Short-duration rainfall frequency model selection in Southern Africa#

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Abstract

Short-duration design rainfall estimates are vital in the design of hydraulic structures and for environmental management. The use of a digitised rainfall database is expected to improve these estimates in Southern Africa. In order to estimate design rainfall intensities, an appropriate probability distribution has to be chosen, which adequately fits the data. This paper investigates the use of goodness-of-fit tests to assess potential probability distributions and makes general recommendations of appropriate probability distributions for estimating short-duration design rainfall intensities in Southern Africa.

Introduction

The concept of depth duration frequency (DDF) analysis of historical rainfall data in order to predict the frequency and magnitude of future rainfall events is not only vital to engineers involved in designing hydraulic structures, but is also essential input in environmental management (Dent and Smithers, 1991). At present, the majority of short-duration (i.e. 24 h and less) design rainfall estimates (DRE) in Southern Africa are derived from a co-axial diagram produced by Midgley and Pitman (1978). This was generated from an analysis of manually extracted data at a time resolution of 15 min from 58 autographic rainfall stations, containing data up to 1976. The log-Gumbel (L-EV1) probability distribution (PD) was fitted to the data and was used to estimate design values.

Improved short-duration DRE are now possible as a result of the availability of considerably longer periods of digitised rainfall records at a time resolution ranging from 1 to 5 min, as well as an increased number of stations. In addition, PDs other than the Gumbel PD are commonly used in the frequency analysis of storm rainfall (Stedinger et al., 1993) and may be more suitable than the log-Gumbel PD in Southern Africa. The spatial variation in the most suitable PD, the extrapolation of estimates to data-sparse regions and the applicability of a regional analysis as compared to single-site analyses are further avenues to be explored in improving DRE. A research project is presently being undertaken which is investigating various techniques to improve DRE and to develop synthetic design rainfall hyetographs in Southern Africa by utilising the database of digitised rainfall and the concepts outlined above.

The objectives of the study reported in this paper are to investigate the use of goodness-of-fit (GOF) tests and use the GOF tests to evaluate and determine appropriate PDs for use in Southern Africa. L-moments (Hosking, 1990) are used to estimate the parameters of the PDs and a brief review of the theory of L-moments is presented. The GOF tests evaluate the descriptive and predictive performance of various PDs and comprise L-moment ratio diagrams, statistical tests, standardised deviations from estimates interpolated from plotting position formulae and a non-parametric evaluation of the performance of the PDs.

Probability distributions used in short-duration rainfall analysis

A number of different PDs have been used for single-site short-duration rainfall probability analysis. In Southern Africa, Midgley and Pitman (1978) used the L-EV1 distribution, Schulze (1984) utilised the Gumbel (EV1), log-Normal (LN2) and log-Pearson type 3 (LP3) and Weddepohl (1988) employed the LN2 distribution.

Internationally, PDs that have been commonly used in short-duration design rainfall estimates in the USA include the general extreme value (GEV) distribution (Hosking and Wallis, 1987, cited by Vogel, Thomas and McMahon, 1993) and the LP3 PD (Aron et al., 1987; Naghavi et al., 1993), while Griffiths and Pearson (1993) used the EV1 PD in New Zealand. Stedinger et al. (1993) report that the EV1, LP3 and GEV PDs are commonly used for short-duration rainfall probability analysis.

In this study the LN2, 3 parameter log-normal (LN3), LP3, Pearson type 3 (PE3), EV1, L-EV1, GEV, generalised Pareto (GPA), generalised logistic (GLO) and Wakeby (WAK) PDs were employed. These PDs are summarised by Stedinger et al. (1993).

Parameter estimation for the PDs by L-moments has several advantages as outlined by Stedinger et al. (1993) and hence was used in this study. In addition, in seeking appropriate PDs, the use of a single parameter-fitting technique would not further confound the GOF tests employed. The computer program developed by Hosking (1991b) was used in the analysis.

Method of L-moments

Similar to ordinary product moments, the purpose of L-moments and probability weighted moments (PWMs) is to summarise theoretical probability distributions and observed samples (Vogel, McMahon and Chiew, 1993). Hence L-moments can be used for parameter estimation, interval estimation and hypothesis testing.

