Permeability prediction for water seepage through low porosity granular porous media

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Abstract

Momentum transport characteristic expressions for saturated water seepage through granular porous media are simplified by approximations for low and very low porosity cases. These simplifications are incorporated in momentum transport equations and result in relatively simple expressions for the hydrodynamic permeability of low porosity granular media. The porosity is allowed to differ spatially, but geometrical isotropy is demanded on an average basis. The results are restricted to laminar flow within the pores, but inertial effects due to hydrodynamic flow development within each pore section are accounted for. Results are quantitatively compared to that of the general granular porous medium model and also to experimentally correlated results.

Nomenclature

A_p	cross-sectional pore area
c	as subscript, central value
d	microscopic characteristic length
d_p	mean diameter of solid particles ($=d_s$)
d_s	cube side width
F	microscopic shear factor
f	fluid phase
g	magnitude of $\boldsymbol{g}$
$\boldsymbol{g}$	gravitational body force per unit mass
K	Darcy hydrodynamic permeability when $Re \ll Re_c$
p	pressure
p_f	$\langle p \rangle_f$
q	magnitude of $\boldsymbol{q}$
$\boldsymbol{q}$	specific discharge, $\langle \boldsymbol{v} \rangle$
Re	pore flow Reynolds number, $2\rho v_p(d-d_s)/\mu$
Re_{qd}	RUC Reynolds number, $\rho qd/\mu$
Re_{qs}	particle Reynolds number, $\rho qd_s/\mu$
S	surface area
S_{fs}	fluid-solid interface
t	time
V_f	fluid filled "void" volume within RUC
V_s	solid volume within RUC
V_o	total volume of RUC
$\boldsymbol{v}$	fluid velocity within V_f
v_f	mean pore velocity within V_f, $\langle \boldsymbol{v} \rangle_f$
v_p	mean pore velocity within pore section
ϵ	porosity (void fraction), V_f/V_o
μ	fluid dynamic viscosity
$\boldsymbol{\nu}$	normal vector on S_{fs} pointing into V_f
ρ	fluid mass density
τ	tortuosity, d_e/d
$\langle \phi \rangle$	volumetric phase average of generic variable ϕ, $\frac{1}{V_o}\int_{V_f} \phi \, dV$
$\langle \phi \rangle_f$	volumetric intrinsic phase average of generic variable ϕ, $\frac{1}{V_f}\int_{V_f} \phi \, dV$
$\mathring{\phi}$	deviation, $\phi - \langle \phi \rangle_f$

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Introduction

In recent years a unified approach towards the modelling of saturated flow through porous media has been developed (Bachmat and Bear, 1986; Du Plessis and Masliyah, 1988) and applied with considerable success to various flow phenomena (Du Plessis and Masliyah, 1987; Du Plessis, 1989). Volumetric averaging of the fluid transport equations, together with explicit assumptions regarding the average geometrical properties of the void passages within a porous medium, leads to powerful transport equations capable of resembling flow conditions within a porous medium. Corresponding permeability coefficients were expressed explicitly in terms of microstructure parameters through introduction of simple rectangular representation of the mean characteristics of the microstructure.

Water filtration, percolation, ground-water flow phenomena and numerous other industrial processes such as flow through synthetic membranes frequently concern Newton fluid flow through granular porous media of very low porosity, i.e. also very low hydrodynamic permeability. The momentum transport equation, developed by Du Plessis and Masliyah (1991), is applicable for all porosity values from zero through unity. In case of low porosity granular porous media some of the expressions in their equations can be simplified extensively. This simplification and the resulting equations will be the prime objective of this paper. Such equations may be very beneficial to numerical simulation of porous flows when the entire computational domain is filled with low porosity media. Although confined to low porosities, the porosity may differ spatially.

The mathematical analysis of any isotropic porous medium requires the qualification of 3 independent parameters. In this paper the porosity, the physical dimension of the granules and the structure (granular, in this case, vs. sponge-like) will therefore be assumed known. The results to be obtained may equally well be expressed in terms of other parameters which prove to be measureable, e.g. permeability, pore length, pore area.

Modelling of the granular microstructure

The analysis of this study is based upon a granular porous medium which is rigid, stationary and locally isotropic with respect to average geometrical properties. Both variation in porosity and characteristic microscopic length are assumed to be continuous variable functions of position.

