Development and verification of hydrograph routing in a daily simulation model+

JC Smithers[1]* and RE Caldecott[2]
[1]*Department of Agricultural Engineering, University of Natal, PO Box 375, Pietermaritzburg 3200, South Africa*
[2]*Eskom Marketing Centre, PO Box 95, Edgemead 7407, South Africa*

Abstract

The ability to simulate, on a continuous basis, the peak flow rate, runoff volume and hydrograph from a catchment is important in planning, design and operation of hydraulic structures and the solution of a wide range of problems associated with water use. The ACRU modelling system, developed originally essentially as a small catchments daily time step hydrological model, is being applied increasingly in distributed mode to larger complex catchments where the river network plays an important role in transporting the water to the catchment outlet.

The assumption in the ACRU model that stormflow generated on a particular day passes the catchment outlet on the same day is valid for small catchments, but is not necessarily true of large catchments. Thus a flow routing submodel has been incorporated into the ACRU model which enables continuous hydrograph simulation to improve the temporal distribution of the simulated flow. Verification of the submodel has taken place on the Ntabamhlope wetland in Western Natal and on the catchment upstream of Henley Dam in Natal.

Introduction

The ability to simulate, on a continuous basis, the peak flow rate, runoff volume and hydrograph from a catchment is important in planning, design and operation of hydraulic structures and in the solution of a wide range of problems associated with water use. In Southern Africa, commonly used techniques for predicting the runoff hydrograph shape include the unit hydrograph, the kinematic and the time-area methods (Campbell et al., 1986).

An advantage of applying any of these methods in a spatially distributed model when simulating the runoff from a large heterogeneously responding catchment, is that such a catchment can be subdivided into relatively homogenous response units, thus accounting for the spatial heterogeneity, rather than operating simply a spatially lumped model. The ACRU modelling system (Schulze, 1989), which uses a Southern African modification to the SCS technique to generate daily stormflow and was developed originally essentially as a small catchments daily time step hydrological model, is being applied increasingly in distributed mode to larger, complex catchments where the river network plays an important role in transporting the water to the catchment outlet.

The assumption in the ACRU model that stormflow generated on a particular day passes the catchment outlet on the same day is valid for small catchments, but is not necessarily true of large catchments. Thus when the ACRU model is applied in distributed mode on large catchments, the temporal distribution of streamflow passing the catchment outlet does not reflect the translation of the hydrograph taking place through river reaches and reservoirs encountered *en route* to the catchment outlet.

The development and verification of a flow routing submodel which has been incorporated as an option in the ACRU model, and which enables continuous hydrograph simulation and flow routing to improve the temporal distribution of the daily runoff generated by the ACRU model, is outlined in this paper. Results of verifications of the flow routing submodel undertaken on the 175 km^2 catchment upstream of the Henley Dam in Natal, as well as on the Ntabamhlope wetland in Western Natal, are presented.

River reach and reservoir flood routing

Several methods have been developed for routing floods in reservoirs and rivers. These methods may be classified broadly as either hydraulic or hydrologic routing techniques (Viessman et al., 1989). Both techniques use the principle of conservation of mass. Hydraulic techniques are more complex and were developed for spatially-varied systems using an equation of motion, customarily the momentum equation. Hydrologic routing techniques, which use a conceptual approach, are typically simpler techniques developed for spatially lumped systems (Wilson and Ruffini, 1988).

The hydraulic methods generally describe the flood wave profile more adequately when compared to hydrologic techniques. However, practical applications of hydraulic methods are restricted because of their high demands on computing technology as well as on quantity and quality of input data (Singh, 1988). In addition, investigations by, *inter alia*, Price (1974), Porter (1975) and Hdromadka and De Vries (1988) have shown that the less data intensive hydrologic techniques perform generally as well as the hydraulic techniques. Thus the simpler and less data intensive hydrological routing techniques were implemented as an option in the ACRU modelling system.

The Muskingum method for channel routing is the most widely used hydrologic routing technique (Wilson and Ruffini, 1988). Cunge (1969) related the parameters required for the Muskingum method to physical characteristics of the channel system by noting similarities between the Muskingum method and the numerical approximation of a diffusion wave model. Thus the Muskingum-Cunge flow routing method, although classified as a hydrologic method, gives results comparable with the hydraulic methods. In using physically based estimates for model parameters, the Muskingum-Cunge method operates in accordance with the ACRU modelling philosophy and was thus considered suitable for incorporation, as an option, in the ACRU

---

*To whom all correspondence should be addressed.

