
6	7	1,1 – 7,0	5,76	0,75	0,999	5,44	2,34	0,914
7	8	1,3 – 10,3	2,21	0,47	0,970	4,01	2,07	0,947
8	5	2,5 – 5,9	8,79	0,94	0,990	29,53	3,73	0,984
9	5	2,2 – 6,7	5,68	0,72	0,991	13,64	2,84	0,962
10	4	2,2 – 7,1	6,16	0,65	0,999	25,09	3,13	0,952
11	4	3,1 – 10,5	3,72	0,53	0,988	32,80	3,16	0,964
12	4	2,8 – 10,5	5,31	0,45	0,993	24,01	2,49	0,923
13	6	1,8 – 11,7	5,54	0,51	0,991	9,10	2,10	0,949
14	6	2,7 – 13,5	5,04	0,43	0,980	44,96	2,88	0,986
15	6	1,7 – 11,2	5,55	0,49	0,986	14,71	2,40	0,887
16	6	1,0 – 9,0	4,89	0,58	0,974	3,92	1,99	0,829
17	5	2,3 – 11,7	7,48	0,55	0,999	46,78	3,16	0,948
18	5	1,4 – 7,0	4,61	0,61	0,983	5,30	2,07	0,935
19	5	2,3 – 6,8	10,46	0,75	0,973	26,71	3,01	0,900
20	8	1,8 – 9,7	4,64	0,59	0,951	11,95	2,70	0,933
21	6	2,0 – 7,2	6,70	0,61	0,991	13,28	2,44	0,926
22	10	1,8 – 7,5	4,91	0,47	0,967	8,73	1,90	0,948
23	4	3,2 – 6,1	8,05	0,56	0,986	25,45	2,49	0,984
24	5	4,5 – 8,3	6,61	0,37	0,946	39,19	2,28	0,912
25	5	1,8 – 6,1	5,44	0,47	0,996	7,18	1,66	0,948
26	5	3,1 – 9,2	5,71	0,31	0,996	18,01	1,74	0,971
27	5	3,4 – 12,4	5,75	0,43	0,991	69,12	3,01	0,976
28	4	4,2 – 16,3	3,98	0,32	0,960	99,10	2,93	0,977
29	8	2,2 – 17,1	3,34	0,14	0,960	6,98	0,99	0,901
30	8	3,0 – 11,3	3,55	0,18	0,939	9,07	1,21	0,979
31	5	2,2 – 4,2	3,85	0,61	0,982	4,59	1,89	0,969
32	4	3,4 – 9,9	4,81	0,23	0,999	12,57	1,37	0,974
33	4	2,7 – 7,6	7,48	0,42	0,984	21,07	2,06	0,999
34	4	1,0 – 4,0	2,50	0,61	0,998	1,52	1,32	0,966
35	6	2,0 – 6,5	13,02	0,71	0,995	25,18	2,67	0,974
36	6	2,1 – 6,7	9,43	0,59	0,969	18,62	2,33	0,938
37	7	2,2 – 7,4	11,30	0,59	0,971	20,82	2,34	0,935
38	6	2,3 – 7,0	11,39	0,55	0,995	27,13	2,34	0,989

and 11 gave higher r^2 for the log-log model. The average correlation coefficient for the semi-log model was 0,979 and for log-log 0,954. Out of the 75 sets of data from literature, 70 gave higher r^2 for the semi-log model and 5 gave higher r^2 for the log-log model. The average correlation coefficient for semi-log model was 0,960 and for log-log 0,920.

Analysis of data obtained from the literature substantiates the conclusion that the semi-log equation fits the experimental data better than the log-log equation.

TABLE 2
DATA FROM EKAMA (1984)