L-moments have several important advantages over ordinary product moments (Vogel, Thomas and McMahon, 1993). In order to estimate the sample variance and sample skew, ordinary product moments require the squaring and cubing of the observations respectively. Sample estimators of L-moments are linear combinations of the ranked observations, and do not require squaring and cubing of the observations. Thus L-moments are subject to less bias than ordinary product moments (Vogel, McMahon and Chiew, 1993).

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					DUI	RATION	S USED	TABLE IN EXTR		OF THE	AMS				
							Du	ration (m	nin)						
5	10	15	30	45	60	90	120	240	360	480	600	720	960	1200	1440

L-moments, defined by Hosking (1990), are linear combinations of PWMs. Greenwood et al. (1979) summarise the theory of PWMs. Unbiased sample estimates for the first four PWMs can be computed from Eq. (1) (Stedinger et al., 1993; Vogel and Fennessey, 1993).

$$b_0 = \frac{1}{n} \sum_{j=1}^{n} x_j$$
 (1a)

$$b_{1} = \sum_{j=1}^{n-1} \left[\frac{(n-j)}{n(n-1)} \right] x_{j}$$
 (1b)

$$b_2 = \sum_{j=1}^{n-2} \left[\frac{(n-j)(n-j-1)}{n(n-1)(n-2)} \right] x_j$$
 (1c)

$$b_3 = \sum_{j=1}^{n-3} \left[\frac{(n-j)(n-j-1)(n-j-2)}{n(n-1)(n-2)(n-3)} \right] x_j$$
 (1d)

where:

b_r = r'th order PWM sample estimate

n = number of observations in the sample

 x_j = ranked observations, with x_j being the largest observation and x_j the smallest observation.

The first four L-moments for a sample can be computed from the first four PWMs using:

$$\lambda_{t} = b_{0} \tag{2a}$$

$$\lambda_2 = 2b_1 - b_0 \tag{2b}$$

$$\lambda_3 = 6b_2 - 6b_1 + b_0 \tag{2c}$$

$$\lambda_4 = 20b_3 - 30b_2 + 12b_1 - b_0 \tag{2d}$$

where:

 $\lambda_r = r'$ th L-moment.

Hosking (1990) defines the L-moment ratios as:

$$\tau_2 = \frac{\lambda_2}{\lambda_1} \equiv \text{L-coefficient of variation}$$
 (3a)

$$\tau_3 = \frac{\lambda_3}{\lambda_2} \equiv L$$
-skewness (3b)

$$\tau_4 = \frac{\lambda_4}{\lambda_2} \equiv L$$
-kurtosis (3c)

Hosking (1990) shows that λ_2 , τ_3 and τ_4 can be thought of as measures of a sample's scale, skewness and kurtosis respectively.

Data utilised

The digitised rainfall data used in the analysis were obtained from the South African Weather Bureau (SAWB), except for autographic rainfall data for Athlone which were obtained from the City Engineer's Department, Cape Town. As pointed out previously by Smithers (1991), the SAWB digitised rainfall database contains numerous inconsistencies such as infinite intensities and negative time steps. Smithers (1993a) showed that these inconsistencies, if excluded from the database, could result in significantly different annual maximum series (AMS) and DRE when compared to the results of three different procedures for correcting the database. In addition, no significant differences were found between data corrected using any of the three correcting procedures. The SAWB digitised rainfall database was thus corrected for inconsistencies using "maximum intensity adjustment" as defined by Smithers (1993a). The AMS were extracted from the corrected database for the durations indicated in Table 1, using a modified version of the program RAINX (Schulze and Dent, 1982).

Vogel, McMahon and Chiew (1993), in a frequency analysis of Australian flood data, found less sampling variability in the estimated L-skewness and L-kurtosis using data with record lengths greater than 30 years when compared with data having record lengths greater than 20 years. Hence only sites with record lengths greater than 30 years were used in the SAWB digitised rainfall database. In total, 38 sites in SA meet this criterion, and they are located as indicated in Fig. 1.

