

# Biological excess phosphorus removal — Steady state process design

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## Abstract

From the Kinetic model describing phosphorus (P) release and P uptake, design equations are developed to determine the release, uptake and removal of P in modified Bardenpho and UCT systems under constant flow and load conditions. Knowing the influent COD and TKN concentrations, for specified sludge age and anaerobic mass fraction, the fraction of influent readily biodegradable COD converted to short-chain fatty acids by the non-polyP organisms (and the associated P release to sequester the short-chain fatty acids), are calculated. In this manner the influent substrate fractions available to the polyP and non-polyP organisms are determined, and the respective masses of organisms generated from the substrate are calculated. From the mass of sludge wasted per day, the concentration of P removed from the influent is calculated. The calculated P removal correlates well with the removal observed in laboratory-scale systems treating municipal waste flows, over wide ranges of sludge ages and influent characteristics.

## Introduction

Although considerable research effort worldwide has been directed towards improving understanding of the biological excess phosphorus (P) removal phenomenon, designs of activated sludge systems to accomplish biological excess P removal still are based on experience and semi-empirical methods (Siebritz *et al.*, 1983 and Wentzel *et al.*, 1985). Clearly, the need exists for design procedures based on more fundamental behavioural patterns and kinetics.

Recently, a model describing the kinetic behaviour of enhanced cultures of the organisms mediating biological excess P removal (generically termed polyP organisms) has been developed (Wentzel *et al.*, 1988a; Wentzel *et al.*, 1989a; Wentzel *et al.*, 1989b). This model is highly complex and incorporates 13 processes and 14 compounds. However, for description of steady state enhanced culture systems, the model can be greatly simplified with only minor concessions to accuracy.

These two models (the kinetic model and its simplified steady state version) apply strictly to description of enhanced cultures of polyP organisms. For descriptions of mixed cultures of polyP and non-polyP organisms, the models require modification: Enhanced culture systems require input of short-chain fatty acids (SCFA) to the anaerobic reactor, a situation not present in "normal" mixed culture biological excess P removal systems. In the mixed culture systems (e.g. in systems treating municipal waste streams) the SCFA need to be produced by the action of non-polyP organisms. Hence, description of the non-polyP organism behaviour also is required in a model for mixed culture biological excess P removal systems. This paper describes the development of a steady state design procedure, with the associated equations, for biological excess P removal in "normal" mixed culture activated sludge systems.

## Enhanced cultures

### Enhanced culture behaviour

In developing simplified steady state equations for the enhanced cultures it is useful first to describe briefly the overall behaviour of the polyP organisms in such cultures.

Development of enhanced cultures of polyP organisms basically requires anaerobic/aerobic sequencing with short-chain fatty acids (SCFA) present in the anaerobic phase (Wentzel *et al.*, 1988a; Wentzel *et al.*, 1988b).

### In the anaerobic phase:

The SCFA are sequestered by the polyP organisms and stored as poly- $\beta$ -hydroxybutyrate (PHB). The stored polyphosphate (polyP) is cleaved to provide energy for sequestration and the cleaved P is released to the bulk liquid. On a molar basis 1 mol P is released/mol of SCFA sequestered ( $\approx 0,5$  mgP released/mg SCFA as COD sequestered).

### In the aerobic phase:

Oxygen is available as an external electron acceptor and the stored PHB is utilised as a substrate source. A fraction of the PHB is oxidised to provide energy for growth and for polyP formation (with associated P uptake); the balance of the PHB is incorporated into new cell mass. From bioenergetics, the energy requirements for polyP formation are minor compared to the requirements for growth. The stored polyP is abstracted from the system via the waste activated sludge, to give a nett P removal.

### Simplified model for P removal in enhanced cultures

It was stated earlier that the kinetic model describing polyP organism behaviour in enhanced cultures is relatively complex with many processes. Some of these processes have a relatively minor influence on the eventual mass of P removal; for the purpose of design these can be omitted. Others, within the usual ranges of system conditions, will not become operational, so that these also can be omitted for the purpose of design.

Two assumptions greatly simplify the development of the steady state model:

- P release for anaerobic maintenance energy requirements is always small compared to P release for sequestration energy requirements, and thus can be neglected. The implication of this assumption is that the polyP content of the polyP organisms in the activated sludge wasted per day is constant. From experimental observations on enhanced cultures, and simulation studies using the polyP organism kinetic model,

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this assumption is completely acceptable.

- All the substrate sequestered in the anaerobic zone, and stored as PHB, is utilised in the subsequent aerobic zone. From simulation studies, using the polyP organism kinetic model, the sequestered substrate remaining at the discharge point of the aerobic zone in UCT and modified Bardenpho systems, is usually very small compared to the influent substrate available for sequestration.

Taking due account of the above two assumptions, and their implications, steady state equations can be developed for the enhanced culture systems.

### Steady state equations

In developing the steady state equations cognisance must be taken of the anaerobic and aerobic states, and their associated processes.

### Anaerobic release of P and sequestration of SCFA

In the enhanced culture systems acetate is the only substrate. (For details of the influent composition see Wentzel *et al.*, 1988b.) The rate of acetate sequestration is zero order with respect to acetate and very rapid. As a consequence, in the usual range of anaerobic mass fractions employed in biological excess P removal systems (anaerobic mass fraction > 5 per cent), all the acetate will be sequestered in the anaerobic zone. Accordingly, there is no necessity for a kinetic expression for the sequestration — the total mass of acetate sequestered and stored as PHB ( $MS_{phb}$ ) equals the mass of acetate supplied ( $MS_{bs,a}$ ), i.e.

$$(MS_{phb}) = (MS_{bs,a}) \quad (\text{both as mgCOD}) \quad (1)$$

The mass of P released ( $MP_{rel}$ ) is proportional to the mass of acetate sequestered, i.e.

$$MP_{rel} = f_{P,rel} MS_{bs,a} \quad (\text{mgP}) \quad (2)$$

where  $f_{P,rel}$  = constant of proportionality,  
P release/acetate sequestered  
= 0,5 mgP/mg acetate as COD.

### Aerobic PHB utilisation and P uptake

The following steady state equations describe the various processes associated with PHB utilisation and P uptake:

Synthesis: Noting the assumption that all the PHB is utilised in the aerobic zone, then the biological active mass of polyP organisms synthesised ( $MX_{B,G}$  synthesis) is expressed as:

$$MX_{B,G}(\text{synthesis}) = Y_G MS_{phb} \quad (\text{mgVASS}) \quad (3)$$

where  $Y_G$  = specific yield constant for polyP organisms  
= 0,45 mgVASS/mgCOD.

From Eq. (1), this equation is reformulated as:

$$MX_{B,G}(\text{synthesis}) = Y_G MS_{bs,a} \quad (\text{mgVASS}) \quad (4)$$

Endogenous mass loss: From the kinetic model, this process occurs during both the anaerobic and aerobic phases. Endogenous mass loss manifests as a decrease in the biological active organism

mass, and, *inter alia*, an accumulation of particulate endogenous residue. Kinetic rate expressions for these are obtained from the kinetic model. For the decrease in biological active mass:

$$\frac{dMX_{B,G}}{dt} = -b_G MX_{B,G} \quad (\text{mgVASS/d}) \quad (5)$$

where  $b_G$  = specific endogenous mass loss rate constant for polyP organisms  
= 0,04/d at 20°C.

For the increase in endogenous mass ( $MX_{E,G}$ )

$$\frac{dMX_{E,G}}{dt} = +b_G f_{Ep,G} MX_{B,G} \quad (\text{mgVESS/d}) \quad (6)$$

where  $f_{Ep,G}$  = fraction of polyP organisms that is unbiodegradable particulate residue  
= 0,25 mgVSS/mgVASS.

Biological active mass. From Eqs. (4) and (5), doing a mass balance over the system, we derive the steady state equation for the biological active mass of polyP organisms in the system ( $MX_{B,G}$ ), i.e.

$$MX_{B,G} = \frac{Y_G MS_{bs,a} R_s}{1 + b_G R_s} \quad (\text{mgVASS}) \quad (7)$$

where  $R_s$  = system sludge age (d)  
 $MS_{bs,a}$  = influent mass of acetate supplied per day (mgCOD/d)  
=  $Q S_{bs,ai}$   
 $Q$  = influent flow rate (ℓ/d)  
 $S_{bs,ai}$  = influent acetate concentration (mgCOD/ℓ)

Endogenous mass generated: From Eq. (6), doing a mass balance over the system, we derive an equation for the endogenous mass in the system generated by the polyP organism endogenous mass loss ( $MX_{E,G}$ ), i.e.

$$MX_{E,G} = f_{Ep,G} b_G MX_{B,G} R_s \quad (\text{mgVESS}) \quad (8)$$

Total mass of sludge: In the enhanced cultures, because there is no inert material in the influent, the total volatile sludge mass in the system ( $MX_{G,t}$ ) is given by:

$$MX_{G,t} = MX_{B,G} + MX_{E,G} \quad (\text{mgVSS}) \quad (9)$$

$$= \frac{Y_G MS_{bs,ai} R_s}{(1 + b_G R_s)} \left[ 1 + f_{Ep,G} b_G R_s \right] \quad (10)$$

Mass of sludge wasted/day: The mass of sludge wasted/day ( $\Delta MX_{G,t}$ ), or equivalently at steady state the sludge production per day, is given by:

$$\Delta MX_{G,t} = MX_{G,t} / R_s \quad (\text{mgVSS/d}) \quad (11)$$

Mass of phosphorus removed/day: The mass of phosphorus removed/day ( $\Delta MP$ ) is via the P contained in the sludge wasted/day, i.e. in the active and endogenous mass wasted/day ( $\Delta MX_{B,G}$  and  $\Delta MX_{E,G}$  respectively):

$$\begin{aligned} \Delta MP &= f_{XBG,P} \Delta MX_{B,G} + f_{XEG,P} \Delta MX_{E,G} \\ &= f_{XBG,P} MX_{B,G}/R_s + f_{XEG,P} MX_{E,G}/R_s \quad (\text{mgP/d}) \quad (12) \end{aligned}$$

where  $f_{XBG,P}$  = fractional P content of biological active mass (mgP/mgVASS)

$f_{XEG,P}$  = fractional P content of endogenous polyP organism mass (mgP/mgVESS).

Hence the removal of P from the waste stream ( $\Delta P$ ) is given by:

$$\Delta P = \Delta MP/Q \quad (\text{mgP/l influent}) \quad (13)$$

where  $Q$  = influent flow rate ( $\theta$ d).

In Eq. (12), the fractional P content of the biological active mass of the polyP organisms ( $f_{XBG,P}$ ) needs to be determined. This is achieved from the steady state experimental response of the enhanced cultures, as follows:

From Eqs.(7), (8), (11), (12) and (13), we solve for  $f_{XBG,P}$ , i.e.

$$f_{XBG,P} = \Delta P \frac{(1 + b_G R_s)}{S_{bs,ai} Y_G} - \frac{f_{XEG,P} f_{Ep,G} b_G R_s}{(\text{mgP/mgVASS})} \quad (14)$$

The fractional P content of the polyP organism endogenous mass ( $f_{XEG,P}$ ) can be accepted at 0,03 mgP/mgVESS. From experimental data, with assumptions made above, all the parameters on the RHS of Eq. (14) are known, and  $f_{XBG,P}$  can be calculated. Applying Eq. (14) to the four steady state enhanced culture systems, as reported by Wentzel *et al.* (1989b), values for  $f_{XBG,P}$  were calculated and are shown in Table 1; the average value for  $f_{XBG,P}$  of 0,41 mgP/mgVASS appeared to be acceptable. However, using this value for  $f_{XBG,P}$  in correlating experimental data and theoretical predictions for mixed cultures (see below) it was found that  $f_{XBG,P}$  is slightly high and that a value of 0,38 mgP/mgVASS gives improved predictions.

## Mixed cultures

Having developed a simplified steady state model for enhanced cultures of polyP organisms the model now will be extended to incorporate mixed cultures of polyP and non-polyP organisms.

From the investigations of Wentzel *et al.* (1989a, 1989b) the polyP and non-polyP organism populations appear to act virtually independently of each other in the "normal" mixed cultures of biological excess P removal activated sludge systems. This allows that analysis of the two population groups can be largely separated. The only significant interaction, a one-directional one, is in the anaerobic zone:

In many "normal" municipal sewages the acetate (or other SCFA) content is small or not present (Wentzel *et al.*, 1988a). Wentzel *et al.* (1985) have shown that in the anaerobic zone the readily biodegradable COD (RBCOD) component of the influent is converted to SCFA by the non-polyP organisms mass, thereby

making SCFA available to the polyP organism mass for sequestration. The rate of conversion is much slower than the rate of sequestration, so that the rate of conversion controls the rate of sequestration. Hence, the mass of SCFA substrate that becomes available in the anaerobic reactor is governed by the kinetics of conversion. Should some SCFA be present in the influent, these SCFA will add to the SCFA generated by conversion (see Appendix A).

