# Secondary settling tank modelling and design Part 2: Linking sludge settleability measures

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## Abstract

In order to bring theoretical and practical developments in secondary settling tank (SST) design closer together, a number of tasks have been identified by the authors in Part 1 of this series. The first of these tasks is to establish relationships that link conventional sludge settleability measures to the flux theory constants, $V_o$ and n, and is presented in this paper. Previous work in establishing relationships between various sludge settleability parameters (SSPs) is reviewed. General forms for the $V_o$-SSP and n-SSP relationships are proposed and two different approaches to fitting the data to the selected functions are evaluated: firstly, a one-step direct correlation of zone settling velocity (ZSV, $V_s$) on SSP and sludge concentration ($X_t$) and secondly, a two-step correlation in which first $V_s$ is correlated with $X_t$ to get $V_o$ and n and then $V_o$ and n are correlated to the SSP. Statistical tests were conducted to check which of the different data sets obtained from the literature could be pooled and treated as one set. One- and two-step linear least squares regression analyses were conducted on the largest family of data for each of the three SSPs to determine the constants in the selected relationships. It was concluded that the relationships based on the stirred specific volume index at 3.5 g TSS/ℓ ($SSVI_{3.5}$) are the most reliable. The relationships must be applied with caution because the pooling statistical analysis indicates that the results appear to be influenced by activated sludge plant type.

## Introduction

In order to accomplish the objective of bringing the theoretical and practical developments in secondary settling tank (SST) modelling and design closer together (see Ozinsky and Ekama (1995), Part 1 of this series), two tasks were identified:

1. To collect, evaluate and analyse as much information on sludge settleability parameters and relationships between them as available in the literature and examine, refine or establish relationships between the sludge volume index (SVI), $SSVI_{3.5}$, diluted SVI (DSVI) and the flux constants $V_o$ and n.
2. To verify the flux theory as a model for the simulation of dynamic behaviour of the settling tank, with varying influent flow rates, reactor concentrations and feed solids concentrations.

Task (1) was identified as necessary in order to use the Dutch full-scale secondary settling tank data set collected by STORA (1981a; b; c; 1983) in order to accomplish Task (2). In the STORA investigation, in which 47 solids loading tests were conducted on 27 full-scale settling tanks in Holland, the sludge settleability was measured in terms of DSVI, $SSVI_{3.5}$ and the $V_o$ and n values. Unfortunately, the zone settling velocity (ZSV) - concentration (X) column tests from which the $V_o$ and n values were obtained were conducted over too narrow a concentration range (1 to 6 g·$\ell^{-1}$). The $V_o$ and n values obtained were therefore not reliable and led to the erroneous conclusion that the flux theory was fundamentally deficient as a design procedure. As a result, STORA (1981b) (see also Stofkoper and Trentelman, 1982) pursued the DSVI-based ATV design approach and developed the STORA design procedure based on it (Epskamp and Van Hernen, 1984). To use the STORA data set, which is the only comprehensive data set available with the necessary detail for the verification of the flux theory to accomplish Task (2), it was necessary to determine the required $V_o$ and n values by some other indirect method. The only viable indirect method that appeared to be available was to establish relationships between the $SSVI_{3.5}$ and DSVI and the $V_o$ and n settleability parameters. These relationships would then serve as a basis to determine the $V_o$ and n values from the DSVI and/or the $SSVI_{3.5}$ measured by STORA. The establishment of relationships between the sludge settleability parameters (SSP) and the flux theory constants $V_o$ and n is presented in this paper. In Ozinsky et al. (in prep.) Part 3, accomplishment of Task (2) above is presented. In Ozinsky and Ekama (In prep.) Part 4, the calibrated dynamic flux theory model is checked to see if it is capable of reproducing important design information included in the ATV and STORA design procedures (ATV, 1973; 1976; 1991; Epskamp and Van Hernen, 1984)

## Review of previous work

The general acceptance of a single simple sludge settleability parameter that defines the settleability of a sludge is a source of much controversy in the field. It is recognised that the flux theory ZSV-X, or its associated $V_o$ and n, is the best sludge settleability description in so far as settling tank design and operation is concerned, but the tediousness of its measurement and the lack of proven reliability of the associated flux theory has led to other simpler sludge settleability parameters (SSPs) being developed in practice. As many as three different SSPs are in current use (SVI, $SSVI_{3.5}$ and DSVI) and there is disagreement as to their relative merits. Nevertheless, despite the obstacles, the desire to use the flux theory for settling tank design and operation has been great in English-speaking countries because many relationships have been proposed by means of which the flux $V_o$ and n values can be calculated from one of the simpler SSPs (*inter alia* Pitman, 1984; Daigger and Roper, 1985; Ekama and Marais, 1986; Koopman and Cadee, 1983 and Wahlberg and Keinath, 1988). However, there appear to be no generally accepted functional relationships linking one SSP to the flux ZSV-X (or associated $V_o$ and n) or even to another SSP. This means that, at present, if data are available in

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## TABLE 1
## SUMMARY OF CONTRIBUTIONS MADE BY PREVIOUS RESEARCHERS IN ESTABLISHING RELATIONSHIPS BETWEEN SSPs AND THE FLUX ZSV-X OR $V_0$ AND N AND BETWEEN ONE ANOTHER

| Researcher(s) | Data source[1] and type | Measured parameter | Equations | Comments |
|---|---|---|---|---|
| Daigger and Roper (1985) | 6 pilot-scale and 2 full-scale activated sludge plants | SVI | $V_0=7.30$ ($r^2 = \pm 0.89$)<br>$n=0.148+0.00210*SVI$ ($r^2=0.997$) | * Limited database<br>* Data exhibited considerable scatter. |
| Hartel and Popel (1992) | Literature data<br>* Wahlberg and Keinath (1988)<br>* Koopman and Cadee (1983)<br>* Morris et al. (1989)<br>* Forster (1982)<br>* Pitman (1980)<br>* Ericsson et al.. (1988)<br>* Knocke (1986)<br>* Chad and Keinath (1979)<br>All data pooled together. | SVI | $V_0=17.4\ \exp(-0.0113*SVI) + 3.931$<br>$n=0.9834\ \exp(-0.00581*SVI) + 1.043$ | * Used incorrect parameter from Wahlberg and Keinath (1988) data<br>* Large degree of scatter in the data<br>* Assumed all data could be pooled |
| Wahlberg and Keinath (1988) | A. 141 sets of literature data<br>* Hartley (1985)<br>* Pitman (1980)<br>* Guthrie (1985)<br>* Grady et al. (1982)<br>* Morris et al. (1986)<br>* Daigger (1985)<br>* Roper (1976)<br>* Dick et al. (1967) | SVI | No clear relationship between $V_0$ and SVI<br>Relationship between n and SVI not linear | * Wide degree of scatter in the data |
| | B. Sampled 21 activated sludge, completely mixed step aeration and contact stabilisation treatment plants. Wide variety of influents and aeration systems. 32 sets of data obtained. All 185 data pooled. | SVI<br>SSVI | $V_0=15.3-0.0615*SSVI$<br>$n=0.426+0.00384*SSVI -0.0000543*SSVI^2$ | * Applicable in the SSVI range 35 to 220 mℓ/g<br>* Found SVI to be a very imprecise measurement<br>* Recommended SSVI as a measurement parameter<br>* Assumed all data could be pooled |
| Pitman (1984) | Measurements from 4 full-scale nutrient removal plants (AX, OF, GK and NW)[2] over 6 years (see also Pitman, 1980) | SVI<br>SSVI | $V_0/n=37.63\ \exp(-0.00388*SVI)$<br>$V_0/n=67.9\ \exp(-0.016*SSVI_{3.5})$ ($r^2=0.968$)<br>$n=0.88-0.393\ \log(V_0/n)$ ($r^2=0.976$) | * Small sludge diversity and limited database.<br>* Relationship between SVI and $V_0$ and n should not be used because of its dependence on conc. at high 30 min settled vols (>400mℓ) |
| Ekama and Marais (1986) | A. * Pitman (1980; 1984)<br>* White (1975)<br>* Rachwal et al. (1982)<br>* Koopman and Cadee (1983) | SSVI | $V_0/n=67.9\exp(-0.0016SSVI_{3.5})$ ($r^2=0.968$)<br>$n=0.88-0.393\log(V_0/n)$ ($r^2=0.976$) | * Confirmed Pitman (1984) relationship with White (1975) and Rachwal et al. (1982) data<br>* Koopman and Cadee (1983) data did not conform |
| | B. Own data<br>* 15 Western Cape plants | SVI<br>SSVI<br>DSVI | $SSVI=0.67*DSVI$ | * Wide scatter in data |
| | * Stofkoper and Trentelman (1982) (STORA, 1981b) | DSVI<br>SSVI | $SSVI = 0.65*DSVI$ | * Wide scatter in data |
| Hartley (1985) | Literature data<br>* Ekama et al. (1984)<br>* Sezgin et al. (1982)<br>* Rachwal et al. (1982)<br>* Werda (1983) | SVI<br>SSVI | | * Small data set<br>* Method lacks statistical rigour |

| **Researcher(s)** | **Data source¹ and type** | **Measured parameter** | **Equations** | **Comments** |
|---|---|---|---|---|
| Koopman and Cadee (1983) | Literature data<br>* Sezgin (1980)<br>* Palm et al. (1981)<br>* Jenkins et al. (1981)<br>* Lee et al. (1981) | DSVI | $n=0.249+0.002191*DSVI$ ($r^2=0.99$)<br>$Ln\ V_o=2.605-0.00365*DSVI$ ($r^2=0.735$) | * Data collected over a very narrow range of concentrations (0.7 to 4.8g·$\ell^{-1}$) |
| Merkel (1971) | Large no. of measurements | DSVI<br>SVI | $DSVI=SVI*(300/SV_{30})^{0.6}$<br>where<br>$SV_{30}$=settled volume at 30 min in the SVI test | * Applies to $300<SV_{30}<850$ m$\ell$ |
| Rachwal et al. (1982) | Averages of a number of years of measurements at full- scale Carrousel type activated sludge plants | SSVI | Gave $V_o$ and n values for different ranges of SSVI | * Individual $v_s$, x and SSVI data not given |

[1] The references in this column are not all given in the reference list; they indicate the data source of the researcher(s) (column 1) and are fully referenced in the researcher(s) papers.

[2] AX = Alexandra;GK = Goudkoppies; NW = Northern Works; OF = Olifantsvlei Works.

terms of only one of the SSPs, the flux ZSV-X (or $V_o$ and n) or other SSPs cannot be confidently derived. Usually it is recommended that further settleability measurements be carried out to determine the particular settleability parameter of interest. A summary of the existing relationships between the various SSPs and flux ZSV-X or $V_o$ and n and one another which have been proposed are presented in Table 1. Before commenting on Table 1, it needs to be pointed out that the flux theory constants ($V_o$ and n) defining the ZSV-X function that appear to have been widely adopted by all researchers listed in Table 1 are based on Vesilind's (1968) semilog model:

$$V_s = V_o e^{(-nX)} \ (m{\cdot}h^{-1}) \qquad (1)$$

where $V_s$ = zone settling velocity (ZSV) ($m{\cdot}h^{-1}$)
$X$ = total settleable solids (TSS) concentration ($kgTSS{\cdot}m^{-3}$)
$V_o$, n = flux theory constants ($m{\cdot}h^{-1}$ and $m^3{\cdot}kgTSS^{-1}$)

The semilog form of the ZSV-X relationship will also be accepted in this analysis because it gives a better correlation between ZSV-X data, leads to a more consistent flux model compared to other forms (Smollen and Ekama, 1984) and is widely accepted so that the form of the function is no longer regarded as a point of debate.