The concept of a representative unit cell (RUC), introduced by Du Plessis and Masliyah (1988), was used to describe the geometrical properties of the porous medium in order to quantify the fluid-solid dynamic interaction. An RUC provides the facility to consider flow conditions within the most elementary control volume of the particular porous medium and still have all the geometrical properties of the medium at hand for modelling of physical phenomena.

The assumption of mean geometrical isotropy allows the introduction of a cubic RUC of linear dimension d and volume V_o, so that its (fluid filled) "void" part can be written in terms of the porosity as:

$$V_f = \epsilon V_o = \epsilon d^3 \tag{1}$$

It is assumed that the average geometrical properties of the granular solid structure within the RUC can be resembled by a cube of solid material, nested centrally within and aligned with the sides of the cubic RUC. Furthermore, the hypothetical arrangement of solid cubes in neighbouring RUCs is required to provide maximum possible staggering of resulting duct sections within the porous medium. This requirement ensures that isotropy is maintained and that fluid is forced to traverse all transverse void sections of the medium.

If the sides of the solid cube are of length d_s, the volume V_s of solid material within the RUC is given by:

$$V_s = d_s^3 = (1-\epsilon)d^3 \tag{2}$$

Introduction of the tortuosity τ in terms of the total tortuous path length d_e, which is available within the RUC for flow under the constant cross-section A_p, and over a streamwise displacement of length d, then leads to:

$$A_p\tau = \epsilon d^2 \tag{3}$$

This correct inclusion of the tortuosity is the key to a satisfactory quantitative agreement between theory and experiment. It should be noted that the tortuosity as defined in this paper is the inverse of its counterpart in most of the references quoted. The following relationship between the porosity and tortuosity of the granular porous medium can also be deduced (Du Plessis and Masliyah, 1991) from Eqs. (2) and (3):

$$\tau = \frac{\epsilon}{1-(1-\epsilon)^{2/3}} \tag{4}$$

It is evident that the average geometry of the porous microstructure is determined by the parameters ϵ and d only.

Momentum transport equations

The traversing fluid is assumed to consist of a single fluid phase with constant physical properties. The flow itself is considered laminar with no local flow separation within the pores. A no-slip boundary condition applies to all fluid-solid interfaces. There is no restriction on averaged flow separation or recirculation due to the presence of external boundaries or regions of different permeabilities.

The continuity equation for conservation of mass of the fluid within the void sections of the porous medium is given by:

$$\nabla \cdot \boldsymbol{v} = 0 \tag{5}$$

with $\boldsymbol{v}$ the actual velocity field present within the pore volume V_f. The Navier-Stokes equation, governing the transport of momentum locally within V_f and under the conditions mentioned above, may be written as follows:

$$\rho\frac{\partial \boldsymbol{v}}{\partial t} + \rho(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} + \nabla p - \rho \boldsymbol{g} - \mu\nabla^2\boldsymbol{v} = \boldsymbol{0} \tag{6}$$

Equations (5) and (6) govern the fluid movement within the pores and they have to be transformed to equations governing the specific fluid discharge $\boldsymbol{q}$ through the porous medium. The latter quantity gives the fluid discharge rate relative to the bulk area of the porous medium and is defined as follows as the volumetric phase average of the actual fluid velocity:

$$\boldsymbol{q} \equiv \frac{1}{V_o}\int_{V_f} \boldsymbol{v}\, dV \equiv \langle \boldsymbol{v} \rangle \tag{7}$$

The velocity variable $\boldsymbol{q}$ is therefore identical to the filter or Darcy velocity used in ground-water literature. Its relationship with the intrinsic phase averaged velocity,

$$\langle \boldsymbol{v} \rangle_f \equiv \frac{1}{V_f}\int_{V_f} \boldsymbol{v}\, dV$$

commonly used in literature to describe the average magnitude of the pore velocity, is given by:

$$\boldsymbol{q} = \epsilon \langle \boldsymbol{v} \rangle_f \tag{8}$$

Volumetric phase averaging (Bachmat and Bear, 1986) of the continuity equation (Eq. 5) yields the following generalised equation for fluid mass conservation during its traversing of a porous medium:

$$\nabla \cdot \boldsymbol{q} = 0 \tag{9}$$

Similarly, and neglecting dispersion of the fluid momentum at large averaged velocity gradients by the solid particles (Du Plessis and Masliyah, 1988; 1991), the volumetrically averaged form of the Navier-Stokes equation (Eq. 6) can be written as:

$$\rho\frac{\partial \boldsymbol{q}}{\partial t} + \rho\boldsymbol{q}\cdot\nabla(\boldsymbol{q}/\epsilon) + \epsilon\nabla p_f - \epsilon\rho\boldsymbol{g} - \mu\nabla^2\boldsymbol{q} + \frac{1}{V_o}\int_{S_{fs}} (\tilde{p}\boldsymbol{\nu} - \mu\boldsymbol{\nu}\cdot\nabla\boldsymbol{v})\, dS = \boldsymbol{0} \tag{10}$$

