A note on a light-temperature dependent model for algal blooms in the Vaal River

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Abstract

A mathematical model describing the behaviour of the total planktonic algal biomass in a river has been developed and tested on existing data from the Vaal River (South Africa). It was shown that qualitative as well as quantitive features of the development of winter algal blooms can be described by taking a few fundamental environmental variables such as light, temperature and suspended solids concentration into account.

Introduction

The Vaal River is one of the principal sources of water for the central part of South Africa. This resource is used for industrial and domestic purposes in different, and often conflicting, ways (Pieterse, 1986). For example, the use of the Vaal River as a pathway for the disposal of industrial and domestic wastes is increasing the concentrations of dissolved phosphorus and nitrogen in the water which leads to the eutrophication of the river and results in the development of algal blooms . Such increases in the algal population density may have many adverse effects on the quality of the water, and results in acute problems for water authorities responsible for the treatment and distribution of potable water to mines, cities and towns. Some problems related to the occurrence of algal growth in rivers are (Walmsley and Butty, 1983; Pieterse, 1986): Increased purification costs of water for potable purposes, tastes and odours produced by algae in water intended for potable purposes, toxins produced by certain algae that can result in losses of livestock and that may also affect humans or that may interfere with irrigation.

Thus, the prediction of the development of winter algal blooms is of importance in water resource management. The mathematical model introduced in this communication is intended to assist the relevant authorities by providing them with a tool which can both lead to a better understanding of aspects of algal population growth, and to enable short- and long-term chlorophyll-*a* concentration predictions.

The model

The model is based on the following three fundamental assumptions:

- The water is eutrophic, i.e. we assume there are more than enough nutrients for algal growth.
- At most N representatives of different algal groups such as e.g. diatoms and green algae, can be responsible for the development of an algal bloom.
- The growth and death of algae are dependent mainly on the available light as well as the average temperature of the water and these driving variables are time dependent.

In addition, we assume that other possible factors influencing algal growth are steady state for the duration of the bloom. Horizontal homogeneity of the water is also assumed in the immediate vicinity of the site at which the model is applied.

Within the boundaries of these assumptions, the scheme illustrated in Fig. 1 may be used as a basis for the algal growth model.

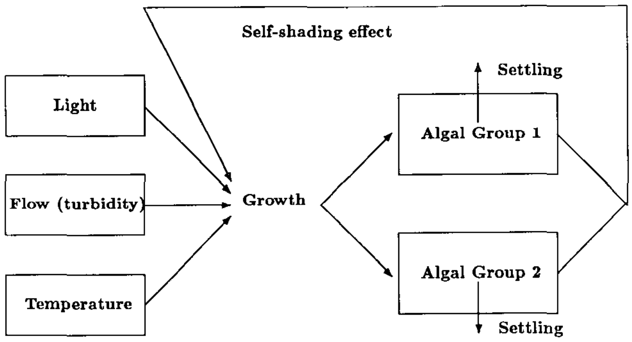

Figure 1
Vaal River N-algae growth model : A schematic representation

The transcription of the scheme in Fig. 1 in terms of mathematical relations leads to a system of $2N$ coupled nonlinear differential equations of the form:

$$\begin{cases} \dot{x}_{1i} = -(k_{Di} + S_i)x_{1i} \\ \qquad\quad k_{Gi}(\underline{E}, x_{1j}, x_{2j}, \underline{K}_j : j = 1, N; t)x_{1i} \\ \dot{x}_{2i} = k_{Di}x_{1i} - S_i x_{2i} \end{cases} \qquad i = 1,N$$

where x_{1j}, x_{2i}; is the concentration of living and dead algae belonging to group "i" respectively, and an upper dot "." stands for the derivative respect to time, t. The vectors $\underline{E}$ and $\underline{K}_j$ contain information on environmental parameters and information specific to a given algal species "j", respectively. The components of vectors $\underline{E}$ and $\underline{K}_i$ are given in Table 1a and b.

More of the explicit form taken by the differential equations and details on the dynamics of the system of equations can be obtained from the authors.

