The problem of sea-water intrusion near Lake Mzingazi at Richards Bay

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Abstract

The problem of sea-water intrusion into the Mzingazi flood plain at Richards Bay was surveyed. Steady state analytical models and a two-dimensional vertical collocation numerical approximation were applied in the study.

No serious danger of sea-water intrusion exists provided that Lake Mzingazi water-levels remain between 3 and 1,5 m above mean sea-level. If, however, the Lake water-level were to drop lower than 1,5 m the toe of the wedge could reach a distance of more than 200 m in the flood plain after 1 200 days.

Introduction

The Institute for Ground-water Studies investigated the problem of salt water intrusion in the Mzingazi flood plain at Richards Bay (Figure 1).

A large urban complex with balanced industrial, commercial and residential components has developed during recent years at Richards Bay. The Richards Bay Town Board planned and effected this development within an area of about 310 km² around the bay. The natural advantages of this area include the scenic freshwater lake, Mzingazi, and its environs.

A fundamental problem associated with industrial developments concerns the provision of an adequate permanent water supply, which in the Richards Bay area, is Lake Mzingazi with a surface area of 10,3 km². The preservation of this water supply is therefore of vital importance. The risk of pollution and replenishment of Lake Mzingazi must therefore be minimized.

During 1973 the Mzingazi channel was dredged from the harbour to a point south-west of Lake Mzingazi. Worthington (1978) identified the flood plain, which lies between this channel and the Lake, to be the area most vulnerable to salt water pollution as a result of the inflow of tidal sea water into the channel.

The main purpose of this investigation was to calculate the position of the sea-water wedge in the flood plain for different water-levels in Lake Mzingazi.

Geohydrology

A comprehensive geohydrologic study of the Richards Bay area was carried out by Worthington (1978). There are few geological outcrops in the Richards Bay area, most of which is covered by late Pleistocene to Holocene alluvial and aeolian deposits.

The ground waters in the Richards Bay area are derived from rainfall which averages 1 000 mm per annum. Geohydrologic data of the Mzingazi flood plain (Table 1) are limited to information deriving from three boreholes P, Q and R (Figure 2).

The conductivity of the sea water in the Mzingazi channel is 3 730 mS/m, compared to the 44,9 mS/m for water of Lake Mzingazi.

Siltation, both at the bottom of the lake and in the channel, has been ignored in this study.

Sea-water intrusion

The phenomenon of sea-water intrusion has received much attention in countries such as the USA, Israel and the Netherlands, due to its potential for contaminating fresh water supplies at developed coastal regions. Of particular importance for this problem is the nature, extent and rate of movement of the fresh/sea-water interface.

TABLE 1
GEOHYDROLOGICAL INFORMATION FOR THE THREE BOREHOLES IN THE MZINGAZI FLOOD PLAIN

Boreholes	Q	R	P
Water-level (m.a.s.l.)	2,24	2,78	3,19
Collar (m.a.s.l.)	5,14	4,15	5,74
Depth (m)	14,6	12,87	18
K (m/d)	22	8	10
S	0,003	0,0045	0,0025
Conductivity (mS/m)	34	27,5	50

K = hydraulic conductivity and S = specific yield

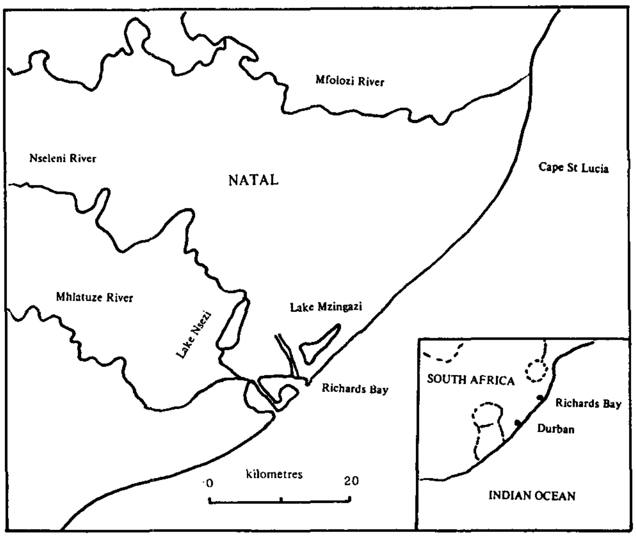

Figure 1
Locality map of Lake Mzingazi at Richards Bay.

