Activated sludge settling Part I: Experimental determination of activated sludge settleability

PFC Catunda and AC van Haandel*
Associate Professors, Dept. of Civil Engineering, Federal University of Paraiba, Rua Aprigio Veloso 882, 58.110 Campina Grande Pb, Brazil

Abstract

Several methods to quantify sludge settleability are reviewed and interrelationships between the zone settling velocity (as defined by Vesilind's (1968) equation), the stirred sludge volume index and the diluted sludge volume index are derived from empirical equations. An experimental investigation showed that activated sludge composition had a marked influence: Sludge settleability increased with increasing sludge age, i.e. as the fraction of active sludge (living organisms) in the sludge decreased.

Introduction

Due to a high suspended solids concentration, sedimentation of mixed liquor occurs in a fashion quite different from that of more dilute solutions like raw sewage. In dilute suspensions, the velocity of a particular floc or particle is determined by the resulting force of gravity and frictional shear. In concentrated suspensions, a matrix of interlinked particles is formed and these settle all with the same velocity: the zone settling velocity (ZSV). The magnitude of ZSV is determined by the total force on the interlinked particles. The minimum total suspended solids (TSS) concentration for zone settling to develop is 0,5 to 1,0 g/ℓ; below this range flocs are dispersed and tend to settle individually. As activated sludge systems are operated with a TSS concentration in the range of 2 to 5 g/ℓ, invariably in final settlers zone settling will occur.

The value of sludge settleability and its eventual fluctuations are of great importance for design and operation of activated sludge systems. Both the system reactor/final settler and the system thickener/sludge digester can be optimised only if the sludge settleability is known. In day-to-day operation of an activated sludge system, variations of sludge settleability are an important indicator to detect at an early stage the onset of sludge bulking, allowing the operator to take measures that may avoid operational problems, or at least reduce these considerably.

In this paper three experimental methods are compared to quantify activated sludge settleability, viz. the determinations of:

- zone settling velocity (ZSV);
- stirred sludge volume index (SSVI); and
- diluted sludge volume index (DSVI).

Empirical relationships are presented to relate the values of these three parameters.

An experimental investiqation was carried out to establish the influence of sludge composition on settleability. The experimental results showed that, as the fraction of active (i.e. life organism) sludge increases, settleability tends to become poorer. It was possible to derive expressions to describe quantitatively the influence of sludge composition on sludge settleability.

Experimental methods to determine sludge settleability

Determination of the zone settling velocity (ZSV).

Zone settling can be observed conveniently in an apparatus described by White (1975) and represented schematically in Fig. 1a. This apparatus consists of a vertical transparent cylinder, in which a batch of activated sludge is placed. The sludge is gently stirred by a vertical rod, connected to a central axis, driven by a low-speed motor. When a batch of sludge is placed in the cylinder, the following behaviour is observed (Fig. 1b):

- A short time (a few minutes) after introducing the sludge, a clearly visible interface is formed between settling sludge in the lower section and a clear supernatant, essentially free of suspended solids, at the upper section.
- In the bottom section, the particles settle with a uniform and constant rate and the interface moves downwards with this same velocity.
- Simultaneously, at the bottom of the cylinder, settled sludge with a higher concentration accumulates and, as settling continues, a steadily increasing fraction of the solids is incorporated in this more concentrated part of the sludge.
- After some time, all the suspended solids will be part of the settled sludge. On approaching this moment, the velocity of interface displacement will gradually decrease.

Figure 1b shows a typical curve of the interface level as a function of settling time. The zone settling velocity (ZSV) is defined as the displacement rate of the interface at the linear (or linearised) section of the curve.

Figure 2 shows a more elaborate version of White's apparatus, which allows the determination of ZSV of six batches simultaneously. If, for example, batches of one sludge with different concentrations are used, then the influence of the concentration on ZSV can be evaluated (of course it is also possible to use one cylinder with sequential batches of sludge with different concentrations).

ZSV is influenced by several factors, but the most important is the initial sludge concentration. Several research workers have investigated the relationship between ZSV and the initial sludge concentrations. The best known models to describe the relationship are those by Vesilind (1968) and by Dick (1972),

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*To whom all correspondence should be addressed.

