A model for batch settling curve

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Abstract

A model is developed for describing sludge blanket interface settling in a batch reactor. The sludge settling curve is considered as one entity, not as a conglomerate of several separate parts, and thus described in one model. The model interprets sludge settling as a time-dependent phenomenon and it describes the settling process from the beginning to the end. Since the whole settling curve is modelled, sludge settleability is accurately assessed. Two parameters of the model can be easily estimated with a non-linear estimation method. The derivative of the model can be used for computing sludge blanket interface settling velocity. The agreement between the observed and the computed activated sludge settling curves shows that the proposed approach is justified.

Nomenclature

α ($kg \cdot m^{-2} \cdot h^{-1}$) and C ($m \cdot h^{-1}$)	are parameters describing sludge settling
$\alpha(X,h_0)$	a function dependent on activated sludge concentration and the initial sludge blanket interface level (h^{-1})
β	a parameter for modelling small concentrations ($kg^2 \cdot m^{-6}$)
h_{long}	the sludge blanket interface level (m) after a long settling period
h_t	the sludge blanket interface level (m) at time t
h_0	the initial sludge blanket interface level (m)
SVI	sludge volume index ($m\ell \cdot g^{-1}$)
t	time (h)
X	activated sludge concentration ($kg \cdot m^{-3}$)
ZSV	zone settling velocity ($m \cdot h^{-1}$).

Introduction

Successful waste-water treatment requires that particles which are formed during biological and chemical treatment should be removed from the water. A predominant unit process for solids separation is gravity sedimentation. Increased influent flow during sustained rainy periods, as well as the tendency for chemical-free waste-water treatment, cause additional requirements for settling tanks operation.

Operation of final settlers can be optimised only when sludge settleability is known. In addition, detection of sludge settleability problems in daily monitoring allows operators of waste-water treatment plants to take measures to avoid or reduce operational problems. The optimisation of clarifier operation and the significance of observation are dependent on the accuracy of the method describing sludge settleability.

Settling properties of activated sludge are generally described by SVI. Despite its widespread use, SVI has been strongly criticised because of its shortcomings: two differently settling sludges can have identical SVIs (Fig. 1), it depends largely on initial sludge concentration (Fig. 2), and has limitations in comparison to settleability of sludges from different plants. Due to these constraints, several alternative methods for settleability description have been suggested and investigated (e.g. Dick and Vesilind, 1969; Fitch and Kos, 1976: Wilson, 1983; Hultman et al., 1991; Catunda and Van Haandel, 1992).

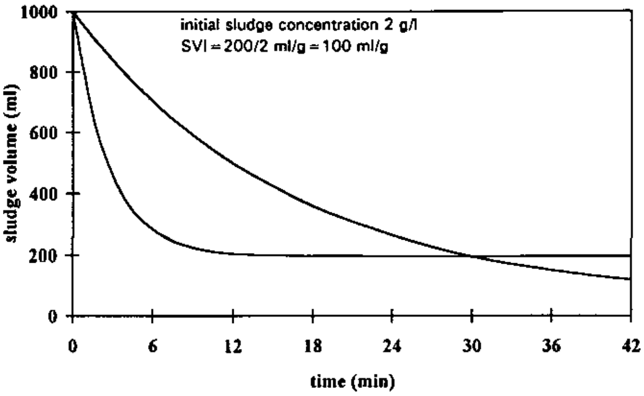

Figure 1
Two sludges with different settling characteristics have identical SVI values

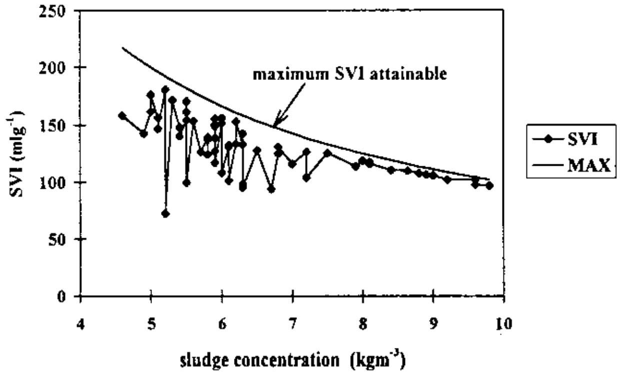

Figure 2
The effect of initial sludge concentration to SVI (Kiuru et al., 1996)

ZSV of activated sludge is one of the experimental methods used for describing sludge settling, instead of SVI. ZSV is well known but not as much used in daily monitoring of treatment plants as the traditional SVI. Measuring for ZSV is laborious and

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Received 2 October 1995; accepted in revised form 8 July 1996.