## Kinetics of conversion of RBCOD to SCFA

From the behavioural pattern of P release in anaerobic reactors, the following conversion model is proposed (Wentzel *et al.*, 1985). It is hypothesised that:

- Only readily biodegradable COD ( $S_{bs}$ ) can be converted to a form suitable for sequestration by the polyP organisms ( $X_{B,G}$ ) within the time scale of residence of the mixed liquor in the anaerobic reactor.
- The conversion is mediated by the non-polyP heterotrophic mass in the anaerobic zone, ( $X_{B,Hn}$ ).
- All  $S_{bs}$  converted is immediately sequestered by the polyP organisms, i.e. the rate of P release is controlled by the rate of conversion.
- All  $S_{bs}$  not converted in the anaerobic reactor is utilised subsequently for non-polyP heterotrophic metabolism.
- The rate of conversion of  $S_{bs}$  is given by

$$dS_{bs}/dt = -K X_{B,Hn} S_{bs} \quad (15)$$

where  $K$  = first order rate constant ( $0,06 \text{ d}^{-1}$ )

$S_{bs}$  = readily biodegradable COD concentration in the anaerobic reactor (mgCOD/l)

- The rate of P release is assumed to be stoichiometrically related to the  $S_{bs}$  sequestered. Hence:

$$\begin{aligned} dP/dt &= -C_{sp} (dS_{bs}/dt) \\ &= C_{sp} K X_{B,Hn} S_{bs} \quad (16) \end{aligned}$$

where  $C_{sp}$  = stoichiometric ratio ( $\Delta P:\Delta S_{bs}$ )  
= 0,5 mg( $\text{PO}_4\text{-P}$ )/mgCOD converted.

Should nitrate be recycled to the anaerobic reactor, the conversion of RBCOD to SCFA is further complicated. It is hypothesised that any nitrate recycled to the anaerobic reactor is utilised as electron acceptor by the non-polyP heterotrophs. The implication is that the SCFA no longer are released, but are metabolised directly by the non-polyP organisms, until the nitrate is depleted. In the model this can be accommodated by reducing the amount of  $S_{bs}$  available for conversion as follows:

$$S_{bsi}^I = S_{bsi} - r.8,6. N_{03,r} \quad (17)$$

where  $S_{bsi}^I$  = Readily biodegradable COD available for conversion per litre influent (mgCOD/l)

$S_{bsi}$  = Readily biodegradable influent COD concentration (mgCOD/l)

$N_{03,r}$  = Nitrate concentration in recycle to anaerobic reactor (mgN/l)

$r$  = Recycle ratio based on influent flow

8,6 = Mass of COD removed per unit nitrate denitrified in synthesis [mgCOD/mg( $\text{NO}_3\text{-N}$ )].

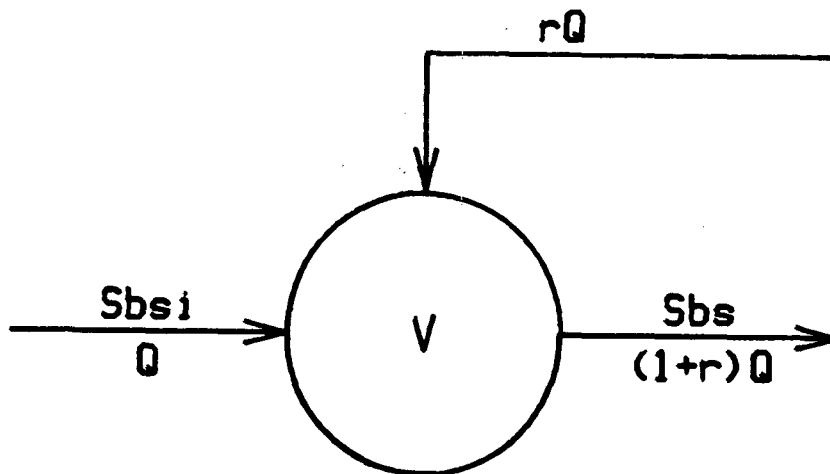


Figure 1

Schematic diagram of a single anaerobic reactor (with volume  $V$ ) receiving influent flow (with flow rate  $Q$ ) and recycle flow (with recycle ratio  $r$ ). The influent readily biodegradable COD (with concentration  $S_{bsi}$ ) is converted in the anaerobic reactor to short-chain fatty acids by the non-polyP heterotrophs. The readily biodegradable COD not converted (with concentration  $S_{bs}$ ) leaves the reactor via the outflow (with flow rate  $(1+r)Q$ ).

Steady state equations for the conversion of RBCOD to SCFA can be developed from the above hypotheses. Consider a single anaerobic reactor with volume  $V$ , influent flow rate  $Q$  and recycle ratio  $r$ , as shown in Fig. 1. A mass balance on  $S_{bs}$  yields:

$$V \frac{dS_{bs}}{dt} = Q S'_{bsi} - (1+r)Q S_{bs} - K X_{B,Hn} S_{bs} V \quad (18)$$

Hence, at steady state ( $dS_{bs}/dt = 0$ ):

$$S_{bs} = \frac{S'_{bsi}/(1+r)}{[1 + K X_{B,Hn} R/(1+r)]} \quad (19)$$

where  $R$  = nominal retention time of anaerobic reactor  
 $Q$  = Influent flow to system ( $\ell/d$ )  
 $V$  = Volume of reactor ( $\ell$ ).

Similarly, from a mass balance on the  $n^{\text{th}}$  reactor in a series of  $N$  anaerobic reactors of equal volume, the  $S_{bs}$  concentration in the  $n^{\text{th}}$  reactor ( $S_{bsn}$ ) is given by:

$$S_{bsn} = \frac{S'_{bsi}/(1+r)}{[1 + K X_{B,Hn} R_N/(1+r)]^n} \quad n = 1, 2, \dots, N \quad (20)$$

where  $R_1 = R_2 = \dots = R_N$

Concentration of non-polyP heterotrophs in the  $n^{\text{th}}$  anaerobic reactor, ( $X_{B,Hn}$ ) is given by:

$$X_{B,Hn} = f_{xa} \cdot MX_{B,H} / V_{at} \quad (21)$$

where  $V_{at}$  = total anaerobic volume ( $\ell$ )  
 $f_{xa}$  = anaerobic mass fraction  
 $MX_{B,H}$  = active mass of non-polyP organisms in system (mgVASS)

Let  $V_{aN} =$  volume of each anaerobic reactor ( $\ell$ ). Using Eq. (21) and noting that

$V_{at} = N \cdot V_{aN}$  and  $R_N = V_{aN}/Q$ , substituting in Eq. (20) gives

$$S_{bsn} = \frac{S'_{bsi}/(1+r)}{[1 + K \frac{f_{xa} MX_{B,H}}{N Q} / (1+r)]^n} \quad (22)$$

where  $MX_{B,H}$  develops from the total mass of biodegradable influent COD less the mass of COD sequestered by the polyP organisms, and  $MX_{B,H}/Q$  is the mass developing per litre influent flow,

$$\frac{MX_{B,H}}{Q} = \frac{[S_{bi} - (S'_{bsi} - (1+r)S_{bsN})] Y_H R_s}{(1 + b_H R_s)} \quad (23)$$

TABLE 1  
 VALUES FOR FRACTIONAL P CONTENT OF PolyP ORGANISM ACTIVE MASS ( $f_{XBG,P}$ ) CALCULATED USING EQ. (14) FOR THE ENHANCED CULTURE SYSTEMS

System	Sludge age (d)	$f_{XBG,P}$ (mgP/mgVASS)
UCT	10	0,44
Modified Bardenpho	7,5	0,39
Modified Bardenpho	10	0,38
Modified Bardenpho	20	0,45
Average value		0,415

where  $(S'_{bsi} - (1+r)S_{bsN})$  = the mass of substrate/l influent sequestered by the polyP organisms through the total anaerobic zone

$S_{bi}$  = biodegradable influent COD (mgCOD/l)

$Y_H$  = heterotrophic organism yield constant (0,45 mgVASS/mgCOD)

$b_H$  = heterotrophic endogenous mass loss rate constant (0,24 mg VASS/mgVASS.d at 20°C)

$R_s$  = system sludge age (d)

$S_{bsN}$  = readily biodegradable COD concentration leaving the last anaerobic reactor (mgCOD/l)

The P release in the  $n^{th}$  reactor (per litre influent flow),  $\Delta P_n$ , is derived from Eq. (16) and (22), i.e.

$$\Delta P_n = C_{sp} S_{bsi} \left[ \frac{1}{\left[ 1 + K \frac{f_{xa}}{N} \frac{MX_{B,H}}{Q} \right]^{(n-1)}} - \frac{1}{\left[ 1 + K \frac{f_{xa}}{N} \frac{MX_{B,H}}{Q} \right]^n} \right] \quad (24)$$

### Implications of conversion theory

The conversion theory set out above provides the means for calculating the mass of SCFA generated per day by the non-polyP organisms for sequestration by the polyP organisms ( $MS_{seq}$ ), i.e.

$$MS_{seq} = (S'_{bsi} - (1+r)S_{bsN})Q \quad (\text{mgCOD/d}) \quad (25)$$

Knowing the mass of substrate sequestered by the polyP organisms, the mass of substrate available per day to the non-polyP organisms ( $MS_{B,H}$ ) is calculated:

$$MS_{B,H} = Q \cdot S_{bi} - MS_{seq} \quad (\text{mgCOD/d}) \quad (26)$$

In effect we have split the influent COD into two fractions, one to be utilised by the polyP organisms and the other to be utilised by the non-polyP organisms. Because of the independence of action of these two groups of organisms we can utilise the simplified enhanced polyP organism culture model (set out earlier) for calculating the biological P removal due to the polyP organisms. This, together with the P content of the endogenous and inert masses, and the P requirements for growth of the non-polyP organisms, allows the total P removal to be calculated for mixed culture biological excess P removal activated sludge systems. This is best illustrated by an example.

### Mixed culture design example

Influent COD raw sewage, 500 mgCOD/l ( $f_{S,us} = 0,07$  mgCOD/mgCOD,  $f_{S,up} = 0,13$  mgCOD/mgCOD,  $f_{S,bs} = 0,24$  mgCOD/mg biodegradable COD); anaerobic mass fraction, 15 per cent, with 2 reactors in-series; sludge age 20 d. Assume that 1

mgN/l of nitrate is present in the recycle to the anaerobic reactor, recycle ratio 1:1 with regard to influent flow. (In practice the actual nitrate in the recycle can be calculated using the procedures set down in WRC Manual, 1984, in which event the TKN/COD ratio also will need to be known).

The calculations set down below apply to both the UCT and modified Bardenpho systems. (For the modified Bardenpho system replace r-recycle data with s-recycle data). In all the calculations below the flow rate (Q) is assumed to be 1 l/d. Because of this assumption, the masses (M) calculated are equivalent to the mass per unit flow. (To obtain the actual masses, the actual influent flow rate should be utilised in the equations). In the design example it is assumed that no acetate is present in the influent. Should acetate be present, refer to Appendix A.

### Sewage characteristics

- Influent COD,  $S_{ti} = 500$  mgCOD/l
- Unbiodegradable particulate COD,  $S_{upi} = f_{S,up} S_{ti} = 0,13 \cdot 500 = 65$  mgCOD/l
- Unbiodegradable soluble COD,  $S_{usi} = f_{S,us} S_{ti} = 0,07 \cdot 500 = 35$  mgCOD/l
- Biodegradable influent COD,  $S_{bi} = S_{ti} (1 - f_{S,us} - f_{S,up}) = 500 (1 - 0,07 - 0,13) = 400$  mgCOD/l
- Readily biodegradable COD,  $S_{bsi} = f_{S,bs} S_{bi} = 0,24 \cdot 400 = 96$  mgCOD/l

### Readily biodegradable COD available for conversion

The influent readily biodegradable COD concentration available for conversion is given by Eq. (15):

$$S'_{bsi} = S_{bsi} - r \cdot 8,6 N_{03,r} = 96 - 1 \cdot 8,6 \cdot 1 = 87,4 \text{ mgCOD/l}$$

### Readily biodegradable COD not converted

The concentration of readily biodegradable COD ( $S_{bsN}$ ) leaving the last anaerobic reactor (N) is calculated from Eqs. (22) and (23) using the following procedure:

- Assume  $S_{bsN} = 0$  mgCOD/l
- Calculate  $\frac{MX_{B,H}}{Q}$ , using Eq. (23)
- Using calculated value for  $\frac{MX_{B,H}}{Q}$ , calculate  $S_{bsN}$  from Eq. (22).
- Recalculate  $\frac{MX_{B,H}}{Q}$  using calculated value for  $S_{bsN}$ .
- Repeat the last two steps until  $S_{bsN}$  and  $\frac{MX_{B,H}}{Q}$  are constant.

