Equilibrium scour in rivers with sandbeds

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Abstract

Severe floods caused extensive scour along sand-bedded rivers in Natal during 1984 and 1987. Recorded information on the extent of scour as well as peak flood discharges was analysed in an attempt to develop criteria which could be used to predict scour depths in future.

Contrary to expectations, all the indications are that laminar rather than turbulent boundary layer conditions prevail when equilibrium scour depths are approached.

By deforming their beds through the formations of dunes and other bed formations, the sediment transporting capacity of rivers is decreased. This means that rivers have a built-in mechanism through which excessive scour is prevented when extreme floods occur.

Introduction

Severe floods caused extensive damage to river systems in south-eastern Africa during 1984 and 1987 (Kovács et al., 1985; Van Bladeren and Burger, 1989). The floods which occurred in the Komati, Mkuze, Black Mfolozi and White Mfolozi Rivers during 1984 together with the 1987 flood in the Mhlatuze River were the largest on record at the gauging stations on these rivers. Their respective estimated return periods ranged from at least 20 years to more than 200 years.

Extensive bed and bank erosion occurred and a large number of bridges were either destroyed or severely damaged.

Shortly after the floods had occurred, the South African Department of Water Affairs performed topographical surveys of specific reaches (Fig. 1) along these rivers. Maximum flood levels that had been reached were recorded at the same time. It was thus possible to calculate the peak discharges that had occurred and compare these values with the depths and widths to which the sandbed river channels had been eroded, in an attempt to establish criteria which could be used in future to predict equilibrium scour depths.

Theoretical background

It might be expected that when extremely large floods with high sediment carrying capacities occur in rivers with erodible bed and bank materials, scour will continue to take place until the erosive capacity of the stream approaches the minimum value required to transport the available material.

A number of criteria have been developed which depict the critical stage where a stream's transporting capacity becomes sufficient to transport the available material. Classical examples of such criteria are represented by the Hjülstrom (1935), Shields (1936) and Liu (1957) diagrams. Whilst these diagrams were developed primarily on an intuitive basis, rigorous theoretical analysis of flow transporting capacity and sediment transportability (Rooseboom, 1974; 1992) leads to the type of relationships represented in the Liu diagram. The success of this (applied power) approach is attributed to the fact that both flow transporting capacity and sediment transportability can be expressed in directly comparable scalar terms. This approach has been developed further in order to quantify the influence that bed roughness has on sediment transporting capacity.

It can be argued that whenever alternative modes of flow exist, that mode which requires the least amount of unit power will be followed. Accordingly fluid flowing over movable material would not transport such material unless this would result in a decrease in the amount of unit power which is being applied. Alternatively, if two modes of yielding exist, yielding will take place according to that mode which offers the least resistance.

Where flow takes place over movable material and the relatively large amount of unit power required to maintain motion along the bed becomes greater than that which would be required in the process of deformation of the bed, the stream should begin to transport the bed material rather than persist in its existing mode of flow. The applied power required per unit volume to suspend a particle with density ρ_S and settling velocity V_{SS}, in a fluid with density ρ, equals $(\rho_S - \rho)\, g\, V_{SS}$.

In rough turbulent flow the unit stream power applied in maintaining motion along a smooth bed consisting of particles with diameter d is proportional to:

$$\frac{\rho g s D \sqrt{gDs}}{d}$$

(representing the applied unit stream power $\tau dv/dy$ along the bed)

with:

ρ	=	fluid density
g	=	acceleration due to gravity
s	=	energy slope
D	=	flow depth
d	=	particle diameter (proportional to the absolute bed roughness for a smooth bed)
τ	=	shear stress
dv/dy	=	velocity gradient

In terms of the concept of minimum applied power, the stream will begin to entrain particles when the power required to suspend the particles becomes less than the power required to maintain the status quo.