SAWB DIGITISED RAINFALL STATIONS WITH > 30 YEARS RECORD

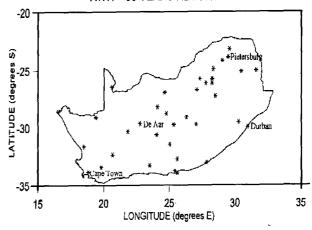


Figure 1

Location of sites with record lengths greater than 30 years in the SAWB digitised database

Goodness-of-fit tests

The use of GOF tests and hypothesis testing does not lead to unique choices of distributions, but could serve to reject unsuitable distributions (Cunnane, 1989). Tests were performed which focused on the ability of alternative PDs to approximate the data as well on the ability of alternate PDs to provide appropriate estimates of design quantiles.

L-moment diagrams

L-moment diagrams have been used extensively in recent research to select appropriate PDs (e.g. Hosking and Wallis, 1987; Vogel, McMahon and Chiew, 1993; Vogel, Thomas and McMahon, 1993). L-moment diagrams are similar to conventional product moment diagrams and compare sample estimates of τ_2 , τ_3 and τ_4 with a range of different theoretical distributions. An advantage of L-moment diagrams is that a range of PDs can be plotted on the same graph and the diagram is thus useful for evaluating which PD provides a satisfactory approximation to the distribution of a particular hydrological variable. Vogel and Fennessey (1993) advocate the replacement of product moment diagrams by L-moment diagrams because, unlike product moment diagrams, L-moment ratio's are nearly unbiased for all underlying distributions.

The theoretical relationships between τ_3 and τ_4 for the PDs shown in Fig. 2 are summarised by Hosking (1991a) and Stedinger et al. (1993). The 2-parameter PDs in an L-moment diagram are represented by a single point, and the 3-parameter PDs by a continuous curve.

It is noted that an L-moment diagram for a single duration from all sites, as shown in Fig. 2, contains limited information to select appropriate PDs. However, by combining all the durations from all the sites, as shown in Fig. 3, clearer trends are observed.

A theoretical relationship between τ_3 and τ_4 for the LP3 distribution does not exist (Vogel, Thomas and McMahon, 1993). Hence the procedure outlined by Vogel, McMahon and Chiew (1993) was followed in Fig. 4, where the log-transformed data are plotted with the PE3 distribution, to check whether the data resemble an LP3 distribution.

From the comparisons of the theoretical relationships between L-kurtosis and L-skewness for the different PDs in Fig. 3, it is expected that the AMS would be well approximated by the LN3 and GEV PDs and to a lesser extent by the PE3, GLO and EV1 PDs. Figure 4 reveals the poor approximation of the AMS by the LP3 distribution. No regionalisation of the sites has been performed and it is assumed that the entire Southern Africa is one homogeneous hydrological region. As indicated by the distribution of the AMS and the broad envelope of values in Figs. 3 and 4, this assumption is not entirely correct.

In order to illustrate the need for regionalisation, four sites from four climatically different regions were selected and L-moment diagrams plotted as depicted in Fig. 5. The locations of the four sites selected (Athlone, Durban airport, De Aar and Pietersburg) are shown in Fig 1.

Chi-squared test

A chi-squared test was employed which utilises 10 equally spaced probability class intervals and either rejects or accepts the null hypothesis that the sample of data could have been drawn from the PD being evaluated (Kite, 1988). The results from the Chi-squared tests performed for all durations and from all 38 sites are contained in Table 2.

The results in Table 2 indicate that no PD performed well for

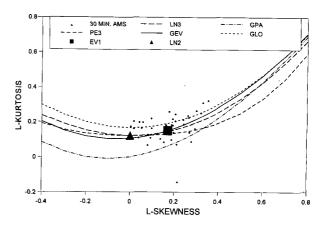


Figure 2
L-moment diagram containing values for 30 min
duration events from all 38 sites

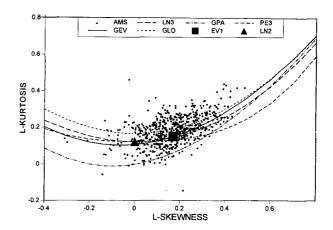


Figure 3 L-moment diagram containing values for all 16 durations from all 38 sites