+Revised paper. Originally presented at the 5th SA Hydrological Sciences Symposium held in Stellenbosch, November 1991.
Received 20 August 1992; accepted in revised form 30 November 1992.

modelling system. The storage indication (modified-Puls) routing method was selected for routing flow through reservoirs.

Muskingum-Cunge river reach routing

The linear Muskingum method models the storage at time=n in a river reach as given in Eq. 1.

$$S_n = K[XI_n+(1-X)Q_n] \tag{1}$$

where:

S_n = storage in river reach at time = n (m^3)
I_n = inflow to river reach at time = n ($m^3 \cdot s^{-1}$)
Q_n = outflow from river reach at time = n ($m^3 \cdot s^{-1}$)
K = the storage time constant for the reach (s)
X = dimensionless weighting factor that varies between 0 and 0,5.

In physical terms K is considered as an average reach travel time for a flood wave, and X indicates the relative importance of I and Q in determining the storage in a reach. Typical values of X for a river reach range between 0,0 and 0,3, with a mean value near 0,2 (Chow et al., 1988; Viessman et al., 1989).

Thus, from Eq. 1, the change in storage over time interval Δt is given in Eq. 2:

$$S_{(n+1)}-S_n=K[XI_{(n+1)}+(1-X)Q_{(n+1)}]-K[XI_n+(1-X)Q_n] \tag{2}$$

The Muskingum equation as given in Eq. 4, is derived by equating the finite difference form of the continuity equation (Eq. 3) with Eq. 2.

$$S_{(n+1)}-S_n = \frac{(I_{(n+1)}+I_n)}{2}\,\Delta t - \frac{(Q_{(n+1)}+Q_n)}{2}\,\Delta t \tag{3}$$

where:

Δt = routing interval

$$Q_{(n+1)} = C_1 I_{(n+1)}+C_2 I_n+C_3 Q_n \tag{4}$$

where:

$$C_1 = \frac{-KX + 0{,}5\ \Delta t}{K-KX + 0{,}5\ \Delta t} \tag{5}$$

$$C_2 = \frac{KX + 0{,}5\ \Delta t}{K-KX + 0{,}5\ \Delta t} \tag{6}$$

$$C_3 = \frac{K-KX + 0{,}5\ \Delta t}{K-KX + 0{,}5\ \Delta t} \tag{7}$$

The computation of negative outflows ($Q_{(n+1)}$) in Eq. 4 is avoided if the limits on Δt given in Eq. 8 are adhered to (Viessman et al., 1989).

$$2KX < \Delta t < 2K(1-X) \tag{8}$$

Compliance with Eq. 8 gives rise to conflicting requirements (Bauer, 1975). The smaller the value of Δt, the closer the finite difference approximation of the continuous hydrograph. However, Eq. 8 requires the value of Δt to be large relative to the value of the wave travel time, K. This conflict may be solved by subdividing the reach into appropriate subreaches and routing the flow through a series of subreaches.

The accuracy of the method relies on the accurate estimation of parameters relating channel storage volume to inflow and outflow rates (Wilson and Ruffini, 1988). If observed inflow and outflow data are available for the reach, a reverse routing procedure may be implemented which solves Eq. 1 for K by assuming different values of X until an optimum solution is found (Chow et al., 1988). In the absence of observed flow data, either the wave travel time down the reach may be estimated and a default value of X assumed, or the physically based Muskingum-Cunge method may be used for parameter estimation, as shown in Eqs. 9 and 10.

$$X = \frac{1}{2}\left(1-\frac{q_0}{S_0 c\, \Delta l}\right) \tag{9}$$

where:

q_0 = reference discharge per unit channel width ($m^3 \cdot s^{-1} \cdot m^{-1}$)
S_0 = dimensionless channel bottom slope
c = wave celerity ($m \cdot s^{-1}$)
= mV
m = constant, depending on channel dimensions
V = reference velocity ($m \cdot s^{-1}$)
Δl = routing reach length (m)

$$K = \frac{\Delta l}{c} \tag{10}$$

A problem in implementing Eq. 9 is deciding on the reference velocity and hence reference discharge to be used in the calculation. Wilson and Ruffini (1988) have suggested using Eq. 11 to calculate the reference flow rate.