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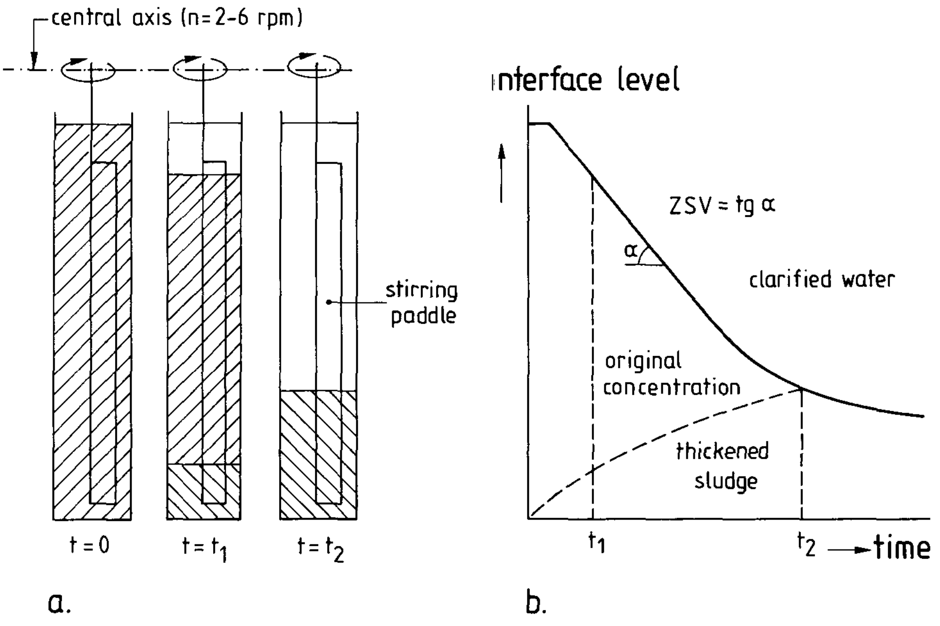

Figure 1
Schematic presentation of the ZSV test (a) and typical curve of the interface level in a batch of settling activated sudge (b)

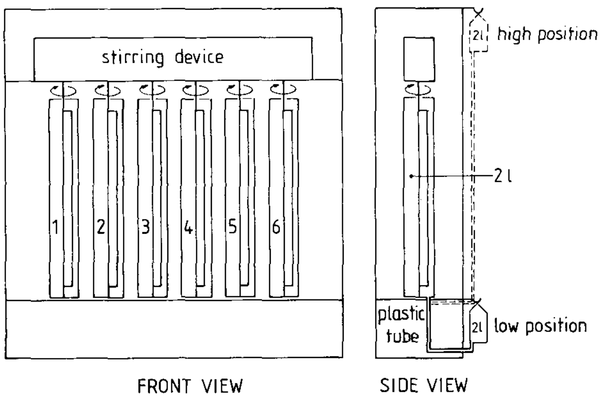

Figure 2
Apparatus for simultaneous ZSV or SSVI determinations

which can be expressed as:

- Vesilind (1968): $v = v_0 \exp(-kX)$ (la)
and

- Dick (1972) : $v = V_0(X)^{-K}$ (lb)

where:

v = zsv
X = sludge concentration
v_0, k, V_0, and K are settling constants.

Smollen and Ekama (1984) analysed their own data as well as those of others (Pitman, 1980 and 1984; Tuntoolavest and Grady, 1982 and Rachwall et al., 1981), to evaluate which of the two models gave a better description. The available data were used to plot log v as a function of X and log v as a function of logX, as indicated in Figs. 3a and 3b. It was found that in all cases the correlation coefficient of the assumed right line was better in the semi log plot (Fig. 3a) than in the log-log plot (Fig. 3b), so that Vesilind's equation gave a better description of the experimental data than Dick's.

Alternative parameters for sludge settleability

Being a complicated and tedious test, the determination of ZSV as described in the preceding section is not very adequate for routine use in waste-water treatment plants. Several authors have tried to develop alternative methods to determine sludge settleability.

The sludge volume index (SVI) is the oldest and probably the most utilised test. Its procedure is very simple: A batch of sludge with known concentration is placed in a vertical transparent cylinder (for example a 1 000 mℓ glass cylinder) and the volume fraction of settled sludge is read off after a fixed period (usually 30 min), whence the volume occupied by a unit mass of suspended solids is calculated. This number expresses the SVI value. Unfortunately the ability of the SVI to characterise sludge settleability is very limited: The result of the test depends to a large degree on the initial concentration. Thus, the SVI test does not give adequate information about sludge settleability.

