Determination of runoff frequencies for ungauged urban catchments

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Abstract

Engineers often intuitively use the recurrence interval of a design storm for runoff recurrence interval. It is suggested that this assumption is not soundly based, as the antecedent moisture content (AMC) or catchment wetness has a significant effect on the storm runoff recurrence interval.

A method is described for generating flood frequency information for a catchment for a design situation. The technique comprises a combination of deterministic and stochastic components. This involves deterministically modelling the response of the catchment together with the use of a stochastic element to derive the conditional probability vector of the outlet hydrograph peaks. The advantage of the method is that the infiltration is modelled using a range of values that is described statistically.

Introduction

The recurrence interval, or the risk of exceedence, of storm runoff requires assessment in order to avoid over or under-design of drainage facilities. Design flood estimation on small catchments, for which streamflow data are generally not available, is commonly based upon the use of design rainfall data together with a hydrological simulation model. There are various methods available to the design engineer for deriving the runoff from an applied design storm. These methods in order of simplicity range from the Rational method (Kuichling, 1889) to the distributed models (e.g. SHE model, Abbott *et al.*, 1986) and in hydraulic accuracy from Time-area to Kinematic methods.

The design engineer is usually interested in the recurrence interval of the peak runoff rate of the storm event as this indicates the risk of failure of the structure being designed. In the case of a smooth impermeable catchment the recurrence interval of the storm and of the runoff can be equated without significant loss of accuracy, provided the storm is defined by more than its rainfall depth. However, when a permeable catchment is considered, then large differences can exist between the recurrence intervals of the rainfall and the runoff and the engineer has no way of allowing for these variations in the design storm. A 10-year design storm on any particular permeable catchment will not necessarily produce the 10-year runoff hydrograph peak because the antecedent soil moisture condition can affect the runoff. Several researchers are in agreement with this conclusion (e.g. Dunsmore *et al.*, 1986; Hughes, 1986; Cordery, 1971; Hope, 1980).

The usual method of hydrograph synthesis for small catchments involves firstly selecting an appropriate design storm duration. The storm of required magnitude for a given frequency of occurrence and duration is distributed in time. Various storm durations are investigated before identifying the critical storm duration which produces the highest peak flow. The storm duration that produces the peak flow will not necessarily produce the greatest volume of runoff. The shape of the intensity distribution has a variable effect on the storm hydrograph peak and distribution (Akan and Yen, 1984, Lambourne and Stephenson, 1987).

Hope (1980) grouped the concepts of runoff generation into two major categories, namely those based on the infiltration and overland flow theory of runoff developed by Horton (1933), and those based on the unit source area theories, which include both the variable source and partial area approaches. The unit source area theories assume that the production of stormflow in the catchment is non-uniform. Either the stormflow is not necessarily surface flow but may be derived from subsurface flow (may be a combination of both), or stormflow occurs from certain areas of the catchment. In engineering practice a combination of Hortonian overland flow and subsurface flow is usually modelled. In urban catchments, subsurface flow is usually ignored (Green and Stephenson, 1986).

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Runoff peak frequency methods

Methods of derivation of the frequency of storm runoff can be subdivided into either deterministic or probabilistic formulations.

Deterministic approach

Packman and Kidd (1980) used deterministic methods to assess the moisture content and storm shape for design applications to derive the design peak runoff with the same recurrence interval as rainfall input.

Constantinides (1982) proposed a concept involving excess storm depth – duration – frequency (D-D-F) curves. These curves are developed for different soil types based on the assumption that Hortonian theory dominates the streamflow process. Excess D-D-F curves are obtained by subtracting losses from actual storm hyetograph data and approximating the excess storms using a triangular shape to derive the duration. An assumption in the technique is that no further losses exist, which is not completely correct as infiltration occurs after the end of the storm, as long as there is water on the surface of the catchment. This assumption will not affect the peak discharge but will result in higher runoff volumes.