$$Q_0 = Q_b+\frac{1}{2}\,(Q_p-Q_b) \tag{11}$$

where:

Q_0 = reference flow rate ($m^3 \cdot s^{-1}$)
Q_p = peak flow rate ($m^3 \cdot s^{-1}$)
Q_b = base flow rate ($m^3 \cdot s^{-1}$)

Reservoir routing by the storage indication method

The storage indication method of routing flows through a reservoir involves solving the continuity equation as given in Eq. 12, where the unknowns for a particular routing interval are written on the left hand side of the equation, in conjunction with a storage-discharge relationship for the reservoir.

$$\frac{2S_{(n+1)}}{\Delta t} + Q_{(n+1)} = I_{(n+1)}+I_n+\left(\frac{2S_n}{\Delta t} - Q_n\right) \tag{12}$$

Eq. 12 is solved in a step-wise manner, using a storage indication relationship ($2S/\Delta t + Q$ vs. Q) as an aid in the solution.

Implementation of flow routing submodel in ACRU

The flow routing submodel would only be invoked when the ACRU model is applied in a distributed mode to a catchment. The daily stormflow generated by the ACRU model is disaggregated by using the SCS unit hydrograph (Schmidt and Schulze, 1987) and one of four regionalised rainfall distribution patterns calculated by Weddepohl (1988). Thus the daily rainfall amount is distributed according to the rainfall distribution type, divided into shorter durations so that 5,5 time increments equal the catchment lag time and incremental triangular hydrographs are computed and summed to give the stormflow hydrograph. The daily amount of baseflow generated by the ACRU model is then added to the storm hydrograph, assuming a linear change in baseflow rate over the day which ranges from the baseflow rate generated on the previous day to baseflow rate generated on the current day. These concepts are illustrated in Fig. 2 for two nested subcatchments with a reservoir located in the downstream subcatchment as shown in Fig. 1.

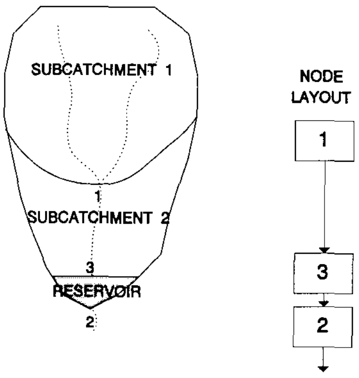

Figure 1
Schematic catchment layout nd node numbering

As illustrated in Fig. 2, the streamflow hydrograph at the outlet to Catchment 1 (node 1) is formed by adding baseflow to the stormflow hydrograph, which was generated using incremental triangular hydrographs. The streamflow hydrograph generated for Catchment 1 (node 1) is routed through subcatchment 2 to node 3, which is the inlet to the reservoir. The hydrograph routed from node 1 to node 3 is then added at node 3 to the streamflow hydrograph generated for the land portion of subcatchment 2. The combined hydrograph at node 3 is then routed through the reservoir to the outlet from Catchment 2 (node 2).

River reach routing

Three parameter estimation options are available to the user for river reach routing and these are outlined briefly below.

- ***Single reach routing: User input Δt, K and X***
  Flow is routed through a single reach, using user-defined values of the routing period (Δt) and the Muskingum parameters (K and X).
- ***Multiple reach routing: User input Δt, K and X***
  Flow is routed through a series of subreaches in order to comply with requirements stipulated by Eq. 8, using user-defined values of Δt, K and X. Thus the number of subreaches used is calculated as shown in Eq. 13.