In an attempt to eliminate the influence of the initial sludge concentration on the result of the test, Stobbe (1964) developed the diluted sludge volume index (DSVI) test. This test is based on the experimental observation that the result of the SVI test is not much influenced by the initial concentration, if the final volume of settled sludge is less than 25% of the original batch volume. Thus, Stobbe (1964) suggested the dilution of the activated sludge batch sufficiently to obtain, after settling, a final sludge volume of less than 20 % of the initial diluted volume. The volume of settled, diluted sludge per unit mass of solids is

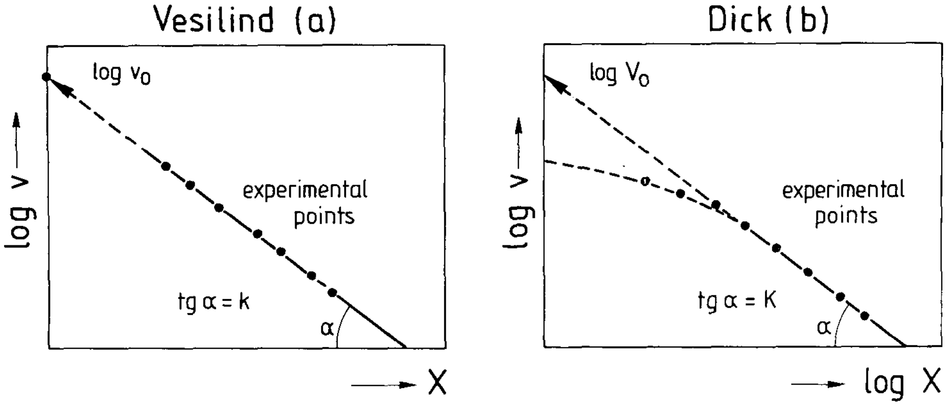

Figure 3

Relationships of ZSV and sludge concentration in accordance with the equations of Vesilind (1968) (a) and Dick (1972) (b)

defined as the DSVI.

White (1975) developed the stirred sludge volume index (SSVI), defined as the sludge volume per unit mass of solids after 30 min of settling in a vertical cylinder while gentle mechanical stirring is applied. This stirring increased the settling velocity and greatly reduced the influence of the initial sludge concentration on the result. Only when SSVI was very small (ZSV< 1 m/h) the initial sludge concentration had a marked influence. For this reason White (1975) suggested to standardise the sludge concentration by taking a value of 3,5 g TSS/ℓ, to be able to compare the settleability of different sludges, introducing thus $SSVI_{3,5}$.

Relationships between sludge settling parameters

Both DSVI and SSVI are much simpler tests than ZSV. However, as will be shown in a following paper, the values of Vesilind's constants k and v_0 are necessary to optimise activated sludge settlers and thickeners. Hence it is of interest to try and correlate the DSVI and/or the SSVI tests to the ZSV test and estimate the k and v_0 values from these.

Pitman (1984) observed an empirical relationship between the constants of Vesilind's equation (v_0/k) and $SSVI_{3,5}$. From the results of six years of full-scale observations the following empirical relationship was established:

$$v_0/k = 68\exp(-0,016\ SSVI_{3,5}) \qquad (2)$$

Ekama and Marais (1986) analysed data of several authors (White, 1975; Rachwall et al., 1981; Koopman and Cadee, 1983) and concluded that Pitman's relationship also applied to these and also noted that there was a relationship between k and v_0/k:

$$k = 0,88 - 0,393 \log (v_0/k) \qquad (3)$$

Thus, by determining $SSVI_{3,5}$ the ratio v_0/k and hence k can be calculated with the aid of Eq. (3). The second constant is then easily determined as:

$$v_0 = k\,(v_0/k) \qquad (4)$$

By substituting Eq. (2) in Eq. (3), an explicit expression for k as a function of $SSVI_{3,5}$ can be obtained:

$$k = 0,16 + 0,0027.SSVI_{3,5} \qquad (5)$$

If Eq. (5) is substituted in Eq. (4) an expression for v_0 is obtained:

$$v_0 = (10,9 + 0,18.SSVI_{3,5})\exp(-0,016\ SSVI_{3,5}) \qquad (6)$$

Equation (6) is shown plotted in Fig. 4. It is clear that the value of v_0 decreases approximately linearly with increasing $SSVI_{3,5}$ values, until a value of 100 mℓ/g is reached, which is generally considered as the maximum value for "normal" or flocculent sludge. Sludges with SSVI>100 mℓ/g are considered as filamentous or bulking (Smollen and Ekama, 1984). Thus the empirical value of v_0 can be approximated as:

$$v_0 = 12 - 0,06.SSVI_{3,5} \text{ with the proviso } SSVI_{3,5}<100 \text{ m}\ell\text{/g} \qquad (7)$$