The evaluation of the surface integral in Eq. (10) is subject to evaluation of the real local velocity gradients and pressure levels at the pore surfaces. This in turn warrants a fairly accurate description of the porous microstructure and the actual velocity field within the pores. At this point the present work deviates from general literature by introduction of a better estimate on the magnitude velocity field vectors than the instrinsic phase average $\langle v \rangle_f$ mentioned above. In accordance with the definition of tortuosity (Eq. (3)) and mass conservation within V_f the following pore-wise mean velocity v_p may be defined (Du Plessis and Masliyah, 1988):

$$\boldsymbol{v}_p \equiv \tau \langle \boldsymbol{v} \rangle_f = \frac{\tau}{\epsilon}\boldsymbol{q} \tag{11}$$

The flow through the porous matrix was modelled (Du Plessis and Masliyah, 1991) according to the assumption of laminar developing flow in each pore section from a uniform pore entry velocity of v_p. The surface integral in Eq. (10) was shown by the full analysis to be expressible as μFq and this reduces the full momentum equation (Eq. (10)) to:

$$\rho\frac{\partial \mathbf{q}}{\partial t} + \rho\mathbf{q}\cdot\nabla(\mathbf{q}/\epsilon) + \epsilon\nabla p_f - \epsilon\rho\mathbf{g} - \mu\nabla^2\mathbf{q} + \mu F\mathbf{q} = \mathbf{0} \quad (12)$$

The frictional effects introduced by the presence of the porous medium are governed by the term μFq, the factor F being given by the following expression (Du Plessis and Masliyah, 1991) for the case of granular porous media:

$$Fd^2 = \frac{36(1-\epsilon)^{2/3}}{\left[1-(1-\epsilon)^{1/3}\right]\left[1-(1-\epsilon)^{2/3}\right]}\left\{1+\frac{0.0822\, Re_{qd}\left[(1-\epsilon)^{-1/3}-1\right]}{\left[1+(1-\epsilon)^{1/3}\right]}\right\}^{1/2} \quad (13)$$

In case of small Reynolds number flow, the square-rooted factor on the RHS of Eq. (13) approaches unity, rendering the term μFq linear in velocity. The hydrodynamic permeability, inclusive of the non-linear microscopic inertial effects, is given by ϵ/F and this leads to a velocity-independent Darcy-permeability for very low Reynolds number flow ($Re_{qd} \ll (Re_{qd})_c$) of:

$$\frac{K}{d^2} = \frac{\epsilon\left[1-(1-\epsilon)^{1/3}\right]\left[1-(1-\epsilon)^{2/3}\right]}{36(1-\epsilon)^{2/3}} \quad (14)$$

In case of higher Reynolds numbers the square-rooted factor becomes significantly larger than unity, causing a non-linear deviation in the dependence of pressure gradient on velocity and which is known as the Forchheimer effect. The critical Reynolds number $(Re_{qd})_c$ gives the location of the centre of the transition region between the Darcy region of velocity-independent F and the Forchheimer region, where non-linearity is introduced by the inertial effects. This critical value corresponds to the case where the two terms of the square-rooted factor in Eq. (13) are equal and it can therefore be expressed explicitly in terms of porosity as follows:

$$(Re_{qd})_c = \frac{1+(1-\epsilon)^{1/3}}{0.0822\left[(1-\epsilon)^{-1/3}-1\right]} \quad (15)$$

The internal pore flow Reynolds number Re, based upon a flat plate configuration of the pore microstructure, is given by:

$$Re = \frac{2}{1+(1-\epsilon)^{1/3}}\, Re_{qd} \quad (16)$$

This Reynolds number is important as it provides a clear indication of the flow conditions present within V_f.