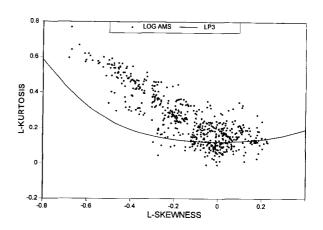


Figure 4 L-moment diagram containing values for all 16 durations from all 38 sites

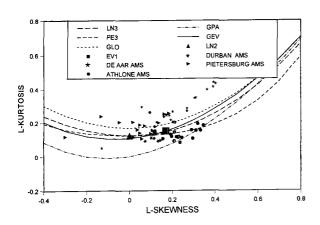


Figure 5 L-moment diagram for 4 sites located in 4 climatically different regions

durations shorter than 30 min. The Chi-squared test indicates that for SA the most appropriate PDs are the LN3, EV1, GEV, PE3 and GLO, determined by these PDs having less than 15% of Chisquared GOF tests rejected.

Standardised deviations

The standardised deviation (SD) GOF method adopted is similar to where: techniques used by Benson (1968), Bobee and Robitaille (1977) and Kite (1988) and is given as:

$$SD_{j} = \frac{1}{df_{i}} \sum_{i=1}^{k} \frac{(x_{i} - y_{i})}{y_{i}}$$
 (4)

where:

standardised deviation of jth PD SD recorded data, interpolated (if necessary) to y_i correspond to the ith return period, with

probabilities assigned to observed data us-

plotting position equation ing a

event magnitude computed from the jth probability distribution for the ith return period

number of recurrence intervals (5) used in the k computation

degrees of freedom used to fit the jth PD. df.

Return periods of 2, 5, 10, 20, 50 and 100 years corresponding to non-exceedance probabilities of 0.50, 0.80, 0.90, 0.95, 0.98 and 0.99 respectively were used in the calculation of the SD. The choice of plotting position equation was shown by the Natural Environment Research Council (1975) and Smithers (1993b) to affect the computed SD, although Kite (1988) expressed the opinion that the relative rankings of PDs would not be influenced by the choice of plotting position.

The Weibull plotting position, as given in Eq. 5, has been shown by means of a survey conducted in different countries, to be the most frequent plotting position used, despite its bias in graphical quantile estimates (Cunnane, 1989).

$$P_e = \frac{i}{N+1}$$

exceedance probability of ith ranked data

rank of data

number of points in the data series.

The results from ranking the PDs according to the SD statistic are presented in Table 3.

TABLE 2 NUMBER OF REJECTIONS OF THE NULL HYPOTHESIS THAT THE SAMPLE COULD HAVE BEEN DRAWN FROM A PARENT DISTRIBUTION, AT 95 CONFIDENCE LEVEL, EXPRESSED AS A PERCENTAGE OF TOTAL NUMBER OF SITES (38)

Duration (min)	Probability distribution												
	LN2	LN3	LP3	L-EV1	EV1	GEV	GPA	PE3	GLO	WAK			
5	61	37	45	87	55	37	50	34	37	37			
10	50	47	55	55	42	42	53	42	32	58			
15	32	21	37	45	21	18	42	21	16	34			
30	29	13	26	39	11	11	16	13	18	24			
45	24	8	37	26	13	11	26	11	5	24			
60	16	8	21	24	11	8	24	13	11	21			
90	24	5	26	29	13	13	16	0	13	24			
120	18	3	21	29	8	3	18	5	8	13			
240	18	3	21	34	3	5	18	3	3	11			
360	18	3	21	29	3	3	11	3	3	8			
480	24	3	21	42	3	0	24	11	5	21			
600	11	3	26	42	3	3	26	11	13	16			
720	13	5	13	34	13	5	26	3	8	24			
960	13	5	24	32	5	3	39	16	0	24			
1 200	16	11	24	29	5	8	37	5	13	18			
1 440	21	11	26	34	11	8	26	11	8	32			
All	24	12	28	38	14	11	28	13	12	24			
durations	1												

TABLE 3

RELATIVE RANKING OF 10 PDS ACCORDING TO COMPUTED SD AT ALL 38 SITES (1 = BEST, 10 = WORST).