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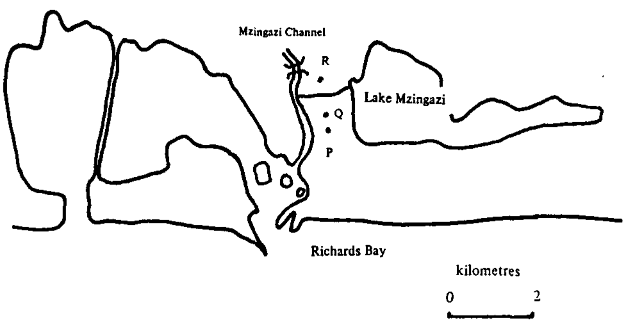

Figure 2
Map showing the position of the three boreholes between Lake Mzingazi and the channel.

Analytical solutions

The earliest attempt at describing the phenomenon of sea-water intrusion is usually attributed to Ghyben and Herzberg (De Wiest, 1965). They discovered that salt water was not observed at sea-level in wells near the coast, but at a depth below sea-level of roughly 40 times the height of the fresh water above sea-level. It can be expressed mathematically in the form:

$$Z = h.\rho_f/(\rho_s - \rho_f)$$

where

Z = depth of the interface below sea-level
h = height of the water-table above sea-level
ρ_f = density of fresh water
ρ_s = density of sea-water

This expression is based on the assumption that the ground water is at a constant potential, whereas it actually is dynamic (otherwise the water-table would have zero slope and the interface would be horizontal). The theory, furthermore, does not allow for seepage interfaces, through which fresh water can escape, below sea-level. Nevertheless, the theory provides at least a first approximation and is still widely in use today. Although some of the constraining assumptions in the Ghyben-Herzberg approximation have been relaxed in the more modern theories, all or at least some of them are still based on the assumption that the system can be adequately described as a steady state aquifer. A few of the more used approximations are:

- The Nomitzu approximation (Lau, 1967)

  $$Q = K\,\delta(\delta - 1)\,(y_i^2 - y_0^2)/2L$$

- The Glover approximation (Glover, 1959)

  $$Q = K\,(\delta - 1)\,(y_i^2 - y_0^2)/2L$$

- The Lau approximation (Lau, 1967)

  $$Q = K(\delta - 1)\,D^2/2L \qquad (1)$$

where

Q = the seaward flux of fresh water per unit width of the aquifer
K = hydraulic conductivity
δ = ρ_s/ρ_f
y_i = depth to interface at a point with distance L from the coast-line (datum : sea-level)
y_o = depth to interface at the coast (datum : sea-level)
L = distance of intrusion (in m), measured along the horizontal.
D = saturated thickness of the aquifer

In deriving these expressions, it is generally assumed that there exists an abrupt interface between the sea and fresh water. However, in reality there always exists a transition zone of varying density that separates the two zones. The net result of this approximation is a solution that over-estimates the actual distance of the toe of the sea-water wedge from the coast.

The seaward flux, Q, can be expressed through Darcy's law as

$$Q = KDi \qquad (2)$$

where i = water-table gradient.

Substitution of (2) in (1) yields:

$$L = 0{,}025D/i \qquad (3)$$

for the usually assumed density values ($\rho_f = 1$ g/cm³ and $\rho_s = 1{,}026$ g/cm³). The distance of intrusion, L, is thus directly proportional to D and inversely proportional to i. The possibility of sea-water intrusion thus increases with increasing aquifer thickness and decreases with increasing water-table gradient.