At that stage:

$$(\rho_S - \rho)\, g\, V_{SS} \;\alpha\; \rho g s D \frac{\sqrt{gDs}}{d} \qquad (1)$$

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TABLE 1
SITE AND FLOOD CHARACTERISTICS

	Unit	Komati	Mkuze	Black Mfolozi	White Mfolozi	Mhlatuze
Site		**Trading site**	**Morgenstond**	**Game reserve**	**Game reserve**	**Riverview (W1H009)**
Catchment area (CA)	km^2	8 040	2 647	3 396	4 776	2 409*
Mean annual runoff (MAR)	x $10^6 m^3$		95	343	255	178
Mean annual precipitation (MAP)	mm		898	965	791	996
Bed slope (from 1:50 000 maps)	m/m	0,00062	0,00125	0,0012	0,00152	0,0013
Flood data						
Date (1984)		31-01-1984	31-10-1984	31-01-1984	31-01-1984	31-01-1984
Method of flood peak measure		SA	SA	SA	SA	SA
Flood peak (Q)	m^3/s	2 640	5 500	10 000	6 500	2 400**
Flood line slope (S)	m/m	0,00061	0,00163	0,0012	0,001	0,003
Storm rain (p)	mm	285	480	580	445	370**
Return period (T)	yr	20-50	50-200	0,93 RMF	50-200	20-50**
Sediment size (mean)	mm	1,33	0,243	0,12	0,38	0,2
Date (1987)		-	29-09-1987	29-09-1987	29-09-1987	29-09-1987
Method of flood peak measurement			SA	SA	SA	SA
Flood peak (Q)	m^3/s	-	1 060	1 740	2 150	3 600
Flood line slope (S)	m/m	-	0,00188	0,00183	0,0022	0,00223
Storm rain (p)	mm	-	165	262	247	436
Return period (T)	yr	-	<10	10	15	50 to 100
Sediment size (mean)	mm	-	0,43	0,425	0,61	0,27

* Catchment excluding Goedertrouw Dam (1980) = 1 336 km^2
** Refer to CA at Goedertrouw Dam
SA = slope area
CA = catchment area

According to the general equation for settling velocity (Graf, 1971):

$$V_{ss} \propto \sqrt{\frac{(\rho_s-\rho)gd}{\rho C_d}} \tag{2}$$

Assuming that C_d, the drag coefficient, is a constant, which is true for larger diameters, then from Eqs. (1) and (2), the condition of incipient sediment motion under rough turbulent flow conditions is depicted by:

$$\frac{\sqrt{gDs}}{V_{ss}} = \text{constant} \tag{3}$$

As can be seen in Fig. 2, this relationship fits measured data as compiled by yang (1973) well, with the value of the constant = 0,12, for values of:

$$\frac{\sqrt{gDs}\,d}{\nu} > 13$$

with:

ν = kinematic viscosity of the fluid

Similarly, with laminar boundary conditions in smooth turbulent flow as well as in completely laminar flow the unit applied stream power equals:

$$\frac{(\rho\, gsD)^2}{\rho\nu}$$

The corresponding equation for settling velocity (Graf, 1971) under viscous conditions states that:

$$V_{ss} \propto d^2 g \frac{(\rho_s-\rho)}{\rho\nu} \tag{4}$$

Accordingly, the relationship for values of $\frac{\sqrt{gDs}}{\nu} < 13$ calibrated with data by Grass (1970) and Yang (1973) is found to be (Fig. 2):

$$\frac{\sqrt{gDs}}{V_{ss}} = \frac{1{,}6}{\frac{\sqrt{gDs}}{\nu}\,.\,d} \tag{5}$$

Two distinct relationships are thus identified (Eqs. 3 and 5) which are valid for describing incipient transport conditions along smooth beds consisting of particles with diameter d. As long as the value of $\frac{\sqrt{gDs}.d}{\nu} > 13$ boundary flow conditions are completely turbulent whilst laminar boundary conditions prevail