Applying the above procedure to the design example we obtain

$$\frac{MX_{B,H}}{Q} = 514,2 \text{ mgVASS/l.d}$$

$$S_{bsN} = 9,4 \text{ mgCOD/l}$$

### SCFA sequestered

The mass of SCFA sequestered per day by the polyP organisms

(MS<sub>seq</sub>) is calculated from Eq. (25):

$$\begin{aligned} MS_{seq} &= [S'_{bsi} - (1+r) S_{bsN}]Q \\ &= [87,4 - (2) \cdot 9,4]1 \\ &= 68,6 \text{ mgCOD/d.} \end{aligned}$$

### Substrate available to non-polyP organisms

The mass of substrate available to the non-polyP organisms per day (MS<sub>B,H</sub>) is calculated from Eq. (26)

$$\begin{aligned} MS_{B,H} &= QS_{bi} - MS_{seq} \\ &= 1.400 - 68,6 \\ &= 331,4 \text{ mgCOD/d.} \end{aligned}$$

From the last two calculations above, the substrate fractions for the polyP and non-polyP organisms are now available, and the masses and P removals of both population groups are calculated separately.

### PolyP organisms

**Biological active mass:** The mass of biological active polyP organisms in the system (MX<sub>B,G</sub>) is calculated from Eq. (7). However, MS<sub>seq</sub> is substituted for MS<sub>bs,ai</sub> because, in the mixed cultures, the substrate available to the polyP organisms includes that generated by conversion in the anaerobic zone:

$$\begin{aligned} MX_{B,G} &= \frac{Y_G MS_{seq} R_s}{(1 + b_G R_s)} \\ &= \frac{0,45 \cdot 68,6 \cdot 20}{(1 + 0,04 \cdot 20)} \\ &= 343 \text{ mgVASS.} \end{aligned}$$

**Endogenous mass:** The endogenous mass in the system for the polyP organisms (MX<sub>E,G</sub>) is calculated from Eq. (8):

$$\begin{aligned} MX_{E,G} &= f_{EB,G} b_G MX_{B,G} R_s \\ &= 0,25 \cdot 0,4 \cdot 343 \cdot 20 \\ &= 68,6 \text{ mgVESS.} \end{aligned}$$

**P removal/l influent:** The P removal for the polyP organisms per litre influent flow (ΔP<sub>G</sub>) is calculated from Eqs. (12) and (13):

$$\begin{aligned} \Delta P_G &= \left[ f_{XBG,P} \frac{MX_{B,G}}{R_s} + f_{XEG,P} \frac{MX_{E,G}}{R_s} \right] \frac{1}{Q} \\ &= \left[ 0,38 \cdot \frac{343}{20} + 0,03 \cdot \frac{68,6}{20} \right] \cdot \frac{1}{1} \\ &= 6,62 \text{ mgP/l influent.} \end{aligned}$$

### Non-polyP organisms

The steady state equations for the non-polyP organisms are given by Marais and Ekama (1976) or WRC Manual (1984).

**Biological active mass in the system (MX<sub>B,H</sub>):**

$$MX_{B,H} = \frac{Y_H \cdot MS_{B,H} R_s}{(1 + b_H R_s)}$$

$$\begin{aligned} &= \frac{0,45 \cdot 331,4 \cdot 20}{(1 + 0,24 \cdot 20)} \\ &= 514,2 \text{ mgVASS.} \end{aligned}$$

**Endogenous mass in the system (MX<sub>E,H</sub>):**

$$\begin{aligned} MX_{E,H} &= f_{EP,H} b_H MX_{B,H} R_s \\ &= 0,2 \cdot 0,24 \cdot 514,2 \cdot 20 \\ &= 493,6 \text{ mgVESS.} \end{aligned}$$

**P removal/l influent (ΔP<sub>H</sub>):**

$$\begin{aligned} \Delta P_H &= \left[ f_{XBH,P} \frac{MX_{B,H}}{R_s} + f_{XEH,P} \frac{MX_{E,H}}{R_s} \right] \cdot \frac{1}{Q} \\ &= \left[ 0,03 \cdot \frac{514,2}{20} + 0,03 \cdot \frac{493,6}{20} \right] \cdot \frac{1}{1} \\ &= 1,51 \text{ mgP/l influent.} \end{aligned}$$

### Inert mass

The steady state equations for the inert mass are given by Marais and Ekama (1976) or WRC Manual (1984).

**Inert mass in system (MX<sub>I</sub>):**

$$\begin{aligned} MX_I &= f_{S,up} QS_{ti} R_s / f_{cv} \\ &= 0,13 \cdot 1.500 \cdot 20 / 1,48 \\ &= 878,4 \text{ mgVISS.} \end{aligned}$$

**P removal/l influent for inert mass (ΔP<sub>I</sub>):**

$$\begin{aligned} \Delta P_I &= (f_{XI,P} \frac{MX_I}{R_s}) \frac{1}{Q} \\ &= (0,03 \cdot 878,4 / 20) \cdot \frac{1}{1} \\ &= 1,32 \text{ mgP/l influent.} \end{aligned}$$

### Total P removal/l influent

The total P removal per litre influent for the system (ΔP) is given by the sum of the component P removals, i.e.

$$\begin{aligned} \Delta P &= \Delta P_G + \Delta P_H + \Delta P_I \\ &= 6,62 + 1,51 + 1,32 \\ &= 9,45 \text{ mgP/l influent.} \end{aligned}$$

### Experimental verification

To test the predictive power of the steady state mixed culture model describing biological excess phosphorus removal (BEPR), data were collected from a number of laboratory-scale systems operated at steady state over the prior six years at the University of Cape Town. System configurations were Phoredox, 3-stage

**TABLE 2**  
Configurations of laboratory-scale systems used to evaluate the steady state BEPR model.  
The system's responses are listed in Table 3.

System code	System type	Sludge age (d)	Influent flow rate (l/d)	Recycle ratios			Number of reactors		Reactor volumes (θ)										Mass fractions			Data source
				r	s	a	Anaerobic	Aerobic	Anaerobic		Aerobic		Anaerobic	Primary anoxic	Secondary anoxic	Aerobic						
									1	2	3	4					1	2	1	2	3	
1	Phoredox	3	30	-	1	-	2	2	-	-	-	-	2	2	-	-	0.50	-	-	0.50	Burke et al (1986)	
2	Phoredox	4	30	-	1	-	2	2	-	-	-	-	2	2	-	-	0.50	-	-	0.50	"	
3	3-stage Barden-pho	6	25	-	1	4	1	2	-	-	-	-	2	2	-	-	0.25	0.25	0.50	0.50	"	
4	Jhb	5	18	-	0.5	-	1	1*	-	-	-	-	1	1.5	1.5	1.5	0.36*	0.36*	0.40	0.40	"	
5	Jhb	5	18	-	1	-	1	1*	-	-	-	-	1.5	1.5	1.5	1.5	0.36*	0.36*	0.40	0.40	"	
6	UCT	6	25	1	1	1	1	2	-	-	-	-	2	2	-	-	0.14	0.29	0.57	0.57	"	
7	UCT	8	15	1	1	1	1	1	1	1	1	1	4	-	-	-	0.09	0.28	0.63	0.63	Wentzel (unpublished)	
8	UCT	8	15	1	1	1	2	1	1	1	1	4	-	-	-	-	0.16	0.26	0.58	0.58	"	
9	UCT	8	15	1	1	1	2	1	1	1	1	2	-	-	-	-	0.28	0.22	0.50	0.50	"	
10	UCT	8	15	1	1	1	2	1	1	1	1	2	-	-	-	-	0.37	0.20	0.43	0.43	"	
11	UCT	10	15	1	1	1	2	1	2	3	3	-	-	-	-	-	0.29	0.20	0.51	0.51	"	
12	UCT	10	15	2	1	1	2	1	2	3	3	-	-	-	-	-	0.36	0.18	0.46	0.46	"	
13	UCT	20	30	1	1	1	1	1	1	3.65	-	-	-	-	-	-	0.10	0.35	0.55	0.55	Siebritz et al (1983)	
14	UCT	20	30	1	1	1	1	1	1	5.47	-	-	-	-	-	-	0.15	0.30	0.55	0.55	"	
15	UCT	20	30	1	1	1	1	1	1	7.27	-	-	-	-	-	-	0.20	0.25	0.55	0.55	"	
16	MUCT	15	30	2	1	4	2	1	1	2.50	3.50	10	-	-	-	-	0.14	0.13	0.54	0.54	Lakay (unpublished)	
17	MUCT	15	30	1	1	4	2	1	2	2.50	3.50	10	-	-	-	-	0.20	0.13	0.50	0.50	"	
18	MUCT	15	30	2	1	4	2	1	1.50	1.50	1.50	2	3	10	-	-	0.21	0.10	0.53	0.53	"	
19	MUCT	15	30	2	1	4	2	1	2	2	2	2	2	2	2	2	0.25	0.12	0.47	0.47	"	
20	MUCT	20	60	1	1	2	2	1	1.50	1.50	1.50	2.50	3.50	10	-	-	0.16	0.13	0.53	0.53	Robinson (unpublished)	
21	MUCT	20	60	1	1	2	2	1	2.50	2.50	2.50	2.50	3.50	10	-	-	0.24	0.12	0.47	0.47	"	
22	MUCT	20	30	1	1	4	1	2	3.65	-	-	-	-	-	-	-	0.10	0.05	0.55	0.55	Siebritz et al (1983)	
23	MUCT	20	30	1	1	2	4	2	1.50	1.50	1.50	1.50	3	3	10	-	0.16	0.16	0.52	0.52	Lakay (unpublished)	
24	MUCT	20	30	1	1	4	1	2	7.27	-	-	-	-	-	-	-	0.20	0.05	0.55	0.55	Siebritz et al (1983)	
25	MUCT	20	18,75	1	1	4	1	2	3.65	-	-	-	-	-	-	-	0.10	0.05	0.55	0.55	"	
26	MUCT	21	15	1	1	4	1	2	6	-	-	-	-	-	-	-	0.16	0.11	0.52	0.52	Lakay (unpublished)	
27	MUCT	25	30	1	1	4	4	2	1	1	1	1	1	1	1	1	0.11	0.14	0.56	0.56	"	
28	MUCT	25	30	2	1	4	4	2	1	1	1	1	1	1	1	1	0.14	0.13	0.54	0.54	"	
29	MUCT	25	30	1	1	4	4	2	1	2	2	2	2	2	2	2	0.20	0.13	0.50	0.50	"	
30	MUCT	28	30	1	1	4	4	2	1.50	1.50	1.50	2.50	3.50	10	-	-	0.16	0.13	0.53	0.53	"	

\* Anoxic reactor in underflow (s-) recycle stream.

modified Bardenpho, UCT, modified UCT (MUCT) and Johannesburg. System sludge ages ranged from 3 to 28 d. The anaerobic zone in these systems consisted of either a single reactor or two or four reactors in series, with total anaerobic mass fraction ranging from 0,09 to 0,5. Recycle ratio to the anaerobic zone was either 0,5 or 1 or 2 with respect to the influent flow. The different system configurations with their associated sludge ages, sludge mass fractions and recycle ratios are listed in Table 2.

The following tests were done daily:

- unfiltered effluent COD
- filtered effluent COD
- unfiltered influent TKN
- filtered effluent TKN
- unfiltered influent total phosphate
- filtered effluent total phosphate
- total phosphate on filtered mixed liquor samples from each of the reactors
- filtered effluent nitrate
- nitrate on filtered mixed liquor samples from all the reactors
- oxygen utilisation rate(s) of the aerobic reactor(s)
- pH of the aerobic reactor(s)
- VSS of the aerobic mixed liquor
- readily biodegradable COD (RBCOD) using the method described in the WRC Manual (1984).