Because there is little agreement on a general form for the $V_o$ - SSP and n - SSP functions, some rationale needed to be developed in order to determine their form. In the end, the functions selected for $V_o$ - SSP and n - SSP relationships were:

$$V_o = \alpha e^{(-\beta * SSP)} \qquad (2)$$

$$n = \gamma + \delta * SSP \qquad (3)$$

on the following basis:

- The trend of decreasing $V_o$ with increasing SSP and increasing n with increasing SSP was observable for most of the data sets listed in Table 1. It also appeared that, for $V_o$, the trend is curvilinear and, for n, the trend is linear. However, no clear indication of the specific form for the functions relating $V_o$ and n to the SSP was apparent from an examination of the collected data.
- The functional forms accepted are sufficiently flexible for the number of constants introduced (ln α, β, γ and δ). In general, it was found that the greater the decrease in $V_o$ with increase in SSP, the more curvilinear the data trend, which is in conformity with the semilog form. The increase in curvilinearity was not evident in the n vs. SSP data, so that the linear form was deemed appropriate.
- The $SSVI_{3.5}$ was identified as the most important settleability parameter: firstly, because it is more reliable than the SVI; and secondly, because the available data set for $SSVI_{3.5}$ is much larger than that for DSVI. It was therefore considered most important that the correct form of the $V_o$-$SSVI_{3.5}$ and n-$SSVI_{3.5}$ functions be identified. From the graphical data presented and from a statistical analysis performed on these data, it was concluded that the semilog and linear forms for the two functions respectively, in conformity with Eqs. (2) and (3), could be accepted confidently for the $SSVI_{3.5}$. Alternative forms of these functions did not lead to a significant improvement in the correlation coefficient, an observation also made by Rachwal et al. (1982) on their own data set.
- As there was no specific evidence to the contrary, these functional forms were also accepted for the SVI and DSVI.

The semilog form for the $V_o$-SSP gives the curvilinearity required without introducing additional constants as would be the case with, for example, a parabolic function. Also, for some cases where $V_o$ is approximately constant with increasing SSP (e.g the $V_o$ vs SVI data of Daigger and Roper, 1985), this could be accommodated by small to zero values of β in Eq (2). With regard to the linear form for the n-SSP curve, most of the data sets gave good correlations with the linear function and, for those cases where the data appeared curvilinear in trend, the correlation coefficient was not significantly improved with a curvilinear function (exponential, log, hyperbolic, parabolic).

## Single- vs. double-step correlation

In establishing the relationships between the three SSPs and the flux constants $V_o$ and n, two approaches can be followed:

> **A double-step approach** where first the $V_o$ and n values for a particular group of ZSV-X data (with an associated SSP) is found by a linear regression on $\ln V_s = \ln V_o - nX$ (Eq.1) and then, in a second step, these $V_o$ and n values are correlated with the SSP values in terms of some relationship describing the form of the $V_o$-SSP and n-SSP data (e.g. Eq. (2) and Eq. (3)). Wahlberg and Keinath (1988) criticised this approach on the basis of the fact that it lacks statistical rigour for two reasons. Firstly, statistical information is lost when the raw settling ZSV-X data are condensed into a single ($V_o$, n) data pair. Secondly, they argued that the log transformation ignores the error associated with the linear least squares estimation of $V_s$. To overcome these shortcomings, they proposed a single-step approach.
>
> **A single-step approach**, in which the correlation of ZSV (or $V_s$) on concentration (X) and SSP is done in a single step, taking all the $V_s$, X and SSP data of the set together without first determining $V_o$ and n of the groups making up the set. To do this requires a knowledge of the form of the functional relationship between the $V_o$ and n values and the SSP. Accepting these as the semilog for the $V_o$-SSP (Eq. (2)) and linear for the n-SSP (Eq. (3)) functions and substituting into Eq. (1) yields:
>
> $$V_s \quad = \quad \alpha\, e^{-\beta * SSP}\, e^{-\gamma * X}\, e^{-\delta * SSP * X} \qquad (4)$$
>
> $$\ln V_s \quad = \quad \ln\alpha - \beta * SSP - \gamma * X - \delta * SSP * X \qquad (5)$$
>
> The constants $\alpha$, $\beta$, $\gamma$ and $\delta$ in Eqs. (4) and (5) can be determined by multiple least squares correlation on the function $\ln V_s$ in terms of X and SSP in a single step instead of two.

All the researchers in Table 1 except Wahlberg and Keinath (1988) and Hartel and Popel (1992) adopted the two-step approach to establish the relationship between $V_s$ and the X and SSP.

In carrying out either of the two approaches described above, an additional constraint needs to be taken into account. Ideally, it would be most convenient if all the data sets for a particular sludge settleability measure could be pooled, in which case the measured parameters account for all the positive variability in the $V_s$. However, it is apparent that considerable differences exist between data sets for the same SSP. Factors such as plant type, temperature, settled or unsettled influent, etc. appear to affect the $V_s$ in a way not accounted for by measurements of X and the SSP. Statistically, it is not valid to randomly pool the different data sets in order to determine the constants ln $\alpha$, $\beta$, $\gamma$ and $\delta$ unless they all originate from the same population. Therefore, statistical tests needed to be undertaken to examine whether or not different data sets could be legitimately pooled.

During the course of the statistical analysis, a mixture of the single- and double-step approaches needed to be adopted for the following reasons:

> In the process of determining whether or not it was statistically permissible to pool particular sets of data and subsequently treat them as a single group, the double-step approach needed to be adopted. This is because the statistical tests that need to be carried out to determine the permissibility of pooling the data do not apply to multilinear regression and therefore cannot be applied to the regression lines obtained with the single-step method. Instead, they have to be carried out in two stages using the two regression lines (one for $V_o$ and one for n) obtained in the double-step method.
>
> After establishing the validity of pooling various data sets and combining them into their permissible groups, the pooled data sets were subjected to a linear regression analysis (Eq. (5)) with the single-step method. The reason for this is that the multiple correlation coefficient ($r^2$) obtained using the double-step method was in no case greater than the coefficient obtained by the single-step method. This is to be expected, because the least squares fitting technique applied in one step to the data will always yield constants which give the highest possible value of $r^2$ i.e. it is not possible to choose another set of constants which give a higher value of $r^2$.

The $r^2$ values obtained with the single- and double-step approaches were compared. The constants obtained with the double step approach were substituted into Eq. (5) for the single step and the $r^2$ value calculated. Even though different values of the constants were obtained with the double-step method compared with the single-step method, the $r^2$ values were not significantly different (<1%). This provided the theoretical substantiation for testing pooling of data sets with the double-step method but determining the constants for the pooled set with the single-step method. In addition, having calculated the constants with the single-step method, Eq. (5) can be separated into Eqs. (2) and (3) for $V_o$ and n individually, leaving the constants unchanged and without loss of predictive accuracy. This implies that, even though the $V_s$ values obtained from the single- and double-step methods for a particular X and SSP pair are different, statistically one is no more reliable than the other.

## Statistical tests for pooling the data

The statistical tests that need to be carried out to determine whether or not a number of data sets belong to a single population consist of establishing the answers to the following questions (Ostle, 1963):

- Can one regression line be fitted to all the data?
- Are all the sample slopes estimates of the same true slope?
- Would a regression fitted to the group means be linear?
- Is the true pooled within groups regression coefficient equal to the true regression coefficient for the means?

If the answer to all of the above questions is positive, then it can be concluded that the grouped samples all originate from the same population and it is valid to perform a linear regression on the pooled data as a single sample. If not, then linear regression may only be carried out on individual samples. The order in which these tests are performed is very important since the assumptions necessary for the later tests are tested as hypotheses in the earlier tests. The tests for pooling the data described here do not apply to multilinear regression and therefore can only be applied to the double-step correlations. Pooling the data was carried out by a trial-and-error procedure. Selected groups of samples were pooled and then the hypothesis that they originated from the same population was tested. This was achieved by calculating the F statistics for the regression line ($F_1$), the slope ($F_2$), the regression of the group means ($F_3$) and the within-groups regression coefficient ($F_4$). The critical F statistics were found from a standard table of cumulative F distribution with degrees of freedom $v_1$ and $v_2$, where each critical F statistic depends on the particular values of $v_1$ and $v_2$. If each of the calculated F statistics are less than each of the critical F statistics, then it can be concluded that the data may be pooled. This process was carried out separately for the $V_0$ data and the n data. In order for the data to be pooled, it needed to be shown via the F statistics that both the $V_0$ and n data groups individually complied with the F statistics for corresponding pooled sets. The results of the statistical pooling tests are described in detail under each settleability parameter below.

## The data sets

The sludge settleability data measured by numerous researchers at a number of different waste-water treatment plants were collected either from the publication, if included, or directly from the researchers when these were not given in the published papers. Each data set comprises a group of zone settling velocity ($V_s$) - concentration (X) measurements with an associated single value for the sludge settleability parameters SVI, $SSVI_{3.5}$ and/or DSVI.

A summary of all the collected data is presented in Table 2. For reference purposes, the data sets have been labelled with the name of the researcher, or the site at which the data were collected, whichever is more convenient. Table 2 lists 16 data sets for which ZSV-X data were measured. Four of the sets (these are listed at the bottom of the table) were rejected because either the individual ZSV-X data could not be obtained or the ZSV-X data were measured over too narrow a range. Of the remaining 12 sets, all 12 were useful for establishing the $V_s$ relationship in terms of the SVI; all 5 data sets [viz. Lukuko (Lk), Wallace (Wl), Dickinson (Dk), Cape Flats (CF) and Mitchell's Plain (MP)] making up the University of Cape Town (UCT) (1986) group, all 4 data sets [viz. Alexandra (AX), Olifantsvlei (OF), Northern Works (NW) and Goudkoppies

**TABLE 2**
**SUMMARY OF THE AVAILABLE DATA SETS**

| | Settleability parameters measured | | | |
|---|---|---|---|---|
| **Reference** | **SVI** | **$SSVI_{3.5}$** | **DSVI** | **ZSV-X** |
| UCT (1986) - Western Cape region | | | | |
| * Lukuko (Lk)[1] | Yes | Yes | Yes | Yes |
| * Wallace (Wl)[1] | Yes | Yes | Yes | Yes |
| * Dickinson (Dk)[1] | Yes | Yes | Yes | Yes |
| * Cape Flats (CF)[2] | Yes | No | Yes | Yes |
| * Mitchell's Plain (MP)[2] | Yes | Yes | No | Yes |
| Daigger and Roper (D&R) (1985) | Yes | No | No | Yes |
| Pitman (1984)- Johannesburg Region | | | | |
| * Alexandra (AX)[3] | Yes | Yes | No | Yes |
| * Olifantsvlei (OF)[3] | Yes | Yes | No | Yes |
| * Northern Works (NW)[4] | Yes | No | No | Yes |
| * Goudkoppies (GK)[4] | Yes | Yes | No | Yes |
| Wahlberg and Keinath (W&K) (1988)[5] | Yes | Yes | No | Yes |
| Tuntoolavest and Grady (T&G) (1982)[6] | Yes | No | No | Yes |