The basic problem with these methods is that runoff and rainfall data have to be available for the catchment being studied. While some autographic rainfall data may be available there will, in all probability, be inadequate data to define a reasonable range of D-D-F curves.

Statistical approach

When only peak flow records are required, then the best technique involves fitting parameters of a theoretical distribution to a flow record using statistical methods. In situations where flow records are not available, other techniques are adopted, based upon regionalised parameters or relationships with physical characteristics of the catchment (Benson and Matalas, 1967; NERC, 1975).

Where storage is required in a design problem, then the peak flow will be inadequate on its own. Hiemstra and Francis (1981) showed that more than one combination of flood peak and associated volume can have the same joint exceedence probability. They developed the 'runhydrograph' method to account for this using gauged runoff data. This is a technique which involves treating peaks and volumes as the two variables of a bivariate log-Normal probability distribution. This method allows for the synthesis of a whole family of design hydrographs for each derived return period.

Hughes (1986) investigated the bivariate probability distribution of the daily rainfall depth and an antecedent moisture index. He found that it was possible to make use of the bivariate normal distribution to represent the probability of one (or two) day rainfall and their associated indices of antecedent precipitation (API), given that suitable transformations were made to the data. The application of the method is a subject of further research.

Combined stochastic – deterministic approach

Most hydrological systems have both stochastic and deterministic components. The stochastic component is primarily defined through the application of probability distributions, whilst the deterministic component can be modelled mathematically without considering the probabilistic nature of the system. If any of the inputs to a hydrological system are stochastic, then the output will also be stochastic.

Most of the applications of combined hydrologic systems have been with storage behavioural patterns in reservoirs and water supply problems. Laurenson (1974) suggested that the marrying of stochastic and deterministic techniques can lead to great advances in many areas of hydrological practice, and detailed a combined approach to systems modelling.

Model formulation

A computer model called WHISPER was developed to derive flood frequency information for small urban catchments. The model is composed of a deterministic element, an AMC probability generator, and a combined deterministic-stochastic element.

Deterministic element

There are two relevant aspects in the deterministic modelling of the rainfall runoff process of concern, namely subcatchment overland flow and routing through channels. The components of the model WITWAT (Green, 1984) were used to describe these processes. WITWAT employs a kinematic routing for the overland flow contribution and time shift routing in channels. WITWAT has been tested (Green and Stephenson, 1985), both on South African and American catchments and has been shown to yield reasonable results.

In the runoff frequency model (described in this paper) excess rainfall hyetographs are calculated, subtracting losses using the Horton's equation. The depth of rainfall for a particular recurrence interval is estimated from D-D-F equations for the inland and coastal regions of South Africa as proposed by Op Ten Noort and Stephenson (1982). For a selected duration, the rainfall depth can be distributed over time using either a square topped or triangular shape (which accounts for the influence of the hyetograph shape on the hydrograph peak and volume). Constantinides (1982) derived a method for calculating excess storm hyetographs assuming a triangular storm hyetograph. With a square topped storm a triangular excess storm can be calculated with the maximum intensity at a time equal to the storm duration. The excess rainfall is then routed across the catchment to derive the runoff.

The infiltration parameters are selected on the basis of soil groups (A, B, C and D) and the AMC class (1, 2, 3 and 4). The loss parameters used in the Horton infiltration model were proposed by Constantinides (1982) and shown in Table 1. The deterministic element of the model produced excess storms for all the moisture condition classes, the probability of a particular moisture class being defined later.

TABLE 1
INFILTRATION VALUES FOR HORTON'S MODEL
(AS RECOMMENDED BY CONSTANTINIDES, 1982)

	Hydrological soil group		A	B	C	D
Hortons	AMC					
Parameter	Bone Dry	(1)	83	67	42	25
Fo	Rather Dry	(2)	54	43	26	15
mm/h	Rather Wet	(3)	28	22	12	3
	Saturated	(4)	14	11	4	2
Fc mm/h			13	8	3	2

Fo = Initial infiltration rate
Fc = Final infiltration rate

Antecedent moisture condition probabilities

In determining the flood frequencies of runoff from a subcatchment, the probability of the antecedent moisture condition of the soil is also used. Gray *et al.* (1982) used simple probability methods for determining the average probability of each AMC class (1, 2, 3 and 4) assuming growing seasons for Indiana, Kentucky and Tennessee in the United States of America. In design practice, the seasonal variation of AMC class probability would be required.