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TABLE 1a
ENVIRONMENTAL PARAMETERS

I_0 : maximal irradiance available under the water surface (cal/cm^2·min)
$\mu(t)$: cosine of solar zenith angel
T : the water temperature (°C)
$c(t)$: concentration of inorganic material suspended in the water (mg/ℓ)
z_0 : depth of the mixed layer (m)
k_w : light extinction coefficient for pure water (m^{-1})
k_c : algal light extinction coefficient for suspended inorganic material (m^{-1})

TABLE 1b
PARAMETER SET GOVERNING ALGAL

I_s : optimal light intensity for growth (cal/cm^2·min)
T_{opt} : optimal temperature for growth (°C)
T_i : minimum temperature for growth (°C)
N_{max} : maximum growth rate (d^{-1})
k_D : dying rate (d^{-1})
k_x : self-shading coefficient for algae ((mg Chl-a/m^3·m)$^{-1}$)
S : algal settling rate (d^{-1})

Model calibration

Prior to any integration of the system of differential equations formally represented in Fig. 1, all the variables contained in the model have to be defined explicitly. It is obvious that once two particular algal groups have been chosen, all the coefficients can be determined. However, in practice, such accurate information is seldom available, rather one has the knowledge of the algal group(s) responsible for the bloom. In this case, one can only define a range of variations for these parameters which can be assumed to be as narrow as possible. The literature on the relevant variables was quite helpful and valuable information on the optimal light intensity, self-shading coefficients and settling velocity was obtained from Eppley (1977), Wofsy (1983) and Smayda (1970), while the temperature dependent growth rate coefficients were obtained from Canale and Vogel (1974) and Castenholz (1969). It should be clear, however, that this reference list is incomplete. With the help of the information on environmental parameters, one was able to reduce strongly the number of degrees of freedom present in the model. The list of the parameters that have been reduced is given in Table 2 .

Moreover, the optimal growth rate coefficients are within the constraints of the statistical relationships determined by Canale and Vogel (1974).

Numerical simulations

The annual distribution of the chlorophyll-a concentration in the Vaal River shows seasonal patterns that are repeated from year to year. An example of this cycle can be found in Fig. 2 (dashed lines) where the chlorophyll-a concentration measured at Stilfontein is illustrated over a three-year period starting January 1985. Clearly the concentration of chlorophyll-a goes through three different phases:

Autumn: a period when the water exhibits usually low concentrations in chlorophyll-a and inorganic suspended material.

Winter and early spring: a period when two algal blooms are observed. From experimental evidence the first bloom is usually dominated mainly by diatoms while the second is quite often dominated by mainly green algae.

Late spring and summer: a period when living organic material is always found in suspension in the river; however, the level of chlorophyll-a is relatively low.

To test the mathematical model, the second phase of the cycle that is characterised by the bloom of two different groups of algae, i.e. diatoms and greens, was considered. Starting in the middle of autumn, the solution of the system of differential equations was computed over a period covering the winter to early spring seasonal pattern, by means of a Runge-Kutta fourth order method with a fixed timestep of 5 min for the years 1985 to 1987. The weekly averaged temperature and the total suspended solids concentration were considered as inputs and were updated daily during the whole computation, by way of a linear interpolation procedure. The remaining free parameters were chosen in such a way that their values satisfy the constraints stated in the previous section. The calculated chlorophyll-a concentration is shown in Figs. 2a to c (solid lines), together with the corresponding measured values (dashed lines) for comparative purposes. The agreement between numerical (calculated) and measured values was fairly good in qualitative as well as quantitative terms during all three years of our investigation.

These results lead to the following conclusions:

- The mathematical model chosen to describe the response of the algae to temperature, light and turbidity variations is consistent with observations in the Vaal River during the winter months.
- The factors presumed to influence the growth (temperature

TABLE 2
ALGAL GROWTH PARAMETER CONSTRAINTS

Parameters	Algal group 1	Algal group 2	Unit
I_s	$\in$ [0,02 — 0,15]	$\in$ [0,02 — 0,15]	cal/cm^2·min
T_i	0,00	5,00	°C
k_D	0,15	0,15	d^{-1}
k_x	0,03	0,03	(mg Chl-a/m^3·m)$^{-1}$
S	0,05	0,05	d^{-1}