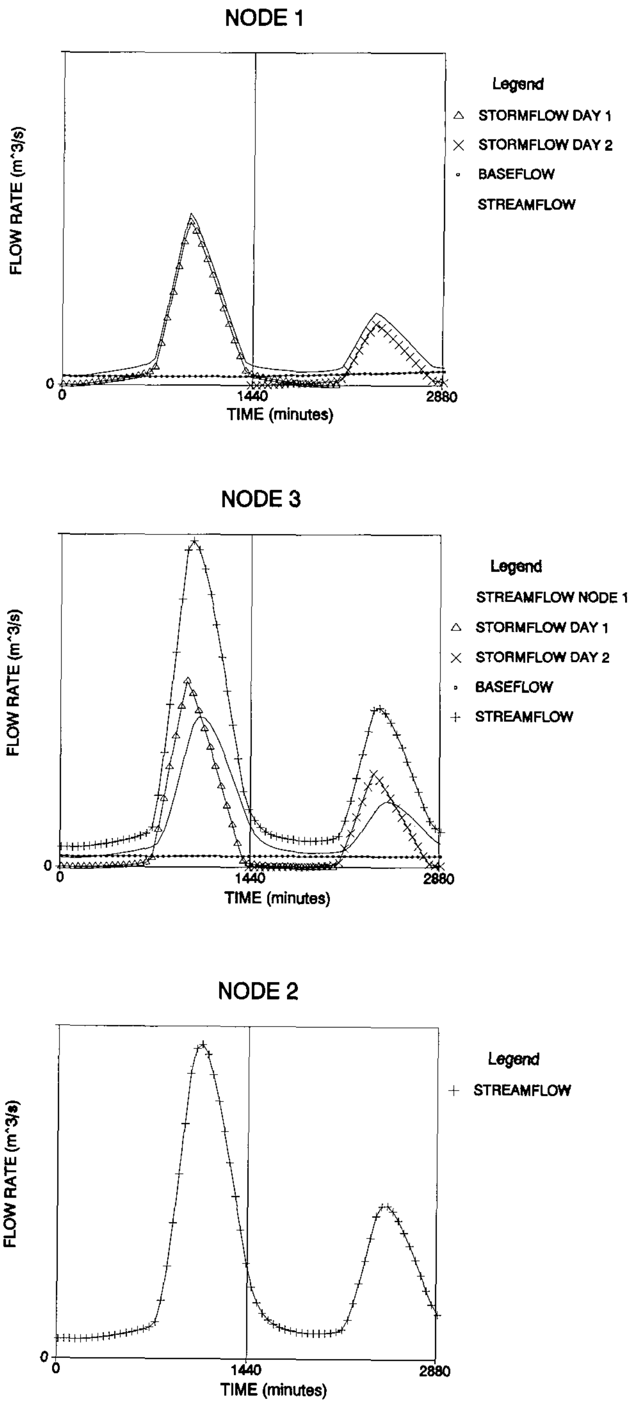

Figure 2
Hydrograph routing at nodes 1, 2, and 3

$$NREACH = \frac{K}{\Delta t} \tag{13}$$

where:

$NREACH$ = number of sub-reaches.

Thus for each subreach $K = \Delta t$, with the routing time being adjusted if necessary for routing through the last remaining subreach.

- **Multiple reach routing: User input Δt and channel dimensions** Flow is routed through a series of subreaches in order to comply with requirements stipulated by Eq. 8, using user-defined values of the routing period (Δt), channel geometry and channel slope. Equations 9 and 10 are used to estimate the Muskingum parameters (K and X) using Eq. 11 to define the reference flow rate. Equation 13 is used to calculate the required number of subreaches.

If the downstream subcatchment contains a reservoir, the hydrograph is routed through the reservoir before it becomes inflow to the next subcatchment.

Reservoir routing

The spillway discharge relationship used is given in Eq. 14 (Chow et al., 1988).

$$Q=C_dBH^{\left(\frac{3}{2}\right)} \qquad (14)$$

where:

Q = flow rate over an uncontrolled spillway ($m^3 \cdot s^{-1}$)
C_d = dimensionless coefficient of discharge, input by the user
B = width of spillway (m), input by the user
H = depth of storage above spillway crest (m).

The reservoir surface area:storage relationship used in the ACRU model may either be input by the user based on a basin survey or a default idealised surface area:storage relationships may be selected (Schulze, 1989). Using the selected surface area:storage relationship, the relationship between storage and depth of flow over the spillway is obtained by calculating the surface area for a particular depth, and then using the surface area to calculate the storage above the spillway crest. By combining Eq. 14 with the derived storage:depth relationship, an exponential equation is fitted to the storage indication curve ($2S/\Delta t$+Q vs. Q) and Eq. 12 is then solved in a sequential step-wise manner to route the flow through the reservoir.