1. Measured at 15 different plants in the Western Cape Province: some with, some without primary sedimentation but all with long sludge ages (>20 d) and with N removal, none exhibiting significant P removal
2. CF: a 5-stage Bardenpho plant, not exhibiting P removal
   MP: modified Ludzack - Ettinger plant for N removal
3. AX, OF: both extended aeration (long sludge age, no primary sedimentation), modified for significant biological phosphorus removal
4. NW, GK: both 3/5 stage Bardenpho plants for N and P removal
5. 21 different treatment plants: conventional activated sludge as well as completely mixed, step aeration and contact stabilisation systems. All fully aerobic.
6. Fully aerobic, 2.8 $m^3 \cdot d^{-1}$ pilot plant

| **Additional data** | **SVI** | **$SSVI_{3.5}$** | **DSVI** | **ZSV-X** |
|---|---|---|---|---|
| Koopman and Cadee (K&C) (1983)[7] | Yes | No | No | No |
| Hartley (1985)[8] | No | Yes | No | No |
| Rachwal et al. (1982)[8] | No | Yes | No | Yes |
| STORA (1981b)[7] | No | Yes | Yes | Yes |

7. ZSV-X data over a very narrow X range (1 to 6 $g \cdot \ell^{-1}$); rejected
8. No ZSV-X data given, only $V_0$ and n vs $SSVI_{3.5}$; only useful for confirming $V_0$-$SSVI_{3.5}$ and n-$SSVI_{3.5}$ relationship

(GK)] making up the Pitman (1984) group and the Daigger and Roper (D&R) (1985), Wahlberg and Keinath (W&K) (1988) and Tuntoolavest and Grady (T&G) (1982) each contributing one data set. For the $SSVI_{3.5}$ and the DSVI only eight and four data sets respectively included the measurements for the $SSVI_{3.5}$ and DSVI viz. for the $SSVI_{3.5}$, the eight are the UCT (1986) Lukuko (Lk), Wallace (Wl), Dickinson (Dk) and Mitchell's Plain (MP) data sets, the Pitman (1984), Alexandra (AX), Olifantsvlei (OF) and Goudkoppies (GK) data sets and the Wahlberg and Keinath (W&K) (1988) data set; for the DSVI the four are the UCT (1986), Lukuko

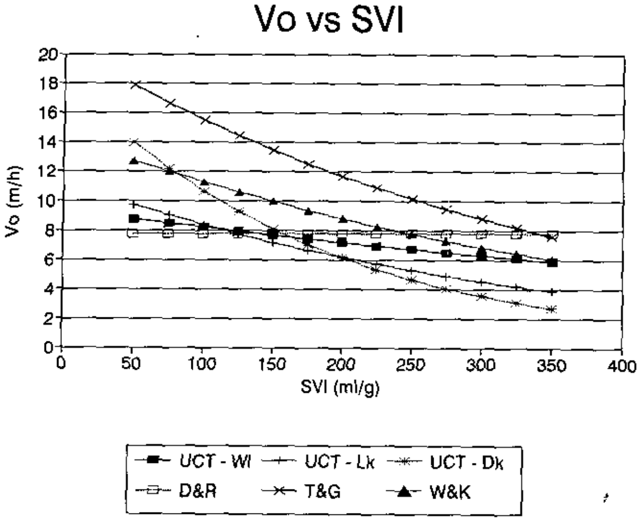

**Figure 1**
*Vo vs. SVI data of six data sets*

UCT (1986) - Wl: $V_o = 9.394e^{(-0.00134*SVI)}$ ($r^2 = 0.241$)
UCT (1986) - Lk: $V_o = 11.345e^{(-0.00304*SVI)}$ ($r^2 = 0.451$)
UCT (1986) - Dk: $V_o = 18.453e^{(-0.00550*SVI)}$ ($r^2 = 0.473$)
D&R (1985): $V_o = 7.80$ ($r^2 = 0.89$)
T&G (1982): $V_o = 20.610e^{(-0.00285*SVI)}$ ($r^2 = 0.08$)
W&K (1988): $V_o = 14.484e^{(-0.00250*SVI)}$ ($r^2 = 0.038$)

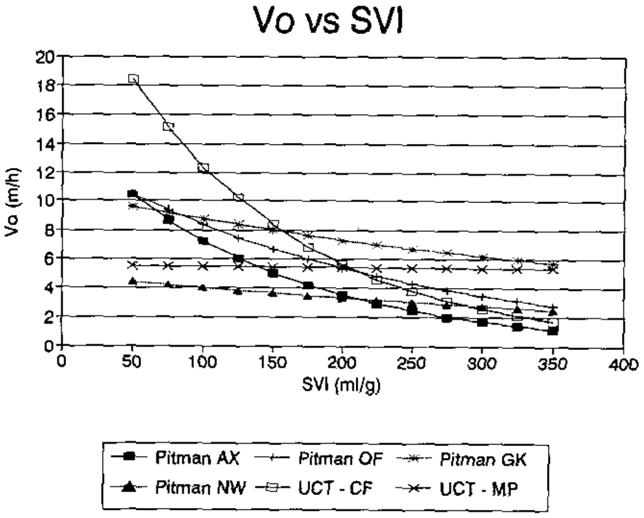

**Figure 2**
*Vo vs. SVI data of six data sets*

Pitman (1984) - AX: $V_o = 14.900e^{(-0.00724*SVI)}$ ($r^2 = 0.699$)
Pitman (1984) - OF: $V_o = 12.992e^{(-0.00440*SVI)}$ ($r^2 = 0.781$)
Pitman (1984) - GK: $V_o = 10.360e^{(-0.00171*SVI)}$ ($r^2 = 0.365$)
Pitman (1984) - NW: $V_o = 4.887e^{(-0.00195*SVI)}$ ($r^2 = 0.095$)
UCT (1986) - CF: $V_o = 27.406e^{(-0.00791*SVI)}$ ($r^2 = 0.605$)
UCT (1986) - MP: $V_o = 5.557e^{(-0.00012*SVI)}$ ($r^2 = 0.0001$)

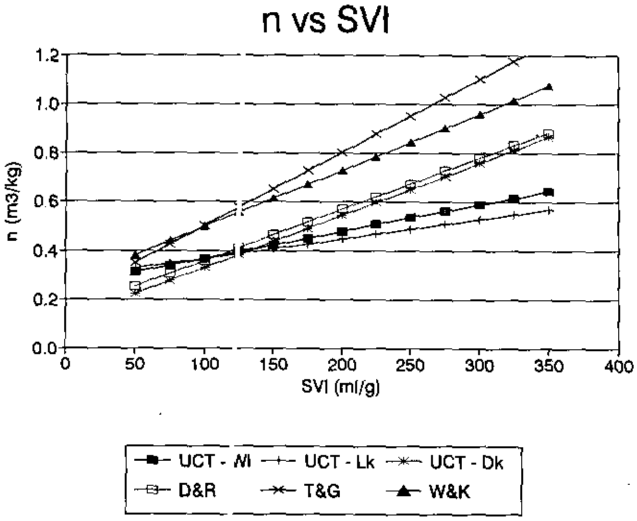

**Figure 3**
*n vs. SVI data of six data sets*

UCT (1986) - Wl: $n = 0.257+0.00110*SVI$ ($r^2 = 0.415$)
UCT (1986) - Lk: $n = 0.288+0.00079*SVI$ ($r^2 = 0.386$)
UCT (1986) - Dk: $n = 0.115+0.00214*SVI$ ($r^2 = 0.714$)
D&R (1985): $n = 0.148+0.00210*SVI$ ($r^2$ not given)
T&G (1982): $n = 0.201+0.00300*SVI$ ($r^2 = 0.260$)
W&K (1988): $n = 0.267+0.00230*SVI$ ($r^2 = 0.257$)

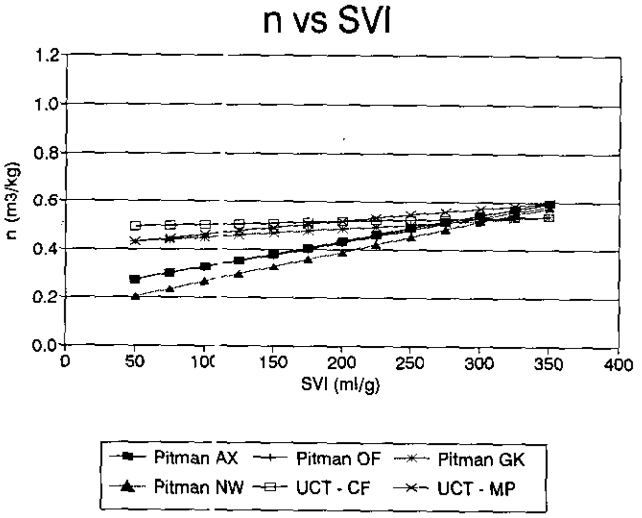

**Figure 4**
*n vs SVI data of six data sets*

Pitman (1984) - AX: $n = 0.218+0.00107*SVI$ ($r^2 = 0.578$)
Pitman (1984) - OF: $n = 0.219+0.00103*SVI$ ($r^2 = 0.807$)
Pitman (1984) - GK: $n = 0.413+0.000340*SVI$ ($r^2 = 0.249$)
Pitman (1984) - NW: $n = 0.139+0.00124*SVI$ ($r^2 = 0.700$)
UCT (1986) - CF: $n = 0.483+0.00014*SVI$ ($r^2 = 0.006$)
UCT (1986) - MP: $n = 0.403+0.00055*SVI$ ($r^2 = 0.049$)

(Lk), Wallace (Wl) and Dickinson (Dk) and Cape Flats (CF) data sets (Table 2). Considering the reliability of the different SSPs and the data available for establishing the relationship between $V_s$ and the X and SSP, it is clear that the $SSVI_{3.5}$ is likely to give the best estimates of $V_s$ because it is far superior to the SVI and has a far larger data set than the DSVI. The ranges of the concentration, ZSV and SSP of the data sets are listed in Table 3. Figures 1 to 10 show the $V_o$ and n vs. SSP relationships of the data in Table 3 in graphical form.