In the current model attention was given to evaluating the probabilities of four AMC classes. This is a very simple approach to the concept and only requires the rainfall depth on the preceding five days to determine the AMC class. A sufficient length of daily rainfall record is required to adequately determine the probabilities for a particular catchment. Where record lengths do not allow the derivation of seasonal AMC probabilities, then an appropriate stochastic daily rainfall generator can be used. In South Africa such a model has been developed by Zucchini and Adamson (1984) using model parameters estimated for all regions of the Republic.

The four class AMC probability approach is very coarse and simplistic, and other methods of deriving the moisture status of the soil profile (e.g. use of ACRU model developed by Schulze, 1984; unsaturated component of the SHE model, Abbott *et al.*, 1986) would provide a more realistic approach.

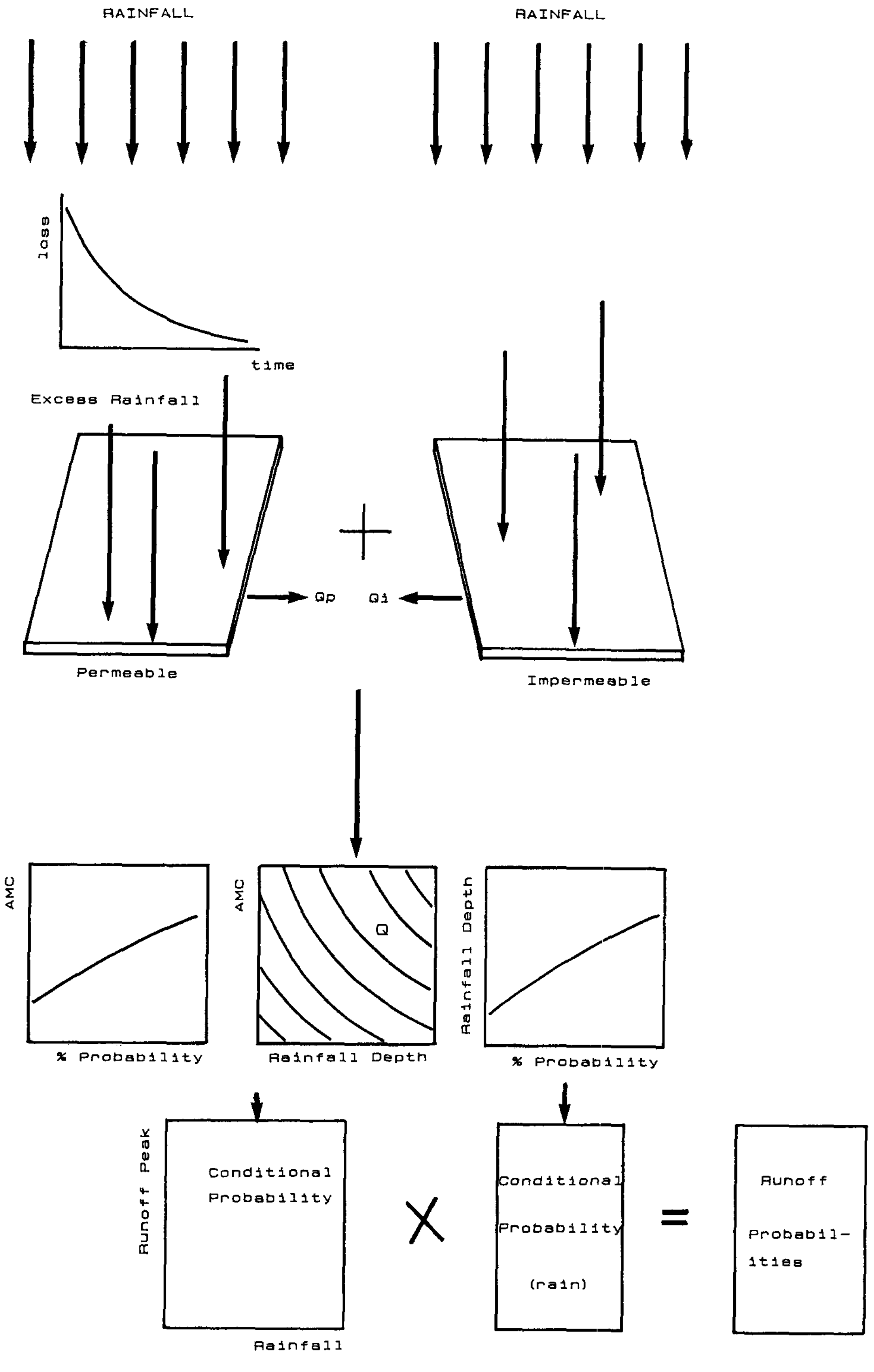

Figure 1
Subcatchment flood frequency procedure

Stochastic – deterministic element

The objective of this element of the model is to determine the probability distribution $\{q_i\}$ of the output y from a known probability distribution $\{p_i\}$ of the input x, the known probability distribution $\{r_i\}$ of a parameter z which may or may not be correlated with x, and a known deterministic matrix transformation T. This objective can be described in matrix notation by

$$Q = A.P \qquad (1)$$

were Q is the column matrix of output probabilities (order m), A is the matrix of transition probabilities (order mxn) and P is the column matrix of input probabilities (order n).

Applied to the rainfall-runoff approach, the probability distribution of flood peaks is derived from a known probability distribution of rainfall of duration D, known probability distribution of AMC and a known deterministic transformation to relate rainfall to runoff.