Data set no.	Number of data	Concentration range gMLSS/ℓ	Exponential equation			Logarithmic equation		
			v_o	n	r^2	v_o^1	n^1	r^2
1	3	1,9 – 4,8	2,9	0,49	0,96	3,4	1,59	0,99
2	6	0,6 – 4,1	9,0	0,77	0,98	3,9	1,16	0,98
3	6	0,5 – 6,7	4,9	0,52	0,96	2,8	1,28	0,95
4	6	2,2 – 6,5	8,4	0,60	0,96	18,5	2,42	0,95
5	6	1,9 – 6,3	4,8	0,58	0,99	7,8	2,18	0,99
6	5	2,8 – 6,3	5,4	0,46	0,99	12,9	1,99	0,99
7	6	2,1 – 6,8	4,3	0,46	0,97	8,1	1,90	0,98
8	6	1,6 – 4,8	4,6	0,52	0,96	4,8	1,56	0,97
9	6	2,1 – 5,3	5,6	0,57	0,98	8,3	2,00	0,94
10	6	1,9 – 5,3	3,9	0,53	0,97	5,2	1,82	0,94
11	5	2,8 – 6,4	8,1	0,56	0,99	17,1	2,27	0,96
12	6	1,9 – 6,5	5,0	0,51	0,99	7,4	1,89	0,97
13	6	1,8 – 6,9	4,2	0,45	0,99	6,3	1,73	0,99
14	6	2,1 – 6,8	6,3	0,51	0,99	12,0	2,07	0,98
15	5	1,1 – 4,9	16,1	0,56	0,98	12,4	1,46	0,97
16	5	1,4 – 4,7	13,8	0,54	0,99	8,9	1,22	0,98
17	4	1,3 – 5,4	13,0	0,45	0,99	11,5	1,27	0,96
18	5	1,2 – 6,3	13,3	0,53	0,99	11,7	1,57	0,90
19	5	1,5 – 6,6	15,0	0,53	0,99	18,8	1,84	0,96
20	5	1,8 – 6,9	12,3	0,40	0,99	17,5	1,52	0,97
21	5	1,4 – 5,8	13,0	0,53	0,98	13,4	1,64	0,97
22	5	0,8 – 5,9	8,7	0,43	0,99	6,4	1,13	0,97
23	5	0,4 – 3,2	6,1	0,47	0,99	3,3	0,62	0,95
24	5	1,0 – 7,8	8,6	0,48	0,99	8,1	1,58	0,92
25	5	1,2 – 7,9	8,0	0,50	0,99	8,7	1,75	0,93
26	5	0,9 – 7,3	7,9	0,49	0,99	6,7	1,51	0,91
27	5	1,1 – 6,9	5,5	0,42	0,96	5,6	1,36	0,98
28	8	1,9 – 6,5	9,2	0,65	0,99	17,4	2,46	0,98
29	6	1,9 – 5,9	6,9	0,58	0,99	10,5	2,07	0,99
30	6	2,0 – 6,2	7,9	0,58	0,99	13,4	2,16	0,98
31	6	1,4 – 5,6	8,6	0,60	0,99	8,3	1,80	0,97
32	6	2,3 – 6,0	6,4	0,55	0,99	12,8	2,17	0,99
33	6	1,5 – 3,0	10,5	0,67	0,99	7,3	1,46	0,99
34	6	1,5 – 8,3	4,6	0,51	0,98	7,5	2,07	0,99
35	16	1,7 – 9,7	6,97	0,37	0,974	15,20	1,83	0,891
36	7	1,3 – 7,2	7,10	0,52	0,962	8,58	1,81	0,910
37	7	1,4 – 6,5	5,69	0,53	0,945	6,60	1,78	0,913
38	6	1,3 – 6,1	4,34	0,56	0,972	4,42	1,76	0,959
39	8	1,7 – 8,2	5,58	0,30	0,982	8,04	1,26	0,978
40	5	1,9 – 6,7	4,20	0,28	0,997	5,40	1,07	0,972
41	4	2,1 – 5,9	6,22	0,41	0,982	8,68	1,49	0,961
42	5	2,7 – 6,9	8,06	0,44	0,952	18,38	1,96	0,918
43	8	2,7 – 3,4	4,41	0,27	0,941	5,75	1,05	0,840
44	9	2,4 – 13,7	6,70	0,36	0,969	42,2	2,38	0,953
45	9	3,1 – 11,2	6,29	0,39	0,993	35,1	2,38	0,977
46	11	2,5 – 13,5	9,59	0,39	0,973	58,7	2,55	0,953

TABLE 3
DATA FROM PITMAN (1980)