Stofkoper and Trentelman (1982) determined $SSVI_{3,5}$ and DSVI at 25 activated sludge plants treating municipal waste in the Netherlands. Although the data exhibited a considerable spread, the relationship between the two parameters was shown to be approximately linear and proportional:

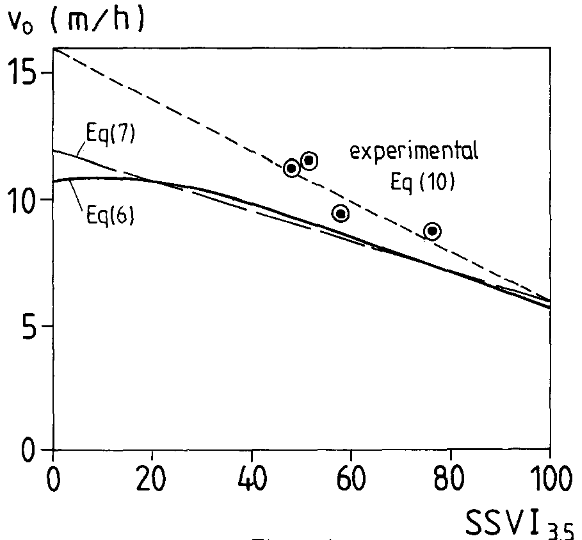

Figure 4

Empiric relationship for v_0 as a function of $SSVI_{3,5}$, derived from Smollen and Ekama (1984)

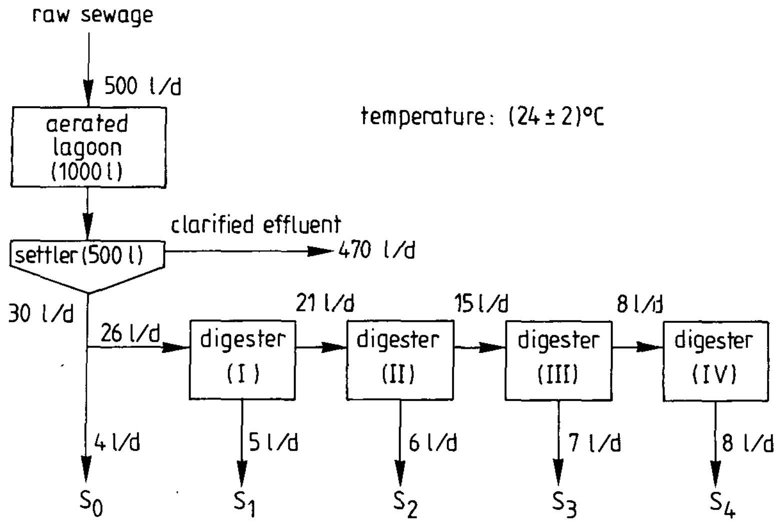

Figure 5
Schematic presentation of the experimental set-up during the investigation of the influence of sludge composition on settling characteristics

TABLE 1
MAIN CHARACTERISTICS OF THE SLUDGES S_0 TO S_4 GENERATED AND USED IN THE EXPERIMENTAL INVESTIGATION

Parameter	Unit	S_0	S_1	S_2	S_3	S_4
Daily produced volume	ℓ/d	4	5	6	7	8
TSS conc.	g/ℓ	4,10	3,66	2,88	2,43	2,01
VSS conc.	g/ℓ	2,93	2,46	1,90	1,56	1,26
0_2 uptake rate	mg/ℓ·h	44	29	16	8	4
Daily TSS mass	g/d	16,4	18,3	17,3	17,0	16,1
Daily VSS mass	g/d	11,7	12,3	11,4	10,9	10,1
Hydraulic ret. time	d	2,00	1,73	2,14	3,00	5,63
Active sludge conc.	g/ℓ	2,31	1,52	0,84	0,42	0,21
Active sludge fraction	-	0,78	0,62	0,44	0,27	0,17
Equivalent sludge age	(d)	2	5	11	26	50

$$DSVI = 0{,}65\ SSVI_{3,5} \qquad (8)$$

Smollen and Ekama (1984) noted almost the same relationship for South African conditions. Their proportionality constant was 0,67.