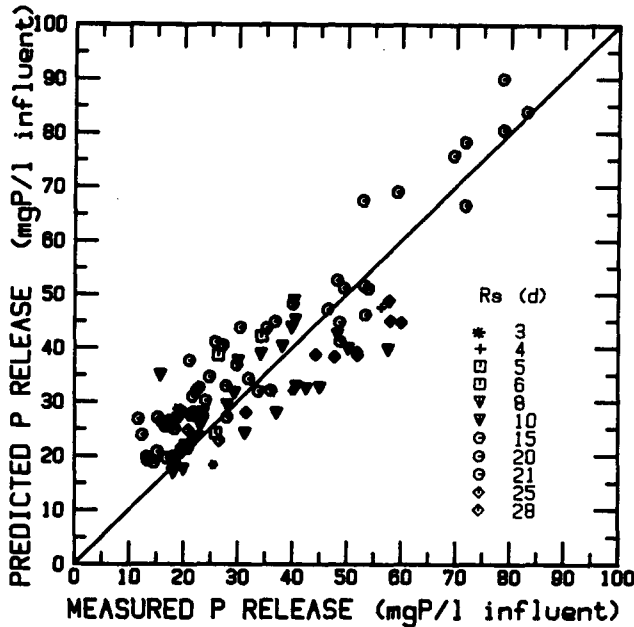


Figure 2

Predicted versus measured P release; data from Tables 2 and 3.

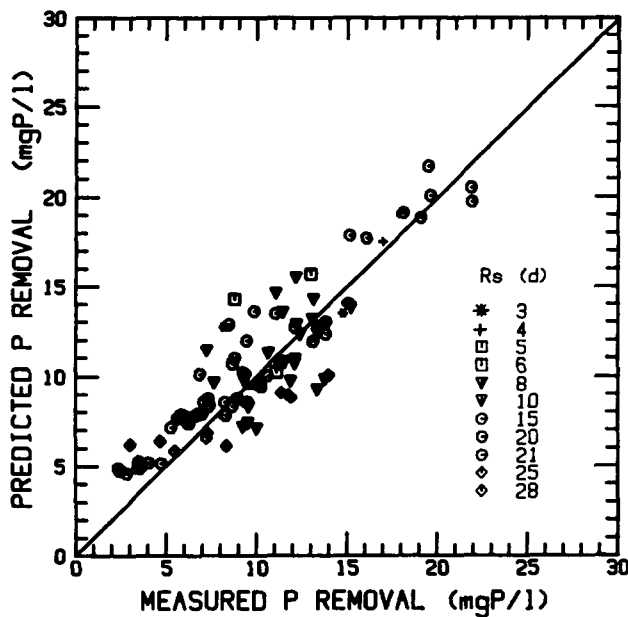


Figure 3

Predicted versus measured P removal; data from Tables 2 and 3.

All the systems received unsettled municipal waste water from Mitchell's Plain, Cape Town. Batches of sewage were collected from the treatment works and stored in stainless steel tanks at 4°C — experience has shown that this minimises changes in sewage characteristics (Wentzel *et al.*, 1988a). Each batch of sewage supplied feed for approximately two weeks. Feed strengths selected were either 250 or 500 or 800 or 1 000 mgCOD/l and were maintained by appropriate dilution of the sewage batch with tap water. P concentration in the influent feed was augmented, by  $\text{KH}_2\text{PO}_4$  addition, to ensure P always was present in the effluent from the laboratory-scale systems. Sewage batches collected at the works often differed quite appreciably with respect to their TKN/COD ratios and RBCOD concentrations; each laboratory-scale system treated a number of such batches. Data on each batch were averaged; provided the batch response exhibited steady state behaviour, the appropriate averages were used as input to the steady state theory and the P release, P removal and VSS concentrations calculated, following the procedures set out in the section above **Mixed culture design example**. The average experimental data obtained on each sewage batch that exhibited steady state behaviour, are listed in Table 3. In Table 3 the number in column 1 reflects the number assigned to each system configuration in column 1 of Table 2.

### Phosphorus release

A plot of predicted versus measured total anaerobic P release is shown in Fig. 2. Constants for conversion of RBCOD to SCFA (Eq. 22) used in the predictions were  $K = 0,06/\text{d}$  and  $C_{sp} = 0,5 \text{ mgP/mgCOD}$ . These values are the same as those used previously by Wentzel *et al.* (1985) to describe the P release down a series of anaerobic reactors in a modified UCT system. With these values for the constants, from Fig. 2, reasonable correlation is obtained between observed and predicted data on P release over the wide range of system configurations, operating conditions and sewage characteristics; this correlation lends strong support to the basic conversion hypothesis, and to the steady state P release theory developed from it.

### Phosphorus removal

Predicted versus measured total P removal is shown in Fig. 3. In predicting the P removal it was found that the value for  $f_{\text{XBG,P}}$  (P content of polyP organism biological active mass) obtained from the enhanced cultures ( $f_{\text{XBG,P}} = 0,41 \text{ mgP/mgVASS}$ ) caused the model to overpredict the measured P removal, by approximately 7 per cent. Accordingly,  $f_{\text{XBG,P}}$  was reduced to 0,38 mgP/mgVASS;







Table 3 (continued)

System code	Sewage code	COD (mg/l)			TKN (mgN/l)		Total soluble P (mg P/l)							E $\Delta$ P	I	Nitrate (mg N/l)							pH							Aerobic OUR (mg O <sub>2</sub> /h)			Suspended solids (mg/l)			
		I	E	CO <sub>5</sub>	I	E	I*	Reactor								I	1	2	3	4	5	6	7	E	1	2	3	1	2	3	VSS	TSS				
								1	2	3	4	5	6																				7			
		1	2	3	4	5	6	7	1	2	3	4	5			6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7					
27	a	497	39	102	50	3.2	16.3	47.2	46.3	49.6	48.1	27.8	14.9	2.8	2.3	14.0	0	0	0	0	0	0	0	0.3	2.5	15.7	15.7	7.70	40						2408	
	b	486	55	104	50	3.3	16.6	45.1	48.0	49.2	48.8	33.2	19.5	5.7	4.7	11.9	0	0	0	0	0	0	0.3	2.0	12.4	12.4	7.80	36						2832		
	c	518	63	119	52	3.0	16.8	48.2	53.4	53.9	54.9	35.0	24.0	3.1	2.8	14.0	0	0	0	0	0	0	0	0	2.2	17.7	17.7	7.65	38						2986	
	d	512	59	102	50	3.6	16.9	44.5	49.7	49.8	51.2	33.6	29.4	5.5	5.5	11.4	0	0	0	0	0	0	0	0	1.1	16.6	16.6	7.58	37						3307	
	e	497	54	106	49	4.2	17.1	40.8	44.0	46.9	47.6	33.8	16.1	5.5	5.3	11.8	0	0	0	0	0	0	0	0.5	5.4	14.1	14.1	7.55	36						3107	
28	a	497	54	106	49	3.1	16.1	41.0	41.4	40.5	42.0	30.0	9.3	2.9	2.3	13.8	0	0	0	0	0	0	1.3	12.4	18.2	18.2	7.39	38								
	b	496	64	97	56	3.4	14.3	39.8	40.2	40.8	40.8	29.6	10.4	3.9	2.6	11.7	0	0	0	0	0	0	1.6	13.9	18.0	18.0	7.23	39								
	c	492	54	109	66	3.3	18.2	49.2	50.4	51.5	52.0	38.5	13.3	5.0	4.2	14.0	0	0	0	0	0	0	1.7	18.0	24.6	24.6	7.17	41								
	d	502	57	112	61	3.5	17.1	41.6	43.4	43.4	44.4	33.4	13.1	6.9	5.2	11.9	0	0	0	0	0	0	2.1	18.4	22.6	22.6	7.13	42								
29	a	504	50	117	54	3.1	17.6	42.1	49.0	50.2	54.0	32.6	13.6	7.0	6.2	11.4	0	0	0	0	0	0	1.0	12.1	18.3	18.3	7.65	40							2074	
	b	497	39	102	50	2.5	17.6	51.3	53.2	55.3	56.6	35.5	10.6	3.0	2.7	14.9	0	0	0	0	0	0	0.4	10.8	16.2	16.2	7.71	41							2289	
30	a	477	36	65	51	4.9	9.7	21.8	25.7	26.6	27.3	18.4	18.9	4.2	4.2	5.5	0	0	0	0	0	0	1.4	5.3	21.5	21.5	6.88	36								
	b	497	56	62	49	3.9	9.9	21.0	23.6	24.9	26.3	19.9	12.5	7.1	6.9	3.0	0	0	0	0	0	0	0.9	1.8	14.6	14.6	7.17	38								
	c	504	52	75	63	3.3	15.3	23.0	26.0	27.7	28.8	21.5	15.6	10.8	10.6	4.7	0	0	0	0	0	0	2.2	11.9	27.1	27.1	7.12	40								
	d	491	52	70	65	3.9	16.6	24.0	26.7	27.6	29.6	20.9	14.3	8.6	8.2	8.4	0	0	0	0	0	0	2.0	12.9	26.8	26.8	7.30	43								
e	a	443	69	92	81	4.5	16.5	22.8	25.4	25.9	27.0	18.1	12.7	9.2	9.2	7.3	0	0	0	0	0	0	2.8	28.5	33.4	33.4	7.21	46								
	f	498	58	79	76	4.0	19.7	31.9	37.4	39.8	40.2	29.4	18.6	12.4	12.4	7.3	0	0	0	0	0	0	1.7	15.4	28.3	28.3	7.27	43								

\* Phosphorus measured in influent is total phosphorus  
\*\* Aerobic reactor subdivided into 2 equal size reactors in series

all other constants obtained from the enhanced cultures remained the same. With the reduced value for  $f_{XBG,P}$  from Fig. 3, it is apparent that reasonable correlation is obtained between predicted and measured P removal, over the wide range of system configurations, operating conditions and sewage characteristics. Therefore, for mixed culture systems, a value for  $f_{XBG,P}$  of 0.38 mgP/mgVASS appears the most appropriate for use in the steady state theory.

### Volatile suspended solids concentration

Predicted versus measured VSS concentration is shown plotted in Fig. 4; again good correlation is obtained over a wide range of conditions.

## Discussion

### Model comparison

The steady state BEPR activated sludge constitutes an extension of the steady state aerobic activated sludge model of Marais and Ekama (1976) and the anoxic/aerobic model of Van Haandel *et al.* (1981) (see also WRC Manual, 1984). The extension is brought into operation when an anaerobic reactor is introduced at the head of the activated sludge system. In this event the extended or BEPR steady state model predicts, in particular, the generation of a new sludge fraction - the polyP organism mass. It is now of interest to enquire briefly into the differences in response invoked with the incorporation of the anaerobic reactor in the system.

### VSS and TSS generation

Effect of incorporating an anaerobic reactor in the system on volatile suspended solids (VSS) and total suspended solids (TSS) is shown in Fig. 5; VSS and TSS produced per unit of COD applied

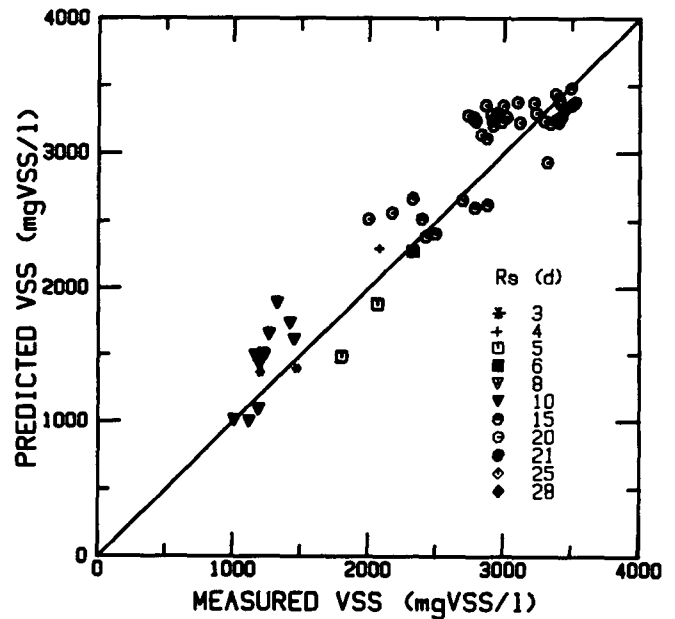


Figure 4

Predicted versus measured VSS concentration; data from Tables 2 and 3.