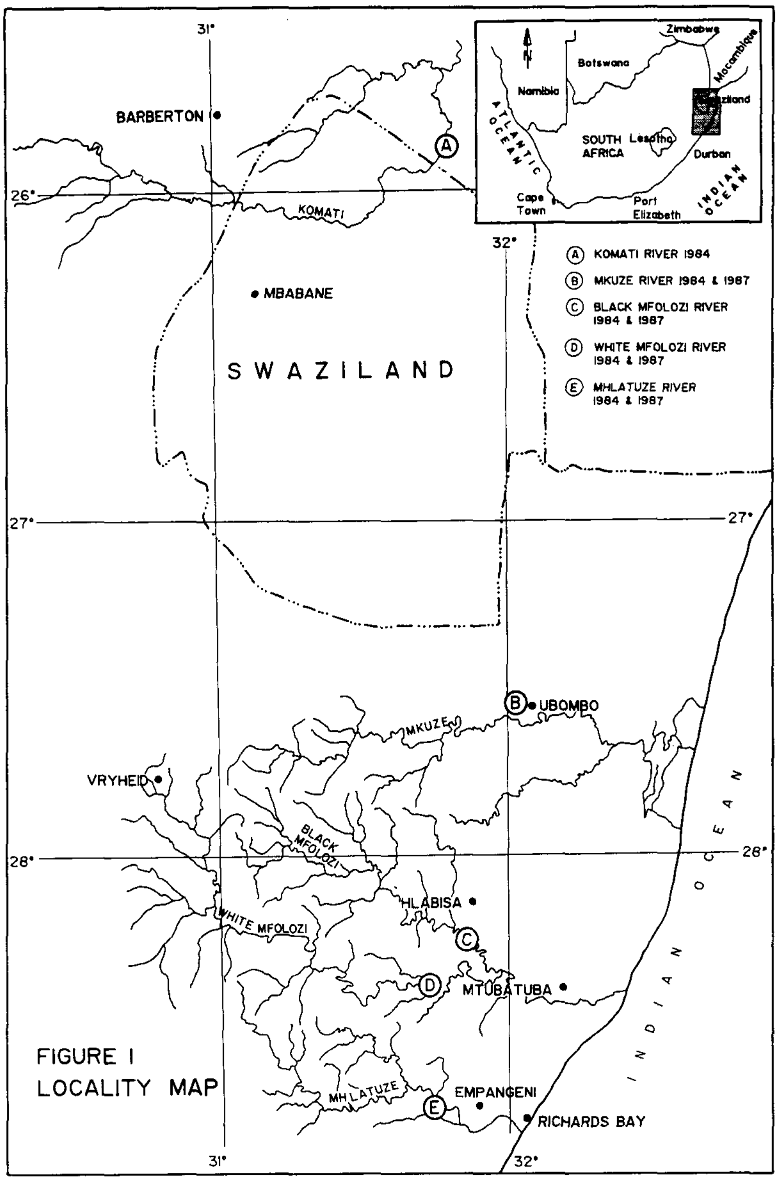

FIGURE I
LOCALITY MAP

Figure 1
Locality map

when $\dfrac{\sqrt{gDs}.d}{\nu} < 13.$

However, when the bed is not smooth, the particle diameter d is no longer representative of the absolute roughness k and the power applied along the bed becomes porportional to:

$$\frac{\rho gsD\sqrt{gDs}}{k}$$

instead of:

$$\frac{\rho gsD\sqrt{gDs}}{d}$$

As the value of the absolute roughness k increases, the transporting capacity, represented by the applied power function, decreases, whilst the unit power required to suspend particles with a given settling velocity remains the same, i.e.:

$$(\rho_s\text{-}\rho)g\ V_{ss}$$

Following the same arguments as before, critical conditions are now represented by:

$$(\rho_s\text{-}\rho)g\ V_{ss}\ \alpha\ gsD.\ \frac{\sqrt{gDs}}{k}$$

and

$$Vss\ \alpha\ \frac{(\rho_s\text{-}\rho)gd}{\rho C_d} \qquad (2)$$

Substitution leads to the result:

$$\frac{\sqrt{gDs}}{V_{ss}} = \text{Constant.}\ (k/d)^{1/2} \qquad (6)$$

TABLE 2 SEDIMENT CHARACTERISTICS AND FLOW PARAMETERS								
Site	**River**	**Year**	**Representative particle diameter d_{85} (mm)**	**Absolute roughness[1] k_1 (m)**	**Settling velocity V_{ss} (m/s)**	$\frac{\sqrt{gDs}}{V_{ss}}$	$\frac{\sqrt{gDs}\, d^{1/3}}{V_{ss}\, k}$ (-)	$\frac{k}{d_{85}}$
A	Komati	1987	2,33	1,3	0,217	1,33	0,167	485
B	Mkuze	1984	0,429	1,5	0,063	6,52	0,423	3 497
		1987	0,88	1,28	0,113	2,53	0,222	1 455
C	Black Mfolozi	1984	0,205	1,48	0,028	16,3	0,803	7 220
		1987	0,530	0,89	0,076	3,84	0,32	1 679
D	White Mfolozi	1984	0,605	1,16	0,085	4,14	0,331	1 917
		1987	1,7	0,8	0,178	1,81	0,228	471
E	Mhlatuze	1984	0,368	1,11	0,055	8,63	0,592	3 016
		1987	0,471	0,87	0,069	6,61	0,528	1 847

1) According to estimates by the Department of Water Affairs and Forestry

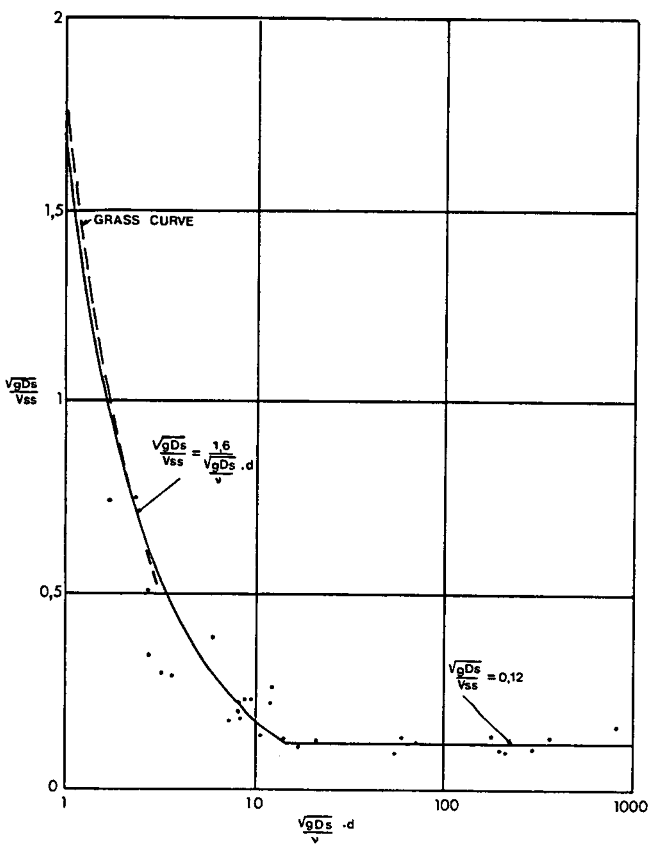

Figure 2
Mathematical relationships for true laminar and turbulent critical boundary layer conditions

Field results

Table 2 contains the most important values which were measured as well as derived for the different river reaches under consideration. More comprehensive information is contained in Le Grange (1992).

Whereas one tends to assume in terms of the parameters in the Liu diagram (Fig. 2) that the value of the $\frac{\sqrt{gDs}}{V_{ss}}$ functions should be constant for cases where turbulent boundary conditions ought to prevail, the recorded values of this function displayed in Table 2 vary significantly.