Data set no.	Number of data	Concentration range gMLSS/ℓ	Exponential equation			Logarithmic equation		
			v_o	n	r^2	v_o^1	n^1	r^2
1	14	3,1 – 15,2	7,43	0,27	0,934	21,39	1,70	0,871
2	5	1,3 – 9,0	6,73	0,35	0,994	7,92	1,36	0,941
3	[illegible]	1,0 – 10,7	8,84	0,35	0,978	11,68	1,49	0,942
4	3	1,4 – 8,0	6,43	0,32	0,943	8,75	1,30	0,943
5	5	2,0 – 10,4	7,48	0,31	0,971	15,82	1,58	0,995
6	7	1,7 – 10,7	11,89	0,27	0,995	21,44	1,39	0,960
7	10	1,3 – 11,6	9,15	0,29	0,953	16,48	1,46	0,948
8	10	1,6 – 13,5	8,53	0,27	0,974	18,58	1,49	0,965
9	10	1,6 – 12,8	7,68	0,28	0,982	17,36	1,53	0,950
10	16	0,7 – 9,8	8,04	0,35	0,969	9,10	1,26	0,830
11	56	0,4 – 11,3	9,40	0,28	0,916	11,21	1,12	0,796
12	54	0,6 – 10,4	8,89	0,28	0,892	13,84	1,25	0,855
13	34	0,4 – 12,8	7,89	0,25	0,949	13,60	1,22	0,825
14	32	0,9 – 10,5	8,26	0,29	0,949	15,67	1,45	0,936
15	30	0,8 – 12,2	11,54	0,26	0,911	13,75	1,07	0,867
16	32	0,5 – 6,6	9,15	0,28	0,922	15,94	1,39	0,901
17	21	0,5 – 5,3	9,24	0,29	0,917	18,94	1,49	0,912
18	40	0,9 – 7,9	8,39	0,44	0,935	17,79	1,87	0,950
19	24	2,0 – 5,5	6,98	0,31	0,852	7,86	1,06	0,935
20	25	1,4 – 5,6	9,31	0,53	0,929	10,08	1,67	0,884
21	28	1,2 – 4,2	5,34	0,55	0,908	4,02	1,33	0,881
22	31	1,3 – 6,0	6,97	0,61	0,879	7,55	1,88	0,861
23	26	1,1 – 5,7	4,18	0,45	0,920	3,80	1,27	0,841
24	26	1,2 – 5,3	7,55	0,48	0,979	6,52	1,31	0,963
25	25	1,4 – 5,3	6,15	0,45	0,962	6,07	1,31	0,912

TABLE 4
DATA FROM TUNTOOLAVEST *et al.* (1980)

Data set no.	Number of data	Concentration range gMLSS/ℓ	Exponential equation			Logarithmic equation		
			v_o	n	r^2	v_o^1	n^1	r^2
1	4	1,3 – 3,8	17,8	0,58	0,997	12,6	1,33	0,956
2	4	1,8 – 3,8	28,3	1,00	0,999	25,6	2,71	0,984
3	4	1,5 – 3,8	13,9	0,43	0,970	15,5	1,04	0,937
4	4	1,3 – 3,6	13,3	0,85	0,957	7,38	1,83	0,898
5	4	1,2 – 3,4	10,08	0,79	0,993	5,58	1,63	0,978
6	4	2,3 – 6,6	14,22	0,46	0,997	26,64	1,88	0,955
7	4	2,4 – 5,6	17,64	0,54	0,990	30,78	2,03	0,983
8	4	2,3 – 6,0	22,32	0,42	0,995	35,46	1,62	0,972
9	4	2,4 – 8,5	18,18	0,41	0,987	46,26	1,99	0,961
10	4	2,2 – 7,3	18,72	0,41	0,993	32,94	1,70	0,944
11	4	2,6 – 6,2	9,00	0,34	0,980	15,12	1,42	0,983
12	4	3,2 – 5,6	20,70	0,58	0,999	62,28	2,52	0,989
13	4	2,8 – 7,9	12,78	0,38	0,985	32,58	1,84	0,995
14	4	2,1 – 6,6	15,84	0,47	0,987	26,82	1,86	0,974
15	4	2,3 – 6,7	14,04	0,43	0,993	24,84	1,75	0,954

TABLE 5
DATA FROM RACHWAL *et al.* (1981)

Data set no.	Number of data	Concentration range gMLSS/ℓ	Exponential equation v_o	a	r^2	Logarithmic equation v_o^1	n^1	r^2
1	83	0,6 – 9,8	6,50	0,36	0,963	–	–	0,916
2	310	1,2 – 11,0	5,83	0,37	0,959	–	–	0,937
3	148	1,5 – 9,6	5,30	0,41	0,937	–	–	0,900
4	168	1,5 – 8,3	4,23	0,42	0,962	–	–	0,942
5	733	0,6 – 11,0	5,18	0,38	0,873	–	–	0,867