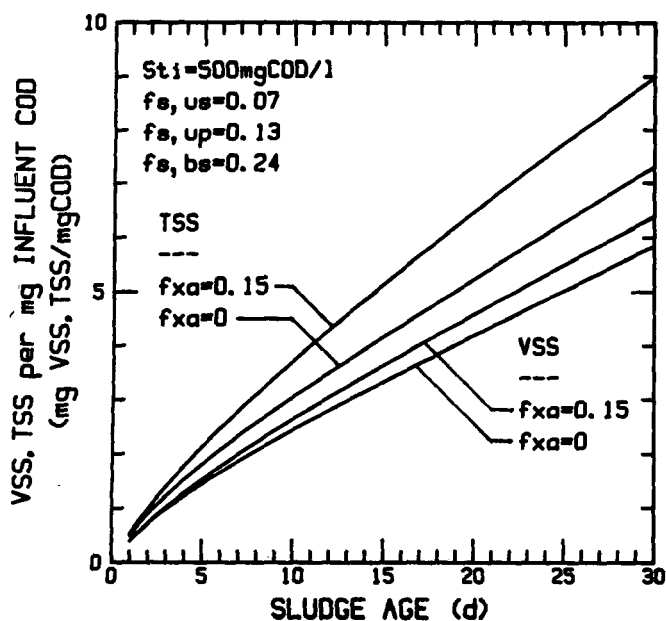


Figure 5

Predicted VSS and TSS generated per unit influent COD versus sludge age for a BEPR single anaerobic reactor ( $f_{xa} = 0,15$ ) and a non-BEPR system ( $f_{xa} = 0$ ) receiving influent as shown.

to the system are shown plotted against sludge age for a system not exhibiting BEPR (anaerobic mass fraction,  $f_{xa} = 0$ ) and a system exhibiting BEPR (anaerobic mass fraction,  $f_{xa} = 0,15$ ) for influent COD of 500 mg/l. The VSS in both systems is calculated from steady state theory (set out earlier) and TSS ( $X_T$ ) from

$$X_T = (X_{B,H} + X_{E,H} + X_I)/f_{V/T,H} + (X_{B,G} + X_{E,G})/f_{V/T,G} \quad (27)$$

where

$$f_{V/T,H} = \text{VSS/TSS ratio for non-polyP organisms} \\ \approx 0,8 \text{ mgVSS/mgTSS}$$

$$f_{V/T,G} = \text{VSS/TSS ratio for polyP organisms} \\ = 0,46 \text{ mgVSS/mgTSS (Wentzel et al., 1989a).}$$

From Fig. 5, the VSS of the BEPR system is only slightly higher than that of the non-BEPR system. However, the TSS of the BEPR system is significantly higher than that of the non-BEPR system. For example, from Fig. 5, at 20 d sludge age and influent COD of 500 mg/l, the TSS in the BEPR system is 6,5 mgTSS/mgCOD applied, and that in the non-BEPR system 5,1 mgTSS/mgCOD applied. This higher TSS production is due to the large quantities of polyP, and associated counterions, stored in the sludge produced by BEPR systems. The increase in TSS production will have significant impact on the design of secondary settling tanks in BEPR systems because the design is based on the TSS not VSS (Ekama and Marais, 1986).

#### VSS fractions

Although there is only small difference in VSS production between a BEPR and a non-BEPR system, the constituent sludge fractions for the two systems differ markedly. This can be il-

lustrated most readily by comparing the sludge fractions generated in the BEPR design example dealt with earlier (anaerobic mass fraction,  $f_{xa} = 0,15$ ; sludge age = 20 d and COD = 500 mg/l of unsettled municipal waste water) with those generated in a non-BEPR system ( $f_{xa} = 0$ ) having the same sludge age and influent COD. The two sets of sludge constituents are listed in Table 4. Note that the BEPR system has a smaller non-polyP organism biological active mass than the non-BEPR system, but that the BEPR system has a significant concentration of polyP organism biological active mass.

#### Denitrification

The differences in sludge constituents between these two systems introduce new problems in the modelling of the denitrification processes in activated sludge systems. In the general (and steady state) nitrification/denitrification activated sludge models (Van Haandel *et al.*, 1981; WRC Manual, 1984), in the primary anoxic reactor of anoxic/aerobic systems, two denitrification rates have been recognised:

- A rapid denitrification rate due to the utilisation of RBCOD; and
- a slower rate due to the utilisation of slowly biodegradable COD (SBCOD).

This approach appears to model behaviour of anoxic/aerobic systems very satisfactorily. To date, the denitrification kinetic model has been applied also to the primary anoxic zone of anaerobic/anoxic/aerobic (BEPR) systems and again the denitrification capacity appears to have been predicted satisfactorily. However, from the research reported in this paper, the RBCOD is virtually totally sequestered in the anaerobic zone of BEPR systems by the polyP organisms and Wentzel *et al.* (1989a) have found that, in enhanced cultures, the polyP organisms have very small denitrification capacities. Thus, in BEPR systems, it would appear that the RBCOD no longer is available for denitrification. Furthermore, from Table 4, the biological active mass of the non-polyP organisms in the BEPR system is smaller than that in the non-BEPR system, that is, in the BEPR system there is a smaller active mass of organisms available for mediating the denitrification. Yet, experimentally the denitrification achieved in the primary anoxic reactor of the BEPR system is approximately the same as that in the primary anoxic reactor of a non-BEPR system with the same primary anoxic mass fraction. Consequently, modelling of the denitrification processes in the primary anoxic zone of BEPR systems needs to be re-examined to find an explanation for the high denitrification capacities observed in BEPR systems. Research into this aspect is at present in progress.

#### Phosphorus response in BEPR systems

By means of the steady state BEPR model, one can examine the phosphorus response (i.e. P release, P uptake and P removal) response in BEPR systems under different modes of operation (e.g. different sludge ages,  $f_{xa}$  ect.) and receiving waste waters with different characteristics (e.g. different  $S_{tp}$ ,  $S_{bsi}$  ect.).

#### P release

Parameters that influence P release can be categorised into two groups, sewage characteristics and system characteristics.

**TABLE 4**  
**CONSTITUENT SLUDGE FRACTIONS PREDICTED BY THE BEPR**  
**STEADY STATE MODEL FOR A SYSTEM NOT EXHIBITING BEPR ( $f_{xa} = 0$ )**  
**AND A SYSTEM EXHIBITING BEPR ( $f_{xa} = 0,15$ ), BOTH SYSTEMS**  
**WITH SLUDGE AGE 20 d AND TOTAL INFLUENT COD 500 mg/l OF**  
**UNSETTLED WASTE WATER.**

Parameter	Symbol	Units	Value	
			$f_{xa} = 0$	$f_{xa} = 0,15$
<b>PolyP organisms</b>				
Biological active mass	$MX_{B,G}$	mgVASS	0	343
Endogenous mass	$MX_{E,G}$	mgVESS	0	68,6
<b>Non-polyP organisms</b>				
Biological active mass	$MX_{B,H}$	mgVASS	620,7	514,2
Endogenous mass	$MX_{E,H}$	mgVESS	595,9	493,6
<b>Inert mass</b>	$MX_I$	mgVISS	878,4	878,4
<b>Total</b>				
Biological active mass	$MX_{B,T}$	mgVASS	620,7	857,2
Endogenous mass	$MX_{E,T}$	mgVESS	595,9	562,2
Inert mass	$MX_I$	mgVISS	878,4	878,4
Volatile solids	$MX_V$	mgVSS	2095	2298
Active fraction	$f_{av}$	mgVASS/mgVSS	0,30	0,37
P removal	$\Delta P$	mgP/l influent	3,1	9,5

Sewage characteristics (WRC Manual, 1984):

- Influent COD,  $S_{ti}$
- Fraction of  $S_{ti}$  which is unbiodegradable particulate, i.e.  $S_{up}$   
 $= f_{S,up} S_{ti}$
- Fraction of  $S_{ti}$  which is unbiodegradable soluble, i.e.  $S_{us} = f_{S,us} S_{ti}$
- Biodegradable influent COD i.e.  $S_{bi} = S_{ti}(1 - f_{S,us} - f_{S,up})$
- Fraction of  $S_{bi}$  which is readily biodegradable, i.e.  $S_{bsi} = f_{S,bs} S_{bi}$

A "normal" unsettled municipal waste water in South Africa would have  $f_{S,up} = 0,13$ ,  $f_{S,us} = 0,07$  and  $f_{S,bs} = 0,24$ ; settled waste water has  $f_{S,up} = 0,04$ ,  $f_{S,us} = 0,12$  and  $f_{S,bs} = 0,34$ . In countries where garbage grinding is permitted preliminary estimates for unsettled municipal waste water give very much higher values for  $f_{S,up}$  of approximately 0,23; no data are available for these waste waters when settled.

System characteristics:

- System sludge age,  $R_s = (\text{mass of sludge in system})/(\text{mass wasted per day})$ .
- The r-recycle in the UCT type system and the s-recycle in the Bardenpho type system.
- The anaerobic mass fraction defined by (mass of sludge in anaerobic zone)/(total mass of sludge in system).
- Series or single anaerobic reactor configuration.

The sewage and system characteristics above affect the release directly. Indirect effects are due to nitrification and denitrification which may affect the nitrate discharged to the anaerobic reactor and hence the  $S_{bsi}$  available for conversion in accordance with Eq.

(15). The nitrate effect in turn is dependent upon the maximum specific growth rate of the nitrifiers and the denitrification design of the plant — location of anoxic reactors, anoxic mass fractions and the s- and a-recycles (WRC Manual, 1984) — and the TKN/COD ratio. Temperature also may have an influence but this has not yet been investigated.

Two preliminary remarks need to be made:

- Anaerobic mass fraction ( $f_{xa}$ ) is the superior parameter in terms of which to evaluate the effects of the various parameters. In many papers the hydraulic retention time in the anaerobic reactor ( $R_N$ ) is used in preference to  $f_{xa}$ . Although this is allowable theoretically, using  $R_N$  introduces complexity in that the volume of the anaerobic reactor, and the flow, need to be known; as a consequence, when using  $R_N$  one usually has to approach the solution by repeated trials, instead of directly.
- The parameter, P release per  $S_{bsi}$ , is a convenient one in terms of which to express the release.

Taking note of the above, the effects of selected parameters on P release can now be investigated.

Sludge age, influent COD and anaerobic mass fraction: Taking note of the above, accepting the characteristics for an unsettled waste water from a South African municipal source and assuming that no nitrate enters the anaerobic reactor i.e.  $S'_{bsi} = S_{bsi}$ , also that  $r = s = 1$ , the P release/100  $mgS'_{bsi}$  versus  $f_{xa}$  is shown in Fig. 6a for a single anaerobic reactor with  $S_{ti}$  of 250, 500 and 1 000 mgCOD/l and  $R_s$  of 10, 15, 20, 25 and 30 d. On the same plots are shown the mass of RBCOD/100  $mgS'_{bsi}$  not converted.

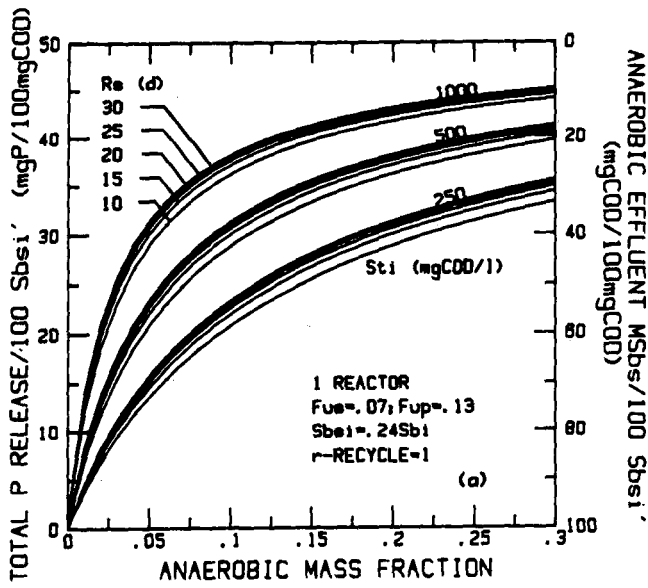


Figure 6a

Predictions of total anaerobic phosphorus release and mass of RBCOD/100 mgS<sub>bsi</sub>' not converted for a single anaerobic reactor system with influent CODs and system parameters shown.

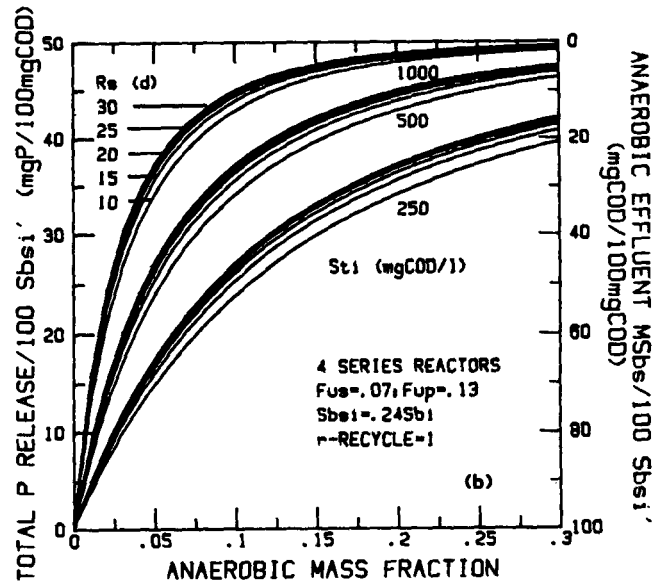


Figure 6b

Predictions of total anaerobic phosphorus release and mass of RBCOD/100 mgS<sub>bsi</sub>' not converted for a four-in-series anaerobic reactor system with influent CODs and system parameters shown.