Results and discussion of verification studies

The flow routing submodel described above was founded on an event-based routing model developed and verified by Caldecott (1989). Initial verification of the continuous hydrograph simulation submodel incorporated in the ACRU model has taken place on the Ntabamhlope wetland in Western Natal and on the 175 km^2 catchment upstream of the Henley Dam in Natal.

Ntabamhlope wetland

The Ntabamhlope wetland is located at a latitude of 29° 04' South and longitude 29° 39' East, at an altitude of 1 440 m above sea level. The wetland surface area is approximately 1,8 km^2 and has an upstream catchment area of approximately 34 km^2. Smithers (1991), in verifying further developments to the ACRU wetland submodel, originally proposed by Schulze et al. (1987), applied the ACRU model to the Ntabamhlope wetland. The results obtained indicate that the model simulated monthly summations of daily flow well, but did not simulate the translation of flow which occurred in the observed daily flow. It was concluded that the addition of a flow routing submodel to the ACRU model would improve the temporal distribution of the simulated daily flow.

The model was re-run with the flow routing submodel invoked and selected statistics of performance are presented in Table 1 for streamflow simulation with and without the routing submodel invoked.

Differences in the simulated runoff depths with and without the routing submodel invoked are due to the finite difference approximation of the runoff hydrograph. The improvement in performance of the model with the flow routing submodel invoked is evident from the improvement in the correlation coefficient and regression coefficient, as shown in Table 1. Figure 3 clearly shows the effect of the routing submodel during a period of high flow conditions.

TABLE 1
STATISTICS OF PERFORMANCE OF DAILY RUNOFF SIMULATION FROM THE NTABAMHLOPE WETLAND BY THE *ACRU* MODEL, WITH AND WITHOUT FLOOD ROUTING SUBMODEL INVOKED (YEARS 1977-1981)

Statistic	*ACRU*	*ACRU* + routing
Total observed runoff (mm)	881	881
Total simulated runoff (mm)	842	854
Correlation coefficient	0,64	0,79
Regression coefficient	0,62	0,81
Regression intercept	0,17	0,08

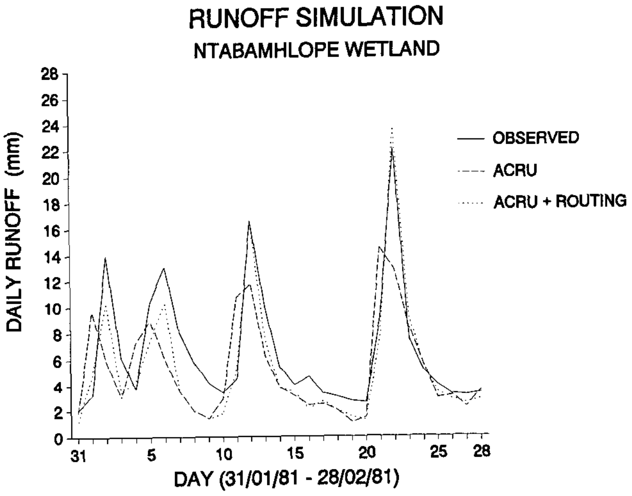

Figure 3
Simulated vs. observed daily values of runoff from the Ntabamhlope wetland

Henley catchment

The ACRU model, in a typical operational type of situation, was applied with and without the flow routing submodel invoked to the 175 km^2 upstream of the Henley Dam catchment. Selected statistics of performance of the runoff simulation are presented in Table 2.

It is evident from Table 2 that the routing submodel had little effect on the statistics of performance. It is suspected that this is a result of poor quality observed data, with the majority of storm events over-topping the gauging weir. However, an example of

TABLE 2
STATISTICS OF PERFORMANCE OF DAILY RUNOFF SIMULATION FROM THE HENLEY DAM CATCHMENT BY THE *ACRU* MODJEL, WITH AND WITHOUT FLOOD ROUTING SUBMODEL INVOKED (YEARS 1975-1980)

Statistic	*ACRU*	*ACRU* + routing
Total observed runoff (mm)	1 221	1 221
Total simulated runoff (mm)	1 046	1 054
Correlation coefficient	0,76	0,76
Regression coefficient	0,96	0,96
Regression intercept	-0,07	-0,07