However, in terms of the arguments leading to Eq. (6), the absolute bed roughness (k) should have a significant influence on the critical transporting condition.

Independent estimates of k-values were used by the Department of Water Affairs to calculate the peak discharge values. These k-values were used to calculate values for the critical condition function depicted in Eq. (6).

It is clear from Table 2 that the values of the $\frac{\sqrt{gDs}}{V_{ss}} (d/k)^{1/3}$ function vary considerably and differ greatly from the expected constant value of 0,12:

$$\left(\frac{\sqrt{gDs}}{V_{ss}} = 0{,}12 \text{ if } d \approx k \right)$$

By plotting the results on the modified Liu-curve (which includes the transition from fully laminar to fully turbulent boundary conditions) (Fig. 3) the variation in $\frac{\sqrt{gDs}}{V_{ss}}$ follows the same pattern as for laminar boundary conditions. All the evidence seems to indicate that somehow, even under such extreme flood conditions, laminar boundary conditions develop below the highly turbulent flows that prevail above.

To test whether this hypothesis is true, the basic relationship for critical conditions with laminar boundary conditions was reconsidered under these conditions:

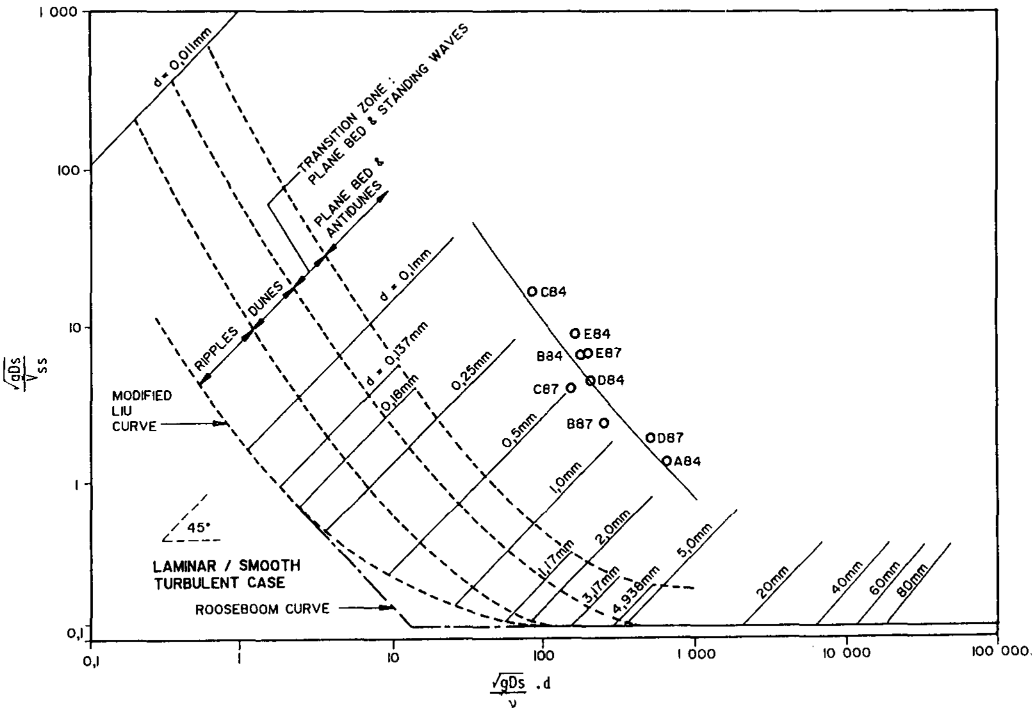

Figure 3
Modified LIU-diagram

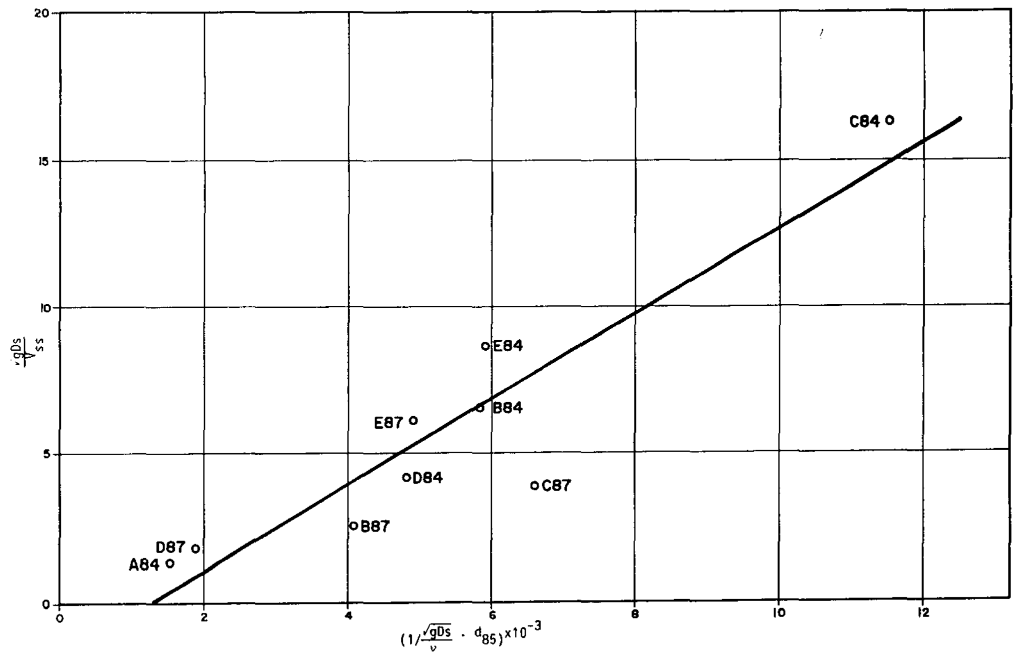

Figure 4
Critical relationship for laminar boundary conditions

$$\frac{\sqrt{gDs}}{Vss} \quad \alpha \quad \frac{1}{\dfrac{\sqrt{gDs}}{\nu} . d}$$

This trend appears evident in Fig. 4.