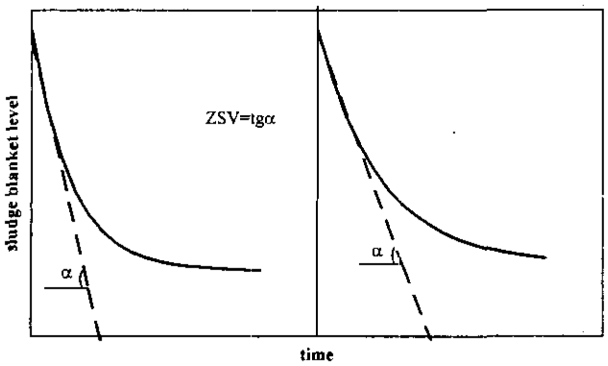

Figure 3
Typical activated sludge blanket interface settling curves in a batch reactor and determination of zone settling velocity

probably for this reason it has not become a standard method for determining sludge settling properties in waste-water treatment plants. Determination of ZSV is illustrated in Fig. 3. This figure also shows the main constraint of ZSV: it totally ignores the end of the settling curve and interprets sludge settling as a constant.

The most important use of ZSV is probably in the settler models, specially in flux theory. Since the pioneer work of Coe and Clevenger (1917), ZSV has been used for decades as a basis for modelling settler operation. However, despite the extensive use of the flux theory, its shortcomings have become evident. For example, Vesilind and Jones (1990) noted that frequent underdesign of continuous thickeners could be due to the incorrect ZSV results obtained for certain concentrations. According to Cacossa and Vaccari (1994) both theoretically and experimentally identified inadequacies associated with this method have become apparent: the theory cannot predict the existence of a sludge blanket in an underloaded thickener and it does not account for the final distribution of concentrations in a completely settled sludge. Ozinsky and Ekama (1995) stated that ZSV had been restricted to research circles and the use of the flux theory had not been widely accepted for design and operation of secondary settling tanks. Krebs (1995) concluded that improvements of 1D settler models required more complex mathematical approaches and 2D had great development potential as a research tool, provided that sludge properties were better characterised. In addition, the theory can be doubted due to the facts that several modifications have been developed (e.g. Laikari, 1988) without final agreement, and the needs for the development of models have been documented (e.g. European Commission, 1995).

It is obvious that sludge settling is not one parameter phenomenon as ZSV suggests. Thus, it might be doubted that the shortcomings of the flux theory and the models based on ZSV are partly due to a simplified description of the sludge settling curve. The recent works by Cacossa and Vaccari (1994), and Tenno and Uronen (1995) are good examples of using the whole settling curve in the modelling of settler operation. However, the idea of using the whole batch curve is not new. Talmadge and Fitch (1955) proposed a graphical method to determine thickener unit areas from the Kynch (1952) theory of sedimentation.

There are well-known methods for describing parts of a settling curve, such as ZSV and the Roberts's (1949) equation for the compression zone, but not for describing the whole settling curve. In order to get comprehensive conception of the phenomenon of sludge settling, and to reduce the number of equations and the parameters (i.e. to simplify the description), a simple model for the sludge settling curve is proposed.

The model

Figure 3 presents typical activated sludge interface settling curves in a test settler. Sludge settling interface level has here been considered to be of similar behaviour as the first rate chemical reaction by taking into account that the curve does not reach the zero level. A model for the sludge settling curve is written as:

$$h_t = h_{long} + (h_0 - h_{long})e^{-\alpha(X,h_0)t} \qquad (1)$$

where:

t is time
h_t is the sludge blanket interface level at time t
h_{long} is the sludge blanket interface level after a long settling period
h_0 is the initial sludge blanket interface level
$\alpha(X,h_0)$ is a function dependent on activated sludge concentration and the initial sludge blanket interface level
X is activated sludge concentration.