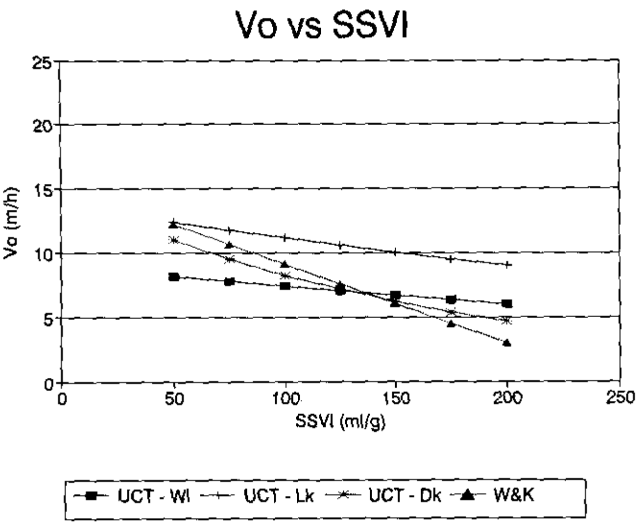

**Figure 5**
*Vo vs. SSVI data of four data sets*

UCT (1986) - Wl: $V_o = 9.107e^{(-0.00208*SSVI3.5)}$ ($r^2 = 0.117$)
UCT (1986) - Lk $V_o = 13.761e^{(-0.00208*SSVI3.5)}$ ($r^2 = 0.524$)
UCT (1986) - Dk $V_o = 14.649e^{(-0.00569*SSVI3.5)}$ ($r^2 = 0.295$)
W&K (1988): $V_o = 15.3-0.0615*SSVI_{3.5}$

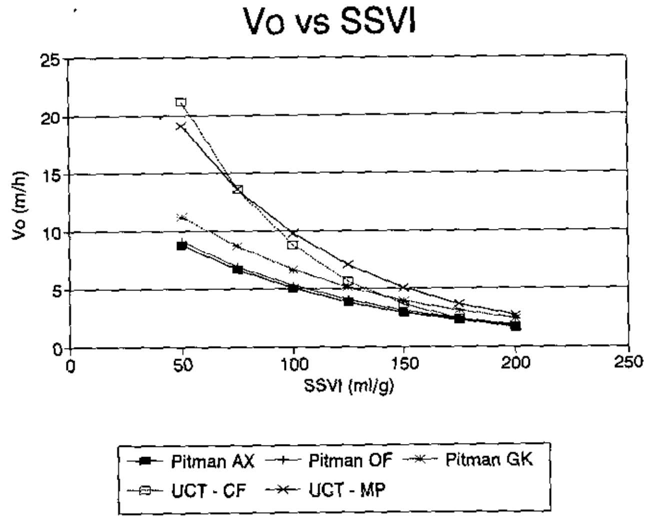

**Figure 6**
*Vo vs. SSVI data of five data sets*

Pitman (1984) - AX: $V_o = 15.405e^{(-0.01114*SSVI3.5)}$ ($r^2 = 0.778$)
Pitman (1984) - OF: $V_o = 15.695e^{(-0.01085*SSVI3.5)}$ ($r^2 = 0.846$)
Pitman (1984) - GK: $V_o = 18.936e^{(-0.01042*SSVI3.5)}$ ($r^2 = 0.713$)
UCT (1986) - CF: $V_o = 51.469e^{(-0.01768*SSVI3.5)}$ ($r^2 = 0.638$)
UCT (1986) - MP: $V_o = 36.856e^{(-0.01322*SSVI3.5)}$ ($r^2 = 0.473$)

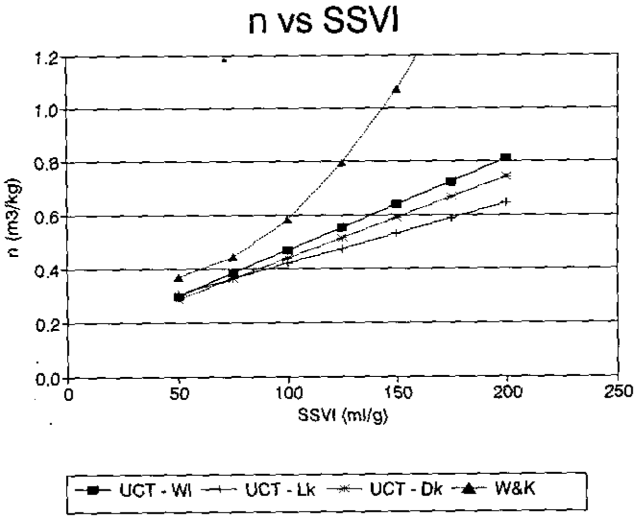

**Figure 7**
*n vs. SSVI data of four data sets*

UCT (1986) - Wl $n = 0.130+0.00340*SSVI_{3.5}$ ($r^2 = 0.798$)
UCT (1986) - Lk $n = 0.195+0.00225*SSVI_{3.5}$ ($r^2 = 0.686$)
UCT (1986) - Dk $n = 0.136+0.00304*SSVI_{3.5}$ ($r^2 = 0.843$)
W&K (1988): $n = 0.426-0.00384*SSVI_{3.5}$
$+0.0000543*SSVI_{3.5}^2$ ($r^2$ not given)

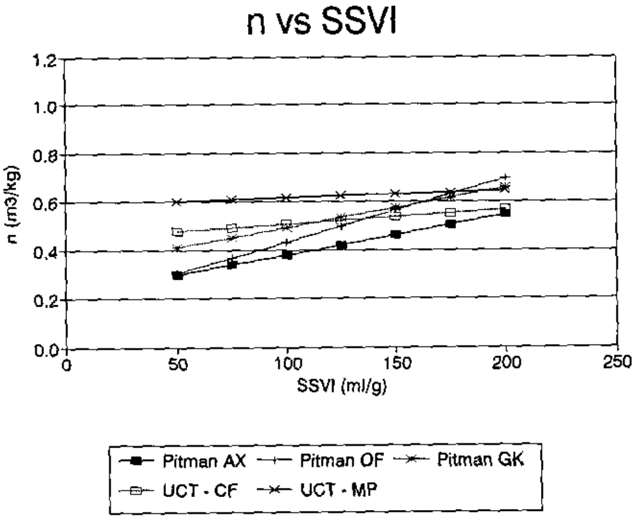

**Figure 8**
*n vs. SSVI data of five data sets*

Pitman (1984) - AX: $n = 0.213+0.00166*SSVI_{3.5}$ ($r^2 = 0.593$)
Pitman (1984) - OF: $n = 0.171+0.00261*SSVI_{3.5}$ ($r^2 = 0.931$)
Pitman (1984) - GK: $n = 0.328+0.00163*SSVI_{3.5}$ ($r^2 = 0.272$)
UCT (1986) - CF: $n = 0.445+0.00060*SSVI_{3.5}$ ($r^2 = 0.024$)
UCT (1986) - MP: $n = 0.585+0.00029*SSVI_{3.5}$ ($r^2 = 0.005$)

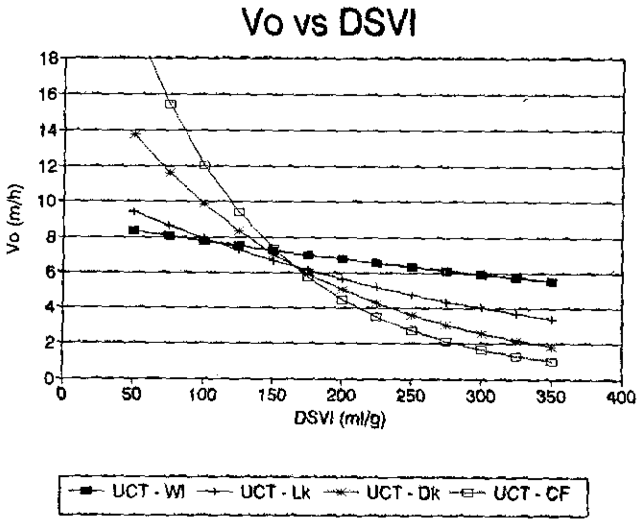

*Figure 9*
*Vo vs. DSVI data of four data sets*

UCT (1986) - Wl: $V_o = 8.918e^{(-0.00136*DSVI)}$ $(r^2 = 0.157)$
UCT (1986) - Lk: $V_o = 11.145e^{(-0.00339*DSVI)}$ $(r^2 = 0.573)$
UCT (1986) - Dk: $V_o = 19.230e^{(-0.00669*DSVI)}$ $(r^2 = 0.461)$
UCT (1986) - CF: $V_o = 32.249e^{(-0.00985*DSVI)}$ $(r^2 = 0.842)$ *

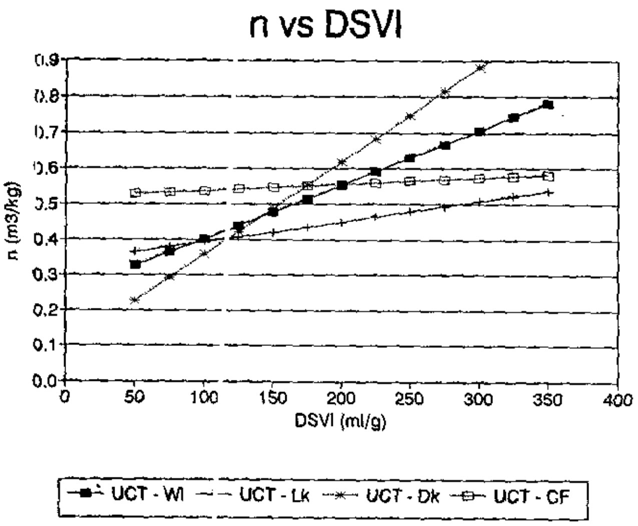

*Figure 10*
*n vs. DSVI data of four data sets*

UCT (1986) - Wl: $n = 0.250+0.00152*DSVI$ $(r^2 = 0.502)$
UCT (1986) - Lk: $n = 0.335+0.00058*DSVI$ $(r^2 = 0.215)$
UCT (1986) - Dk: $n = 0.097+0.00261*DSVI$ $(r^2 = 0.698)$
UCT (1986) - CF: $n = 0.520+0.00018*DSVI$ $(r^2 = 0.009)$

**TABLE 3**
**RANGES OF ALL AVAILABLE DATA SETS**

| **Reference** | **Ranges of data sets** | | | | |
|---|---|---|---|---|---|
| | **X (kgTSS·m$^{-3}$)** | **$V_s$ (m·h$^{-1}$)** | **SVI (m$\ell$·g$^{-1}$)** | **SSVI$_{3.5}$ (m$\ell$·g$^{-1}$)** | **DSVI (m$\ell$·g$^{-1}$)** |
| UCT (1986) data group | | | | | |
| * Lukuko (Lk) | 0.97 - 12.08 | 0.18 - 7.56 | 29 - 310 | 37 - 168 | 59 - 357 |
| * Wallace (Wl) | 0.510 - 12.176 | 0.15 - 6.66 | 44 - 347 | 39 - 167 | 44 - 314 |
| * Dickinson (Dk) | 0.92 - 15.50 | 0.13 - 15.84 | 44 - 267 | 33 - 209 | 47 - 228 |
| * Cape Flats (CF) | 0.41 - 7.91 | 0.15 - 9.30 | 76 - 180 | 64 - 110 | 76 - 176 |
| * Mitchell's Plain (MP) | SVI: 0.5 - 6.85<br>SSVI: 0.5 - 6.94 | SVI: 0.12 - 6.11<br>SSVI: 0.12 - 6.11 | 182 - 273 | 124 - 174 | |
| Daigger and Roper (D&R)(1985) | 1.60 - 19 | 0.03 - 5.00 | 36 - 402 | - | - |
| Pitman (1984) data group | | | | | |
| * Alexandra (AX) | 1.03 - 13.52 | 0.20 - 7.99 | 45 - 120 | 75 - 80 | - |
| * Olifantsvlei (OF) | 0.7 - 12.80 | 0.3 - 10.9 | 45 - 140 | 35 - 75 | - |
| * Northern Works (NW) | 1.08 - 11.68 | 0.14 - 6.09 | 147 - 218 | | - |
| * Goudkoppies (GK) | 0.95 - 8.54 | 0.14 - 6.2 | 80 - 360 | 65 - 125 | - |
| Wahlberg and Keinath (W&K)(1988) | 0.7 - 16.6 | 0.02 - 8.04 | 47.9 - 235 | 40.2 - 230 | - |
| Tuntoolavest and Grady (T&G)(1982) | 1.17 - 8.46 | 0.55 - 8.23 | 76 - 166 | - | - |
| **Rejected sets** | | | | | |
| Koopman and Cadee (K&C)(1983) | - | - | - | - | 65 - 300 |
| Hartley (1985) | - | - | - | 60 - 330 | - |
| Rachwal et al. (1982) | 0.65 - 11.0 | 0.1 - 6.6 | - | 57 - 146 | - |
| STORA (1981b) | 0.3 - 5.9 | 0.1 - 1.82 | - | 40 - 300 | 50 - 440 |

## Pooling the SVI data

On the basis of the statistical tests, the biggest possible sample of SVI data that could be legitimately pooled was one comprising the data collected by Pitman (1984) at the Alexandra (AX), Olifantsvlei (OF), Goudkoppies (GK) and Northern Works (NW) plants as well as that collected by UCT (1986) -Wl. These five data sets are henceforth called the Pitman family because most of the data are from the Pitman group. The pooled sample, or Pitman family, then consisted of 190 groups of SVI and $V_0$ and n pairs from a total number of 713 $V_s$-X data points. The Pitman family was the only group for which both the $V_0$ and n-based F statistics satisfied the critical F statistics for pooling.