Considering the fact that turbulent conditions would have prevailed and that the size of the boundary eddies would have been of similar diameter as the absolute roughness it is clear that as the river bed is deformed from being flat to one with sand waves, some more than a meter high, the applied (turbulent) power along the bed will decrease from being proportional to 1/d to proportional to 1/k. The parameter k/d is therefore representative of the ratio between the very high value of the applied turbulent stream power which is found along a smooth bed compared to the value along a bed with absolute roughness value k. As can be seen in Table 2, these ratios were very high which means that the transporting capacity became very much lower than the equivalent values along smooth beds.

It is interesting to note in Fig. 3 that it is not possible to have turbulent boundary conditions along a smooth bed with particles smaller than say 2 mm. This raises the question as to whether erosion of such small particles can only cease when laminar boundary conditions develop.

Discussion

It is clear from the foregoing that large increases in the absolute roughness of river beds leads to vastly decreased transporting capacities. By creating large sand waves along its bed a river virtually armours itself and prevents the much deeper scour that should have taken place if the bed had remained smooth. The size of the sand waves can be calculated by equating the functions which represent applied unit power for critical laminar and turbulent boundary conditions.

All the evidence seem to indicate that equilibrium scour depths are approached only when the applied stream power drops below the critical value for laminar boundary conditions. Whether a laminar boundary actually exists and whether the limiting turbulent applied power merely reaches the corresponding value is still debatable.

This new approach, based on applied stream-power theory, can be used to predict equilibrium scour conditions in sandbed rivers.

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