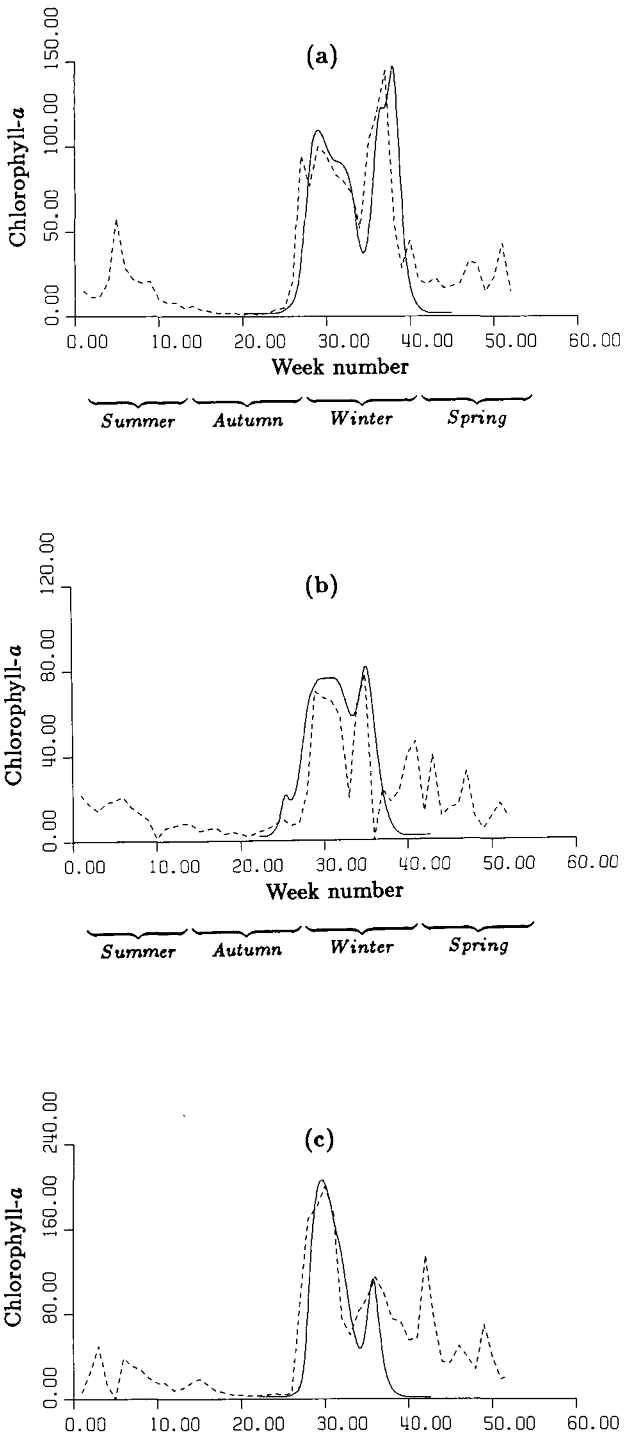

Figure 2

*Total chlorophyll-*a *concentration (mg Chl-a/m³) in the Vaal River at Stilfontein :---: observed values; —— : computed values, during the winter-early spring period. (a) for 1985; (b) for 1986; (c) for 1987*

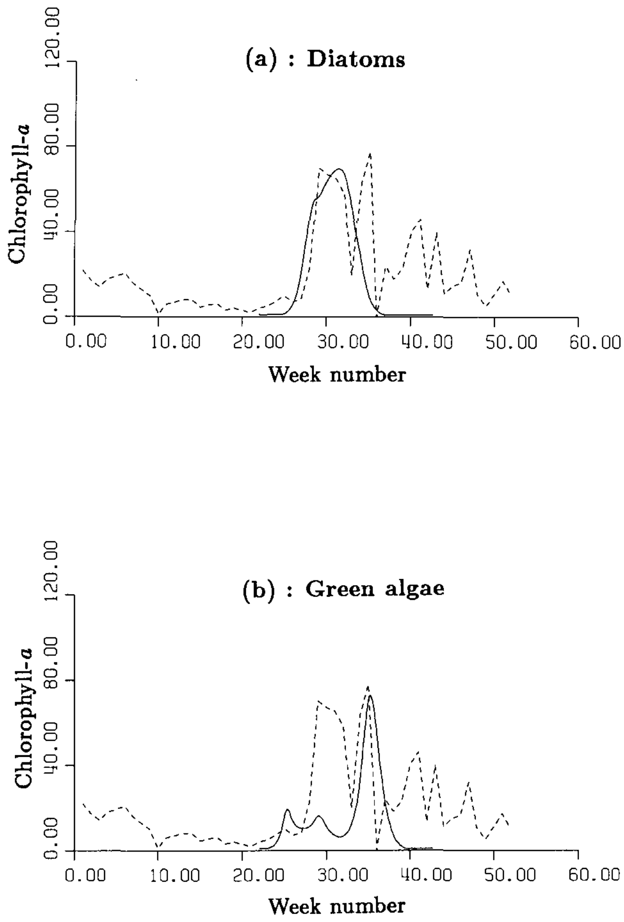

Figure 3

Analysis of the computed total biomass for the period winter-early spring 1986: (a) : diatom concentration; (b) : green algal concentration. Concentrations are expressed in mg Chl-a/m³

and underwater light) of the algae (in the Vaal River context) seemed to be the dominant ones for the specific sampling site and during the period of our investigation.

Two major fields of application for such a mathematical model are possible.

In the first application, when starting off with existing data, the model can be used by biologists to support their investigations by:

- providing a consistent splitting of the global data between algal taxa such as species assumed to be responsible for blooms;
- assisting to identify important environmental factors of algae involved in a bloom; and
- assisting the researcher to describe the mechanisms involved in the development of a bloom by providing numerical information about experimental data that are not available and by providing an easy way to test new hypotheses suggested by other investigations.