The term h_{long} brings additional information to the model and that is not the aim of a well-posed model. The derivative of Eq. (1) can be written as:

$$\frac{dh_t}{dt} = -\alpha(X,h_0)\,h_t + h_{long}\,\alpha(X,h_0) \qquad (2)$$

By replacing $h_{long}\ \alpha(X,h_0)$ with C, the sludge blanket interface level after a long settling period can be written as:

$$h_{long} = \frac{C}{\alpha(X,h_0)} \qquad (3)$$

With this transformation, h_{long} can be replaced and its use avoided in the model.

The function $\alpha(X,h_0)$ is proposed to have the form:

$$\alpha(X,h_0) = \frac{\alpha}{X\,h_0} \qquad (4)$$

Equation 4 indicates that the higher the sludge concentration and the initial sludge blanket interface level are, the more slowly the sludge settles and the higher the ultimate sludge blanket level is. By taking into account Eqs. (1) to (4), the model can be written into the final form as:

$$h_t = \frac{CXh_0}{\alpha} + (h_0 - \frac{CXh_0}{\alpha})e^{-t\alpha/(Xh_0)} \qquad (5)$$

According to Coulson et al. (1991), if the position of the sludge line is plotted as a function of time for two different heights, the curves of the form shown in Fig. 4 are obtained in which the ratio OA':OA" is constant everywhere. Thus, if the settling curve is obtained for any initial height, the curves can be drawn for any other height. It is easy to prove that the proposed model agrees with this feature of the settling curve (see **Appendix**).

Velocity can be thought of as a rate of change and it can be calculated as a derivative of the h_t. Hence, sludge settling velocity

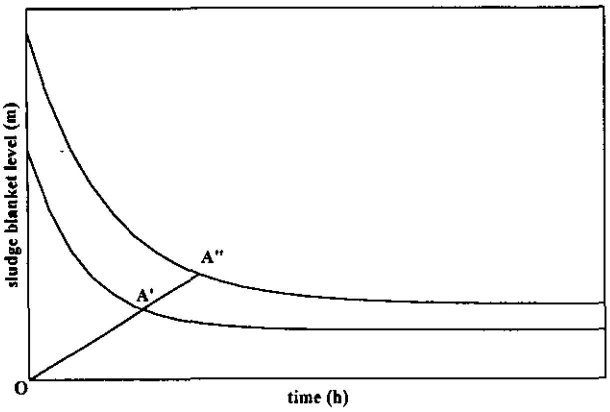

Figure 4
The effect of the initial sludge blanket height on sedimentation

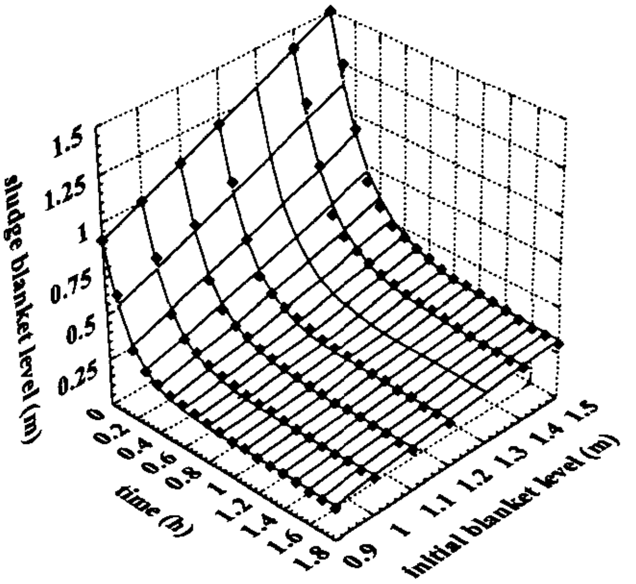

Figure 5
Measured and computed (by Eq. (5)) sludge blanket interface settling curves at Suomenoja waste-water treatment plant on June 13, 1995

can be determined as a derivative of Eq. (5):

$$\frac{dh_t}{dt} = -\left(\frac{\alpha}{X} - C\right)e^{-t\alpha/(Xh_0)} \tag{6}$$

Equation (6) describes sludge blanket interface settling velocity as a function of time. By rearranging the Eq. (6) and taking into account Eq. (5) the sludge settling velocity can be written as a function of sludge blanket level as follows:

$$\frac{dh_t}{dt} = -\frac{\alpha}{X\,h_0}\,h_t + C \tag{7}$$

The minus sign in Eqs. (6) and (7) indicates that the sludge blanket level is decreasing and it is a mathematical result of the presumption in the settling model (Eq. (1)).