WEIBULL PLOTTING POSITION WAS USED TO ASSIGN PROBABILITIES TO OBSERVED DATA.

Duration		Probability distribution												
(min)	LN2	LN3	LP3	L-EV1	EV1	GEV	GPA	PE3	GLO	WAK				
5	9	3	6	10	8	4	7	2	5	1				
10	9	5	8	10	3	2	6	4	7	ī				
15	9	3	8	10	5	4	7	2	6	i				
30	10	3	6	9	7	5	4	2	8	1 1				
45	9	2	8	10	5	4	6	3	7	1				
60	9	3	6	10	7	4	5	2	8	1 1				
90	9	2	5	10	8	3	6	4	7	1				
120	10	3	6	9	5	2	7	4	8	1				
240	8	2	6	10	5	1	9	3	7	4				
360	9	2	6	10	8	1	5	4	7	3				
480	9	2	8	10	6	3	5	1	7	4				
600	9	4	7	10	5	1	6	2	8	3				
720	9	4	7	10	5	2	6	3	8	1				
960	8	3	7	10	5	2	9	4	6	1				
1 200	8	3	7	10	5	1	9	4	7	2				
1 440	7	3	6	10	5	1	9	2	8	4				
All durations	9	3	7	10	5	2	6	4	8	1				

PDs that performed consistently well over all durations based on the SD statistic are the WAK, GEV, LN3 and PE3. The GEV PD is seen to be ranked highly for durations greater than 240 min.

Non-parametric evaluation

A non-parametric test was performed to evaluate the ability of the different PDs to provide estimates of the 100-year return period event. Similar tests have been performed on floodflow data in the USA by Vogel, Thomas and McMahon (1993) and in Australia by Vogel, McMahon and Chiew (1993). The test assumes that the data from the sites are independent and comprises counting, for each PD and at each site, the number of times an observed value exceeds the estimated 100-year return period event. A total of 1329 site years of data were used from the 38 sites. Thus the 100 year return period event is expected to be exceeded 13 times within the 1329 site years of data.

Based on the assumption that the 38 sites are independent and that the rainfall events occur independently from year to year, then it can be assumed that the number of exceedances follow a binomial distribution (Vogel, McMahon and Chiew, 1993). A 95% confidence interval was computed for the expected number of exceedances using the normal approximation of the binomial distribution, as described by Steel and Torrie (1980). The computed 95% confidence interval for the expected 13 exceedances from the 1 329 site years of data was 6 to 20 events. Results from the tests based on the above assumptions are contained in Table 4. No expected probability adjustment was used in generating the results in Table 4.

The values in Table 4 indicate that the LN3, EV1 and PE3 PDs perform well for all durations, while the LN2 and GEV only fall outside the 95 confidence limits for the 5 min duration. A summary of the relative ranking of the PDs for the various GOF tests employed is contained in Table 5.

Discussion and conclusions

The power of the L-moment ratio diagram is clearly illustrated by considering the performance of the PE3 PD in Fig. 3. All the computational tests (Tables 2, 3 and 4) indicate that the PE3 is a reasonable distribution to use. However, the L-moment diagram for all sites and durations (Fig. 3) clearly indicates the poor descriptive performance of the PE3 PD for L-skewness greater than 0.2. Similarly, the reasons for the poor performance of the LP3 PD is also explicitly depicted by the L-moment diagram in Fig.4.

The Chi-squared test of the null hypothesis that the samples have been drawn from the parent distribution, gave results consistent with the other GOF tests. However, since 10 equally spaced class intervals were used, not sufficient weight is given to the extremes of the distribution.

Although the standardised deviations GOF test suffers from the disadvantage of the relative ranking of the PDs changing according to the plotting position used, the results of the test are generally consistent with the other tests.

The non-parametric test which counts the number of exceedances of a specified exceedance probability evaluates the ability of different PDs to provide estimates of design quantiles. This test is a pragmatic and robust approach, and again, gave results consistent with the other tests. The validity of the assumption of independent sites and independent events occurring from year to year needs further investigation.

Of the four GOF tests used, it is advocated that no PD selection should be based on any single test, but that a combination of tests, which includes the graphical L-moment diagram, should be performed.