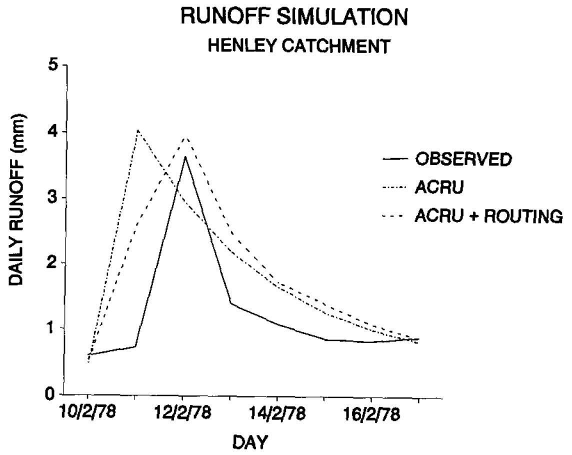

Figure 4
Simulated vs. observed daily values of runoff from the enley Dam catchment

the improvement in the temporal distribution of the simulated daily runoff is given in Fig. 4.

Conclusions

It is evident from the verification studies that the effect of invoking the routing submodel is not always apparent in the statistics of performance. However, from the Ntabamhlope wetland application it is concluded that the routing submodel can be used to improve the temporal distribution of simulated daily runoff. Further operational type of verifications are necessary where the data are not from research catchments.

Improvement to the runoff hydrograph generated for each subcatchment is possible by refining the 4 rainfall distribution types currently available. In addition, the model should have an option where, if available, digitised rainfall could be used in the generation of the runoff hydrograph instead of assuming an average storm distribution type.

In the current state of development, the flow routing submodel assumes no transmission losses from the river channel. The addition of infiltration and evaporation losses directly from the river channel should improve runoff simulation from catchments where channel losses are appreciable.

Acknowledgements

This paper emanated from research which is funded by the Water Research Commission in a project entitled "Hydrological Systems Model Development". The computing facilities provided by the Computing Centre for Water Research and the assistance of its staff are acknowledged gratefully.

References

BAUER, SW (1975) Multiple Muskingum flood routing including flow losses and reservoir storages. HRU Report 3/75, Dept. of Civ. Eng., Univ. of Witwatersrand, Johannesburg.

CALDECOTT, RE (1989) A distributed model for hydrograph simulation and routing. Unpublished M.Sc. dissertation, Dept. of Agric. Eng., Univ. of Natal, Pietermaritzburg.

CAMPBELL, GV, WARD, AD and MIDDLETON, BJ (1987) An evaluation of hydrological techniques for estimating floods from small ungauged catchments. WRC Report 139/2/87.

CHOW, VT, MAIDMENT, DR and MAYS, LW (1988) *Applied Hydrology*. McGraw-Hill, New York, USA.

CUNGE, JA (1969) On the subject of a flood propagation computational method (Muskingum method). *J. Hydraul. Res.* **7** 205-230.

HDROMADKA, TV and DE VRIES, JD (1988) Kinematic wave routing and computational error. *J. Hydraulic Eng.* **114** 207-217.

PORTER, JW (1975) A comparison of hydrologic and hydraulic catchment routing procedures. *J. Hydrol.* **24** 333-349.

PRICE, RK (1974) Comparison of four numerical methods for flood routing. *J. Hydraulic Div. ASCE* **100** 879-899.

SCHMIDT, EJ and SCHULZE, RE (1987) SCS-based design runoff. Dept. of Agric. Eng., Univ. of Natal, Pietermaritzburg. ACRU Report 24.

SCHULZE, RE, CHAPMAN, RA, ANGUS, GR and SCHMIDT, EJ (1987) Distributed model simulation of impacts of upstream reservoirs on wetlands. In: Walmsley, R D and Botten, M L (compilers) *Ecology and Conservation of Wetlands in South Africa*. Occasional Report Series No. 28, Ecosystems Programmes, Foundation for Research Development, CSIR, Pretoria. 115-123.

SCHULZE, RE (1989) ACRU: Background, concepts and theory. Dept. of Agric. Eng., Univ. of Natal, Pietermaritzburg. ACRU Report 35.

SINGH, VP (1988) *Hydrologic Sytems. Rainfall-Runoff Modelling, Volume 1*. Prentice Hall, New Jersey, USA.

SMITHERS, JC (1991) Modelling the Ntabamhlope wetland: Initial results. *Agric. Eng. in SA* **23** 440-449.