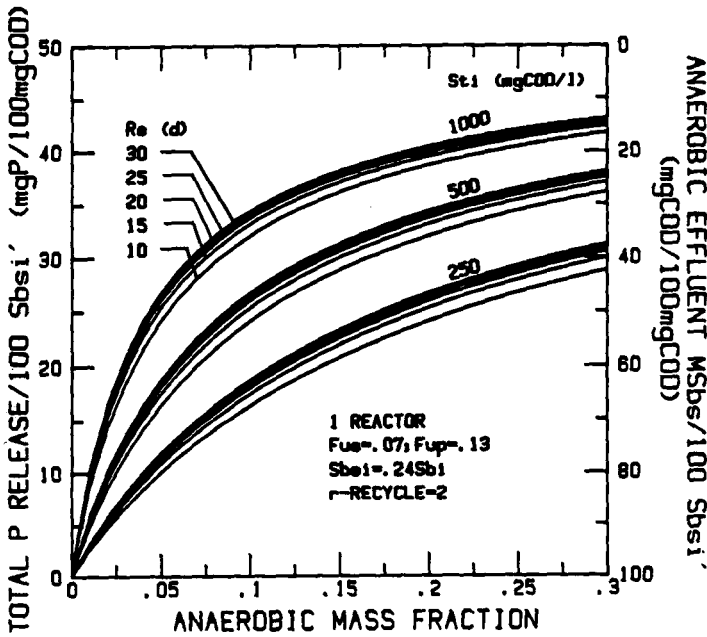


Figure 7

Predictions of total anaerobic phosphorus release and mass of RBCOD/100 mgS<sub>bsi</sub>' not converted for the influent CODs and system parameters shown, with a single anaerobic reactor and r-recycle of 2 (cf Fig. 6a for r = 1).

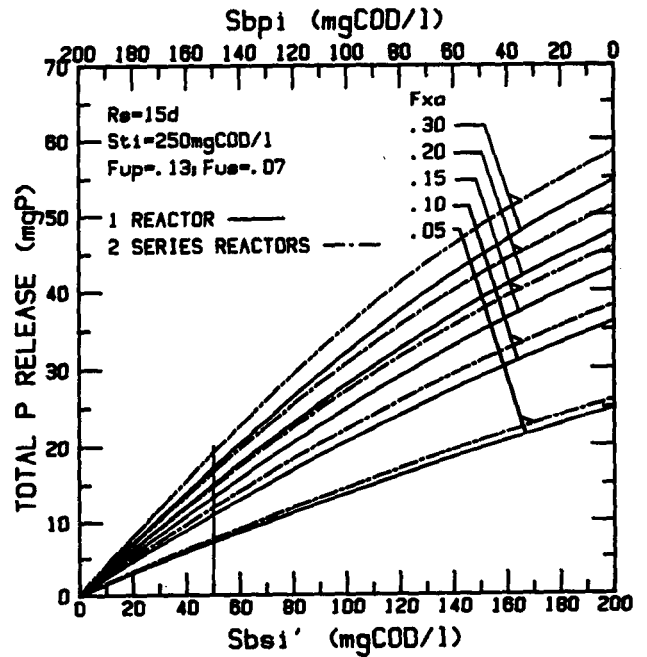


Figure 8

Effect of the magnitude of S<sub>bsi</sub>' on anaerobic phosphorus release for a fixed total influent COD and system sludge age for selected anaerobic mass fractions.

The plots indicate the following:

- $R_s$  has a relatively minor effect on P release.
- $S_{ti}$  has a marked effect on the P release — for a fixed anaerobic mass fraction the higher the  $S_{ti}$  the higher the P release per 100  $S'_{bsi}$  or conversely, the higher the  $S_{ti}$  the lower the anaerobic mass fraction to obtain the same release per  $S'_{bsi}$ .
- $f_{xa}$  also has a marked effect on P release — for a fixed  $S_{ti}$  the higher the  $f_{xa}$  the higher the P release per 100  $S'_{bsi}$ . From the plot it would seem that with a single anaerobic reactor, one should select  $f_{xa} > 0,15$  for design.

**Subdivision of  $f_{xa}$ :** The effect of subdividing the anaerobic reactor is shown in Fig. 6b. The plot is similar to that in Fig. 6a but with the anaerobic mass subdivided into a series configuration of four. Comparing the P release behaviour between Figs 6a and 6b, series anaerobic reactor operation significantly improves the P release per 100  $S'_{bsi}$ , (particularly at higher  $f_{xa}$ ) or conversely, to give the same release a lower  $f_{xa}$  fraction is needed with an in-series anaerobic reactor than with a single reactor. A comparison (not shown) between single, two-in-series and four-in-series anaerobic reactors indicates that the main improvement is from single to two-in-series reactors.

**r-Recycle:** The effect of the r-recycle ratio in the UCT system is illustrated in Fig. 7 where the release in a single reactor system for  $r = 2$  is plotted. Comparing Figs 6a and 7,  $r = 1$  is superior to  $r = 2$ . However in the UCT system the lower recycle requires a larger anaerobic volume to give the same  $f_{xa}$  as the higher recycle (WRC Manual, 1984).

**$S_{bsi}/S_{bi}$  ratio:** The effect of the  $S_{bsi}/S_{bi}$  ratio is illustrated in Fig. 8 for a single and a two-in-series anaerobic reactors,  $R_s = 15d$ , and  $f_{xa}$  of 0,05; 0,10; 0,15; 0,20 and 0,30;  $S_{ti} = 250$  and  $S_{bi} = 200$  mgCOD/l.  $S_{bsi}$  is varied from zero up to  $S_{bi}$ , the  $S_{bpi}$  varying correspondingly from  $S_{bi}$  to zero, ( $S_{bpi} + S_{bsi} = S_{bi}$ ). For this sewage  $S_{bsi}$  normally is approximately 50 mgCOD/l and for a single anaerobic reactor with  $f_{xa} = 0,15$ , the P release is approximately 14 mgP/l influent, i.e. 0,28 mgP/mg $S_{bsi}$ . Clearly for any selected  $f_{xa}$ , if the  $S_{bsi}$  is increased, the P release also increases but the increase in P release has a decreasing tendency so that even if the  $S_{bsi} = 200$  (i.e.  $S_{bsi}/S_{bi} = 1$ ) the release obtained for a single anaerobic reactor with  $f_{xa} = 0,15$  would be only 43 mgP/l influent flow, i.e. 0,22 mgP/mg $S_{bsi}$ . The reason for the low release is that in this case  $S_{bsi} = S_{bpi}$ , so that a large fraction of  $S_{bsi}$  is utilised to generate non-polyP heterotrophic mass. If, however, the  $S_{bsi}$  is increased by adding SCFA, the improvement will be linear (i.e. not with a decreasing tendency as with complex RBCOD). This is so because the SCFA do not require conversion by the non-polyP organisms.

### P uptake

The most important single factor that influences the magnitude of P uptake is the magnitude of PHB substrate sequestered by the polyP organisms in the anaerobic reactor, or equivalently (and more practical) the magnitude of the P release in the anaerobic reactor. Once the magnitude of P release has been determined, only two parameters have any marked direct influence on the magnitude P uptake — sludge age ( $R_s$ ) and total influent COD ( $S_{ti}$ ). However, both these parameters also have an influence on the magnitude of P release, and thus indirectly on P uptake. Accordingly, to enquire into the direct effect of these parameters on P uptake, their effect on P release has to be excluded. This is achieved

by plotting P uptake versus P release, for various values of sludge age and total influent COD respectively.

**Sludge age:** Influence of  $R_s$  on P uptake is shown in Fig. 9 where P uptake is plotted against P release for different  $R_s$  and a fixed  $S_{ti}$  of 500 mg/l. For any given P release, as  $R_s$  increases, P uptake decreases. This is due to the decrease in active mass (and associated P content) wasted per day with increased  $R_s$ .

**Influent COD:** Influence of  $S_{ti}$  on P uptake is shown in Fig. 10

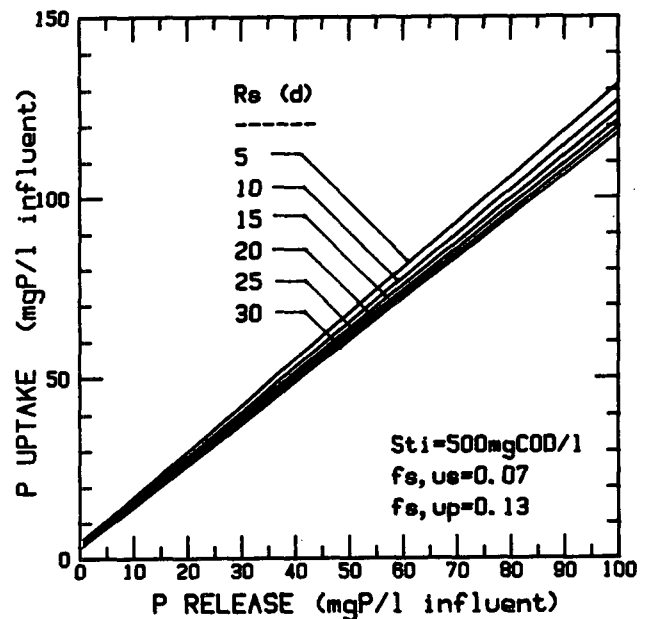


Figure 9  
Predicted P uptake versus P release for various  $R_s$ , as shown.

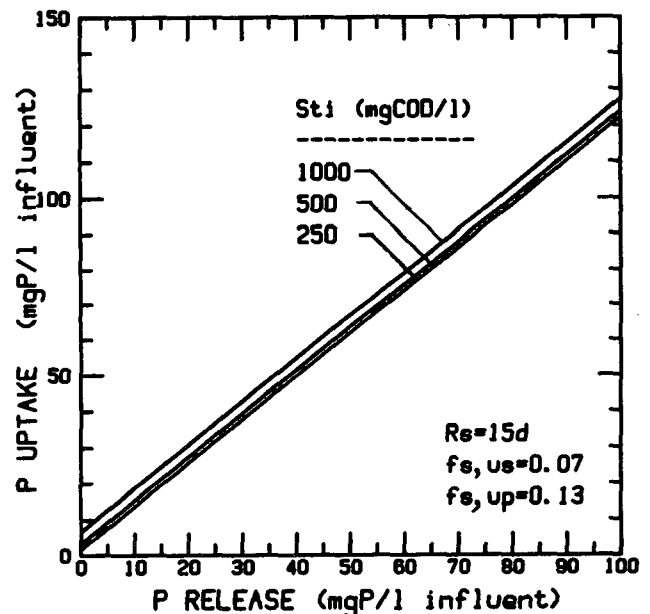


Figure 10  
Predicted P uptake versus P release for various  $S_{ti}$ , as shown.

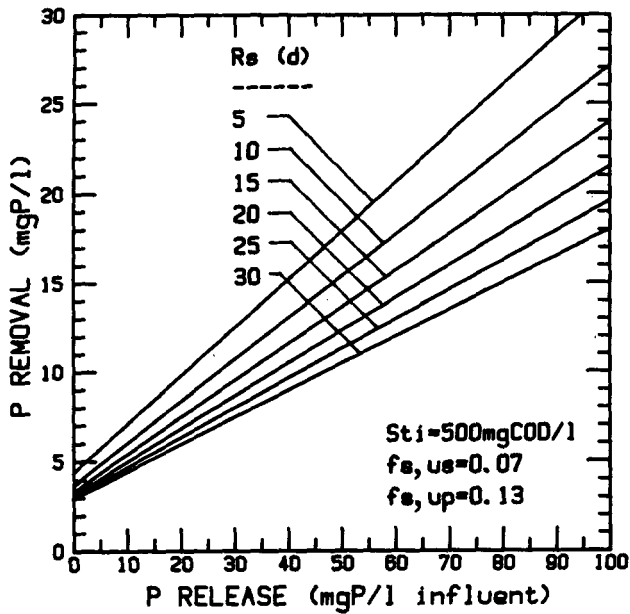


Figure 11  
Predicted P removal versus P release for various  $R_s$ , as shown (cf. Fig. 9).