In the evaluations of the different PDs the results from the WAK PD are excluded owing to the uncertainty in parameter estimation on relatively short data sets. It is reassuring to note that the LN3 and GEV PD performed consistently well in all the GOF

TABLE 4

NUMBER OF DATA VALUES IN THE AMS THAT EXCEED THE 100-YEAR RETURN PERIOD EVENT, AS ESTIMATED BY DIFFERENT PROBABILITY DISTRIBUTIONS, FITTED TO THE DATA USING L-MOMENTS, FROM 1 329 SITE YEARS OF DATA.* INDICATES RESULTS FALLING WITHIN THE 95% CONFIDENCE INTERVAL (6 TO 20 EXCEEDANCES)

Duration	Probability distribution											
(min)	LN2	LN3	LP3	L-EV1	EV1	GEV	GPA	PE3	GLO	WAK		
5	5	14*	71	1	6*	24	53	17*	12*	18*		
10	6*	7*	64	i	9*	10*	41	9*	5	8		
15	9*	12*	60	1	8*	16*	40	13*	7*	8*		
	12*	9*	46	1	11*	9*	28	10*	3	3		
30	11*	14*	56	1	13*	15*	33	16*	5	7*		
45	14*	11*	48	3	15*	10*	33	13*	6*	5		
60	12*	15*	44	1	17*	11*	33	17*	3	6*		
90		12*	42	3	18*	11*	32	14*	6*	5		
120	14*	15*	44	1	17*	11*	33	17*	3	6*		
240	12* 9*	12*	43	1	14*	12*	29	17*	6*	7*		
360	1	13*	37	2	14*	12*	30	16*	5	7*		
480	11*		36	2	13*	10*	32	15*	5	6*		
600	8*	12* 11*	34	1	13*	12*	34	15*	6*	6*		
720	10*		38	0	12*	13*	33	18*	6*	10*		
960	9*	14*	1	0	15*	12*	33	14*	5	7*		
1 200	9*	13*	40	0	15*	14*	31	14*	7*	5		
1 440	9*	14*	38		1.5	- 14				-		
No. of							16	0	8	5		
durations	1	0	16	16	0	1	16	0	6	3		
falling outside												
of 95%			1						1			
confidence												
interval	1	l			<u> </u>			1				

	TABLE 5
	SUMMARY OF RELATIVE RANKING OF PDS ACCORDING TO DIFFERENT GOF TESTS FOR ALL DURATIONS
;	SUMMARY OF RELATIVE RANKING OF PDS ACCORDING TO BIT EXECUTED TO STATE OF THE STATE
	(1 = BEST, 10 = WORST)

GOF	Probability distribution												
test	LN2	LN3	LP3	L-EV1	EV1	GEV	GPA	PE3	GLO	WAK			
Chi- squared	6	2	8	10	5	1	8	4	2	6			
Std deviations	9	3	7	10	5	2	6	4	8	1			
Non- parametric	4	1	8	8	1	4	8	1	7	6			

tests employed. Of the PDs used previously in Southern Africa (LN2, EV1, L-EV1 and LP3), only the EV1 could possibly be considered for general use with short-duration rainfall frequency analysis. Hence, based on the GOF tests performed and using L-moments to estimate parameters of distributions, and considering the entire region and all durations, it is recommended that the LN3 and GEV distributions be used for future short duration rainfall probability analysis in Southern Africa. However, it is conceded that at a local scale, these recommendations may change substantially. Hence future research is required to investigate quantitative discordancy and heterogeneity measures to evaluate which regions in Southern Africa are hydrologically homogenous.

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References

ARON G, WALL DJ, WHITE EL and DUNN CN (1987) Design rainfall for Pennsylvania. In: Feldman AD (ed.) *Engineering Hydrology*. Hydraulics Division, ASCE, Williamsburg, Virginia, USA.