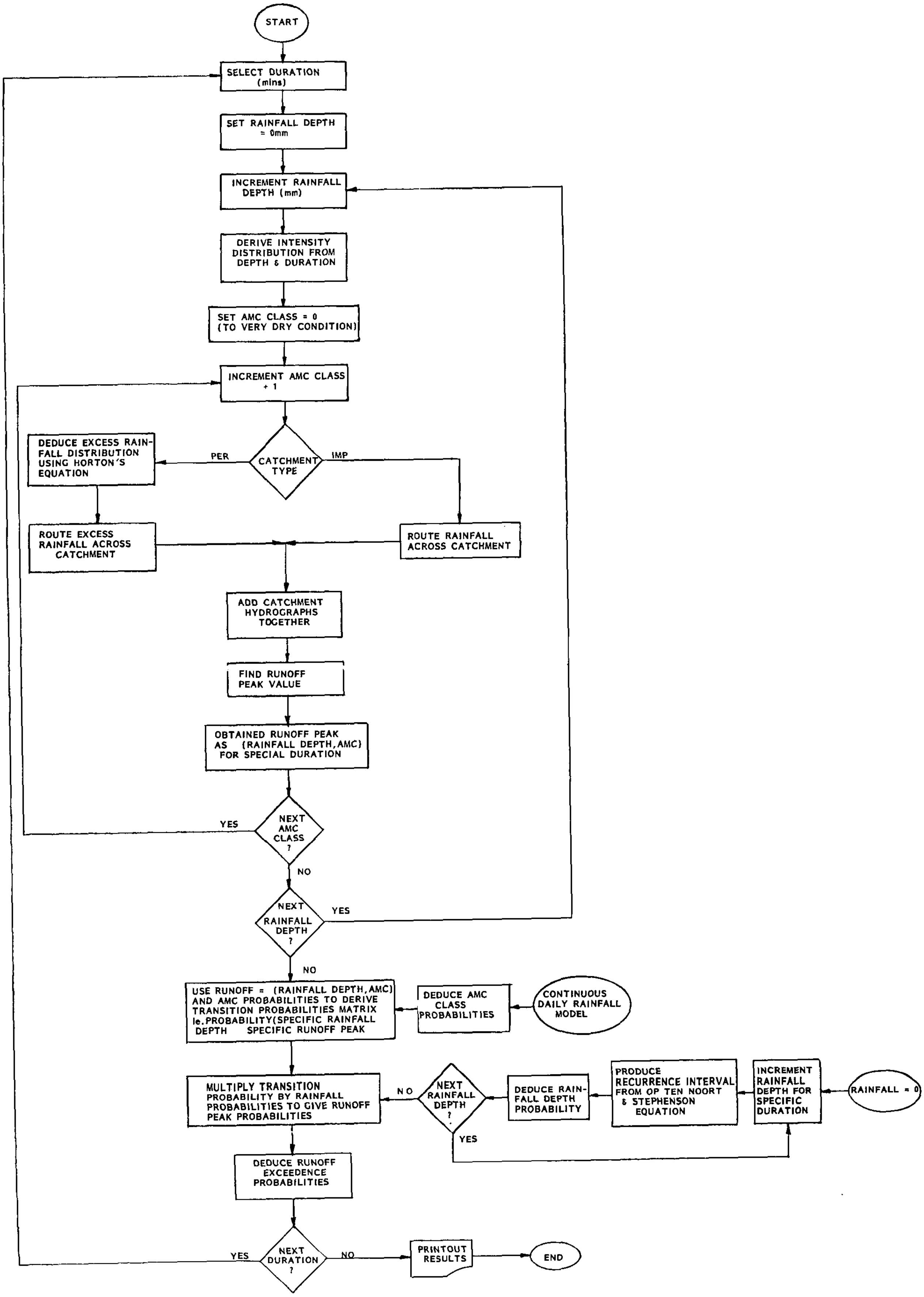

Figure 2
Flow-chart of flood frequency procedure

Let us first consider a catchment draining into an outlet. Goyen (1983) suggested that this was a 'concentrative' process with one independent parameter. The independent parameter is the AMC class. Based on the work of Beran and Sutcliffe (1972), it is assumed that the antecedent moisture content of the soil and the following rainfall distribution are independent events. For any particular value of the independent parameter (AMC), there is a one to one transformation from the input (rainfall) to the output (runoff). The conditional distribution of runoff, given a particular rainfall, consists of unity and a set of zeros (a 'concentrative' process as defined by Laurenson, 1974). However, there is a conditional distribution of runoff, not only for each rainfall but also for each possible value of the AMC. Each value of AMC has attached a certain probability m_k. So each element of the transition matrix (Equation 1) can be represented as

$$a_{ji} = \Sigma b_{jik} r_k \qquad (2)$$

where r_k is the probability of an AMC class and b_{jik} is the conditional probability of an output Y_j, given an input X_i and a parameter Z_k.

$$b_{jik} = \Pr(y = Y_j \mid x = X_i, z = Z_k) \qquad (3)$$

A diagrammatic representation of the logic presented above and a flow-chart is given in Figs. 1 and 2 respectively. The transition probability from one rainfall depth (state i) to that of the runoff peak (state j) is calculated for each of the states based on the AMC probabilities. This probability transition matrix is then multiplied by the conditional rainfall probability array to give the peak runoff conditional probability vector. The corresponding exceedence probabilities are then computed and graphically displayed. In the present application the rainfall depth probabilities were determined from the use of D-D-F relationships.

