Use of the Einstein entrainment function for predicting selective entrainment and deposition of heavy minerals

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Abstract

Einstein's entrainment probability function is investigated as a means for describing size-density sorting of sediment during entrainment into and deposition from transport by intermittent suspension. The function is first compared with the Shields entrainment criterion and shows good agreement in the hydraulically rough zone. Results in the transition zone are sensitive to description of the hiding factor, which remains uncertain. The predicted effect of bed roughness on entrainment from a non-uniformly sized bed is shown to be consistent with observed behaviour. Application of the model to describe sorting by selective deposition and entrainment is illustrated. The Einstein function is incorporated as a boundary condition in a numerical diffusion-convection model for describing the distribution of suspended load under steady, longitudinally uniform flow conditions. This model can be used to predict the spatial distribution and local composition of sediment deposited from suspended load. Hypothetical applications of the model show that it is meaningless to apply sorting criteria at isolated sections in a stream. Temporal and spatial variations of bed material and hydraulic conditions should also be accounted for.

Introduction

The concentration of heavy minerals in sedimentary deposits is a result of segregation during transport and deposition, or of sorting by differential entrainment, transport and deposition during subsequent flow events. Slingerland (1984) has identified four distinct storing phenomena associated with local hydraulic processes, viz. entrainment sorting, suspension sorting, shear sorting and transport sorting. These processes occur throughout active fluvial systems and result in distribution patterns of concentrated heavy minerals on a wide range of scales (Bateman, 1950; McQuivey and Keefer, 1969; McGowan and Groat, 1971; Slingerland, 1977; 1984; Adams *et al.*, 1978; Minter, 1978; Kuzvart and Bohmer, 1978; Toh, 1978; Smith and Minter, 1980).

Prediction of the spatial distribution of sediment deposits in terms of particle size and density requires quantitative descriptions of the sorting process and the factors affecting them. Various attempts have been made to describe sorting by particle size at different scales. On a large scale Rana *et al.* (1973) and Deigaard (1980) developed models to predict bed material size variations along the lengths of alluvial channels. Deigaard (1980), Bridge (1982) and Parker and Andrews (1985) have described sorting by size in channel bends. The role of sorting during suspended transport has been discussed by Brush (1965) and the processes described quantitatively for different situations by Rouse (1937), Jobson and Sayre (1970), Sarikaya (1977), Sengupta (1975, 1979), James (1985) and others. Sorting by size during bed load transport has been modelled by Bridge (1981) and transport velocities have been related to flow and particle characteristics (Kalinske, 1947; Bagnold, 1973; Francis, 1973; Luque and van Beek, 1976; Engelund and Fredsoe, 1976; Fleming and Hunt, 1976; Bridge and Dominic, 1984).

The effect of particle density on sorting has been investigated experimentally and in the field (Rubey, 1933; Hand, 1967; Grigg and Rathbun, 1969; Brady and Jobson, 1973; Steidtmann, 1982; Komar and Wang, 1984) but relatively little has been achieved in the development of predictive models. Effects of differential entrainment and transport have, however, been quantitatively described by Slingerland (1977, 1984) and Komar and Wang (1984).

This paper presents approaches for describing local sorting by size and density and for predicting the spatial distribution and local composition of sediment deposited from suspension on the bed of a channel. Local sorting is described by applying Einstein's (1950) entrainment probability function. Spatial distributions of deposits are predicted by incorporating this function in a numerical model describing the distribution and sorting of material during suspended transport. This model will assist in predicting the locations of concentrations of minerals with specific size and density characteristics in fluvial systems under known hydraulic conditions. In its present formulation the model is able to predict relative rates of deposition of particles with different characteristics for steady longitudinally uniform flow. Bed forms and bed load are not accounted for and the bed material is not considered as a potential source of suspended material.

Selective entrainment and deposition

Sediment in transport in an alluvial channel is involved in continuous interaction with the bed material (Einstein, 1950; Graf, 1971; Leeder, 1983). This is most intense for bed load but applies to suspended load as well. Net deposition of material from suspension depends on spatial and temporal variations in flow conditions and trapping of suspended material during bed interaction. It can be described by considering the frequency of interaction of suspended particles with the bed, which is proportional to local concentration, and the ease of re-entrainment, which can be expressed as a probability. Particles of different size and density with similar fall velocities would tend to have similar bed-interaction frequencies but different probabilities of re-entrainment, resulting in different rates of net deposition, i.e. segregation.

Sumer (1974) described two different modes of suspended transport over a smooth bed viz. where particles would be suspended continuously and where they would be periodically deposited and re-entrained. The mode of transport for a particular particle is determined by local hydraulic conditions and the particle's characteristics. Sumer defined two criteria for continuous suspension, one relating particle fall velocity to shear velocity and the other relating the particle size to the laminar boundary sublayer thickness. The first criterion determines if the turbulence intensity permits particles to reach the bed and the second determines if particles having reached the bed are exposed to re-entraining forces.

James (1984) applied Sumer's criteria to mixtures of gold and sand and showed that the hydraulic conditions defining the

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limit of continuous suspension would be different for particles of different size and density even if their fall velocities were identical. A range of hydraulic conditions was identified where gold particles would be periodically deposited and re-entrained while sand particles with the same fall velocity would be continuously suspended. Some gold but no sand would therefore be expected to become permanently deposited. This approach serves as a good illustration of the segregation processes but suffers from the major drawback that Sumer's (1974) criteria were developed for smooth beds and cannot be accepted for rough granular surfaces. Also it is not realistic to define a rigid threshold condition for deposition and re-entrainment; a probabilistic description would be more satisfactory.

The most important modification needed in the approach described above is replacement of the smooth bed condition by an entrainment criterion generally applicable to granular beds. The most widely accepted entrainment criterion is that developed by Shields (1936), which gives the dimensionless shear stress required to move a given particle and can be used to define critical flow conditions in terms of hydraulic radius and energy gradient. This criterion has been extended and modified by Grass (1970), Mantz (1977), Miller *et al.* (1977) and Yalin and Karahan (1979). The Shields criterion is unsuitable for describing heavy mineral concentration because it was developed for uniform bed material and cannot account for non-uniform sizes in the bed. This is important because sand and heavy mineral grains in transport together would be significantly different in size.

Slingerland (1977) and Komar and Wang (1984) use an entrainment equation which considers binary mixtures. While this is an improvement the model considers pivoting of particles only and ignores hydrodynamic lift, which is very important for small particles in mixtures. Neither this approach nor the Shields criterion expresses entrainment conditions probabilistically. Grass (1970) considered entrainment in terms of probability but not in a form suitable for differentiating between different particles and bed surfaces.