Materials and methods

The data needed for the parameter estimation are activated sludge concentration at the commencement of the batch test and sludge blanket interface level as a function of time (see Eq. (5)).

The experimental data were collected at the Suomenoja waste-water treatment plant in Espoo on June 13 and 26, 1995. The plant treats a domestic waste-water load of 230 000 inhabitants by an activated sludge process with simultaneous precipitation of phosphorus.

The settling of the activated sludge was measured manually at six-minute intervals in graduated cylinders. For testing the effect of the initial heights on the settling curve, sludge settling was measured at the initial levels of 1 500, 1 400, 1 200, 1 100, 1 000 and 900 mm.

For testing the effect of activated sludge concentration on the settling curve, experiments with sludge dilutions of 150:150 (proportion of the sludge to the total sample volume), 143:150, 140:150, 135:150, 130:150: 125:150, and 120:150 were carried out. The initial sludge blanket level was 1 500 mm.

The dia. of the test cylinders was 200 mm and the tolerance of sludge blanket interface height readings 10 mm. The sludge in the test cylinders was allowed to settle without being stirred. The sludge samples were pumped into the cylinders from the aeration tank with a subaqueous pump through a 25 mm hose. Activated sludge concentration was determined at the start of the experimental periods.

The parameters were estimated by using Eq. (5) as a nonlinear regression model. The method used to optimise the parameters was the multivariate secant method (DUD method).

Results

For testing the effect of the initial sludge blanket level, the parameters were estimated by using parallel measurements in the same model (i.e. considering the measurement result as a surface which has three axes: t, h_t and h_0) instead of estimating the parameters separately for each curve. The estimation resulted in 13.94 $kg\cdot m^{-2}\cdot h^{-1}$ for α and 1.16 $m\cdot h^{-1}$ for C. The sum of squares for the regression was 25.9099 and for the residual 0.0673 m^2. Figure 5 shows the measured and the computed sludge settling curves. Figure 6 shows the sludge settling velocities as a function of time computed from the experimental data and with the model.

The parameter estimation for the data measured with different sludge concentration gave the values of 13.14 $kg\cdot m^{-2}\cdot h^{-1}$ for α and 1.27 $m\cdot h^{-1}$ for C. The sum of squares for the regression was 47.6917 and for the residual 0.1310 m^2. Figure 7 shows the measured and the computed sludge settling curves. Figure 8 shows the sludge settling velocities as a function of time computed from the experimental data and with the model.

Discussion and conclusions

The research on sludge settleability and settler operation modelling has been going on for decades. Despite its long history, agreement on the topic has not been reached. Even if the research goes on actively, the major part of the work has been rather conservative: the old approaches are repeated and modified, and it seems that only a few new ideas have been proposed and adopted. For example, a great number of studies on SVI have been carried out, even through the use of SVI for scientific purposes has been criticised by outstanding researchers, such as Dick and

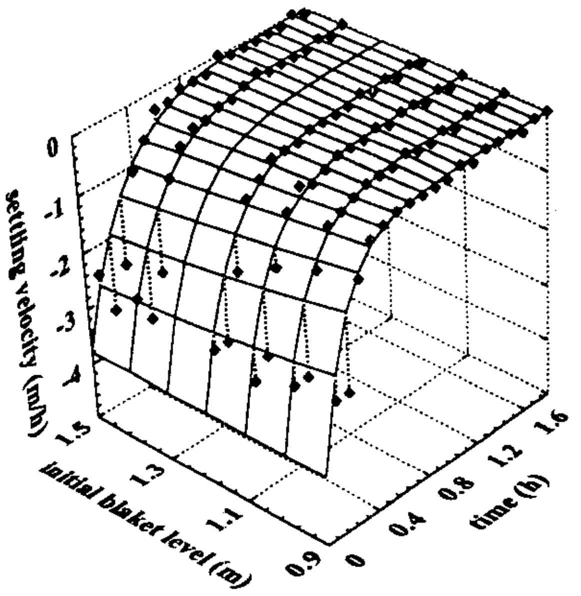

Figure 6
The sludge settling velocities as a function of time computed from the experimental data and with the model