BENSON MA (1968) Uniform flood-frequency estimating methods for

- BOBEE BB and ROBITAILLE R (1977) The use of the Pearson Type 3 and log-Pearson Type 3 distribution revisited. *Water Resour. Res.* 13(2) 427-443.
- CUNNANE C (1989) Statistical Distributions for Flood Frequency Analysis. WMO Report No. **718**, Geneva, Switzerland.
- DENT MC and SMITHERS JC (1991) Quantification of high intensity rainfall: An important input to environmental impact assessment. Agric. Eng. in SA 23 60-66.
- GREENWOOD JA, LANDWEHR JM, MATALAS NC and WALLIS JR (1979) Probability weighted moments: Definition and relation to parameters of several distributions expressible in inverse form. Water Resour. Res. 15(5) 1049-1054.
- GRIFFITHS GA and PEARSON CP (1993) Distribution of high intensity rainfalls in metropolitan Christchurch, New Zealand. *J. Hydrol.* (N.Z.), 31(1)5-22.
- HOSKING JRM (1990) L-moments: Analysis and estimation of distributions using linear combinations of order statistics. J. R. Stat. Soc. B. 52(1) 105-124.
- HOSKING JRM (1991a) Approximations for Use in Constructing Lmoment Ratio Diagrams. Research Report RC-16635, IBM Research Division, New York, USA.
- HOSKING JRM (1991b) Fortran Routines for Use with L-moments, Version 2. Research Report RC-17097, IBM Research Division, New York, USA
- HOSKING JRM and WALLIS JR (1987) An index flood procedure for regional rainfall frequency analysis. EOS Trans. Am. Geophys. Union 68 312.
- KITE GW (1988) Frequency and Risk Analysis in Hydrology. Water Resources Publications, Littleton, USA.
- MIDGLEY DC and PITMAN WV (1978) A Depth-Duration-Frequency Diagram for Point Rainfall in Southern Africa. HRU Report No. 2/78, University of Witwatersrand, Johannesburg, SA.
- NAGHAVI BN, YU FX and SINGH VP (1993) Comparative evaluations of frequency distributions for Louisiana extreme rainfall. *Water Resour. Bull.* **29**(2) 211-219.

- NATURAL ENVIRONMENT RESEARCH COUNCIL (1975) Flood Studies Report. NERC, London, UK.
- SCHULZE RE (1984) Depth-duration-frequency studies in Natal based on digitised data. In: Maaren H (ed.) South African National Hydrology Symposium. Technical Report TR 119, 214-235. Dept. of Environment Affairs, Pretoria, SA.
- SCHULZE RE and DENT MC (1982) Development of a Data File of Autographic Raingauge Records in Southern Africa. ACRU Report No. 16. Department of Agricultural Engineering, University of Natal, Pietermaritzburg, SA.
- SMITHERS JC (1991) Short duration extreme value rainfall analysis using a digitised database. Paper presented at 8th SASAS Conf., Pretoria. SA.
- SMITHERS JC (1993a) The effect on design rainfall estimates of errors in the digitised rainfall database. *Proc. 6th SA Natl. Hydrol. Symp.*, Pietermaritzburg, 95 102.
- SMITHERS JC (1993b) Rainfall probability analysis as an aid to decision making in agriculture. *Agric. Eng. in SA* **25** 245-355.
- STEDINGER JR, VOGEL RM and FOUFOULA-GEORGIOU E (1993) Frequency analysis of extreme events. In: Maidment DR (ed.) *Handbook of Hydrology*. McGraw-Hill, New York, USA.
- STEEL RGD and TORRIE JH (1980) Principles and Procedures of Statistics: A Biometrical Approach. McGraw-Hill, London, UK.
- VOGEL RM and FENNESSEY NM (1993) L-Moment diagrams should replace product moment diagrams. Water Resour. Res. 29(6) 1745-1752.
- VOGEL RM, MCMAHON TA and CHIEW FHS (1993) Floodflow frequency model selection in Australia. *J. Hydrol.* **146** 421-449.
- VOGEL RM, THOMAS WO and McMAHON TA (1993) Flood-flow frequency model selection in southwestern United States. *J. Water Resour. Planning and Manage*. **119** (3) 353-366.
- WEDDEPOHL JP (1988) Design Rainfall Distributions for Southern Africa. Unpublished M.Sc. Thesis. Department of Agricultural Engineering, University of Natal, Pietermaritzburg, SA.