Einstein's (1950) bed load model incorporates the probability of entrainment of particles with particular characteristics from a non-uniformly sized bed. Segregation between sand and heavy mineral particles can be described by using the Einstein probability concept to relate the relative ease of entrainment of particles with different characteristics. The probability of entrainment is represented by the probability that the instantaneous lift force exerted on a particle exceeds its effective weight at any instant, and is given by

$$p = 1 - \frac{1}{\sqrt{\pi}} \int_{-B_*\Psi_* - 1/\eta_\circ}^{B_*\Psi_* - 1/\eta_\circ} e^{-t^2} dt \tag{1}$$

where

$B_* = \frac{B}{(\log 10{,}6)^2 \eta_\circ} = 0{,}143$ for uniform materials

B = constant

$\Psi_* = \zeta Y\left(\frac{\beta^2}{\beta_X^2}\right)\Psi$

ζ = hiding factor
Y = pressure correction factor
$\beta = \log 10{,}6$
$\beta_X = \log (10{,}6\, X/\Delta)$
X = characteristic grain size of mixture
$= 0{,}77\Delta$ if $\Delta/\delta > 1{,}80$
$= 1{,}39\Delta$ if $\Delta/\delta < 1{,}80$
δ = viscous sublayer thickness
$= 11{,}5\ \nu/u_*$
ν = kinematic viscosity
u_* = shear velocity
Δ = apparent roughness diameter

$\Psi = \frac{\rho_s - \rho}{\rho} \frac{d}{SR'}$

S = energy gradient
d = particle size
R' = hydraulic radius associated with grain roughness
ρ = fluid density
ρ_s = particle density
$\eta_\circ$ = standard deviation of η which is distributed according to the normal error law
$= 0{,}5$
t = integration variable

Einstein (1950) presented empirically-based diagrams for estimating the hiding factor, pressure correction factor and the apparent roughness diameter. Various modifications to the hiding factor function have been proposed on the basis of investigations by Einstein and Chien (1953), Egiazaroff (1965), Hirano (1971), Pemberton (1972), Hayashi *et al.* (1980), Shen and Lu (1983) and Misri *et al.* (1984). The implications of different estimates of this factor are discussed in the following section.

The Einstein entrainment probability model has been adopted to describe the deposition of heterogeneous size-density mixtures from suspension. It must be appreciated that this model was intended for use in estimating bed load transport rates and that the empirical constant B_* and other correction factors may need refining for accurate results in a sorting model, particularly to account for non-spherical shapes of some heavy mineral particles. However, probabilities are required in a relative sense only and available information has been accepted provisionally.

Results

Comparison of the Einstein and Shields entrainment conditions

Because Einstein's (1950) probability of entrainment is not usually used as an entrainment criterion, results are first compared with the Shields criterion as presented by Yalin and Karahan (1979). For different particle sizes and bed roughnesses the hydraulic conditions required to produce a particular value of the Einstein probability of entrainment were determined. These conditions were expressed in terms of the Shields parameter and grain Reynolds number and plotted to give a Shields-type diagram for the required probability.

Fig. 1 shows the results for entrainment with a probability of 0,5 of uniform bed material, and the effects of some different estimates of the hiding factor. In the hydraulically rough zone changes in the hiding factor displace the curve without changing its shape. In the transition between the hydraulically rough and smooth zones the position and form of the diagram are affected significantly. The data of Misri *et al.* (1984) are the best available and are used to estimate the hiding factor for $d/X > 1{,}0$. The hiding factor for very small particles is poorly defined. The data of Misri *et al.* for small d/X are sparse and do not include any results for sizes less than 0,5 mm. The scatter of Pemberton's (1972) data for particles smaller than 0,5 mm indicates that the

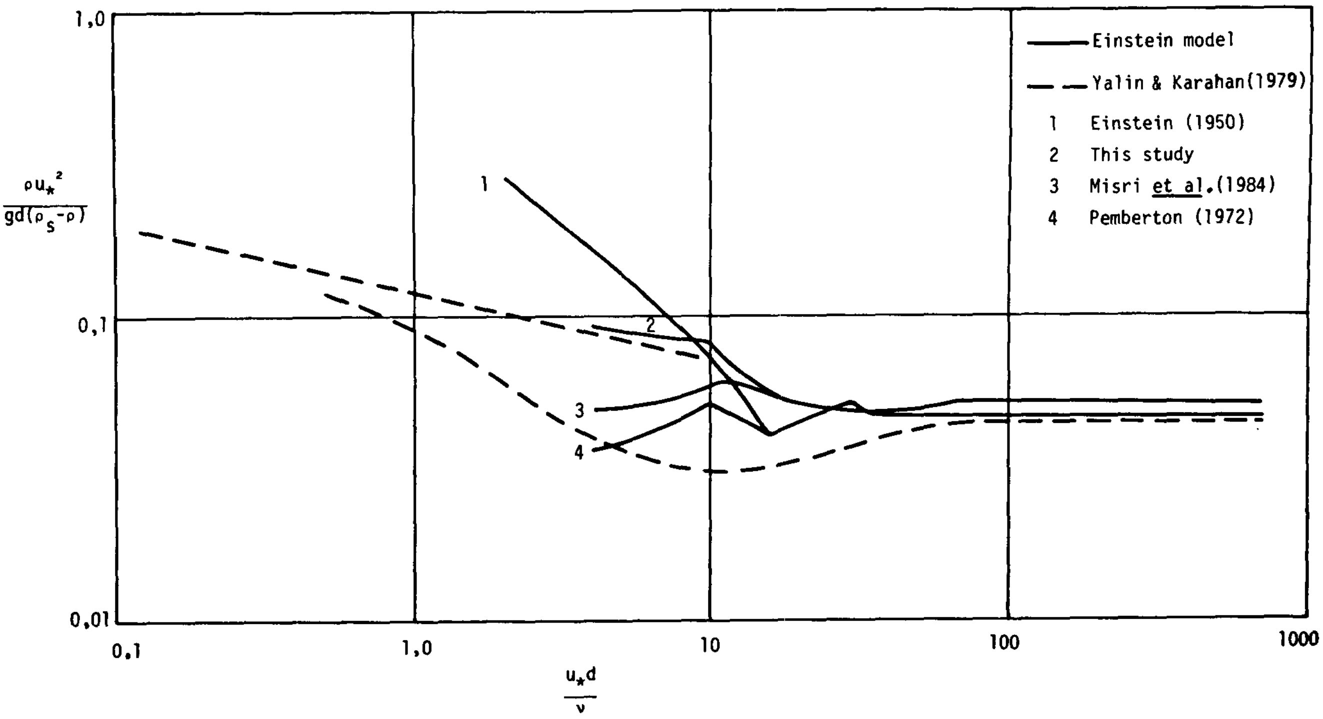

Figure 1
Comparison of Einstein's entrainment probability model with the Shields entrainment criterion, as presented by Yalin and Karahan (1979), for uniformly sized bed material. The Einstein curves all represent a probability of entrainment of 0,5 and are plotted for the different hiding factor functions indicated.

hiding factor curve could lie much closer to the original Einstein curve than as assumed by him. Because of the uncertainty regarding the hiding factor for very small particles and its significant effect, a function has been chosen to give reasonable agreement with the Shields diagram.