Additional statistical tests showed that the remaining data sets [viz. UCT (1986) - Lk, UCT (1986) - Dk, D&R (1985), W&K (1988), T&G (1982), UCT (1986) - CF and UCT (1986) - MP] could not be pooled in any combination (even in pairs) and it was concluded that each had to be treated individually. It was curious to note that the UCT (1986) - Wl set could be pooled together with the four Pitman (1984) sets, while the UCT (1986) - Lk and UCT (1986) - Dk sets could not, even though the latter two sets were measured on the same 15 Western Cape plants at the same time of year as the UCT (1986) - Wl data, albeit one and two years later respectively. That the remaining sets could not be pooled seems reasonable in that the Wahlberg and Keinath (1988) and Daigger and Roper (1985) sets were measured at different plants and that the Cape Flats and Mitchell's Plain sets were each measured at the same plant over a 10-week period and therefore are fairly narrow in SVI or $V_0$ and n range.

The Pitman family set was subjected to a linear regression analysis (Eq. (5)) with the single-step method. The constants, multiple correlation coefficient and significance levels (t) of the four constants set out in Table 4 were obtained.

All the t (significance level) values for the estimates of the fitted constants (ln α, β, γ and δ) were found to be greater than the critical t values, indicating that the null hypothesis (that there is no relationship between the selected variables and the settling velocity) can be rejected. These t values give significance levels that are all <0.001, indicating that, at the 99.9% level, all terms in the regression are significant. The F ratio for the full regression is F = 1259.73. This indicates that, on the basis of a risk of a 1% error (i.e. in only 1% of similar cases will the conclusion be wrong), the least squares equation is a good predictor because the calculated F value is greater than the tabulated F value F(3,710,0.99) = 3.78.

The fitted constants (ln α, β, γ and δ) calculated by the single-step method for the data sets that could not be pooled are detailed in Table 5. Table 6 shows the significance levels for each of the constants.

The data sets that have been marked by an asterix in Table 6 are those that have one or more constants with high significance levels i.e. constants which contribute negligibly to the multiple regression coefficient and therefore contribute negligibly to explaining the variance in the data. These terms can therefore be eliminated from Eq. (5) without a great reduction in correlation coefficient. Eliminating the β and δ terms from Eq. (5) results in $V_0$ and n respectively being independent of the SSP i.e. horizontal lines for $V_0$ and n on the $V_0$ and n vs. SSP plots (Figs. 1 to 10). Eliminating γ from Eq. (5) results in n being proportional to the SSP i.e. the line passes through the origin of the n vs SSP plot. Even though both the UCT (1986) - Lk and the Daigger and Roper (D&R) (1985) data show that the β term may be eliminated as making a negligible contribution to the correlation, these two data sets could not be pooled because, although the form of the regression lines for the

**TABLE 4**
**CONSTANTS, MULTIPLE CORRELATION COEFFICIENT AND SIGNIFICANCE LEVELS FOR THE POOLED SET OF SVI DATA**

| Reference | ln α | β | γ | δ | $r^2$ |
|---|---|---|---|---|---|
| Pitman family set<br>Significance level* | 2.14370<br><0.001 | 0.00165<br><0.001 | 0.20036<br><0.001 | 0.00091<br><0.001 | 0.8418 |

* where the significance levels (t) of the intercept (α) and each of the slopes (β, γ and δ) indicate each of their contributions to the multiple correlation coefficient. A small significance level indicates a large contribution i.e. if one of these parameters were excluded from the regression, then the multiple correlation coefficient would decrease significantly.

**TABLE 5**
**FITTED CONSTANTS FOR SVI OBTAINED USING THE SINGLE STEP METHOD FOR THE UNPOOLED GROUPS**

| Reference | ln α | β | γ | δ | $r^2$ |
|---|---|---|---|---|---|
| UCT (1986) - Lk | 2.009951 | 0.00101 | 0.19571 | 0.00122 | 0.782 |
| UCT (1986) - Dk | 2.59644 | 0.00438 | 0.09836 | 0.00199 | 0.841 |
| D&R (1985) | 1.80826 | -0.00021 | 0.18169 | 0.00141 | 0.828 |
| UCT (1986) - CF | 3.35105 | 0.00820 | 0.46845 | 0.00021 | 0.967 |
| UCT (1986) - MP | 1.48218 | -0.00076 | 0.35095 | 0.00072 | 0.928 |
| W&K (1988) | 2.38040 | 0.00496 | 0.30985 | 0.00036 | 0.641 |
| T&G (1982) | 4.00338 | 0.01641 | 0.59060 | -0.00210 | 0.660 |

**TABLE 6**
**SIGNIFICANCE LEVELS FOR THE FITTED CONSTANTS OF THE UNPOOLED SVI GROUPS**

| Reference | ln α Sig. level | β Sig. level | γ Sig. level | δ Sig. level | $r^2$ |
|---|---|---|---|---|---|
| UCT (1986) - Lk | <0.001 | 0.647 | <0.001 | <0.001 | * |
| UCT (1986) - Dk | <0.001 | <0.001 | 0.002 | <0.001 | |
| D&R (1985) | <0.001 | 0.79 | <0.001 | <0.001 | * |
| UCT (1986) - CF | <0.001 | <0.001 | <0.001 | 0.603 | * |
| UCT (1986) - MP | 0.046 | 0.799 | 0.055 | 0.656 | * |
| W&K (1988) | <0.001 | 0.006 | <0.001 | 0.187 | * |
| T&G (1982) | <0.001 | 0.143 | <0.001 | 0.152 | * |

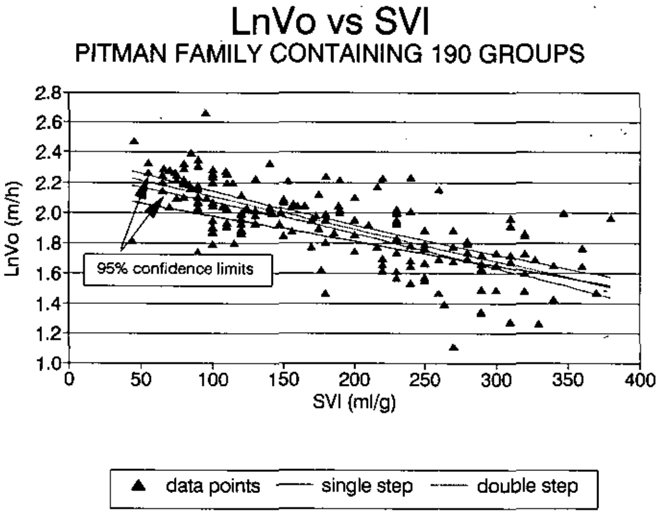

**Figure 11**
*Ln $V_o$ vs. SVI for the pooled Pitman family data set showing the 190 data groups and the best fitted and 95% confidence limit lines*

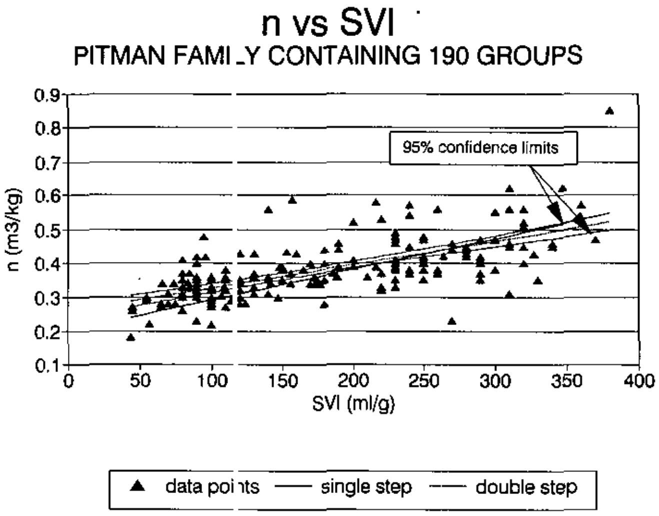

**Figure 12**
*n vs. SVI for the pooled Pitman family data set showing the 190 data groups and the best fitted and 95% confidence limit lines*

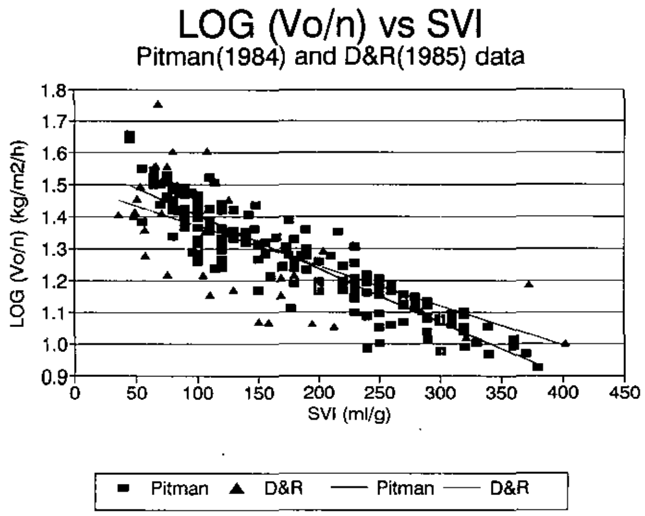

**Figure 13**
*Log ( $V_o$/n) vs. SVI for the Pitman (1984) and Daigger and Roper (1985) data sets*

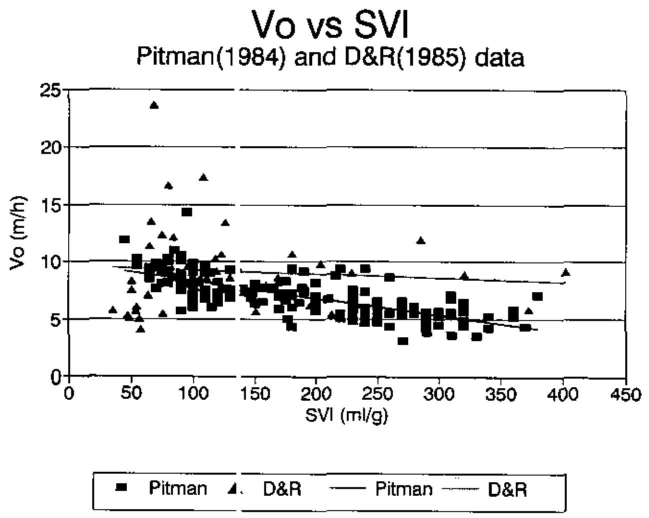

**Figure 14**
*$V_o$ vs. SVI for the Pitman (1984) and Daigger and Roper (1985) data sets*

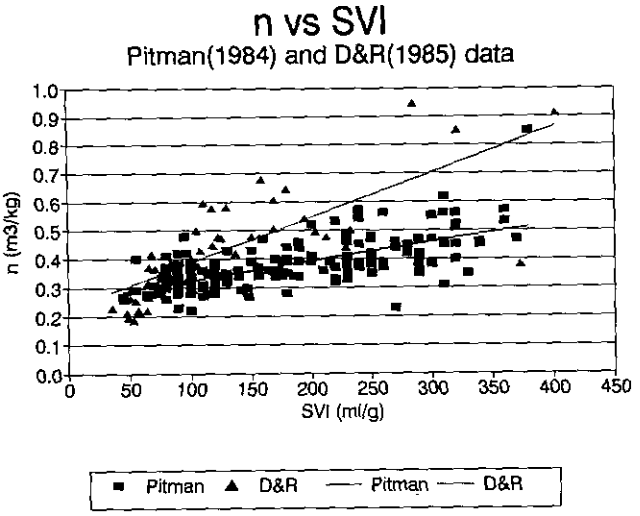

*Figure 15*
*n vs. SVI for the Pitman (1984) and Daigger and Roper (1985) data sets*

two data sets is the same, the values of the constants describing the lines are notably different. The remaining data sets in Table 6 could not be pooled because each shows a different combination of terms that contribute negligibly to the correlation. The UCT (1986) - MP data set has no terms with a low significance level because these data plot in an almost circular domain in the $V_0$ and n vs. SSP plots.