Experimental investigation

Description of the experimental procedure

The empirical relationships for the different parameters expressing sludge settleability do not incorporate explicitly the influence of sludge composition. To evaluate to what extent sludge composition influences settling characteristics, an experimental investigation was carried out in which settleability was determined of sludges generated from the same sewage but under different operational conditions, leading to different sludge compositions.

The sludges were generated in the system presented schematically in Fig. 5. Once per day a volume of 500 ℓ of raw sewage of the City of Campina Grande was pumped into a 1 000 ℓ completely mixed aerated lagoon, from which 500 ℓ of mixed liquor had been discharged previously to a batch settler. After settling of the mixed liquor, 470 ℓ of supernatant was withdrawn, leaving a batch of 30 ℓ of sludge. This sludge was fed to a series of four aerobic digesters with capacities of 45 ℓ each, in the following manner:

- From the last digester(IV in Fig. 5) a daily volume of 8 ℓ of mixed liquor (sludge S_4) was withdrawn and substituted by 8 ℓ of mixed liquor from the mixed liquor in digester III.
- From digester III, along with the 8 ℓ for digester IV, an additional volume of 7 ℓ (sludge S_3) was withdrawn and the sum of these, 8+7 = 15 ℓ was substituted by mixed liquor from digester II.
- From digester II along with the 15 ℓ for digester III, an additional volume of 6 ℓ (sludge S_2) was withdrawn and the total volume 15+6 = 21 ℓ was substituted with mixed liquor from digester I.
- From digester I another 5 ℓ (sludge S_1) were withdrawn and the total volume 21+5 = 26 ℓ were substituted by an equal volume of the batch, obtained from the aerated lagoon, leaving a balance of 4 ℓ of this mixed liquor (sludge S_0).

Thus daily batches of five different sludges were generated: 4 ℓ from the aerated lagoon, 5 ℓ from digester I, 6 ℓ from digester II, 7 ℓ from digester III and 8 ℓ from digester IV. These batches were utilised to carry out experiments to determine their settling characteristics. The daily volumes were chosen in such a fashion that the sludge mass in each was approximately the same. Table 1 shows data characterising the different sludges. The most important parameter to characterise the difference of sludge composition in the batches is the fraction of life (active) material in these. Unfortunately this fraction cannot be measured directly, but it can be calculated from the endogenous respiration rate (Marais and Ekama, 1976):

$$\begin{aligned} O_e &= (P + 4,6*f_n)(1 - f)\, b_h X_a \qquad (9)\\ &= (1,5 + 4,6*0,1)(1 - 0,2)*0,24*1,04^{(t-20)}*X_a\\ &= 0,458\, X_a \end{aligned}$$

where:

O_e = endogenous respiration rate (mg/ℓ·d) (including nitrification of mineralised ammonia)
X_a = active sludge concentration (mg VSS/ℓ)
p = COD/VSS ratio = 1,5 mg COD/mg VSS
f_n = nitrogen mass fraction in sludge = 0,1 mgN/mgVSS
f = endogenous fraction = 0,2 (-)
b_n = endogenous decay rate = $0,24\,(1,04)^{(T-20)}\, d^{-1}$
T = operational temperature = 25°C

The values of the active sludge fractions, f_a, can be calculated as the ratio of the active sludge concentration obtained from Eq. (9) and the experimental volatile sludge concentration shown in Table 1. The calculated values of X_a and f_a are also indicated in Table 1. As can be expected the active sludge fraction varies enormously from the "freshest" sludge generated in the aerated lagoon (f_a = 0,78) to the most digested one in digester IV: f_a = 0,17. To put these values in perspective, the sludge ages to generate these sludges in an activated sludge process were calculated from the theory presented by Marais and Ekama (1976). These values are also indicated in Table 1. The calculations for the equivalent sludge age and the corresponding sludge loading factor are discussed in Appendix A.