VIESSMAN, W, LEWIS, GL and KNAPP, JW (1989) *Introduction to Hydrology*. Harper and Row, New York, USA.

WEDDEPOHL, JP (1988) Design rainfall distributions for Southern Africa. Unpublished M.Sc. dissertation, Dept. of Agric. Eng., Univ. of Natal, Pietermaritzburg.

WILSON, BN and RUFFINI, JR (1988) Comparison of physically base Muskingum methods. *Trans. ASAE* **31** 91-97.

GUIDE TO AUTHORS

AIMS AND SCOPE

This journal publishes refereed, original work in all branches of water science, technology and engineering. This includes water resources development; the hydrological cycle; surface hydrology; geohydrology and hydrometeorology; limnology; mineralisation; treatment and management of municipal and industrial water and wastewater; treatment and disposal of sewage sludge; environmental pollution control; water quality and treatment; aquaculture; agricultural water science; etc.

Contributions may take the form of a paper, a critical review or a short communication. A **paper** is a comprehensive contribution to the subject, including introduction, experimental information and discussion of results. A **review** may be prepared by invitation or authors may submit it for consideration to the Editor. A **review** is an authoritative, critical account of recent and current research in a specific field to which the author has made notable contributions. A **short communication** is a concise account of new and significant findings.

GENERAL

Submission of manuscripts

The submission of a paper will be taken to indicate that it has not, and will not, without the consent of the Editor, be submitted for publication elsewhere. Manuscripts should be submitted to: **The Editor, WATER SA, PO Box 824, Pretoria, 0001, South Africa.**

Reprints

One hundred free reprints of each paper will be provided. Any additional copies or reprints must be ordered from the printer (address available on request).

Language

Papers will be accepted in English or Afrikaans. Papers written in Afrikaans should carry an extended English summary to facilitate information retrieval by international abstracting agencies.

Abstracts

Papers should be accompanied by an abstract. Abstracts have become increasingly important with the growth of electronic data storage. In preparing abstracts, authors should give brief, factual information about the objectives, methods, results and conclusions of the work. Unsubstantiated viewpoints should not be included.

Refereeing

Manuscripts will be submitted to and assessed by referees. Authors bear sole responsibility for the factual accuracy of their publications.

Correspondence

State the name and address of the author to whom correspondence should be addressed on the title page.

SCRIPT REQUIREMENTS

Lay-out of manuscripts

An original typed script in double spacing together with three copies should be submitted. Words normally italicised should be typed in italics or underlined. The **title** should be concise and followed by authors' names and complete addresses. A paper may be organised under main headings such as **Introduction, Experimental, Results, Discussion** (or **Results** and **Discussion**), **Conclusions, Acknowledgements** and **References.**

Contents of manuscripts

The International System of Units (SI) applies. Technical and familiar abbreviations may be used, but must be defined if any doubt exists.

Tables

Tables are numbered in arabic numerals (Table 1) and should bear a short but adequate descriptive caption. Their appropriate position in the text should be indicated.

Illustrations and line drawings

One set of original figures and two sets of copies should accompany each submission. Photographs should be on glossy paper (half-tone illustrations should be kept to a minimum) and enlarged sufficiently to permit clear reproduction in half-tone. All illustrations, line-drawings and photographs must be fully identified on the back, numbered consecutively and be provided with descriptive captions typed on a separate sheet. Authors are requested to use proper drawing equipment for uniform lines and lettering of a size which will be clearly legible after reduction. Freehand or typewritten lettering and lines are not acceptable. The originals should be packed carefully, with cardboard backing, to avoid damage in transit.

References

Authors are responsible for the accuracy of references. References to published literature should be quoted in the text as follows: Smith (1982) or (Smith, 1982). Where more than two authors are involved, the first author's name followed by et al. and the date should be used.

All references are listed alphabetically at the end of each paper and not given as footnotes. The names of all authors should be given in the list of references. Titles of journals of periodicals are abbreviated according to **Chemical Abstracts Service Source Index** (Cassi).

Two examples of the presentation of references are the following:

Grabow, WOK, Coubrough, P, Nupen, EM and Bateman, BW (1984) Evaluations of coliphages as indicators of the virological quality of sewage-polluted water. *Water SA* **10**(1) 7-14.

Wetzel, RG (1975) *Limnology*. WB Saunders Company, Philadelphia. 324.

BERIA