For values of d/X less than unity the hiding factor function can be expressed as

$$\xi = a(^{d}/x)^{b} \qquad (2)$$

A value for a of 1,3 has been used to conform with the data of Misri *et al.* for d/X greater than unity. Values for b of −2,13, −1,09 and −0,94 correspond to the curves of Einstein (1950), Pemberton (1972) and Misri *et al.* (1984) respectively. A value of −1,55 was found to produce a reasonable transition to the laminar curve of Yalin and Karahan (1979) and has been adopted provisionally.

None of the Einstein entrainment curves presented extend to grain Reynolds numbers less than about 3 because Einstein's (1950) pressure correction factor and his parameter for estimating the apparent roughness diameter are not defined for lower values. No attempts have been made to extend the Einstein approach into the laminar region because the entrainment mechanism here is fundamentally different from Einstein's description. The correction factors are provided for the transition zone only.

Einstein's (1950) recommendation to use unity for both the hiding factor and pressure correction factor in the hydraulically rough zone for uniform material produces inconsistent results and has not been applied.

The Einstein approach does not provide an entrainment threshold in the same sense as the Shields criterion and the position of the entrainment curve depends on the probability specified. Fig. 2 shows the positions of the curves for probabilities of 0,9; 0,5 and 0,1.

The effect of bed roughness on entrainment is illustrated in Fig. 3 where curves for a probability of entrainment of 0,5 are plotted for ratios of particle size to bed roughness of 0,5; 1,0 and 2,0. Also plotted in the hydraulically rough zone on Fig. 3 are the Shields parameter values for the same size ratios according to Andrews (1983). Although the corresponding curves do not coincide, the displacements associated with changes in bed roughness are similar. The relationship developed by Andrews for the critical dimensionless shear stress (τ_c^*) required to entrain particles from a heterogeneously sized gravel bed is compared in Fig. 4 with the Einstein results for probabilities of entrainment of 0,1; 0,5 and 0,9. Although Andrews used the d_{50} size to represent bed roughness whereas Einstein used the d_{65} size, agreement between the two approaches is reasonable, with Andrew's data lying between the 0,5 and 0,9 probability curves over their whole range. The Einstein curves do not, however, tend to a minimum critical dimensionless shear stress, as suggested by Andrews.

The shapes of Einstein entrainment curves in the transition zone (Figs. 1, 2 and 3) reflect variations in the correction factors. As the grain Reynolds number decreases the hiding factor first decreases, causing a dip in the curve, and then both the hiding factor and pressure correction factor increase, causing the curve to rise. The break in slope on the rising portion occurs when the pressure correction factor begins to decrease rapidly. The irregularities apparent in Fig. 3 occur because of the different relationship between hiding factors and grain Reynolds number for different ratios of particle size to bed roughness.

Application to sorting

The Einstein entrainment probability model has been applied to determine the hydraulic conditions of a fictive stream that are conducive to sorting gold (ρ_s = 19 300 kg/m^3) and quartz (ρ_s = 2 650 kg/m^3) particles with similar fall velocities. Spherical gold particles with diameter 0,10 mm are considered,

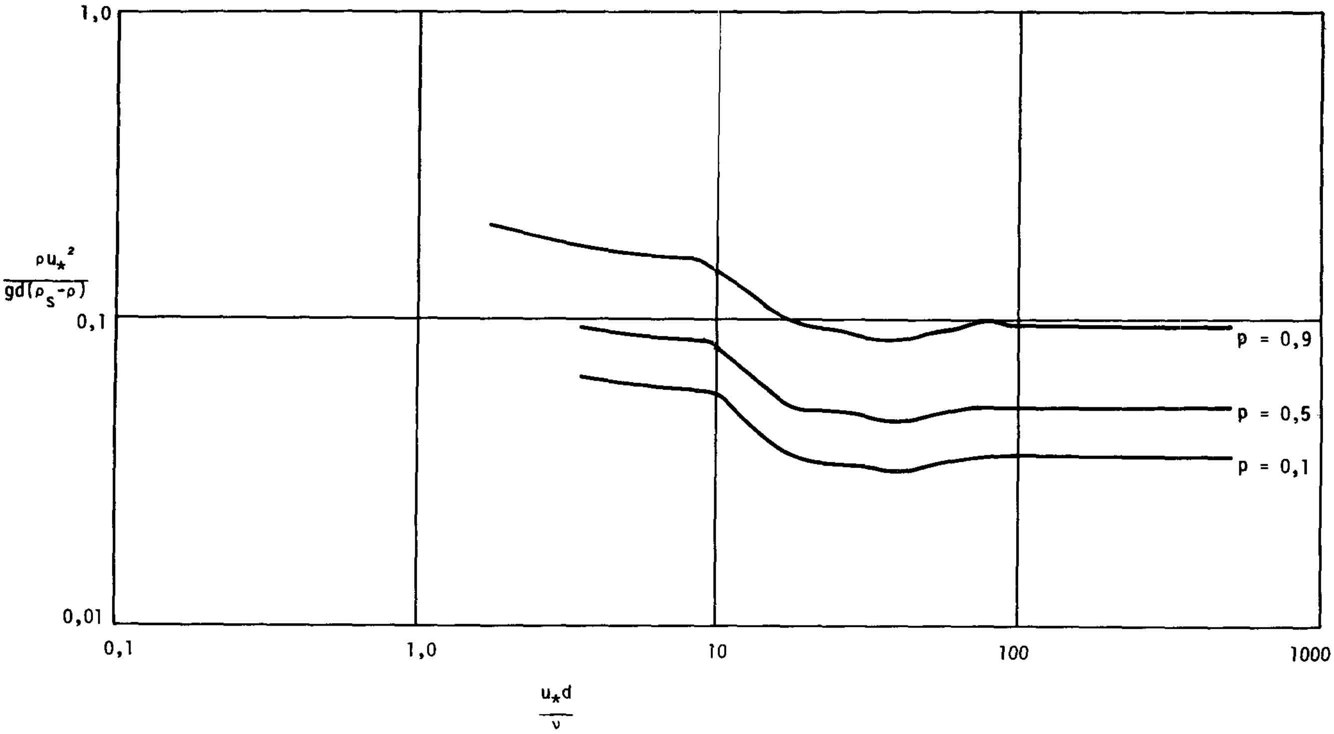

Figure 2
Hydraulic conditions representing different probabilities of entrainment (p) from uniformly sized bed material, according to the Einstein model.