Depending on the application it may sometimes be more convenient to express Eqs. (13) through (16) in terms of the locally averaged linear dimension d_s of the solid granules. To this end the following expressions can be derived readily (Du Plessis and Masliyah, 1991) from those presented above:

$$Fd_s^2 = \frac{36(1-\epsilon)^{4/3}}{\left[1-(1-\epsilon)^{1/3}\right]\left[1-(1-\epsilon)^{2/3}\right]}\left\{1+\frac{0.0822\, Re_{qs}\left[(1-\epsilon)^{-1/3}-1\right]}{(1-\epsilon)^{1/3}\left[1+(1-\epsilon)^{1/3}\right]}\right\}^{1/2} \quad (17)$$

$$(Re_{qs})_c = \frac{(1-\epsilon)^{1/3}\left[1+(1-\epsilon)^{1/3}\right]}{0.0822\left[(1-\epsilon)^{-1/3}-1\right]} \quad (18)$$

$$\frac{K}{d_s^2} = \frac{\epsilon\left[1-(1-\epsilon)^{1/3}\right]\left[1-(1-\epsilon)^{2/3}\right]}{36(1-\epsilon)^{4/3}} \quad (19)$$

Equation (19) is graphically represented in Figs. 1 and 2, illustrating the dependence of the hydrodynamic permeability on porosity.

The intra-pore Reynolds number, i.e. the Reynolds number for the flow conditions locally within a pore, can be written as follows:

$$Re = \frac{2}{1+(1-\epsilon)^{1/3}}\cdot\frac{1}{(1-\epsilon)^{1/3}}\, Re_{qs} \quad (20)$$

Equations (13) through (20) are cumbersome to evaluate and the last 4 equations, written in terms of d_s, will be simplified in the following sections for cases when the porosity remains low enough to allow binomial series truncations to within tolerable accuracy.

Low porosity porous media

In case of low porosity porous media a truncated binomial series expansion may be applied (Roos, 1992) to all the factors involving fractional powers of (1-ϵ). If first order accuracy of these factors is sufficient, the following approximation may be used:

$$(1-\epsilon)^x \approx 1 - x\epsilon \quad (21)$$

Using Eq. (21), Eqs. (17) through (20) simplify to:

$$Fd_s^2 = \frac{27}{\epsilon^2}(6-11\epsilon)\left\{1+0.0822\, Re_{qs}\cdot\frac{\epsilon}{36}(6+7\epsilon)\right\}^{1/2} \quad (22)$$

$$(Re_{qs})_c = \frac{(6-7\epsilon)}{0.0822\ \epsilon} \quad (23)$$

$$\frac{K}{d_s^2} = \frac{\epsilon^3}{972}(6+11\epsilon) \quad (24)$$

$$Re = \left[1+\frac{\epsilon}{2}\right] Re_{qs} \quad (25)$$

The result of Eq. (24) is graphically compared with Eq. (19) in Fig. 1. It is clear that very good agreement with the full expression is obtained up to a porosity of 0,2.

The error introduced by truncation of the binomial series

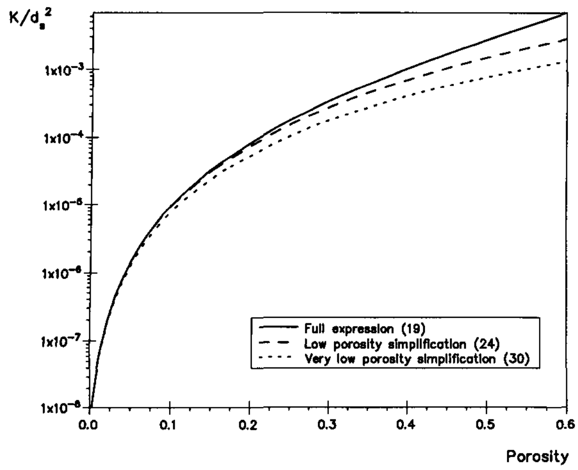

Figure 1
Low porosity and very low porosity approximations of the hydrodynamic permeability