Application of equation (3) for the Mzingazi flood plain for water-levels of 3 to 0,2 m in the Mzingazi Lake and D = 16 m yields the answers shown in Figure 3. The intrusion distances corresponding to water-levels of 3, 0,5 and 0,2 m are 113, 680 and 1 250 m respectively.

To assure that the salt-water wedge does not intrude further than a certain distance, x, a freshwater head, not less than D/40 (m.a.s.l.), must be maintained at the point x. For the Mzingazi flood plain it means that a freshwater head of 0,4 m above sea-level will be sufficient for this purpose.

The water-levels of boreholes P, Q and R are more than 0,4 m, which implies that no sea-water will enter these boreholes as long as a minimum value of 0,4 m is maintained.

Numerical solution

The difficulty of solving the dynamic equations describing sea-water intrusion can be avoided by using numerical methods. In this investigation use was made of a two-dimensional vertical model based on the alternating direction collocation approximation (ADC) proposed by Botha and Celia (1981).

The basis of the model (Celia, 1984) is the equation of mass conservation which can be expressed in the case of a fluid as:

$$D_t\,(\varepsilon\,\rho) + \nabla\,(\varepsilon\,\rho v) = 0$$

$$q = \varepsilon v = -k_1/\mu\,(\nabla p - \rho g)$$

and for the solute (salt) as:

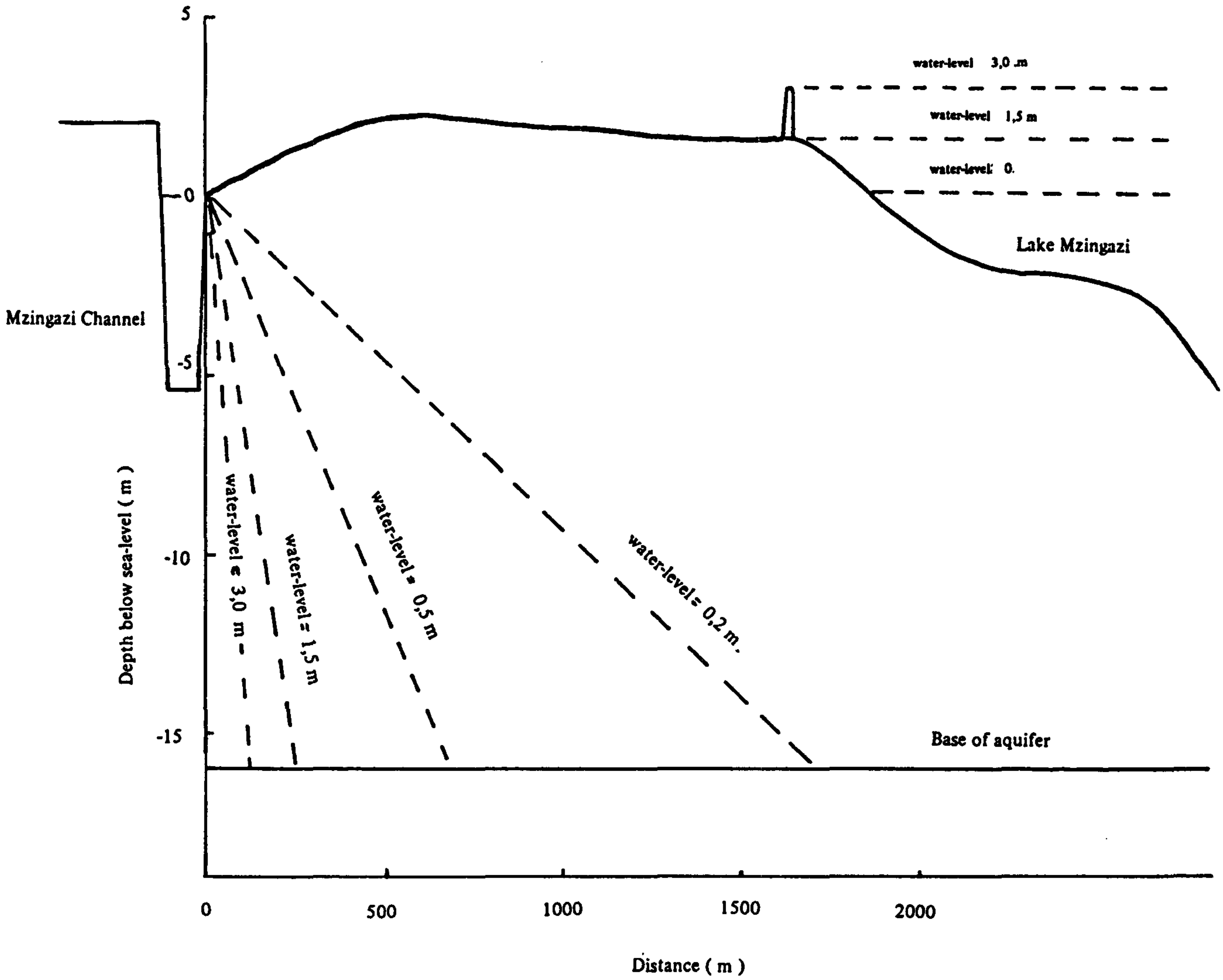

Figure 3
Distance of intrusion of sea-water as obtained by the analytical steady state solution for different Lake water-levels.

$$D_t(\varepsilon c) + \nabla(\varepsilon c v) - \nabla J_s = 0$$

where

ε = void fraction (L^3/L^3)
ρ = fluid density ($m.L^3$)
v = fluid (pore) velocity (L/T)
c = concentration of salt (m/L^3)
J_s = diffusive/dispersive flux vector (m/L^2T)
$\mathbf{k}_1$ = intrinsic permeability tensor (L^2)
μ = dynamic viscosity (m/LT)
p = fluid pressure (m/LT^2)
g = acceleration due to gravity (L/T^2)
q = Darcy velocity (specific discharge) (L/T)
∇ = normal gradient operator
D_t = d/dt

By augmenting this equation with appropriate constitutive equations of state such as

$$\rho = \rho(c) = \rho_0 + ac$$

and

$$J_s = \varepsilon \mathbf{D}_1 . \nabla c$$

where

ρ_0 = density at zero concentration
a = constant
$\mathbf{D}_1$ = hydrodynamic dispersion tensor

the final equations utilized in the numerical model can be obtained. They are

$$\rho S_{op} D_t p + \varepsilon D_c \rho D_t c - \nabla . [\rho(k_1/\mu).(\nabla p - \varrho g)] = 0 \quad (4)$$

$$D_t c + \nabla.(cv) - \nabla . \mathbf{D}_1 . \nabla c = 0 \quad (5)$$

$$\rho = \rho_0 + ac \quad (6)$$

Equations (4) to (6) can only be solved when proper initial and boundary conditions are given. For initial conditions the initial state of the physical system is used. The only restriction placed on the initial conditions is that they must, in fact, be solutions to the governing equations.

Typical boundary conditions are shown in Figure 4. The following should be noted.

- The conditions of no flow across the confining layers are time-varying boundary conditions in that $\rho = \rho(c) = \rho$ (c(t)). Thus the conditions must be updated as the solution proceeds.

- At the opening to the sea, it is assumed that the dispersion in the porous medium is much greater than the diffusion in the sea, implying that $D_n c = 0$. This follows from equating mass fluxes on either side of the boundary (Botha and Celia, 1981).

- For water-table conditions, no provision is made to accommodate a moving boundary, not for any but the simplest approximation ($D_Y p = -\rho g$) to the non-linear boundary conditions. A constant (in time) pressure could also be input at this boundary.

Figure 5 shows the finite element mesh for the Mzingazi flood plain. The computer model was run with K = 20 m/d; D = 16 m; S = 0,003 for different Lake water-levels of 3; 1,5; 0,5 and 0,2 m above sea-level for a period of 1 200 days. The intrusion distance stabilized more or less at this time. The results for the 0,1 and 0,01 isochlors (an isochlor of 1 means a TDS value of 26 000 mg/ℓ) are shown in Table 2.