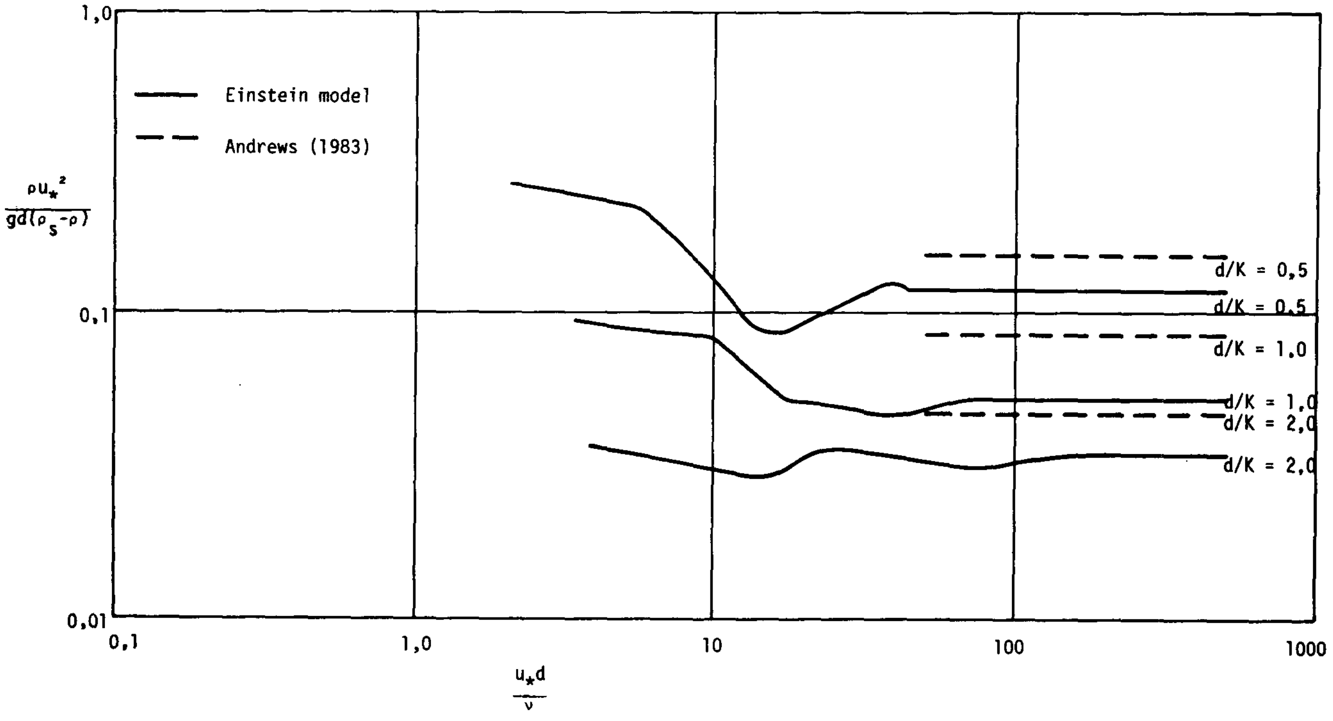

Figure 3
The effect of bed roughness on entrainment conditions. The Einstein curves all represent a probability of entrainment of 0,5 and are plotted for the ratios of particle size (d) to representative bed roughness (K) indicated. The entrainment conditions according to the relationship of Andrews (1983) are also plotted in the hydraulically rough zone and show comparable displacements with changes of d/K.

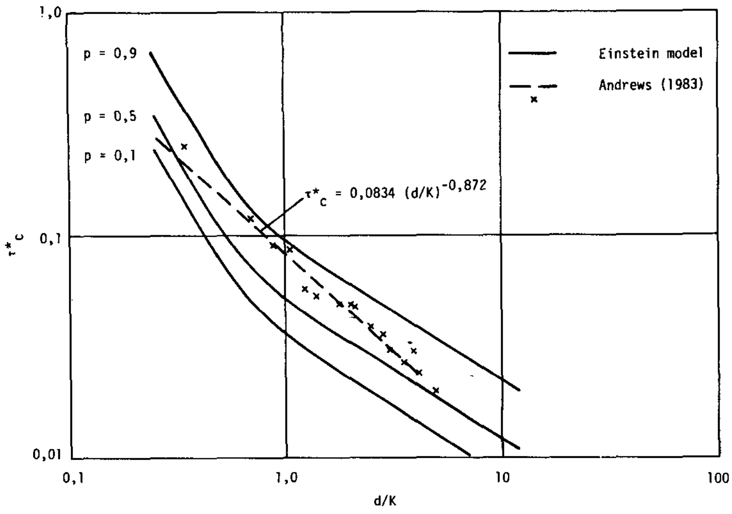

Figure 4
Comparison of the Einstein model with the results of Andrews (1983). Ordinate values represent the critical dimensionless shear stress (τ^*_c) required to entrain a particle of size d from a nonuniformly sized bed material with representative roughness K. Andrews used the median particle size of the subsurface bed material for K.

which have a fall velocity of 0,07 m/s (De Villiers, 1986). Spherical quartz particles with diameter 0,48 mm would have approximately the same fall velocity (Graf and Acaroglu, 1966). Water density and viscosity are assumed to be 1 000 kg/m^3 and 10^{-6}m^2/s respectively.

The shear velocities required to entrain each particle type from a bed with a range of representative roughness (K) values were determined and plotted in Fig. 5. Because of the probabilistic nature of entrainment a unique entrainment condition cannot be defined and conditions representing probabilities of entrainment of 0,1 and 0,9 are plotted. For each particle type, the band defined by the curves for these entrainment probabilities represents conditions for which transport involving significant bed interaction would take place. For flow conditions above the band there would be little bed interaction during transport and particles on the bed would be easily entrained. For flow conditions below the band most particles in transport would deposit and most particles on the bed would not be entrained. The conditions necessary for the gold and quartz particles to be transported in suspension according to the criterion of Engelund (1973) are also indicated on Fig. 5. This criterion states that particles can be suspended if

$$w/u_*' < 0{,}8$$

in which w is the particle fall velocity and u_*' is the shear velocity associated with grain roughness of the bed.