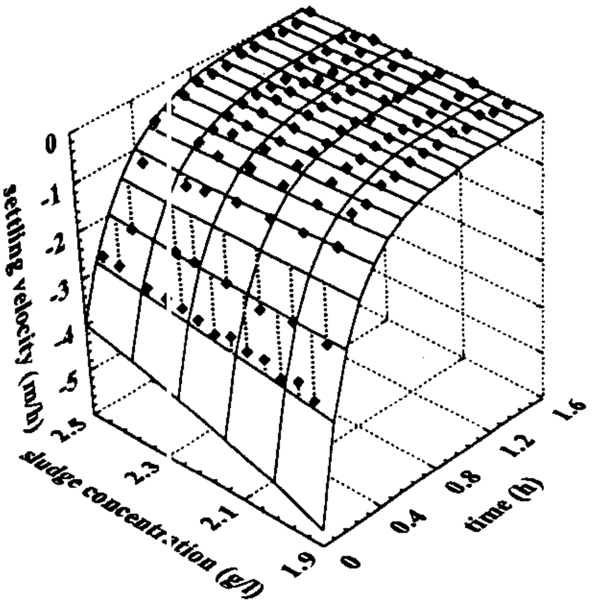

Figure 8
The sludge settling velocities as a function of time computed from the experimental data and with the model

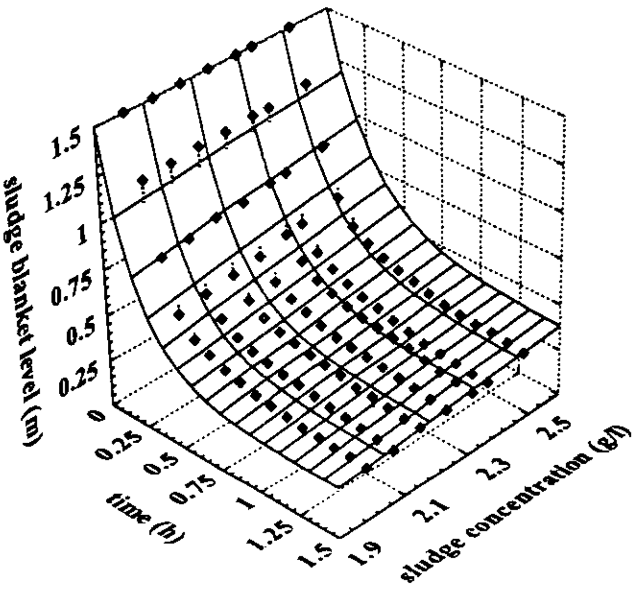

Figure 7
Measured and computed (by Eq. (5)) sludge blanket interface settling curves at Suomenoja waste-water treatment plant on June 26, 1995

Vesilind (1969) and despite its well-known shortcomings. The interpretation might be that better alternative approaches are not available.

The present work suggests a new method for the description of sludge settling. The basic ideas of the work were:

- to interpret sludge settleability as a non-constant, dynamic phenomenon;
- to consider sludge settleability as more than one parameter property; and
- to develop one simple model for describing accurately the whole settling curve. Indeed, sludge settling has been thought of as one entity, not as a conglomerate of several separate parts.

The test results from the Suomenoja waste-water treatment plant (Figs. 5 and 7) demonstrate that the activated sludge settling curve can be accurately described with the model. The sum of squares undoubtedly shows that the model is accurate.

Other apparent advantages of the model are easy and quick determination of parameters and the simple equation.

It is possible to build up a model when taking into account the second derivative of the sludge blanket level, but this does not give better curve fittings and it has the disadvantage of a more complicated structure of the model and more parameters. Therefore, one of the advantages of the proposed model is that it describes settleability with only two parameters. The low number of parameters makes it easier to comprehend the phenomenon on the basis of the parameters.

Activated sludge blanket interface settling velocity is traditionally described as the settling velocity of the linear part of the activated sludge settling curve, i.e. zone settling velocity. To enlarge the definition of the activated sludge settling velocity, a model is suggested, where sludge blanket interface settling velocity is described as a non-constant phenomenon from the beginning to the end. Since the velocity is a rate of change, it can be computed as a derivative of the sludge blanket level h_t.