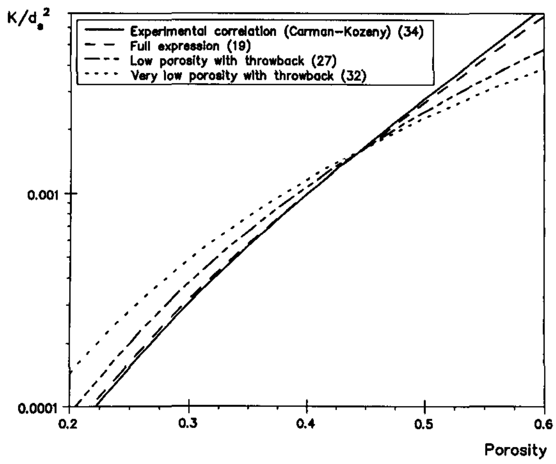

Figure 2
Hydrodynamic permeability with throwback at porosity 0,45

expansion as shown above, can be minimised for a specific small range of porosity by applying a throwback method. In this method the coefficient of the highest retained power of ϵ is changed to incorporate the neglected higher-order terms of the series. This is accomplished through the difference between the original equation and the approximation, at a specific porosity. Fixed permeable beds constructed of granular material normally yield porosity values of about 0,45. Applying throwback to Eq. (24) at ϵ = 0,45 yields:

$$\frac{K}{d_s^2} = \frac{\epsilon^3}{972}\left(6+25.72\epsilon\right) \tag{26}$$

$$= \frac{\epsilon^3}{180}\left(1.11+4.76\epsilon\right) \tag{27}$$

In Fig. 2 the result of throwback at ϵ = 0,45 is shown graphically and compares favourably with the full expression for a range of porosity values around the throwback porosity.

For low porosity values, the tortuosity τ approaches 3/2, as can be seen by application of the truncation (21) to (4). The tortuosity (τ) decreases monotonically with increase in porosity and at ϵ = 0,45 its value is 1,37.

Very low porosity porous media

In case of very low porous media the expressions given above for low porosity media could be truncated even further to yield very simple expressions for the various quantities of Eqs. (13) through (20). Eqs. (13) through (16) and also (17) through (20) are thus both approximated by the following:

$$Fd^2 = Fd_s^2 = \frac{162}{\epsilon^2}\{1 + 0.0137\ \epsilon\ Re_{qs}\}^{1/2} \tag{28}$$

$$\left(Re_{qd}\right)_c = \left(Re_{qs}\right)_c = \frac{73}{\epsilon} \tag{29}$$

$$\frac{K}{d^2} = \frac{K}{d_s^2} = \frac{\epsilon^3}{162} \tag{30}$$

$$Re = Re_{qs} \tag{31}$$

Figure 1 presents a graphical comparison between Eq. (30) and Eqs. (19) and (24). This simple expression is clearly quite usable up to a porosity of 0,1.

Applying throwback at ϵ = 0,45 to Eq. (30) yields:

$$\frac{K}{d_s^2} = \frac{\epsilon^3}{55.29} \tag{32}$$

In Fig. 2 Eq. (32) is compared to the results of Eqs. (19) and (27).

The full three-dimensional momentum equation for seepage through an isotropic porous medium is given by substitution of F in Eq. (12) by its value from Eq. (28) when $Re_{qs} \rightarrow 0$, yielding:

$$\rho\frac{\partial \mathbf{q}}{\partial t} + \rho\mathbf{q}\cdot\nabla(\mathbf{q}/\epsilon) + \epsilon\nabla p_f - \epsilon\rho\mathbf{g} - \mu\nabla^2\mathbf{q} + \frac{162\mu\mathbf{q}}{\epsilon^2 d_s^2} = \mathbf{0} \tag{33}$$

Discussion

To compare the results obtained above to quasi-experimental results, the following form of the well-known Blake-Carman-Kozeny equation (Bird et al., 1960) with the Macdonald correction (Dullien, 1979) is used:

$$\frac{K}{d_p^2} = \frac{\epsilon^3}{180}\left(1-\epsilon\right)^{-2} \tag{34}$$

It should be noted that this equation was derived on the assumption of a non-tortuous void structure necessitating the results to be multiplied by a factor of 2,5 to correspond with experimental data (Bird et al., 1960). $\frac{\tau}{d_p^2}$-values for different porosities according to Eq. (33) are compared in Fig. 2 to the results obtained earlier in this paper. It is clear that all the present results yield the experimentally observed hydrodynamic permeability at a porosity of 0,45.

If the truncation-cum-throwback treatment described above (ϵ = 0,45) is applied to Eq. (33), the following first-order result is obtained for low Reynolds number flow through packed beds:

$$\frac{K}{d_p^2} = \frac{\epsilon^3}{180}(1+5.12\epsilon) \quad (35)$$

where $d_p = d_s$ is the mean diameter of the solid particles. The close qualitative and quantitative correspondence between the presently derived expression (27) and the quasi-experimental result (34) and (35) for packed beds confirms the practical applicability of the present analysis.