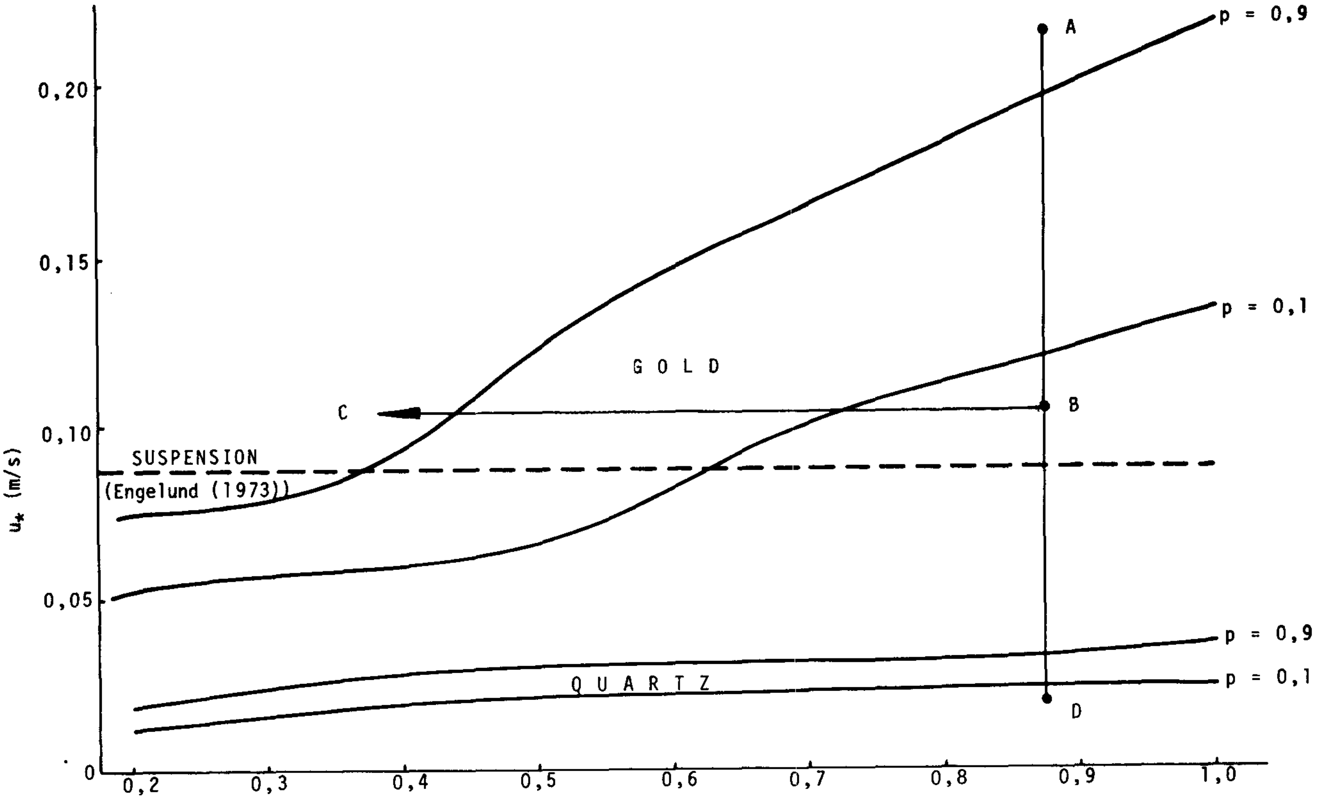

Figure 5
Hydraulic conditions required for suspension and entrainment from a plane bed with roughness K of 0,10mm gold and 0,48mm quartz spheres, both of which have fall velocities of 0,07 m/s. The areas bounded by the 0,1 and 0,9 entrainment probability curves for each particle type represent conditions of transport with significant bed interaction. The flow conditions between the bands defined by the probability curves for the particle types represent the greatest difference between entrainment probability and hence the most favourable conditions for segregation. A temporal or spatial change of flow conditions from point A to point B would increase deposition from transport more for gold than for quartz because of the greater reduction in entrainment probability, although the change in transport capacity would be similar for both particle types. A decrease of bed roughness, along B – C, would reduce gold deposition but would have little effect on quartz deposition. A change from A to D would result in significant deposition of both types, with little segregation.