The position of the sea-water transition zone after 1 200 days with a Lake water-level of 0,5 m above sea-level is shown in Figure 6.

TABLE 2
DISTANCE OF SALTWATER INTRUSION FOR DIFFERENT LAKE WATER-LEVELS AFTER 1 200 DAYS

Lake Water-level (m.a.s.l.)	Distance of intrusion (m)	
	0,1 isochlor	0,01 isochlor
3,0	67	75
1,5	70	100
0,5	90	120
0,2	100	200

Conclusions

A comparison between the analytical and numerical solution confirms that the analytical steady state solution over-estimates the distance of sea-water intrusion.

There is little danger of sea-water intrusion in the Mzingazi flood plain for Lake Mzingazi water-levels of between 3 and 1,5 m above sea-level. For Lake water-levels lower than 1,5 m sea-water intrusion becomes more evident because of the hyperbolic relationship between the freshwater gradient and the distance of sea-water intrusion.

A fresh water head of at least 0,4 m above sea-level must be maintained in boreholes P, Q and R. This implies that there must

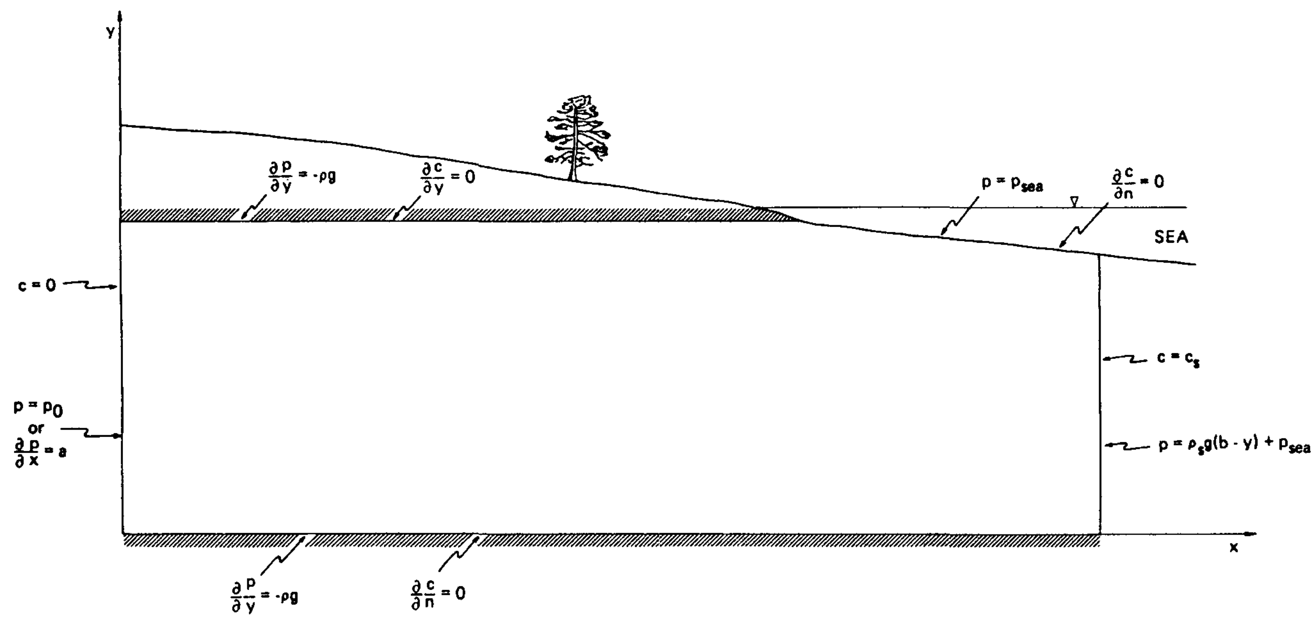

Figure 4
Typical boundary conditions. p_0, a are constants.
$\frac{\partial}{\partial} = -\rho g$ is implied from $q_n = 0$. b is thickness of aquifer.