Substitution of F in Eq. (10) through Eqs. (22) and (28) respectively for the low and very low approximations leads to full momentum transport equations for flow through granular porous media.

Whenever the macro-scale inertial effects may be disregarded, i.e. when the average flow is predominantly unidirectional (x-direction), the momentum transport for flow in case of the very low porosity approximation is given by:

$$\frac{dp_f}{dx} = \rho g + \frac{\mu}{\epsilon}\nabla_\perp^2 q - \mu q \frac{162}{\epsilon^3 d_s^2}\sqrt{1+0.0137\epsilon Re_{qs}} \quad (36)$$

where $\nabla_\perp^2$ refers to the Laplacian operator in a plane normal to the x-direction. If, furthermore, no solid boundaries are present, the so-called Brinkman term may also be dropped, leaving the following Darcy equation for low and intermediate Reynolds number flows:

$$-\frac{dp_f}{dx} + \rho g = \frac{162\mu q}{\epsilon^3 d_s^2}\sqrt{1+0.0137\epsilon Re_{qs}} \quad (37)$$

suggesting an effective hydrodynamic permeability of:

$$K_{eff} = \frac{\epsilon^3 d_s^2}{162\sqrt{1+0.0137 Re_{qs}}} \quad (38)$$

In cases of low Reynolds number flow this equation simplifies to the expression given in (30) and therefore presents an analytical proof of the "Darcy Law". For high Reynolds number flow, the expressions for F may be simplified as illustrated in the following example for Eq. (37):

$$-\frac{dp_f}{dx} + \rho g = \frac{162\mu q}{\epsilon^3 d_s^2}\sqrt{0.0137\epsilon Re_{qs}}$$

$$= \frac{162\mu q}{\epsilon^3 d_s^2}\sqrt{0.0137\epsilon\frac{\rho q d_s}{\mu}}$$

$$= \frac{162\mu q^{1.5}}{\epsilon^3 d_s^2}\sqrt{0.0137\epsilon\frac{\rho d_s}{\mu}} \quad (39)$$

Experimental results seem to be pointing towards a squared dependence on q (e.g. Bird et al., 1960) as opposed to a power of 1,5 in present results as is clear from Eq. (39). It is evident that the introduction of flow development in pore sections does not produce satisfactory results in the higher Reynolds region where the so-called Forchheimer effect signifies onset of a non-linearity in the velocity-pressure gradient relationship. This matter is currently being investigated in order to extend the applicability of the present results to high Reynolds number flow.

As a last comment it should be stressed that the philosophy of the present approach is to provide a unified set of momentum transport equations, which comply with the severe needs of satisfactory computational work for flow through porous media. Such equations should be robust and simple, yet they should be of sufficient accuracy over the entire spectrum of variable values encountered during the numerical computational process, so as to aid convergence and physical plausibility. Invariably the comparison of computational results to experimental results often necessitates the introduction of coefficients or value changes of existing coefficients to enhance correlation efficiency. Such changes may have adverse effects on the behaviour of the differential equations under computation. Although the present equations were derived from the assumption of a rectangular geometric model, it is constructed in such a way that, if needed, any multiplicative shape or "fudge" factors introduced should be near to unity or at least of the order of unity. This will keep the transport equations as close as possible to their original nature over a wide range of water-related applications. Current experiences indicate that the model performs so well that such a shape factor need not be introduced.

Conclusions

Substantial simplification of the expressions regarding saturated flow through granular porous media was obtained by imposing a restriction to low and to very low porosity values respectively. These simplifications will improve the efficiency of large-scale numerical analyses of porous flows, because of the simpler expressions to be computed during iterative numerical procedures, when the porosity values comply with these restrictions temporally and spatially over the computational domain.

The extremely favourable comparison with the well-known Blake-Carman-Kozeny equation provides confidence in the quantitative as well as qualitative plausibility of the present type of analysis. It is clear that the correct introduction of tortuosity (Du Plessis and Masliyah, 1988) as portrayed here in Eq. (3) and the corresponding pore velocity v_p according to Eq. (11) is the key towards rendering quantitative agreement between theory and experiment without the need for an arbitrary correlative adjustment.

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