In conformity with the conclusions above, the reason that the UCT (1986) - WI and four Pitman data sets (AX, OF, NW, GK) could be pooled, is that each of these sets yield not only a low significance level for each of the ln $\alpha$, $\beta$, $\gamma$ and $\delta$ terms but also manifest the same degree in this trend.

Figures 11 and 12 show the ln $V_0$ and n values vs. SVI data for the 190 groups of the Pitman data family as well as the best (single-step) fitted lines with the 95% confidence interval lines. Although this is the largest family of data that can be pooled, this does not mean that this is the best empirical relationship between $V_0$ and n and the SVI. The fact that the Daigger and Roper (1985), Tuntoolavest and Grady (1982), Wahlberg and Keinath (1988) and other sets could not be pooled into the Pitman family indicates that factors other than the concentration (X) and SVI measurements affect the $V_0$ and n values. These factors could be the inherent problems in the SVI test itself (dependence on concentration, etc.), or differences in activated sludge plant type and filamentous organism populations that develop in them. The UCT (1986) - WI and four Pitman data sets were all measured on activated sludges incorporating either biological N or N and P removal whereas the Daigger and Roper (1985), Tuntoolavest and Grady (1982) and Wahlberg and Keinath (1988) data sets were all measured on fully aerobic activated sludges (see Table 2).

Figures 13, 14 and 15 illustrate the differences in the data sets of Pitman (1984) and of Daigger and Roper(1985). It would appear from Fig.13 that these two data sets are similar in that the log of the $V_0$/n ratio (a ratio with units of flux ($kg \cdot m^{-2} \cdot h^{-1}$)) which Pitman (1984) suggests is a useful single ratio indicative of sludge settleability for the flux theory) vs. SVI lines plot close together. However, evaluation of the individual $V_0$ and n values vs. SVI (Figs. 14 and 15) demonstrates the differences in these two data sets. Pitman's (1984) data show a decrease in $V_0$ as SVI increases while Daigger and Roper's (1985) data show $V_0$ to be constant with SVI increase. Both Pitman's (1984) and Daigger and Roper's (1985) data sets show that n increases with increase in SVI, but Daigger and Roper's much more strongly than Pitman's. Clearly, these different changes with SVI compensate each other to give a similar trend in $V_0$/n ratio (Fig. 13).

## Pooling the $SSVI_{3.5}$ data

It was found that the biggest possible sample that could legitimately be pooled was one comprising the UCT (1986) Lukuko, Wallace and Dickinson sets and the Pitman AX and OF sets. These sets together are called the UCT SSVI family. In the pooling analysis it was found that the Pitman GK, UCT (1986) - MP and - CF, and Wahlberg and Keinath (1988) data sets could not be pooled in any combination (even pairs) and consequently are sets that need to be treated as statistically distinct. The UCT family was the only group for which both the $V_0$ and n-based F statistics satisfied the critical F statistics for pooling.

The UCT SSVI family of data comprised 68 groups of $SSVI_{3.5}$ and $V_0$ and n pairs from a total number of 603 $V_s$-X data points. This family was subjected to a linear regression analysis on Eq. (5) with the single-step method. The constants, multiple correlation coefficient and the significance levels obtained for each term are listed in Table 7.

All the t (significance level) values for the estimates of the fitted constants (ln $\alpha$, $\beta$, $\gamma$ and $\delta$) were found to be greater than the critical t values, indicating that each of the parameters makes a significant contribution to the multiple correlation coefficient. The F ratio for the full regression was found to be F = 1124.26. This indicates that, on the basis of a risk of 1%, the least squares equation is a good predictor because the calculated F value is greater than the tabulated F value F(3,599,0.99) = 3.78.

The fitted constants (ln $\alpha$, $\beta$, $\gamma$ and $\delta$) for the remaining data sets that could not be pooled into pairs and needed to be treated individually are detailed in Table 8. These constants are those obtained by the single-step method of regression. Table 9 shows the significance levels for each of the constants for each of these individual sets. The data sets that have been marked by an asterix are those that have high significance levels for one or more of the fitted constants. The only data set for which all four fitted constants have acceptable significance levels for the chosen form of the

**TABLE 7**
**CONSTANTS, MULTIPLE CORRELATION COEFFICIENT AND SIGNIFICANCE LEVELS FOR THE UCT SSVI FAMILY DATA ON THE BASIS OF THE SINGLE STEP CORRELATION**

| Reference | ln $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | $r^2$ |
|---|---|---|---|---|---|
| UCT SSVI family | 2.45095 | 0.00636 | 0.15128 | 0.00287 | 0.849 |
| Significance level | <0.001 | <0.001 | <0.001 | <0.001 | |

**TABLE 8**
**FITTED CONSTANTS FOR SSVI OBTAINED USING THE SINGLE STEP METHOD FOR THE UNPOOLED**

| Reference | ln $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | $r^2$ |
|---|---|---|---|---|---|
| UCT (1986) - CF | 4.18488 | 0.02020 | 0.46101 | 0.00027 | 0.982 |
| UCT (1986) - MP | 3.27466 | 0.13965 | 0.54118 | -0.00008 | 0.960 |
| W&K (1988) | 2.41894 | 0.00487 | 0.22175 | 0.00236 | 0.744 |
| Pitman (1984) - GK | 2.70065 | 0.00808 | 0.22632 | 0.00264 | 0.916 |

**TABLE 9**
**SIGNIFICANCE LEVELS FOR THE FITTED CONSTANTS OF THE UNPOOLED SSVI$_{3.5}$ GROUPS**

| Reference | ln $\alpha$ Sig. level | $\beta$ Sig. level | $\gamma$ Sig. level | $\delta$ Sig. level | |
|---|---|---|---|---|---|
| UCT (1986) - CF | <0.001 | <0.001 | <0.001 | 0.666 | * |
| UCT (1986) - MP | <0.001 | 0.004 | 0.003 | 0.940 | * |
| W&K (1988) | <0.001 | 0.0178 | <0.001 | <0.001 | * |
| Pitman (1984) - GK | <0.001 | <0.001 | <0.001 | <0.001 | |

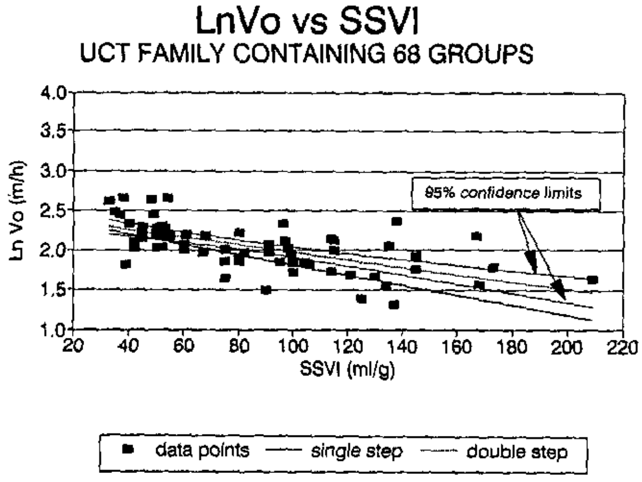

**Figure 16**
*Ln $V_0$ vs. SSVI for the UCT SSVI family data set showing the 68 data groups and the best fitted and 95% confidence limit lines*

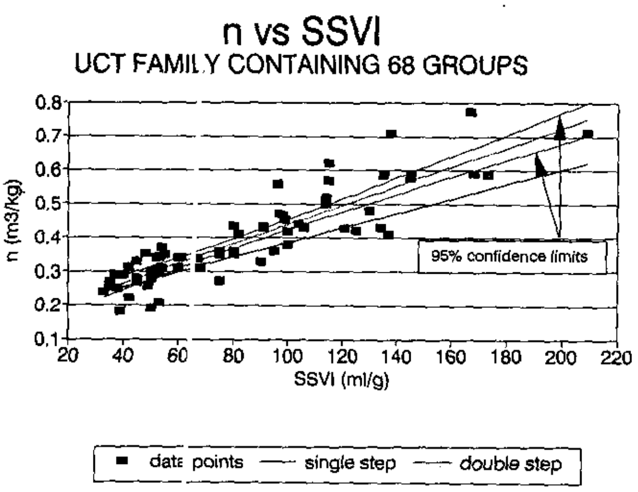

**Figure 17**
*n vs. SSVI for the UCT SSVI family data set showing the 68 data groups and the best fitted and 95% confidence limit lines*

equation is that collected by Pitman (1984) at the Goudkoppies (GK) plant. The fact that the Pitman - GK set also has all four terms significant and yet could not be pooled into the UCT SSVI family supports the previous conclusion regarding the necessity of all pooled sets exhibiting both the same trend and the same degree of the trend. The remaining three data sets (UCT (1986) - CF, UCT (1986) - MP and Wahlberg and Keinath (1988)) could not be pooled because each has different terms that are not significant in the correlation.

With regard to the Pitman - GK set, this set was measured on a 5-stage Bardenpho biological N and P removal plant and is clearly distinct from the Pitman OF and Pitman AX nitrification - denitrification plants (see Table 2). These differences in system design and operation would probably cause the differences in the data sets and indicate that the factors that influence ZSV are not fully described in the measurement of only X and SSVI$_{3.5}$. The Wahlberg and Keinath (1988) data set, obtained from fully aerobic plants, and the UCT (1986) - MP and - CF data sets, obtained from nitrification-denitrification plants, were all measured over a very narrow range of SSVI$_{3.5}$, V$_0$ and n values (Table 3).

Accepting the UCT SSVI family as the basic data set for linking the SSVI$_{3.5}$ to the V$_0$ and n values, Figs. 16 and 17 show plots of Ln V$_0$ and n against SSVI$_{3.5}$, illustrating both the data points and the fitted line with its 95% confidence limit lines.