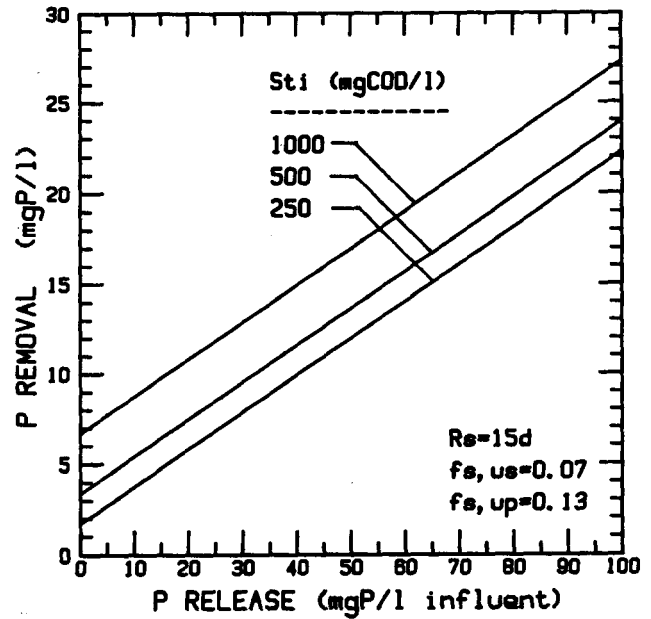


Figure 12  
Predicted P removal versus P release for various  $S_{t_i}$ , as shown (cf. Fig. 10).

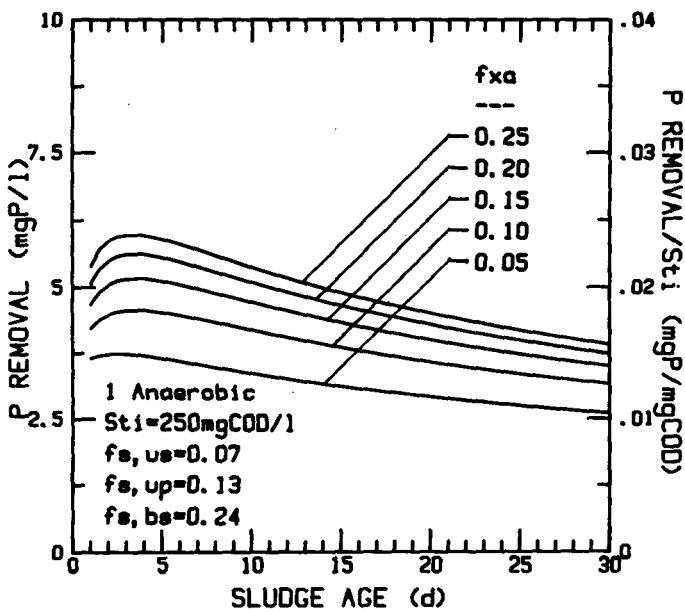


Figure 13  
Predicted P removal versus sludge age for various  $f_{x_a}$ , for a single anaerobic reactor system receiving unsettled waste water of 250 mgCOD/l, with characteristics as shown.

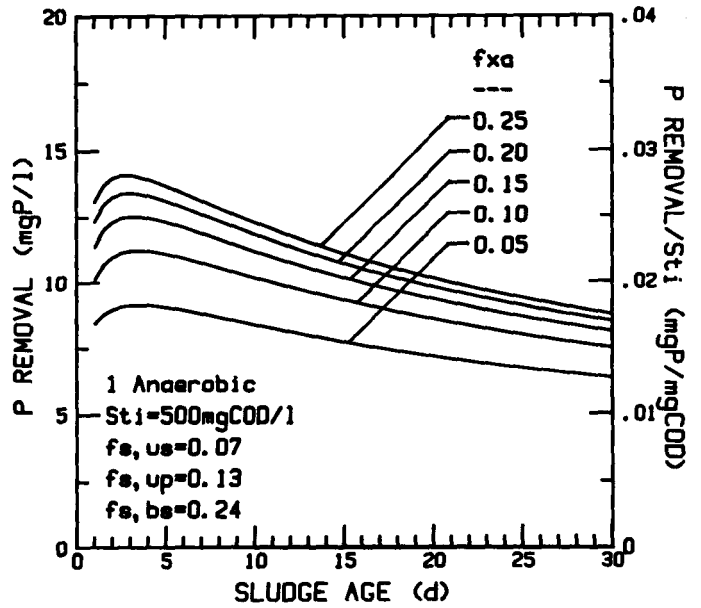


Figure 14  
Predicted P removal versus sludge age for various  $f_{x_a}$ , for a single anaerobic reactor system receiving unsettled waste water of 500 mgCOD/l, with characteristics as shown.



where P uptake is plotted against P release for different  $S_{ti}$  and a fixed  $R_s$  of 15 d. From the plot is evident that for any given P release, as  $S_{ti}$  increases, the P uptake also increases — the influence of  $S_{ti}$  on P uptake is not via the polyP organisms, but is the result of the increase in non-polyP organism mass (and its associated P content) with increased  $S_{ti}$ .

From the plots in Figs 9 and 10, for a fixed P release the effect of  $R_s$  and  $S_{ti}$  on P uptake would not appear to be marked. However, for the same P release the relative effect on the P removal, which is the difference between P uptake and P release, is significant. To illustrate this, the same plots as in Figs 9 and 10 are shown in Figs 11 and 12 respectively, but with P removal plotted against P release. It now is apparent that, for the same P release, the P removal is markedly affected by both  $R_s$  and  $S_{ti}$ .

### P removal

The influence of various factors on P release and P uptake has been described above. Since P removal is the difference between P uptake and P release, all these factors also will exert an influence on the P removal. However, a single factor may influence both the P release and P uptake so that the net effect, on the P removal, may not be immediately clear. It is of interest therefore to investigate the influence of the main design orientated parameters on P removal; these are sludge age ( $R_s$ ), anaerobic mass fraction ( $f_{xa}$ ), total influent COD ( $S_{ti}$ ), number of anaerobic reactors (N) and raw or settled sewage.

**Sludge age and anaerobic mass fraction:** Accepting the characteristics for an unsettled waste water from a South African municipal source with total influent COD of 250 mgCOD/l, and assuming that no nitrate enters the anaerobic reactor and that a recycle ratio to the anaerobic of 1:1 is present, P removal versus sludge age is shown in Fig. 13 for a single anaerobic reactor with  $f_{xa}$  of 0,05; 0,10; 0,15; 0,20 and 0,25. On the same plots P removal /  $S_{ti}$  also are shown. The plots indicate the following:

- Effect of  $R_s$  on P removal is complex. For  $R_s < 3$  d the P removal increases with increase in  $R_s$ ; however for  $R_s > 3$  d P removal decreases with increase in  $R_s$ . The reason for this is that increase in  $R_s$  causes an increase in the RBCOD conversion and therefore increased P release and P uptake, however the increased  $R_s$  also causes a decrease in P uptake due to the lower active mass (and its associated P content) wasted per day. At  $R_s < 3$  d, the former effect dominates the P removal, while at  $R_s > 3$  d the latter dominates, giving rise to the shape of the curve. The latter effect, that is the decrease in both polyP and non-polyP organism active masses with increase in sludge age, would be crucially affected by the specific endogenous mass loss rate of the polyP organism mass — should the endogenous mass loss rate of the polyP organisms (0,04/d) have been the same as that of the non-polyP organism mass (0,24/d), virtually no BEPR would have been obtained.
- Effect of  $f_{xa}$  on P removal also is shown in Fig. 13. For a selected  $R_s$ , increase in  $f_{xa}$  gives rise to an increase in P removal. This is due to the increased conversion of RBCOD with larger anaerobic mass fractions. The improvement in P removal however diminishes with each step increase in  $f_{xa}$  [cf section **P release**]

**Influent COD:** In Figs. 14 and 15 plots similar to Fig. 13 are given, except that  $S_{ti}$  is 500 mgCOD/l (Fig. 14) and 1 000 mg COD/l (Fig. 15). To assist comparison between the different  $S_{ti}$ 's, the right axis is given as P removal/ $S_{ti}$ . Comparing Figs. 13, 14 and 15, it is evi-

dent that with increase in  $S_{ti}$ , P removal efficiency (i.e. P removal/ $S_{ti}$ ) increases. This is due to the increased magnitude of RBCOD conversion with increased  $S_{ti}$  [cf section **P release**].

**Subdivision of  $f_{xa}$ :** Effect of subdividing the anaerobic reactor is

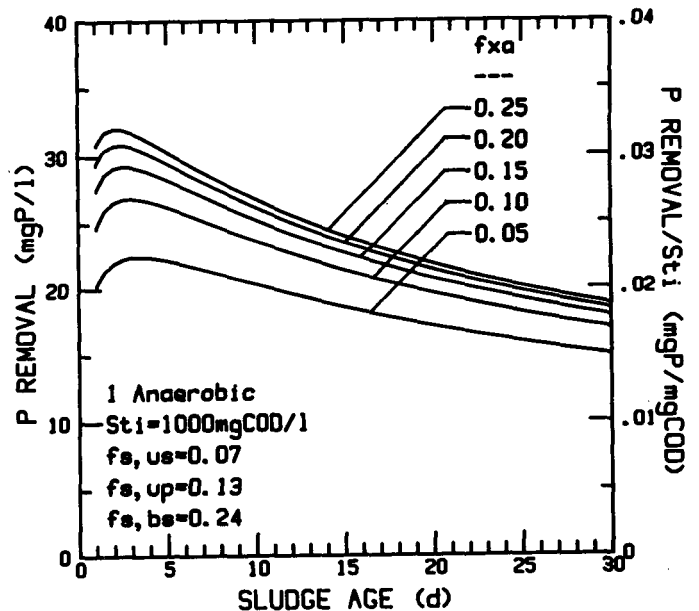


Figure 15  
Predicted P removal versus sludge age for various  $f_{xa}$ , for a single anaerobic reactor system receiving unsettled waste water of 1 000 mgCOD/l, with characteristics as shown.

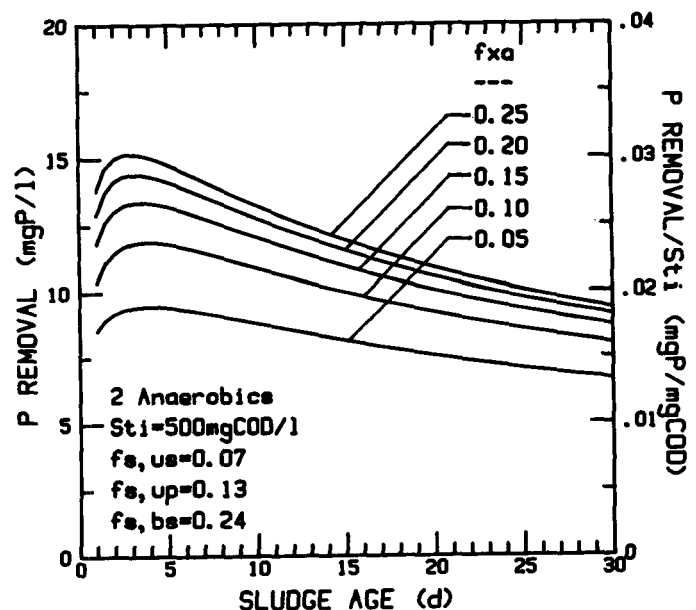


Figure 16  
Predicted P removal versus sludge age for various  $f_{xa}$ , for a two-in-series anaerobic reactor system receiving unsettled waste water of 500 mgCOD/l, with characteristics as shown (cf Fig. 14 for single anaerobic reactor system).

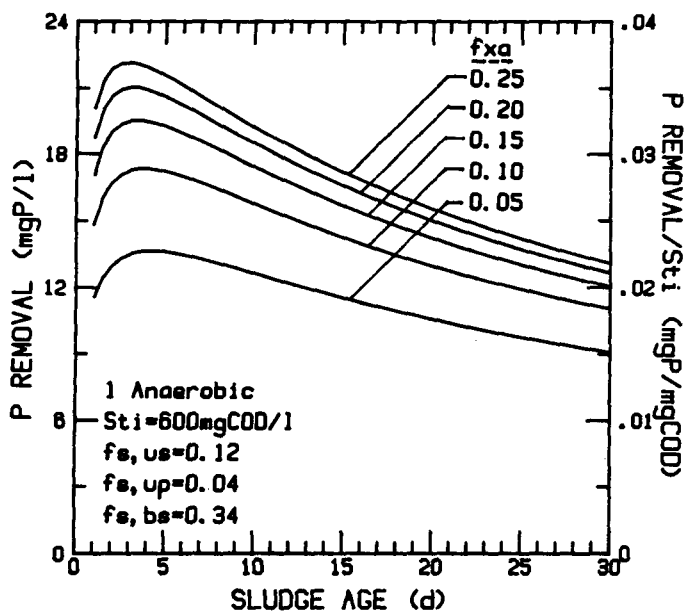


Figure 17

Predicted P removal versus sludge age for various  $f_{xa}$ , for a single anaerobic reactor system receiving settled waste water of 600 mgCOD/l, with characteristics as shown (cf. Fig. 15 for predicted P removal for original unsettled waste water). Note that for P removal/ $S_{ti}$ , the  $S_{ti}$  refers to the settled waste water.

shown in Fig. 16. The plot is similar to Fig. 14, but with the anaerobic zone subdivided into two equal reactors. Comparing the P removal behaviour in Fig. 14 and 16, series operation of the anaerobic zone significantly improves the P removal. This improvement is due to the increased RBCOD conversion with in-series anaerobic reactor operation as a result of the first order nature of the conversion kinetics.