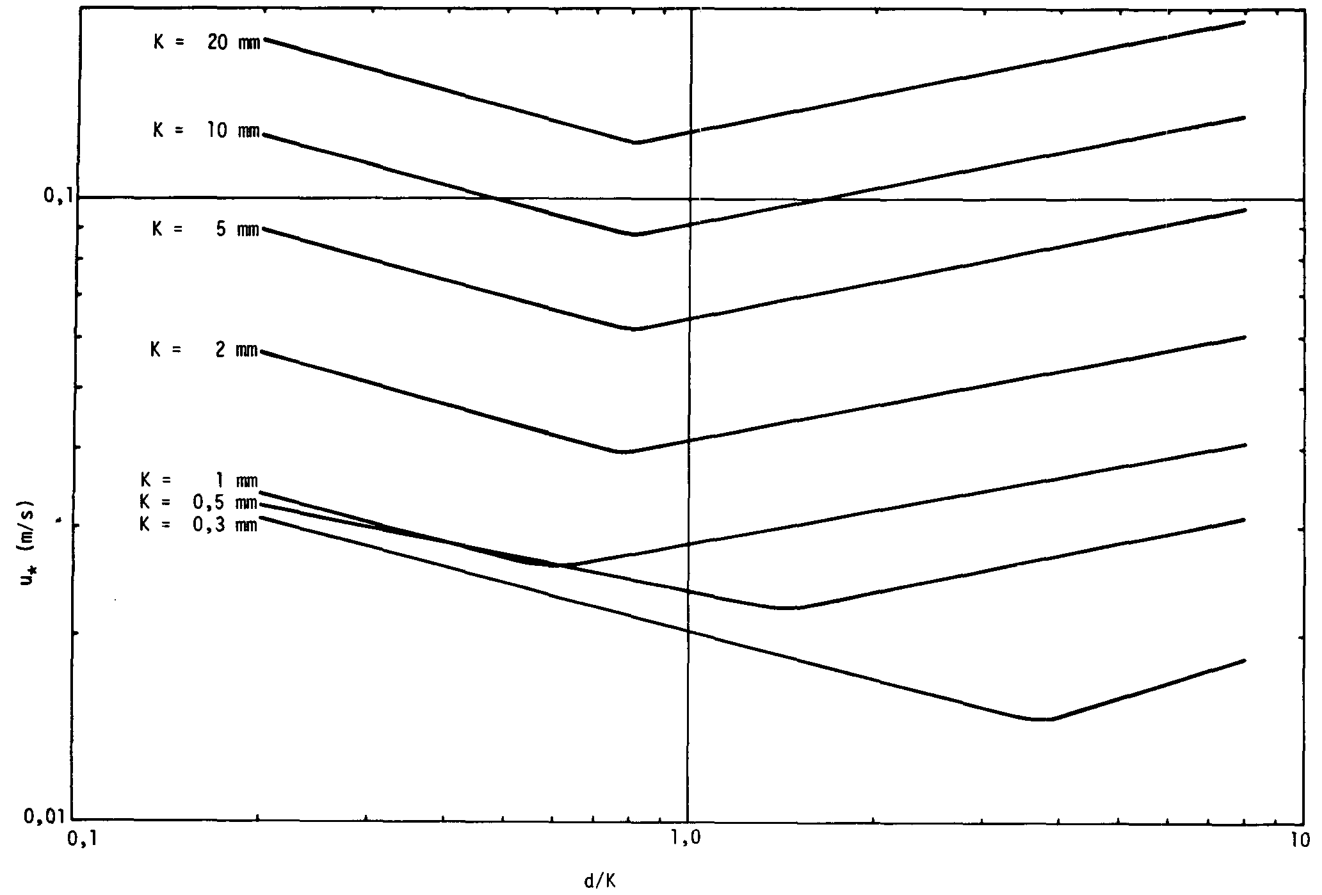

Figure 6

Conditions for entrainment of quartz sand particles of size d from a nonuniformly sized bed material with representative roughness K. The curves represent probabilities of entrainment of 0,5 according to the Einstein model and indicate preferential entrainment of intermediate sizes.

Fig. 5 shows that the entrainment probability for the gold particles is less than that for the quartz particles for all hydraulic conditions and bed roughness values. For some conditions, however, there would be very little bed interaction and deposition from transport of either particle type, while for other conditions most particles of both types would deposit. Concentration of gold would take place most readily at those conditions where the probability difference is greatest, which occurs between the bands shown.

Fig. 5 can be used to describe the changes of relative deposition rates of gold and quartz particles as flow conditions (represented by u_*) or bed conditions (represented by K) vary with time or distance. Suppose, for example, that gold and quartz particles are transported together in flow represented by point A on the diagram. For these conditions little bed interaction and deposition of either type of particle would take place. If flow conditions were to change (either with time or distance) to point B, gold and quartz would both deposit more rapidly as a result of reduced transport capacity. More gold than quartz would deposit, however, because the low entrainment probability would result in trapping during bed interactions; transport of quartz would continue to take place with little bed interaction. Gold enrichment of the bed would therefore take place. If the flow conditions represented by point B were to persist, the representative roughness of the bed would decrease progressively with time (along B – C) due to deposition of the fine gold grains. The probability of re-entrainment of gold would increase and eventually gold deposition would virtually cease although transport capacity remains unchanged. If flow conditions changed from point A to point D both quartz and gold particles would deposit with ease and no significant sorting would occur.

Sorting can result from selective entrainment as well as from differential net deposition. If flow conditions represented by point D were to occur over a bed on which both gold and quartz grains were present, very little entrainment of either would take place. If flow conditions changed to point B the quartz grains would be entrained while gold remained on the bed. If flow conditions changed further to point A, both gold and quartz particles would be easily entrained and removed. The flow conditions between the entrainment bands of the two particle types are potentially the most favourable for their sorting.

A more general analysis of sorting by selective entrainment is shown in Fig. 6. Here the Eisntein model has been applied to beds of non-uniformly sized quartz-density material. The shear velocities necessary to entrain (with a probability of 0,5) different sized quartz particles from beds with a range of representative roughness values (K) are shown. The results indicate distinct preferential entrainment of intermediate sized particles. The most easily entrained particles have d/K values of about 0,8 for beds with representative roughnesses of 2 mm and greater, but this increases considerably for fine beds where hiding by the viscous sublayer becomes effective. The results imply that flow over a mixed grain bed will tend to remove intermediate size fractions, leaving both larger and smaller sizes. This requires experimental verification but the bimodal nature of some fluvial gravel with significant deficiencies of intermediate fractions (e.g. Dyer, 1970; Kellerhals and Church, 1977) could be the result of such selective removal. The results presented in Fig. 6 should be interpreted in a qualitative sense only; they cannot easily be applied quantitatively because of problems related to the dynamic nature of armouring and the definition of representative roughness.

The comparison of the Einstein model with the Shields criterion shows that it is suitable for use as an entrainment criterion in the hydraulically rough zone. Its application in the transition zone would require further clarification of the variation of the hiding factor through this zone. The consistence of the model predictions with the result of Andrews (1983) confirms the model's validity in accounting for the effects of bed roughness on entrainment in non-uniformly sized bed material. The usefulness of the model for describing sorting by size and density during entrainment and deposition has been illustrated. A distinct advantage of the Einstein approach is that entrainment is expressed probabilistically and relative rates of entrainment of different particles can be predicted at any specified hydraulic conditions. Criteria such as those of Shields (1936) and Andrews (1983) define only critical threshold conditions.

Prediction of heavy mineral concentration

The formation of mineral-enriched deposit depends not only on relative probabilities of entrainment as described in the previous section, but also on the relative frequencies of bed interaction and variations of these with time and space. The probability of entrainment can be interpreted in terms of the time a particular particle rests on the bed between successive periods of suspension. A low probability means that the particle will spend a relatively long time on the bed between settling and resuspension. Therefore, if two particles have different probabilities of entrainment it may be expected that more of those with the lower probability than with the higher probability will be present on the bed at any given instant. This only applies, however, if the concentrations in suspension near the bed are similar for the two particles, as this determines the frequency of bed interaction. Net deposition will occur only if the frequency of entrainment is less than the frequency of deposition. Direct application of Einstein's model for predicting segregation, as represented by Fig. 6 is therefore restricted to particles with similar fall velocities and suspension concentrations. A generally applicable description of sorting requires knowledge of particle characteristics as well as local concentrations and entrainment probabilities and their spatial and temporal variations.