Model application on a local catchment

The model was used to derive the flood frequency curve for a catchment in Montgomery Park, Johannesburg (Lat 26° 9,5'S Long 27° 59'E). Unfortunatly there is only a 3-year record of peak flows from the catchment during a time of drought in South Africa. The catchment is 1 036 ha in extent and is at a mean altitude of 1 695 m above mean sea level. The predominant land use is the natural soil land vegetation, although building programmes are underway in most sectors of the catchment which will increase the urbanised tracts of land and associated service roads and paved areas. The topography is fairly hilly with surface slopes ranging from 0,02 to 0,15 m/m. The highest elevation on the boundary of the catchment is 1 800 m above sea level and the outlet is about 1 600 m. The main drainage system of the catchment comprises both natural and artificial channel sections.

The Montgomery Park catchment information was applied to the model together with rainfall parameters, to determine the AMC probabilities. The peak runoff conditional probability vector was computed using the technique described in the previous section. The runoff peak frequency curve for the Montgomery Park catchment is presented (Fig. 3) as a function of either return period or percentage probability of exceedence and runoff peak discharge. The average of the observed peak flows in the catchment was 1,5 m^3/s and flows of 6,6 and 8,7 m^3/s were observed once in the record.

In order to test or verify a model of this type a considerable length of rainfall and runoff data would be needed (at least 50 years would provide adequate verification). This data however, is not readily available. The deterministic component of the model is based on the model WITWAT (Green, 1984) which has been tested satisfactorily on several catchments. The subcatchment