Experimental results

During the four-month period of investigation, the parameters to characterise settling of the five generated sludges were determined as a function of sludge concentration. The most important results from the experimental investigation are:

- Vesilind's equation gave an adequate prediction of the zone settling velocity. The constants k and v_0 of the equation were not influenced by the initial sludge concentration, but they depended on the active sludge fraction. The data show a large spread, but there are approximately linear relationships between the constants and the active sludge fraction. Table 2 gives the experimental values of Vesilind's constants k and v_0 obtained for the five different sludges characterised in Table 1. The number of tests carried out to determine the constants and the magnitude of the standard deviations are also indicated. The averages of the experimental values for k and v_0 are shown plotted in Figs. 6a and 6b. The experimental data suggest the following equations for the average values:

$$k = 0,30 + 0,08 f_a \qquad (10)$$

and

$$v_0 = 12 - 3\, f_a \qquad (11)$$

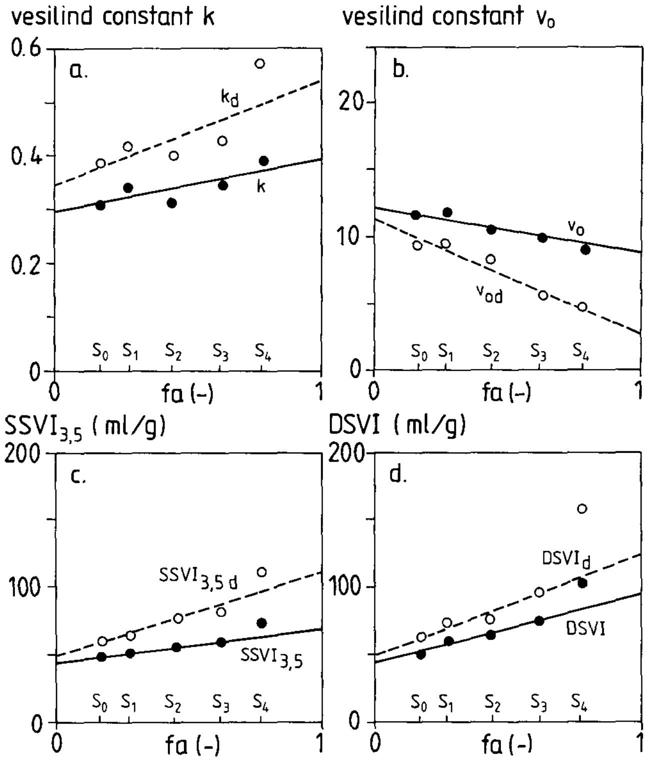

Figure 6
Experimental average values of the different settling characteristics of activated sludge as a function of the active sludge fraction (Extra index d refers to design value for 95% confidence interval)

- Simultaneously with the ZSV tests, SSVI tests were also carried out as a function of the solids concentration and for the different sludges. It was noted that the SSVI value of a sludge tended to depend on the active sludge fraction as well as the suspended solids concentration. Figure 7 shows the experimental results of SSVI tests for the five different sludges. Again, there is a considerable spread in the experimental data, particularly when the active sludge concentration is high, but there is a clear tendency of SSVI to increase with increasing concentration and decrease with increasing active sludge concentration. The following empirical relationship was derived from the experimental results.

$$SSVI = 25 + 5X_t + 25\, f_a \qquad (12)$$

The determinations of SSVI as a function of the concentration were used to estimate $SSVI_{3,5}$ for the different sludges by interpolation of the results. From the results, shown in Table 2, the influence of the active sludge fraction on $SSVI_{3,5}$ can be evaluated. The respective standard deviations are also indicated. Figure 7c shows a plot of the average $SSVI_{3,5}$ values as a function of the active sludge fraction. In conformity with Eq.(12) the influence of the active sludge fraction on the average value of $SSVI_{3,5}$ can be expressed as:

$$SSVI_{3,5} = 43 + 25 f_a \qquad (13)$$

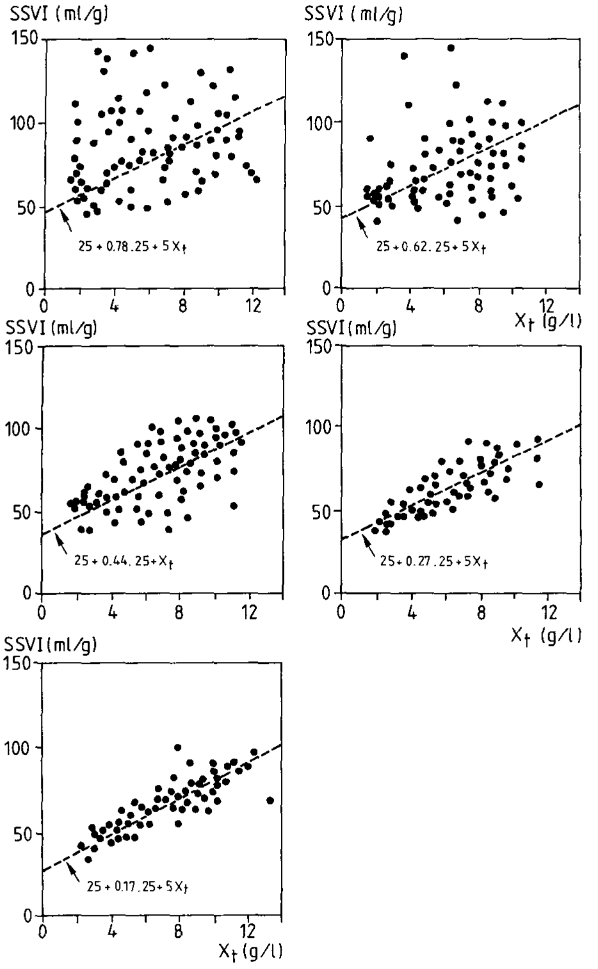

Figure 7
Values of SSVI as a function of the sludge concentration in g TSS/ℓ for the five investigated sludges

- Experimental data of DSVI were obtained, and corresponding $SSVI_{3,5}$/DSVI ratios were determined. The relationship between DSVI and f_a is approximately linear as can be noted from Fig. 6d, although there is a considerable deviation when the active sludge concentration is high. The relationship between DSVI and f_a can be approximated by:

$$DSVI = 43 + 50\,f_a \qquad (14)$$

- From the empirical equations for $SSVI_{3,5}$ and DSVI, it can be noted that the average value of the ratio $SSVI_{3,5}$/DSVI varied from a value of 1,00 for sludges with a low active fraction (f_a = 0,16) to 0,73 for sludges with a high sludge fraction (f_a = 1). The latter value compares well with the value of Stofkoper and Trentelman (1982) who found a ratio of 0,65 (It must be considered that in Holland many activated sludge plants operate at a short sludge age (i.e. f_a is high).
- The fluctuations of the settleability parameters did not follow a trend with time, i.e. there were no periods of systematically increasing or decreasing settleability. No true bulking sludge with the characteristic "hairy" aspect of sludge developed during the test period.
- Upon occasional microscopic examinations, it was found that the sludges had very different compositions of micro-organisms: sludge S_0 practically contained only bacteria; sludge S_1 also contained protozoa, whereas in sludges S_3 to S_5 also ciliates and rotifers developed.

It should be emphasised that the experimental results were obtained, using only the municipal sewage of Campina Grande. In view of the differences in the values for settling constants that have been reported by research workers who used sewage from different sources, it is likely that, along with operational conditions at the treatment site (notably the sludge age), the origin of the sewage also influences the sludge settleability characteristics.

Discussion

It is apparent from the experimental results that the values of the parameters that characterise settleability exhibit considerable oscillations around average values. These "spontaneous" fluctuations of the settling characteristics increase when the active sludge fraction increases, or, equivalently, the standard deviations of the settleability parameters increase as the sludge age of the process is shorter. It would appear that variations in the composition and nature of the sewage feed are the main cause of these fluctuations.

If the experimental results are to be used for design, the values attributed to the settling characteristics need to be conservative, so that the treatment system, resulting from the design will function well, also under adverse circumstances. Taking the parameter $SSVI_{3,5}$ as an example, the standard deviation is 13 % when the active sludge concentration is 17 % (S_4), but it is much larger when the active sludge fraction is high: 27 % when f_a = 0,77 (S_0, Table 2). For the mean values and the standard deviations of $SSVI_{3,5}$ for the different sludges, the design values for a 95 % confidence interval can be calculated. This value is given by the average value plus the double of the standard deviation. If this value is accepted as the design value, this means that in 95 % of the cases the actual $SSVI_{3,5}$ value will be smaller than the design value. Consequently the resulting settler will function well and produce an effluent essentially free of suspended solids in 95 % of the time:

$$SSVI_{3,5d} = SSVI_{3,5} + 2s \qquad (15)$$

where:
$SSVI_{3,5}$ = average value of $SSVI_{3,5}$
$SSVI_{3,5d}$ = design value of $SSVI_{3,5}$ for a 95 % confidence interval
s = standard deviation.