For comparative purposes, Figs. 18 and 19 show to the same scale plots of Ln V$_0$ and n against SSVI$_{3.5}$ for the data collected by

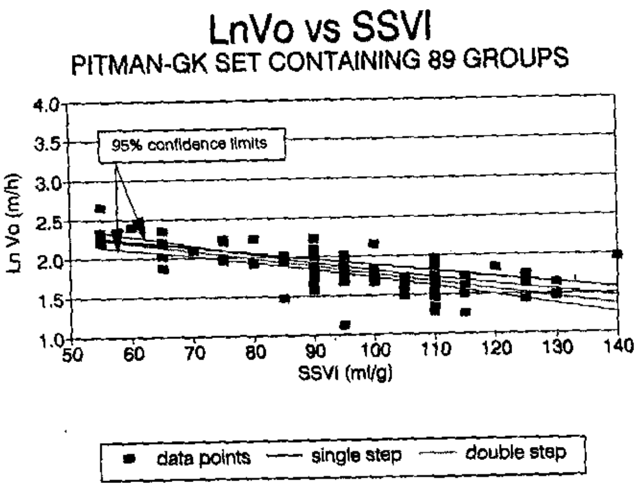

**Figure 18**

*Ln $V_o$ vs. SSVI for the data set collected by Pitman (1984) at the Goudkoppies plant showing the 89 data groups and the best fitted and 95% confidence limit lines*

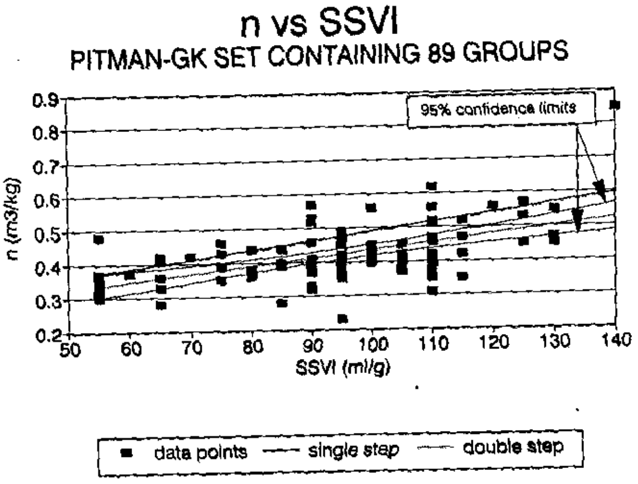

**Figure 19**

*n vs. $SSVI_{3.5}$ for the data set collected by Pitman (1984) at the Goudkoppies plant showing the 89 data groups and the best fitted and 95% confidence limit lines*

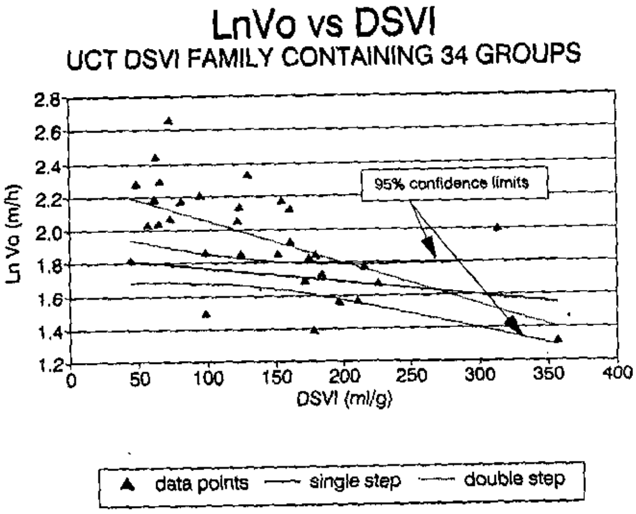

**Figure 20**

*Ln $V_o$ vs. DSVI for the UCT DSVI family data set*

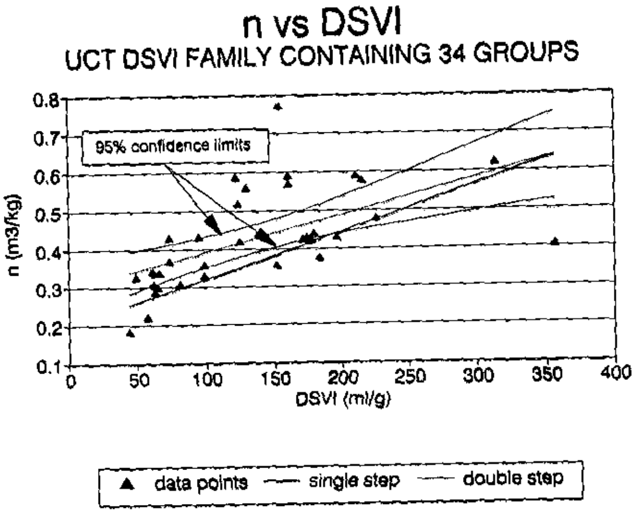

**Figure 21**

*n vs. DSVI for the UCT DSVI family data set*

Pitman (1984) at the GK plant which, as stated before, has all four terms significant but cannot be pooled with the UCT SSVI family.

## Pooling the DSVI data

The only group of data that could with reasonable reliability form the basis of a function between $V_s$ and the DSVI and X was the UCT (1986) group; both the Koopman and Cadee (1983) and STORA (1981b) data had to be rejected because their concentration ranges were too narrow (Table 3). Of the UCT group, the biggest possible sample that could legitimately be pooled was one comprising the Lukuko (Lk) and Wallace (Wl) data sets. The pooled data set, called the UCT DSVI family, consisted of 34 groups of DSVI and $V_o$ and n pairs from a total number of 239 $V_s$-X data points.

The statistical pooling tests showed that the UCT (1986) - Dk and UCT (1986) - CF data sets could not be pooled into a separate family and had to be treated individually.

The UCT DSVI family comprising the UCT (1986) - Lk and - Wl data sets was subjected to a linear regression analysis on Eq. (5). The constants, multiple correlation coefficient and the significance levels obtained are listed in Table 10.

All t (significance level) values for the estimates of the fitted constants (ln α, β, γ and δ) were found to be greater than the critical t values, indicating that each of the terms is significant in contributing to the multiple correlation coefficient. The F ratio for

### TABLE 10
### CONSTANTS, MULTIPLE CORRELATION COEFFICIENT AND SIGNIFICANCE LEVELS FOR THE UCT DSVI FAMILY DATA

| Reference | ln α | β | γ | δ | r² |
|---|---|---|---|---|---|
| Pooled DSVI group<br>Significance level | 1.84931<br><0.001 | 0.00084<br><0.001 | 0.19818<br><0.001 | 0.00123<br><0.001 | 0.776 |

### TABLE 11
### FITTED CONSTANTS FOR DSVI OBTAINED USING THE SINGLE STEP METHOD FOR THE UNPOOLED DSVI GROUPS

| Reference | ln α | β | γ | δ | r² |
|---|---|---|---|---|---|
| UCT (1986) - Dk | 2.57637 | 0.00395 | 0.04934 | 0.00304 | 0.841 |
| UCT (1986) - CF | 3.35136 | 0.00088 | 0.45652 | 0.00029 | 0.974 |

### TABLE 12
### SIGNIFICANCE LEVELS FOR THE FITTED CONSTANTS OF THE UNPOOLED DSVI GROUPS

| Reference | ln α Sig. level | β Sig. level | γ Sig. level | δ Sig. level | |
|---|---|---|---|---|---|
| UCT (1986) - Dk | <0.001 | 0.012 | 0.139 | <0.001 | * |
| UCT (1986) - CF | <0.001 | <0.001 | <0.001 | 0.557 | * |

the full regression was found to be F = 221.925. This indicates that, on the basis of a 1% risk, the least squares equation is a good predictor because the calculated F statistic is greater than the tabulated F statistic F(3,195,0.99) = 3.78.

The fitted constants for the remaining data sets that could not be pooled into one group are detailed in Table 11. These constants are those obtained by the single-step method of regression. Table 12 shows the significance levels for each of the constants.

The high significance levels for β and γ for the UCT (1986) - Dk data and δ for the UCT (1986) - CF data support the finding that these data sets could not be pooled with the rest of the DSVI group.

Figures 20 and 21 show plots of Ln $V_0$ and n against DSVI for the pooled UCT DSVI family data set.

## Discussion and conclusions

The principal objective of the statistical evaluation was to derive empirical relationships linking the zone settling velocity, $V_s$, and $V_0$ and n values in the $V_s=V_0e^{-nX}$ function to the SSVI$_{3.5}$, DSVI or SVI sludge settleability parameters. These relationships would enable the $V_0$ and n values to be calculated from the measured SSVI$_{3.5}$ or DSVI so that the full-scale settling tank solids loading tests undertaken by STORA (1981b) could be used for verification of the flux theory as a reliable dynamic simulation model for secondary settling tanks (Ozinsky et al. (in prep.), Part 3); and to establish to what degree the SST behaviour embodied in the empirical relationships of the ATV (1973; 1976; 1991) and STORA (1981b; Stofkoper and Trentelman, 1982; Epskamp and Van Hemen, 1984) design procedures flows naturally from a flux theory-based dynamic activated sludge - SST model (Ozinsky and Ekama (in prep.), Part 4). The relationships established are summarised in Table 13. In order to compare these relationships with those obtained earlier by other researchers, see Figs. 22 and 23.

While the use of the $V_0$ and n vs SSVI$_{3.5}$ or DSVI relationships may be appropriate for our particular objectives, caution should be exercised in using these relationships because not all of the available data could be pooled into one family. This indicates that factors other than concentration (X) and sludge settleability parameter (SSP) influence the zone-settling velocity ($V_s$). It appears that activated sludge plant type is a factor which may cover a number of subfactors such as temperature, filamentous organism types, biological N and P removal, fully aerated, etc. Use of the proposed relationships therefore should be restricted as far as possible to the type of activated sludge plants on which the data set was measured. The SSVI$_{3.5}$ relationship is superior because it is a better measure for sludge settleability than the SVI and it is based on a larger data set than the DSVI relationship. Consequently, in the simulation of the STORA full-scale settling tank solids loading tests (see Ozinsky et al. (in prep.) Part 3), the $V_0$ and n values were calculated with the aid of Eqs. (2) and (3) where the ln α, β, γ and δ values are those for the UCT SSVI family given in Table 7.