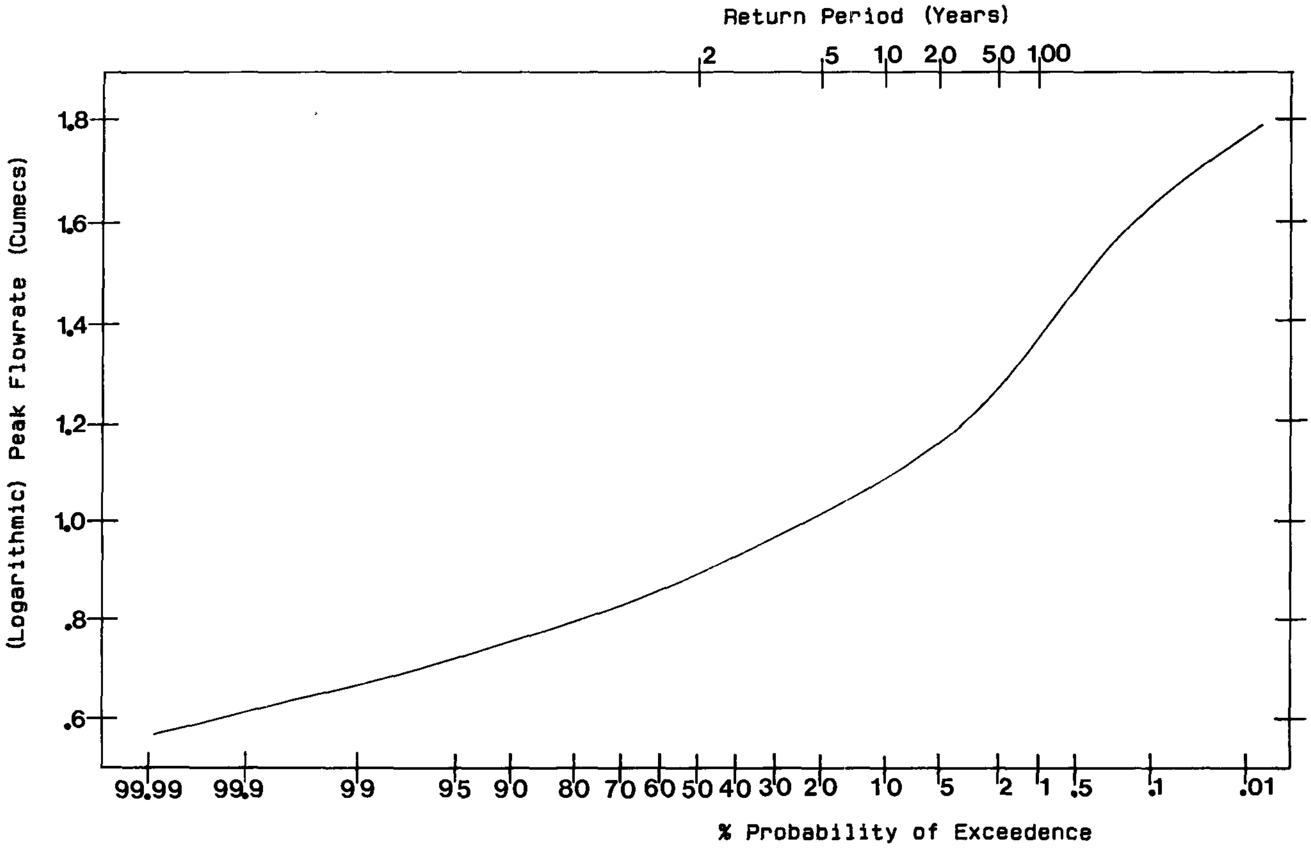

Figure 3
Flood frequency information for Montgomery Park catchment.

overland flow simulated by WITWAT was tested prior to its inclusion in the WITWAT model (Green, 1987). Another factor is that WITWAT performs to the same capability as SWMM (Huber *et al.*, 1982) which has been extensively tested. Laurenson (1974) tested the stochastic approach to deriving flood frequency information downstream of a confluence and found good agreement with that which was observed.

Discussion

A combined stochastic – deterministic procedure to overcome the shortcomings of both the excess D-D-F curves and excess storm recurrence interval methods was devised. It also overcomes the problems of transposition of parameters from gauged to ungauged catchments, which occur with statistical approaches to flood frequency estimation (e.g. the runhydrograph technique). This involves a catchment and routing procedure to derive the conditional probabilities of the outlet hydrograph peak. The deterministic component of the model is built around a tested simulation model. The method has the advantage of a traditional simulation model, of a modular construction for simulating land use effects and an output parameter (exceedence probability or return period) which is physically defined. Risk analysis can be easily applied to the result of this procedural technique. A program was written to perform the extensive calculations required with this type of technique.

The use of a range of infiltration values for different moisture conditions (the probabilities of occurrence being determined within the procedure) overcomes a major problem faced by engineers, of the correct selection of one infiltration value for a single event model.