A model has been developed to describe the local composition and spatial distribution of deposits originating from a suspended mixture of particles in steady longitudinally uniform flow.

The vertical and longitudinal distribution of sediment in suspension can be described by the following two-dimensional diffusion-convection equation (Jobson and Sayre, 1970; Sarikaya, 1977; Kerssens *et al.*, 1979).

$$0 = -v\frac{\partial C}{\partial x} + w\frac{\partial C}{\partial y} + \frac{\partial}{\partial y}(\varepsilon_y\frac{\partial C}{\partial y}) \tag{3}$$

in which C is sediment concentration, x and y are the longitudinal and vertical directions in a flow, v and w are the corresponding particle convection velocities (w is the particle fall velocity and is assumed positive downwards), and ε_y is the sediment diffusivity in the vertical direction. Longitudinal diffusion is assumed to be negligible compared with longitudinal convection and is ignored.

The vertical diffusivity for sediment is assumed to be equal to the diffusivity for momentum and is distributed in the vertical according to the parabolic distribution derived from Prandtl's mixing length theory assuming a linear distribution of shear stress (Graf, 1971), i.e.

$$\varepsilon_y = \kappa u_* (D - y) - \frac{y}{D} \tag{4}$$

in which κ is the Von Karman constant (= 0,4 for clear water) and D is the total flow depth.

The boundary conditions to be satisfied in the solution of Eq. (3) are, at the water surface

$$\varepsilon_y\frac{\partial C}{\partial y} + wC = 0 \tag{5}$$

and at the bed

$$\varepsilon_y\frac{\partial C}{\partial y} + (1 - P)wC = 0 \tag{6}$$

in which P is the probability that a particle settling to the bed is permanently deposited. This probability can be expressed as 1 - p, where p is Einstein's probability of entrainment.

For any specified vertical distribution of sediment concentration at the head of a channel reach, the solution to Eqs. (3), (5) and (6) will describe the vertical and longitudinal distribution of concentration through the reach and enable the longitudinal distribution of deposits to be calculated. Solutions can be obtained separately for different particle types to determine the local composition of deposits at any distance along the reach.

Eqs. (3), (5) and (6) can be expressed in finite difference form and solved by an explicit method as, for example, by Sarikaya (1977).

The model described above does not account for bed load and this precludes consideration of the bed as a source of suspended material and spatial variation of the bed boundary condition due to bed load sorting. This limits application of the model to depositional systems in which changes of bed characteristics are gradual. The model also does not consider unsteady conditions and cannot account for the changing character of the bed material with time as deposition and bed load sorting takes place. It is reasonable to neglect time when considering suspended load only as the time scales of suspended transport and bed material changes are significantly different. Time would have to be considered, however, in a description of sorting in bed load. A realistic, generally applicable model would therefore have to consider bed load and unsteady conditions.

The model can be applied to describe the distribution of deposition rates of any number of different particle types in transport together so that the composition of deposits at any location along a channel can be predicted. For illustration, the deposition in a hypothetical channel reach of gold and quartz particles with the same fall velocity is considered. The channel is assumed to have a width of 5m, a length of 50m and a flow depth of 0,40m. The longitudinal slope and representative bed

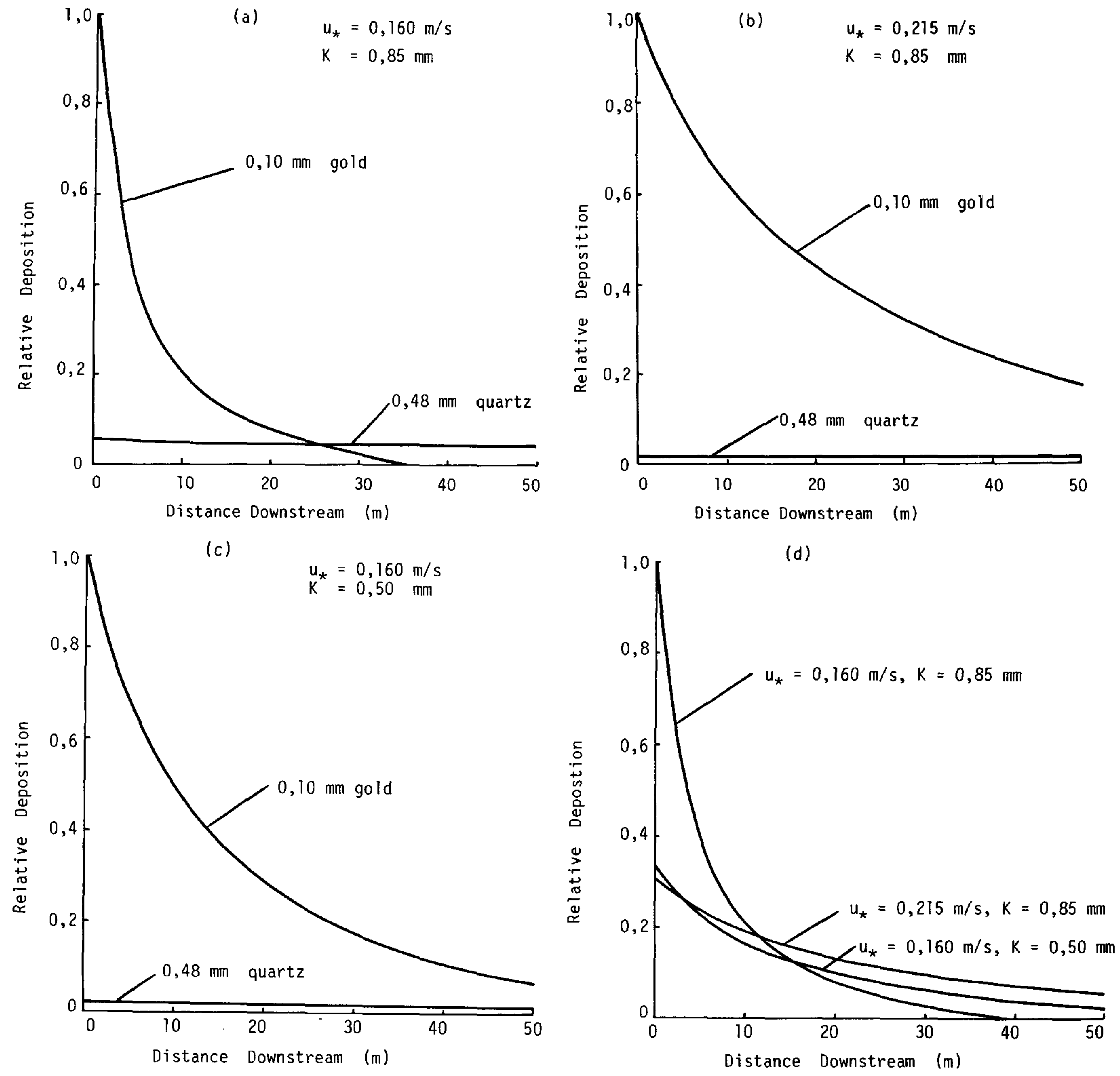

Figure 7

The distributions of predicted deposition rates of 0,10mm gold and 0,48mm quartz along a hypothetical channel with a width of 5mm, a length of 50m and flow depth of 0,40m. In Figs. a, b and c the deposition rates of gold and quartz are plotted relative to the rates for gold at the beginning of the channel and compared to the conditions indicated. In Fig. d the deposition rates for gold for the three conditions are plotted relative to the rate for gold at the beginning of the channel for the conditions of Fig. a and compared.