Comparing the UCT SSVI family $V_0$ and n vs. SSVI$_{3.5}$ relationships, it can be seen that these compare very closely with those developed earlier by Ekama and Marais (1986) (see Figs. 22 and 23 respectively). This close correspondence implies that the permissible overflow rate and applied solids loading vs. sludge settleability graphs developed by Ekama and Marais (1986) (in particular, their Figs. 11 and 12) will not be significantly different,

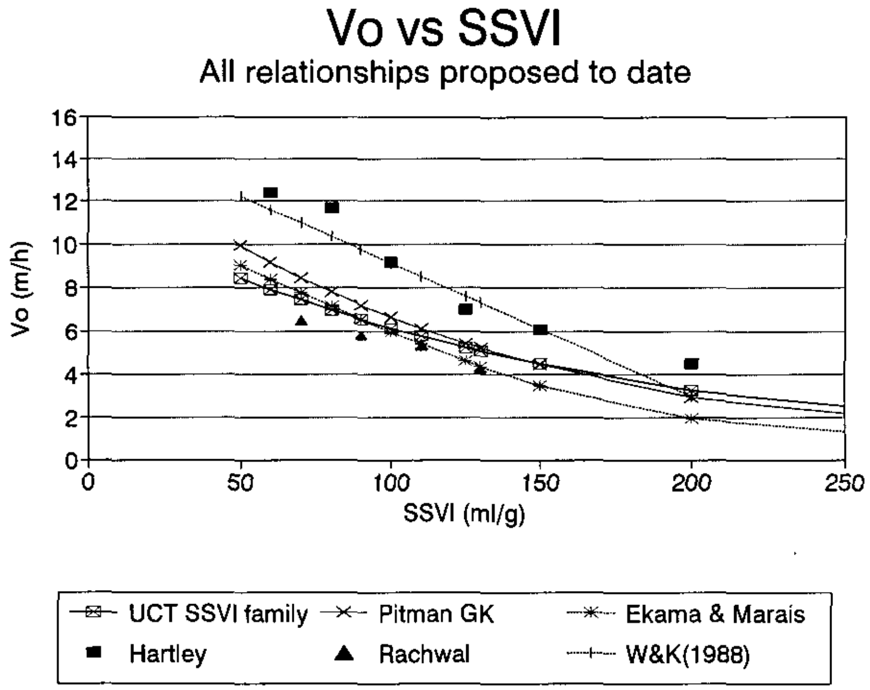

***Figure 22***

*All relationships for $V_o$ vs. $SSVI_{3.5}$ proposed to date*

UCT SSVI family: $V_o = 11.599e^{(-0.00636*SSVI)}$ ($r^2$(overall) = 0.849)
Pitman (1984) - GK: $V_o = 14.889e^{(-0.00808*SSVI)}$ ($r^2$(overall) = 0.916)
Ekama and Marais (1986): $V_o = e^{(-0.0016*SSVI)}*(10.8779+0.01854*SSVI)$ ($r^2$= 0.976)
Hartley (1985): 6 data points
Rachwal et al. (1982): 4 data points
Wahlberg and Keinath (1988): $V_o = 15.3 - 0.00615*SSVI$

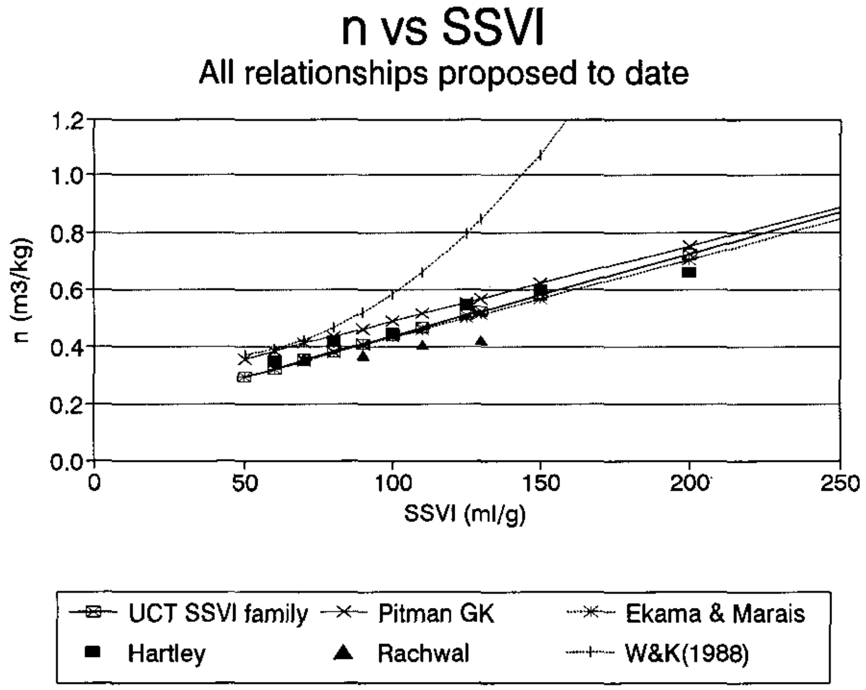

***Figure 23***

*All relationships for n vs. $SSVI_{3.5}$ proposed to date*

UCT SSVI family: $n = 0.15128+0.00287*SSVI$ ($r^2$(overall) = 0.849)
Pitman (1984) - GK: $n = 0.22632+0.00264*SSVI$ ($r^2$(overall) = 0.916)
Ekama and Marais (1986): $n = 0.1602+0.00273*SSVI$ ($r^2$ = 0.976)
*Hartley (1985): 6 data points*
Rachwal et al. (1982): 4 data points
Wahlberg and Keinath (1988): $n = 0.426 + 0.00384*SSVI - 0.0000543*SSVI^2$

TABLE 13
SUMMARY OF RELATIONSHIPS ESTABLISHED BETWEEN SLUDGE SETTLEABILITY PARAMETERS

| SSP | SSVI | | DSVI | SVI |
|---|---|---|---|---|
| ln $\alpha$ | 2.45095 | 2.70065 | 2.30854 | 2.14370 |
| $\beta$ | 0.00636 | 0.00808 | 0.00297 | 0.00165 |
| $\gamma$ | 0.15128 | 0.22632 | 0.29721 | 0.20036 |
| $\delta$ | 0.00287 | 0.00264 | 0.00095 | 0.00091 |
| $r^2$ | 0.849 | 0.916 | 0.776 | 0.8418 |
| **Data set** | UCT (1986) - Lk<br>UCT (1986) - Wl<br>UCT (1986) - Dk<br>Pitman (1984) (AX)<br>Pitman (1984) (OF) | Pitman (1984) -GK | UCT (1986) - Lk<br>UCT (1986) - Wl | Pitman - AX<br>Pitman - OF<br>Pitman - GK<br>Pitman - NW<br>UCT (1986) - Wl |
| **Data range** | | | | |
| **SSP** | 33 - 209 | 65 - 125 | 44 - 357 | 44 - 360 |
| **X** | 0.510 - 15.50 | 0.95 - 8.54 | 0.51 - 12.18 | 0.510 - 12.80 |
| **$V_s$** | 0.13 - 15.84 | 0.14 - 6.2 | 0.15 - 7.56 | 0.14 - 10.9 |
| **No of data** | | | | |
| **$V_s$, X** | 603 | 225 | 239 | 713 |
| **SSP** | 68 | 89 | 34 | 190 |
| **Plant characteristic** | long sludge age, nitrification - denitrification (ND) | 3/5 stage Bardenpho plant for N and P removal | 15 Western Cape plants, long sludge age ND | No common characteristics |

**ln $V_s$ = ln $\alpha$ - $\beta$.SSP - $\gamma$.X - $\delta$.SSP.X** (Eq. (5))
or
**$V_0 = \alpha$ exp (-$\beta$.SSP) and n = $\gamma + \delta$.SSP**
in
**ln $V_s = V_0 e^{(-nX)}$** (Eqs. (1) - (3))
It is statistically permissable to use either form because Eq. (5) (single-step method) does not give a significantly improved $r^2$ correlation compared with Eqs. (1) - (3) (double-step method)

and can be used with the same margin of error as earlier.

Because the various data sets for a particular sludge settleability parameter (SSP) could not all be pooled into a single family, it was concluded that differences exist in the data sets which are not accounted for in the measured parameters X and SSP. It is interesting to note that, for the SVI, all the Pitman (1984) data (AX, OF, NW, GK) could be pooled but, for the $SSVI_{3.5}$, the GK data set could not be pooled with the AX and OF sets. Because the latter two plants are extended aeration with N removal and the former a biological N and P removal plant, the $SSVI_{3.5}$ appears sufficiently sensitive in distinguishing between these two plant operating conditions whereas the SVI is not. Also, all three of the UCT (1986) data sets measured on 15 Western Cape plants (Wl, Dk, Lk - all long sludge age, biological N removal, no P removal) could be pooled with the Pitman (1984) OF and AX data sets. Since all these plants are similar in design i.e. long sludge age nitrification-denitrification, it seems reasonable that their data sets could be pooled. In contrast, for the SVI, while all the Pitman (1984) data sets could be pooled, (even though these are from two different plant types), the UCT Western Cape plant SVI data could neither be pooled into a family of their own nor with Pitman's (1984).

A possible reason for certain data sets not pooling with others is the effect of nitrate concentration on the flux constant n. Denitrification of nitrate may commence at high concentration in the multiple zone settling batch test which settles for a long time. The nitrogen gas generated would retard the zone-settling velocity at high concentrations, which would cause the n value to increase. This does not affect the $V_0$ value much because, at low concentrations, the test reaches completion relatively quickly (<1h) before significant ($>10\,mgNO_3\text{-}N\cdot\ell^{-1}$) denitrification can take place. Curiously though, the n values for plants where this could have been a problem (UCT (1986) and Pitman (1984) groups - see Table 13) are not high, in fact most are lower than for fully aerobic plants (Daigger and Roper, 1985). The potential effect of denitrification on the $V_0$ and n value was tested as part of the UCT (1986) - Dk data set, which was measured on 15 Western Cape plants. At all long sludge age (>20 d) nitrification-denitrification plants (ND) plants, no effect was observed and the nitrate concentration was virtually identical before and after high concentration ZSV tests. Only at one plant with a short sludge age (~12 d) was denitrification a problem, but high concentration ZSV tests could not be completed because the sludge floated in under 1 h. The measure with which denitrification may have influenced all the different researchers' ZSV tests cannot be established. However, it would appear that, for high temperature (>20°C), intermediate sludge age (5 to 12 d) plants where nitrification is possible and denitrification not complete, the effect may be severe. While it is likely that not many of these kinds of plants are included in the data sets considered in this paper, the effect of denitrification on the ZSV tests requires further investigation.

The above discussion serves to demonstrate that a consistent pattern with regard to plant type for the $SSVI_{3.5}$ emerges, which gives some confidence to the empirical relationships developed for the pooled data sets. Furthermore, because the relationship was developed on the basis of the UCT (1986) Western Cape Plant and Pitman (1984) AX and OF data sets, all of which are long sludge age nitrification-denitrification plants, the developed relationship is particularly suited to the STORA (1981b) results because these plants also were mainly long sludge age nitrification plants (with some denitrification inevitably taking place in the poorer aerated and mixed zones). It was accepted, therefore, that the flux theory constants $V_o$ and n could be calculated with reasonable reliability from the measured $SSVI_{3.5}$ data for the simulation study which is discussed in Part 3 (Ozinsky et al., in prep.). Even though the DSVI also is a good sludge settleability parameter, because the empirical relationship based on it was established on a rather limited data set and has a lower multiple correlation coefficient than the $SSVI_{3.5}$, it was not regarded as being as reliable as the $SSVI_{3.5}$ to estimate the $V_o$ and n for the flux theory verification against the STORA (1981b) results.

In conclusion, even though the empirical relationships developed in this paper are based on a large data set obtained from different countries, these relationships should be used with caution for a particular design application. The fact that not all the data sets for a particular SSP could be pooled into one family indicates that the zone-settling velocity $V_s$ is influenced by factors not incorporated in the X and SSP measurements. The statistical analysis in this paper indicates that plant type possibly affects the results. In this regard, the $SSVI_{3.5}$ proved more discerning than the SVI, which appeared to allow and disallow pooling of data sets more randomly. This possibly arises in part from the weakness of the SVI as a sludge settleability measure.

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