The application of the method in its present form does use the Horton equation for derivation of losses, with no allowance for interflow mechanisms. Water movement within the soil matrix (an interflow component) could be easily added to the subcatchment procedure, provided a sufficient time-delay was incorporated. A further development of the method would be the replacement of both the Horton equation and the moisture condition classification by a procedure that would more adequately describe the infiltration process and moisture budget (as a function of moisture content and not classes). The rainfall input to the model is assumed to be spatially uniform and therefore the introduction of spatially varied rainfall would enable the method to be used for larger catchment areas.

The stochastic – deterministic technique presented in this paper goes some of the way to overcoming some of the problems identified with regard to obtaining flood frequency information on an ungauged catchment.

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References

ABBOTT, M.B., BATHURST, J.C., CUNGE, J.A., O'CONNELL, P.E. and RASMUSSEN, J. (1986) An introduction to the European hydrological system – Systeme Hydrologique Europeen (SHE), 2 : Structure of a physically based distributed modelling system. *J. Hydrol.* **87** (1/2) 61-78.

AKAN, A.O. and YEN, B.C. (1984) Effect of time distribution of rainfall on overland flow. *Proceedings of the 3rd International Conference on Urban Storm Drainage,* Göteborg, Sweden. 193-202.

BENSON, M.A. and MATALAS, N.C. (1967) Synthetic hydrology based on regional statistical parameters. *Water Resources Research* **3**(4) 931-935.

BERAN, M.A. and SUTCLIFFE, J.V. (1972) An index of flood-producing rainfall based on rainfall and soil moisture deficit. *J. Hydrol.* **17** 229-236.

CONSTANTINIDES, C.A. (1982) Two-dimensional kinematic modelling of the rainfall – runoff process. Report 1/82, Water Systems Research Programme, Department of Civil Engineering, University of the Witwatersrand.