An analysis was done of the numerical data provided by the model for the year 1986. Figures 3a and b illustrate the separation between diatom and green algal dominated assemblages.

Table 3 shows the order of magnitude for the characteristics of both algal groups, while Figs. 4a to c show the concentrations of dead suspended organic material present in the river, as predicted by the model. This last result illustrates the kind of information that can be reached by means of the mathematical model which is not usually available from direct experimental measurements.

In the second application, mathematical models of this nature can be used in water resource management. Sometimes it is necessary to know in advance the possible influence of the current state of the river on the water quality in future. In this

TABLE 3 PARTICULARS OF ALGAL GROWTH PARAMETERS FOR 1987		
Parameters	**Diatoms**	**Green algae**
I_S	0,115	0,08
T_{opt}	10	13
N_{max}	1,28	1,45

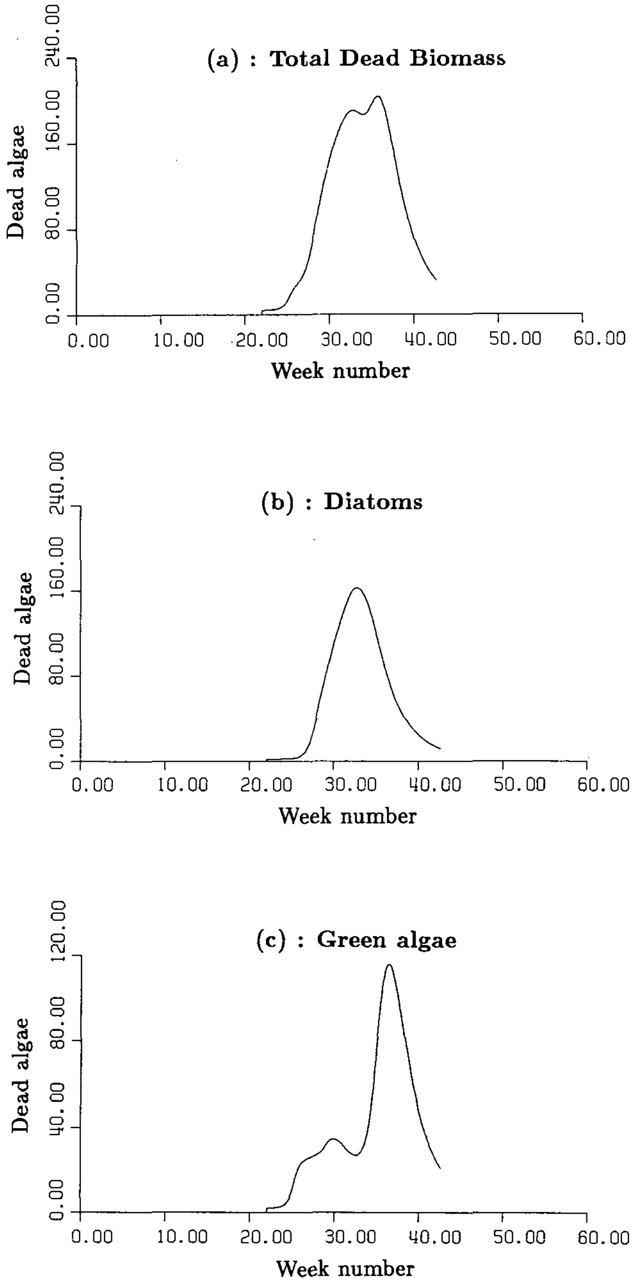

Figure 4
Projected concentration of suspended dead organic material (in Chl-a/m ^{3}) at Stilfontein during the winter-early spring blooms in 1987: (a) : Total suspended dead organic material; (b) : fraction of dead suspended material belonging to the diatom group; (c) : same as (b) for the green algal group

case, the mathematical model can be used as a predictive tool. For example, by making reasonable assumptions concerning the inputs such as temperature and discharge, the behaviour of different algal groups can be simulated numerically in order to predict when a possible bloom will occur, and what its intensity and duration might be. The success of such predictions will depend strongly on the nature of the available environmental information used for calibration, as well as on the nature of projected environmental information used for the simulation and not just on the quality of the model.

Conclusions

A model has been developed to simulate the winter algal blooms of the diatom and green algal dominated assemblages in the Vaal River at Stilfontein (South Africa). The model takes into account light and temperature variations as well as the effect of discharge fluctuations on suspended sediment concentrations. Comparison between the model simulations and existing environmental data for the period between 1985 and 1987 showed a good agreement with the algal behaviour observed on site. Improvements and extensions of this basic model are possible and are being investigated. The results of these modifications will be reported on in due course.

Acknowledgements

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