The experimental value can now be used to estimate the design value $SSVI_{3,5d}$ as a function of sludge composition. From Fig. 6c:

$$SSVI_{3,5d} = 48 + 66f_a \qquad (16)$$

Following a similar procedure for the other settleability parameters and using the data of Table 2 plotted in Fig. 6, the following design value expressions can be derived:

TABLE 2
EXPERIMENTAL VALUES OF THE SETTLING CHARACTERISTICS FOR THE DIFFERENT UTILISED SLUDGES (THE STANDARD DEVIATIONS ARE INDICATED BETWEEN BRACKETS)

Parameter	S_0	S_1	S_2	S_3	S_4
Number of tests	12	12	12	12	12
Constant k (ℓ/g)	0,39(22)	0,34(20)	0,31(14)	0,34(14)	0,32(11)
Constant v_0 (m/h)	8,9(27)	9,5(20)	10,4(11)	11,7(10)	11,4(9)
$SSVI_{3,5}$ (mℓ/g)	72(27)	58(20)	55(20)	51(12)	48(13)
DSVI (mℓ/g)	101(26)	73(13)	74(11)	60(11)	51(10)

$$K_d = 0{,}35 + 0{,}19f_a \qquad (17)$$
$$v_{od} = 11 - 8f_a \qquad (18)$$
$$DSVI_d = 50 + 80f_a \qquad (19)$$

where:
index d refers to design value for 95 % confidence interval.

Conclusions

- Activated sludge settleability can be expressed by several parameters: Zone settling velocity characterised by Vesilind's (1968) equation and its constants k and v_0, stirred sludge volume index (SSVI) and diluted sludge volume index (DSVI). The values of all these depend upon the composition of activated sludge i.e. on the active (living organism) sludge fraction. When the active sludge fraction was used to characterise sludge composition, the following empirical relationships were established for the averages of the settling characteristics:
    - Vesilind's constants:
      $k = 0{,}30 + 0{,}08f_a$
      $v_0 = 12{,}0 - 3f_a$
    - Stirred sludge volume index
      $SSVI_{3,5} = 43 + 25f_a$
    - Diluted sludge volume index
      $DSVI = 43 + 50f_a$
- All sludge settling parameters exhibit considerable fluctuations. For 95 % confidence design the following empirical expression was observed for the different parameters. The constants of Vesilind's equation for 95 % confidence were determined as:

$$k_d = 0{,}\,30 + 0{,}\,21\,f_a$$
$$V_{od} = 11{,}5 - 7{,}9\,f_a$$
$$SSVI_{3,5d} = 48 + 66\,f_a$$
$$DSVI_d = 50 + 80\,f_a$$

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Appendix A

The composition of sludge changes considerably as a function of the solids retention time or sludge age in an activated sludge sustem. Marais and Ekama (1976) presented a model of the activated sludge process, in which the organic fraction of the sludge is assumed to be composed of three fractions:

- active sludge: the living organism mass, responsible for the biological activity;
- endogenous residue, originated from the active sludge : it is the unbiodegradable fraction that remains after decay of living organisms; and
- the inert sludge generated by flocculation from unbiodegradable particulate material in the influent.

While the active sludge concentration cannot be measured directly, Marais and Ekama (1976) developed a consistent model, based on measurements of indirect parameters such as the rate of oxygen consumption. In this model the active sludge fraction is given by:

$$f_a = X_a/X_v$$
$$= (1-f_{us}-f_{up})YR_s/(1+b_hR_s)/[(1+fb_hR_s)(1-f_{us}-f_{up})YR_s/(1+b_hR_s)+ f_{up}R_s/p]$$

where:

X_a = active sludge concentration
X_v = volatile sludge concentration
f_{us} = influent COD fraction that is unbiodegradable and soluble
f_{up} = influent COD fraction that is unbiodegradable and particulate
Y = yield coefficient
p = COD/VSS ratio of sludge
f = fraction of endogenous sludge that becomes endogenous residue
R_s = sludge age

The above equation can be used to calculate the sludge composition in an activated sludge system operating at a particular sludge age, or conversely, the sludge age necessary to obtain a particular sludge composition may be calculated.

The sludge loading factor is an alternative (although inferior) parameter to characterise the operational condition of an activated sludge plant. This parameter can be expressed as the ratio of the mass of daily removed organic material and the (volatile)sludge mass in the reactor:

$$F/M = (1-f_{us})\, Q_l S_{ti}/(V_r X_v)$$
$$= (1-f_{us})/[(1+fb_hR_s)(1-f_{us}-f_{up})YR_s/(1+b_hR_s) + f_{up}R_s/p]$$