roughness are varied to illustrate the effects of different hydraulic and bed conditions. Gold particles with diameter 0,10mm and quartz particles with diameter 0,48mm (both having fall velocities of 0,07 m/s) are introduced with equal depth-averaged concentrations at the head of the channel reach. The variation of concentration over the flow depth is specified in accordance with the equilibrium profile defined by the hydraulic conditions in the channel. At the beginning of the reach there is therefore no material in suspension in excess of the carrying capacity of the stream, and deposition must result purely from the trapping effect of the bed during interaction with the suspended material. This phenomenon is described by Einstein (1968) for the extreme case of very fine (3,5μ to 30μ) quartz grains depositing gravel bed with particle sizes up to 150mm.

Fig. 7a shows the deposition pattern with a channel gradient of 0,0065 and a representative bed roughness of 0,85 mm. Deposition rates are plotted relative to the rate for gold particles at the beginning of the reach. The shear velocity of this condition is 0,160 m/s, well in excess of the 0,088 m/s required for suspension according to Engelund's (1973) criterion. For these conditions the deposition probability of the gold is relatively high (0,27) and the gold deposits rapidly near the beginning of the reach. This reduces the availability of gold further downstream and deposition therefore decreases although the probability of deposition is constant along the reach. The deposition probability of the quartz particles is low (0,0013) and the deposition rate is therefore low but practically uniform along the whole reach. The different rates of deposition result in varying composition of deposits along the reach with a preponderance of gold over the first half and of quartz over the second half. All of the gold introduced at the beginning of the reach has deposited after about 35m but 95 per cent of the quartz is still in suspended transport after 50m.

Fig. 7b shows the effect on the distribution of deposition rate of an increase in shear velocity. The channel gradient has been increased to 0,0118 resulting in a shear velocity of 0,215 m/s. For these conditions the deposition probabilities of both the gold and quartz particles have reduced (0,0555 and 0,0007 respectively). Deposition rates are therefore low for both types of particles but large enough in the case of the gold still to affect significantly the downstream availability. The composition of deposits therefore varies along the reach but is everywhere preponderant in gold.

Fig. 7c shows the effect of representative roughness of the bed. Flow conditions are the same as for Fig. 7a but the representative roughness is reduced to 0,50mm. This has a similar effect to increasing the shear velocity as deposition probabilities are reduced to 0,0367 and 0,0007 for gold and quartz particles respectively.

In Fig. 7d the distributions of deposition rates of the gold particles are compared for the different hydraulic and bed conditions. The highest rate of deposition of gold occurs for the combination of hydraulic and bed conditions that results in the highest deposition probability. This concentration also results in the most rapidly varying and least extensive distribution of deposition rate because of the effect of deposition on downstream availability.

The above applications illustrate that significantly different spatial distributions of deposition from suspension can occur for particles with the same fall velocity but different size and density under uniform flow conditions. The differences in distribution are due to the differential deposition probabilities which are determined by particle, flow and bed characteristics. It is important to note that the deposition probability for each particle type is constant along the reach and varying deposition rates are related directly to the changing concentration near the bed which is itself determined by the deposition probability. The significant variation of the composition of the deposits along a channel with uniform hydraulic conditions and bed roughness demonstrates that it is meaningless to apply sorting criteria to isolated sections. The composition at any section depends on conditions for a considerable distance upstream as much as on local hydraulics and bed characteristics.

It is important to appreciate that the results presented above represent distributions of deposition rates at an instant in time and the ultimate distribution of deposits could be significantly different. For example, if the situation represented by Fig. 7a persisted for a long period of time the representative roughness near the head of the reach would decrease due to deposition of fine gold. The deposition probability and deposition rate of gold would then decrease and the availability and deposition of gold downstream would increase. Gold enrichment would therefore progress downstream with time. An extensive, uniform deposit could therefore be the result of a long period of deposition with a high or low deposition probability. Variations of flow conditions with time would obviously also have significant effects. It is therefore essential to consider time effects when simulating heavy mineral deposition in a channel.

The above applications are restricted to uniform flow conditions. The effects of spatial variations of hydraulic conditions could be considered by extending the model to account for non-uniform flow. Also neglected in this study are the effects of bed load in determining variations of representative roughness through sorting and providing a source for suspended material, and the effects of bed forms.

The results presented here illustrate the concentrating effects of differential entrainment and deposition for the relatively simple case of suspended transport over a fixed bed. A realistic, general description of sorting in a channel would require a comprehensive model that accounts, in addition, for bed load movement and unsteady, non-uniform flow conditions.

Conclusions

Einstein's (1950) entrainment probability function can be used satisfactorily as an entrainment criterion for hydraulically rough conditions. For uniformly sized bed material the hydraulic conditions resulting in entrainment with a probability of 0,50 as predicted by this function agree closely with the Shields criterion for grain Reynolds numbers greater than about 40. In the transition zone between hydraulically rough and hydraulically smooth or laminar conditions the prediction of entrainment by the Einstein function is sensitive to the description of the hiding factor, which requires further investigation. Einstein's theory is not applicable to laminar flow conditions. For heterogeneously sized bed material the predicted effects of bed roughness on entrainment are consistent with observed behaviour. The function can be applied to particles with different size and density characteristics to describe sorting under any hydraulic conditions.

The incorporation of the Einstein function in a simple suspended load model has been used to describe the segregation of gold and quartz particles with the same settling velocity by bed interaction during transport under steady, uniform flow conditions. This application demonstrates that it is meaningless to apply sorting criteria to isolated sections in a stream. It is also essential to account for temporal and spatial variations of bed material and hydraulic conditions.

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