Figures 6 and 8 show the sludge settling velocities as a function of time computed from the experimental data and with the model. The biggest difference between the settling velocities is in the beginning of the curve. This part is also the most sensitive to the measurement errors. The settling of the sludge is the fastest at that point and as the sludge velocity is computed from four measured values errors will occur. On the other hand, due to the filling arrangements, the sludge sample "flows" in the test cylinder before it starts to settle. This type of mixing before settling can hardly be avoided and it is difficult to estimate its duration and effect on the settling curve with the data examined here. Therefore, more accurate measurements (shorter reading intervals) are needed, if the purpose is to study the settling velocity. This paper focuses on the settling curve: the settling velocity description can be regarded as a side output.

The values of the parameters are estimated with a non-linear approach. The parameters can also be estimated in the linear regression using the least-square-sum approach, if an appropriate statistical program for non-linear estimation is not available. In the linear approach, sludge settling velocity (dh_t/dt) is plotted as a function of the sludge blanket level h_t and the values of the parameters are determined by using Eq. (7) as the linear regression approach. The parameter C and the function $-\alpha(X,h_0)$ can be obtained as the intersection of the line with the Y-axis and the slope of the line, respectively. However, this approach is not as accurate as the non-linear approach and gives systematic errors due to the computed term dh_t/dt. In order to reduce these errors, the reading intervals should be shortened.

The parameters can also be estimated with a real-time method which can be utilised in advanced control systems. The real-time estimation method is developed by using Eq. (7) as a basic equation in modified Kalman filtering. Detailed description of the real-time estimation is presented by Renko (1995).

Activated sludge settling curve and velocity as a function of time can be described by the parameters α and C having no clear physical interpretation. However, simple physical interpretation can be reached: the sludge blanket level after a long settling period can be computed as $C/\alpha(X,h_0)$, and the theoretical maximum settling velocity as $-\alpha/X + C$.

Parameters α and C can be used as indexes characterising settleability, where high value of α indicates well-settling and low value poorly-settling sludge. Respectively, the high value of C indicates poorly-settling and the low value well-settling sludge. The values of the parameters vary considerably depending on the material examined. For example, the parameter values 13.94 $kg{\cdot}m^{-2}{\cdot}h^{-1}$ for α and 1.16 $m{\cdot}h^{-1}$ indicate well-settling activated sludge, whereas the same values for calcium carbonate slurry might not be realistic.

The fact that the model is more sensitive to the changes of the parameter α can be utilised by using α alone as an index describing the settleability. Then, the limitations of this approximation should be understood: α alone does not give an accurate result, it just gives a quick indicator of sludge settleability. For the purposes of accurate settleability analysis and a process model, both of the parameters should be applied.

Coulson et al. (1991) described the procedure of transforming a sludge settling curve into any different initial height. The model and the experimental data are in accordance with Coulson's approach. Thus, the settling parameters determined in a certain initial sludge blanket level can also be applied in curve description with any other initial level. However, problems in accuracy might occur if the test column is not high enough. Cacossa and Vaccari (1994) suggested a minimum depth of 1 200 mm for batch tests in their studies. The present study gave reliable results at the examined initial heights (900 to 1 500 mm).

The effect of sludge concentration is not as simple as the effect of initial sludge level. The literature does not present a clear relationship between the sludge concentration and the settling curve, which could be written into a mathematical form. For that reason the judgement of the proposed relationship has to be done on the basis of the obtained results. The test results show that the effect of sludge concentration can be taken into account in the model when sludge concentration does not vary significantly.

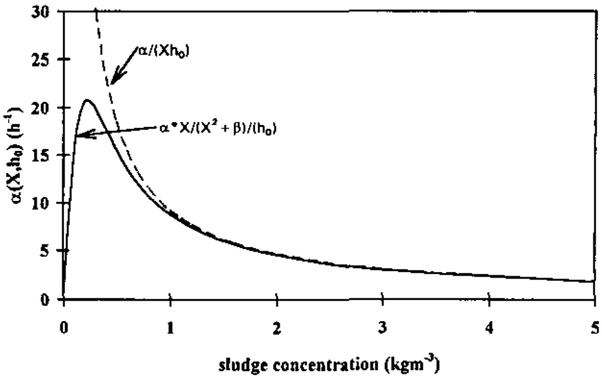

Figure 9

$\alpha(X,h_0)$ as a function of sludge concentration

This means that the settling parameters from different treatment plants operating under normal conditions can be compared.