**Settled and unsettled influent:** Effect of settling the waste water on P removal is illustrated in Fig. 17 where P removal is shown plotted against sludge age for various  $f_{xa}$ , for a waste water of original  $S_{ti} = 1000$  mgCOD/l and subject to primary sedimentation to give a settled waste water with strength 600 mgCOD/l. Comparing the P removal for the original unsettled waste (Fig. 15) with that for the settled waste (Fig. 17), it is evident that settling will reduce the P removal by the system. This reduction is due to the decrease in the mass of biodegradable COD entering the activated sludge system, which causes a reduction in the RBCOD converted and in the mass of non-polyP organisms generated. However, P removal per influent COD entering the **biological** reactor is higher for the settled than for the unsettled waste water. This is apparent from Figs. 15 and 17, by comparing the P removal/ $S_{ti}$  on the right hand axes. This arises because the ratio  $S_{bsi}/S_{ti}$  is higher for settled than for unsettled waste water (It should be noted that it is assumed no  $S_{bsi}$  is removed in settling; this will not be strictly correct but the  $S_{bsi}$  removal in settling appears to be minimal).

## Conclusion

This paper has presented a mechanistically based model for describing BEPR in single sludge activated sludge systems under steady state conditions. It supersedes the largely empirical models previously presented (Siebritz *et al.*, 1983; Wentzel *et al.*, 1985). A

cardinal aspect in maximising the P removal is to minimise, or completely eliminate, entry of the electron acceptors nitrate or oxygen to the anaerobic reactor. Where nitrification will take place in the BEPR system, either by deliberate design or occasionally, prevention of the nitrate entry to the anaerobic reactor dominates the design of the system. Nitrification/denitrification aspects have been extensively investigated and the design procedures are well established, in the WRC Manual (1984). In this paper some questions have been raised about the adequacy of the existing procedure for determining the denitrification capacity of the primary anoxic reactor in anaerobic/anoxic/aerobic systems. However, for the purpose of design the existing procedure can be accepted as adequate.

## Acknowledgements

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## Appendix A

### Effect of acid fermentation on steady state design procedure

The influent RBCOD ( $S_{bsi}$ ) may contain a SCFA fraction. This always will be the case if the underflow from the primary settling tank is acid fermented and the supernatant, or the elutriated supernatant, or the entire fermented underflow, is added back to the influent.

In the anaerobic reactor sequestration of the SCFA from the influent is rapid and always can be taken to be complete. For the purpose of design it is convenient to separate the influent RBCOD into two fractions; one due to SCFA in the influent (this fraction does not require conversion in the anaerobic reactor) and the other due to the normal complex RBCOD in the influent (this fraction requires conversion in the anaerobic reactor to SCFA before sequestration). Let

$$\begin{aligned} S_{bs,ai} &= \text{RBCOD due to SCFA in the influent} \\ S_{bs,ci} &= \text{complex RBCOD in the influent, i.e.} \\ S_{bsi} &= S_{bs,ai} + S_{bs,ci} \end{aligned} \quad (\text{A.1})$$

Should nitrate enter the anaerobic reactor it is not clear whether  $S_{bs,ai}$  or  $S_{bs,ci}$  will be used preferentially as the electron donor. For the purpose of design it is assumed that the influent complex RBCOD ( $S_{bs,ci}$ ) will serve as the electron donor.

Taking note of the above, the design procedure, set out in the body of this paper, can be modified so as to include situations where SCFA are present in the influent. The presence of SCFA in the influent effects the division of substrate between the polyP and non-polyP organism groups. This can be accounted for as follows:

#### RBCOD available for conversion

The influent RBCOD concentration available for conversion ( $S'_{bs,ci}$ ) is given by:

$$S'_{bs,ci} = S_{bs,ci} - r_{8,6} N_{o3,r} \quad (\text{A.2})$$

$$= (S_{bsi} - S_{bs,ai}) - r_{8,6} N_{o3,r} \quad (\text{A.3})$$

#### RBCOD not converted

The concentration of complex RBCOD leaving the last anaerobic reactor ( $S_{bs,cN}$ ) is calculated using the reiteration procedure set down in the body of the paper except that  $S'_{bs,ci}$  is substituted for  $S'_{bsi}$ , and  $S_{bs,cN}$  for  $S_{bsN}$ , i.e. the following two equations are used in the procedure,

$$S_{bs,cN} = \frac{S'_{bs,ci} / (1+r)}{\left[ 1 + K \frac{f_{xa} \text{MX}_{B,H}}{N Q} / (1+r) \right]^N} \quad (\text{A.4})$$

$$\text{MX}_{B,H} = \frac{[S_{bsi} - (S'_{bs,ci} - (1+r)S_{bs,cN}) - S_{bs,ai}] Y_H R_s}{(1 + b_H R_s)} \quad (\text{A.5})$$

#### SCFA sequestered

The mass of SCFA sequestered per day by the polyP organisms ( $MS_{seq}$ ) is the SCFA in the influent, plus the SCFA generated in the anaerobic reactor by the non-polyP organisms, i.e.

$$MS_{seq} = QS_{bs,ai} + [S'_{bs,ci} - (1+r)S_{bs,cN}]Q \quad (\text{A.6})$$

#### Substrate available to non-polyP organisms

The mass of substrate available to the non-polyP organisms per day ( $MS_{B,H}$ ) can be calculated as in the design example, i.e.

$$MS_{B,H} = QS_{bi} - MS_{seq} \quad (\text{A.7})$$

Having divided the influent RBCOD into the fractions available to the polyP organisms and that available to the non-polyP organisms, the rest of the calculation procedure set down in the design example [i.e. from *polyP organisms*] can be followed.

## Appendix B

### List of symbols

Symbol	Description
$b_G$	Specific endogenous mass loss rate constant for polyP organisms (0,04/d)
$b_H$	Specific endogenous mass loss rate constant for heterotrophic non-polyP organisms (0,24/d)
BEPR	Biological excess phosphorus removal
$C_{sp}$	Stoichiometric ratio P release per COD converted (0,5 mgP/mgCOD)
$f_{Ep,G}$	Fraction of polyP organisms that is unbiodegradable particulate residue (0,25 mgVSS/mgVASS)
$f_{Ep,H}$	Fraction of heterotrophic non-polyP organisms that is unbiodegradable particulate residue (0,2 mgVSS/mgVASS)
$f_{P,rel}$	Stoichiometric ratio P release acetate sequestered (0,5 mgP/mgCOD)
$f_{S,bs}$	Fraction of biodegradable substrate that is readily biodegradable (mgCOD/mgCOD)
$f_{S,up}$	Fraction of substrate that is unbiodegradable particulate (mgCOD/mgCOD)
$f_{S,us}$	Fraction of substrate that is unbiodegradable soluble (mgCOD/mgCOD)
$f_{VT,G}$	VSS/TSS ratio for polyP organisms (0,46 mgVSS/mgTSS)
$f_{VT,H}$	VSS/TSS ratio for heterotrophic non-polyP organisms (0,80 mgVSS/mgTSS)
$f_{xa}$	Anaerobic mass fraction
$f_{XBG,P}$	Fraction of polyP organism biological active mass that is phosphorus (0,38 mgP/mgVASS)
$f_{XBH,P}$	Fraction of heterotrophic non-polyP organism biological active mass that is phosphorus (0,03 mgP/mgVASS)
$f_{XEG,P}$	Fraction of polyP organism endogenous mass that is phosphorus (0,03 mgP/mgVSS)
$f_{XEH,P}$	Fraction of heterotrophic non-polyP organism endogenous mass that is phosphorus (0,03 mgP/mgVSS)
G	Subscript for polyP organisms
H	Subscript for heterotroph non-polyP organisms
Jhb	Johannesburg
K	First order rate constant for conversion of RBCOD to SCFA (0,06/d)
M	Prefix to denote mass
$MP_{rel}$	Mass of phosphorus released (mgP/l influent)
$N_{o3,r}$	Nitrate concentration recycled to anaerobic reactor (mgN/l)
P	Phosphorus
PHB	Poly-β-hydroxybutyrate
Q	Influent flow rate (l/d)

R	Nominal hydraulic retention time of anaerobic reactor (d)	$S_{usi}$	Influent unbiodegradable soluble COD concentration (mgCOD/l)
$R_N$	Nominal hydraulic retention time of a single anaerobic reactor in a series of anaerobic reactors (d)	SBCOD	Slowly biodegradable COD
$R_s$	Sludge age (d)	SCFA	Short-chain fatty acids
r	Recycle ratio of r-recycle with respect to influent flow	TSS	Total suspended solids
RBCOD	Readily biodegradable COD	UCT	University of Cape Town
$S_{B,H}$	Concentration of biodegradable COD available to heterotrophic non-polyP organisms (mgCOD/l)	V	Reactor volume (l)
$S_{bi}$	Influent biodegradable COD concentration (mgCOD/l)	$V_{aN}$	Volume of each anaerobic reactor in a series of anaerobic reactors (l)
$S_{bs}$	Readily biodegradable COD concentration (mgCOD/l)	$V_{at}$	Total anaerobic volume (l)
$S_{bsi}$	Influent readily biodegradable COD concentration (mgCOD/l)	VSS	Volatile suspended solids
$S'_{bsi}$	Influent readily biodegradable COD concentration available for conversion (mgCOD/l)	WRC	Water Research Commission
$S_{bsn}$	Readily biodegradable COD concentration in the n <sup>th</sup> reactor of a series of anaerobic reactors (mgCOD/l)	$X_{B,G}$	Concentration of polyP organism biological active mass (mgVASS/l)
$S_{bs,a}$	SCFA concentration (mgCOD/l)	$X_{B,H}$	Concentration of heterotroph non-polyP organism biological active mass (mgVASS/l)
$S_{bs,ai}$	Influent SCFA concentration (mgCOD/l)	$X_{B,Hn}$	Concentration of heterotroph non-polyP organism biological active mass in the n <sup>th</sup> reactor of a series of anaerobic reactors (mgVASS/l)
$S_{bs,c}$	"Complex" readily biodegradable COD concentration (mgCOD/l)	$X_{E,G}$	Concentration of polyP organism endogenous mass (mgVSS/l)
$S_{bs,ci}$	Influent "complex" readily biodegradable COD concentration (mgCOD/l)	$X_{E,H}$	Concentration of heterotroph non-polyP organism endogenous mass (mgVSS/l)
$S'_{bs,ci}$	Influent "complex" readily biodegradable COD concentration available for conversion (mgCOD/l)	$X_I$	Concentration of inert volatile solids (mgVSS/l)
$S_{bs,cN}$	"Complex" readily biodegradable COD concentration in the last anaerobic reactor of a series of N anaerobic reactors (mgCOD/l)	$X_T$	Concentration of total suspended solids (mgTSS/l)
$S_{phb}$	Stored poly- $\beta$ -hydroxybutyrate concentration (mgCOD/l)	$Y_G$	Yield constant for polyP organisms (0,45 mgVASS/mgCOD)
$S_{seq}$	Concentration of substrate available to polyP organisms for sequestration (mgCOD/l)	$Y_H$	Yield constant for heterotroph non-polyP organisms (0,45 mgVASS/mgCOD)
$S_{ti}$	Total influent COD concentration (mgCOD/l)	$\Delta MP$	Mass of P removed per day (mgP/d)
$S_{upi}$	Influent unbiodegradable particulate COD concentration (mgCOD/l)	$\Delta MX_{B,G}$	Mass of polyP organism biological active mass wasted per day (mgVSS/d)
		$\Delta MX_{E,G}$	Mass of polyP organism endogenous mass wasted per day (mgVSS/d)
		$\Delta P$	Concentration of P removed (mgP/l)
		$\Delta P_G$	P removal by polyP organisms (mgP/l)
		$\Delta P_H$	P removal by heterotrophic non-polyP organisms (mgP/l)
		$\Delta P_I$	P removal by inert material (mgP/l)