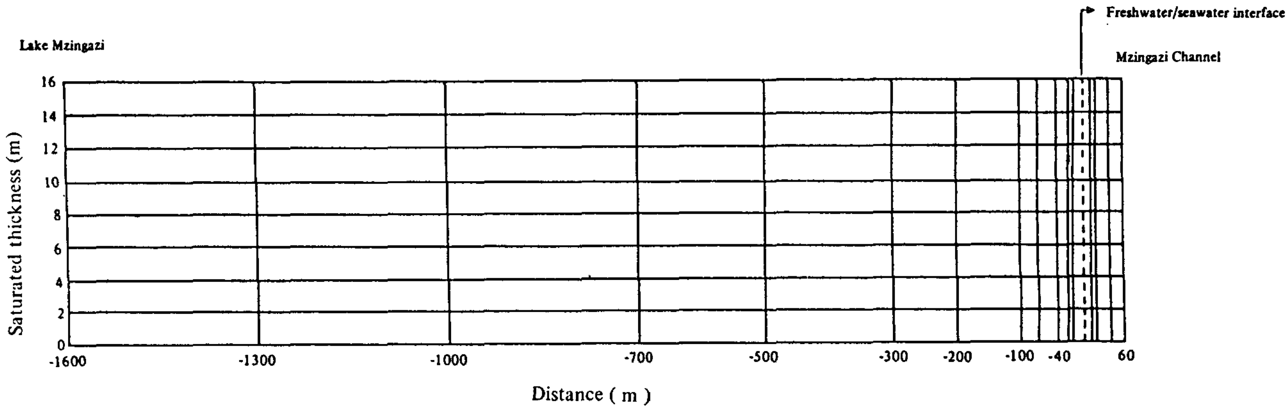

Figure 5
Vertical finite element mesh used for the numerical solution of sea-water intrusion.

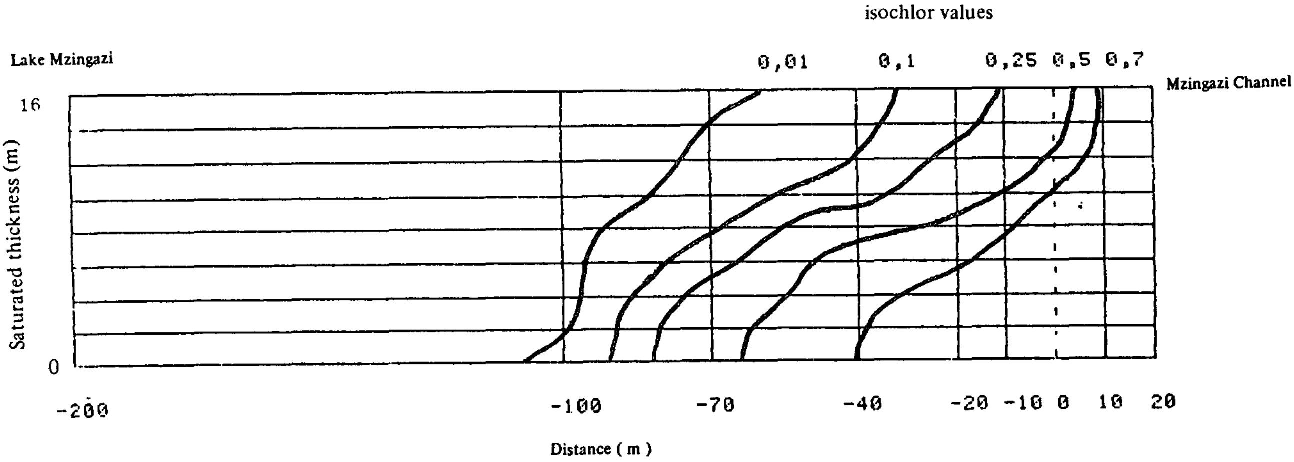

Figure 6
The position of the different salt isochlors after 1 200 days.

be no withdrawal of groundwater in the Mzingazi flood plain through boreholes.

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