When modelling settler operation, sludge concentration might be considerably lower. In such a case, the proposed relationship is not applicable and will probably give inaccurate results. One possible equation to take into account the small concentrations might be:

$$\alpha(X,h_0) = \frac{\alpha X}{(X^2 + \beta)h_0} \tag{8}$$

where β is a parameter ($kg^2{\cdot}m^{-6}$) for modelling small concentrations.

Figure 9 shows the effect of β on $\alpha(X,h_0)$ as a function of sludge concentration where $\alpha = 13.94\ kg{\cdot}m^{-2}{\cdot}h^{-1}$, h_0=1.5 m and $\beta = 0.05\ kg^2{\cdot}m^{-6}$.

This figure reveals that Eqs. (8) and (4) give practically the same result when sludge concentration is higher than 1.5 kg/m^3 and the effect of the parameter β on the function $\alpha(X,h_0)$ can only be recognised with lower sludge concentrations. In normal operation ranges, β could be ignored without any risk. However, a low concentration range is not examined here and further research on that matter is needed.

The presented model can be used in modelling the sludge settling curve and the parameters as sludge indexes. Krebs (1995) sought better sludge settling characterisation; the model does that, and offers a simple tool to characterise the sludge settling process accurately. The model can also be utilised in writing the graphical solutions of the Kynch theory of sedimentation (Kynch, 1952) and Talmadge's method (Talmadge and Fitch, 1955) to determine the settler area in a mathematical form. Since the proposed model describes the whole sludge settling process, it opens possibilities to more profound modelling of operation of secondary settling basins. Accurate description of settleability also enables more specific determination of the relationship between the state of the activated sludge process and the sludge settling properties.

Acknowledgements

I want to thank Prof R Laukkanen, Dr R Tenno, Mr M Pelkonen, Ms C McCullough, Mr A Järvinen and the staff of Suomenoja waste-water treatment plant for their valuable help. The English was revised by Ms NF Mai.

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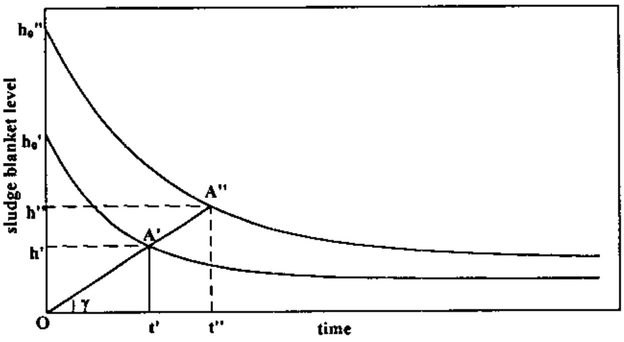
Appendix

Let us assume OA':OA'' is constant for each γ:



It is true that $\cos\gamma = t'/OA' = t''/OA''$
it is also true that $\sin\gamma = h_t'/OA' = h_t''/OA''$ and
$OA'/OA'' = h_0'/h_0''$, when $\gamma = 90°$,

therefore $OA'/OA'' = t'/t''\quad h_t'/h_t'' = h_0'/h_0''$.

Thus, it can be written $h_t'/h_t'' = h_0'/h_0''$.

By replacing h_t' and h_t'' with the proposed model we get

$$\frac{CXh_0'/\alpha+(h_0'-CXh_0'/\alpha)\exp(-t'\alpha/(Xh_0'))}{CXh_0''/\alpha+(h_0''-CXh_0'/\alpha)\exp(-t''\alpha/(Xh_0''))} = \frac{h_0'}{h_0''}$$

and by replacing t'' with $h_0''t'/h_0'$ in the exponent, correspondingly

$$\frac{CXh_0'/\alpha+(h_0'-CXh_0'/\alpha)\exp(-t'\alpha/(Xh_0'))}{CXh_0''/\alpha+(h_0''-CXh_0'/\alpha)\exp(-t'\alpha/(Xh_0'))} = \frac{h_0'}{h_0''}$$

which is equivalent to

$$CXh_0'h_0''/\alpha+(h_0'h_0''-CXh_0'h_0''/\alpha)\exp(-t'\alpha/(Xh_0'))$$
$$= CXh_0'h_0''/\alpha+(h_0'h_0'-CXh_0'h_0''/\alpha)\exp(-t'\alpha/(Xh_0')).$$