CORDERY, I. (1971) Estimation of design hydrographs for small rural catchments. *J. Hydrol.* **13** 263-277.

DUNSMORE, S.J., SCHULZE, R.E. and SCHMIDT, E.J. (1986) Antecedent soil moisture in design runoff volume estimation. Report to the Water Research Commission by the Department of Agricultural Engineering, University of Natal, Pietermaritzburg. Report 155/1/86.

GRAY, D.D., KATZ, P.G., DEMONSABERT, S.M. and COGO, N.P. (1982) Antecedent moisture condition probabilities. *Journal of Irrigation and Drainage Div.* ASCE **2** 107-14.

GREEN, I.R.A. (1984) WITWAT stormwater drainage program – version II. Report 2/84, Water Systems Research Programme, Department of Civil Engineering, University of the Witwatersrand.

GREEN, I.R.A. (1987) Personal communication. Department of Civil Engineering, University of the Witwatersrand.

GREEN, I.R.A. and STEPHENSON, D. (1986) A comparison of urban drainage models. Report to the Water Research Commission by the Water Systems Research Programme of the University of the Witwatersrand. Report 115/6/86.

GOYEN, A.G. (1983) A model to statistically derive rainfall losses. *Proc. Hydrology and Water Resources Symposium,* Hobart. 220-225.

HIEMSTRA, L.A.V. and FRANCIS, D.M. (1981) Runhydrographs for prediction of flood hydrographs. *Journal of Hydraulics Div.* ASCE **107**(6) 759-775.

HOPE, A.S. (1980) Estimation of catchment moisture status for the SCS stormflow model. Unpubl. M.Sc. Eng. Thesis. Department of Agricultural Engineering, University of Natal, Pietermaritzburg.

HORTON, R.E. (1933) The role of infiltration in the hydrologic cycle. *Trans. Am. Geophys. Union* **14** 446-460.

HUBER, W.C., HEANEY, J.P., NIX, S.J., DICKENSON, R.E. and POLMAN, D.J. (1982) Stormwater management model : Users manual. Department of Environmental Engineering Sciences, University of Florida, Gainesville, Florida, USA.

HUGHES, D.A. (1986) The use of the bivariate Normal distribution function for the analysis of extreme rainfalls and associated antecedent catchment moisture status. *Proc. Fourth International Hydrological Symposium on Multivariate Analysis of Hydrologic Processes,* CSU, Colorado, USA, July 1985. 519-530.

KUICHLING, E. (1889) The relation between the rainfall and the discharge of sewers in populous districts. *Transactions,* ASCE. **20** 1.

LAMBOURNE, J.J. and STEPHENSON, D. (1987) Model study of the effect of temporal storm distributions on peak discharges and volumes. *Hydrological Sciences Journal* **32**(2) 215-226.

LAURENSON, E.M. (1974) Modelling of stochastic – deterministic hydrologic systems. *Water Resources Research* **10**(5) 955-961.

NERC (1975) *Flood Studies Report* Volume 1, *Hydrological Studies,* London, United Kingdom.

OP TEN NOORT, T. and STEPHENSON, D. (1982) Flood peak calculation in South Africa. Report 2/82, Water Systems Research Programme, Department of Civil Engineering, University of the Witwatersrand.

PACKMAN, J.C. and KIDD, C.H.R. 61980) A logical approach to the design storm concept. *Water Resources Research* **16**(6) 994-1 000.

SCHULZE, R.E. (1984) Hydrological models for application to small rural catchments in Southern Africa : Refinements and development. ACRU Report 19, Department of Agricultural Engineering, University of Natal, Pietermaritzburg.

ZUCCHINI, W. and ADAMSON, P.T. (1984) The occurrence and severity of droughts in South Africa. Report to the Water Research Commission by the Department of Civil Engineering, University of Stellenbosch and the Department of Water Affairs. Report 91/1/84.