TABLE 6
DATA FROM PITMAN (1984)

Data set no.	Number of data	Concentration range gMLSS/ℓ	Exponential equation v_o	n	r^2	Logarithmic equation v_o^1	n^1	r^2
1	13	0,8 – 10,6	7,02	0,40	0,993	8,85	1,66	0,933
2	16	1,0 – 11,0	9,30	0,36	0,994	10,97	1,40	0,890
3	11	1,4 – 12,3	18,13	0,29	0,992	18,14	1,60	0,940
4	12	2,4 – 11,9	9,49	0,29	0,992	14,17	1,36	0,882
5	13	1,6 – 9,4	9,59	0,33	0,992	14,08	1,46	0,910
6	11	2,3 – 12,2	8,22	0,31	0,994	30,67	1,94	0,972
7	13	1,5 – 11,7	9,07	0,34	0,997	12,19	1,46	0,888
8	7	0,8 – 10,8	9,84	0,34	0,997	8,99	1,20	0,812
9	9	1,2 – 11,2	9,79	0,34	0,996	12,59	1,52	0,906
10	8	1,5 – 11,6	6,90	0,30	0,986	16,60	1,66	0,940
11	9	1,2 – 6,7	7,44	0,47	0,993	7,77	1,51	0,968
12	10	1,3 – 7,5	7,73	0,43	0,993	9,49	1,54	0,952
13	9	1,3 – 11,9	6,35	0,35	0,997	14,47	1,65	0,933
14	10	1,9 – 11,4	7,46	0,35	0,995	18,38	1,85	0,949
15	10	2,5 – 12,9	9,85	0,27	0,996	33,06	1,76	0,919
16	10	1,6 – 8,7	8,07	0,39	0,994	12,38	1,65	0,936
17	8	1,2 – 7,4	4,76	0,33	0,962	5,00	1,09	0,945
18	8	0,9 – 7,5	7,35	0,33	0,975	6,74	1,03	0,923
19	8	1,5 – 8,0	9,02	0,33	0,986	11,58	1,26	0,961
20	8	1,0 – 7,2	7,52	0,44	0,993	11,72	1,75	0,943
21	8	0,5 – 8,1	5,34	0,32	0,950	4,13	0,89	0,875
22	8	1,3 – 10,2	3,04	0,23	0,977	6,26	1,24	0,957
23	8	1,2 – 9,2	5,41	0,37	0,981	6,74	1,40	0,946
24	7	0,8 – 5,1	6,59	0,41	0,985	4,86	0,99	0,969
25	7	1,2 – 5,6	4,73	0,38	0,976	4,45	1,09	0,961
26	7	1,1 – 9,4	9,17	0,46	0,985	10,70	1,72	0,821
27	8	1,5 – 7,8	7,39	0,38	0,993	9,06	1,39	0,943
28	8	0,7 – 6,8	6,85	0,38	0,984	5,20	1,02	0,888
29	8	2,0 – 5,9	6,81	0,56	0,997	11,00	2,09	0,981
30	8	1,0 – 5,7	7,31	0,41	0,982	6,48	1,21	0,964

Conclusions

The relationship between settling velocity and solids concentration as represented by the equation

$$v_s = v_o e^{-nx}$$

fits best the experimental data under consideration and leads to a more intuitively satisfying flux theory model for analysing secondary settling tank behaviour.

The applicability of the flux theory incorporating the semi-log expression as a descriptive model for secondary settling tanks is a matter not addressed in this paper. Verification of the flux theory incorporating the semi-log expression is considered in detail by Ekama and Marais (1984). From an analysis of extensive investigations of the performance of full-scale settling tanks (diameter >30 m) in the Netherlands (Stofkoper and Trentelman, 1982) and Great Britain (White, 1975; Rachwal *et al.*, 1982) it appears that the flux theory overpredicts the maximum permissible solids loading by about 20%. The above authors speculated that over prediction is not due to a deficiency in the basic flux theory, but due to deviations in the basic flux pattern of full-scale tanks (both horizontal and vertical) from that assumed in the ideal settling tank (vertical only).

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