An analytical method for the analysis of pumping tests in fractured aquifers

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Abstract

The formula of Cooper-Jacob has been applied to constant-rate pumping tests in fractured aquifers, by considering the fracture and matrix as two separate systems. Some time-drawdown curves which are obtained from the constant-discharge pumping tests display two distinct slopes. The slopes are representative of the fracture and the aquifer matrix systems. The fracture has a high transmissivity and low storativity and the matrix has a low transmissivity and a high storativity. The analytical method considers flow to take place firstly from the fracture to the borehole. The flow from the matrix to the fracture (leakage) will depend on the piezometric gradient and the yield of the matrix is determined from Darcy's law. The yield of the matrix is then used to calculate its storativity. From the fracture flow equations, the fracture flow velocity and also the distance to the boundary are determined. A pumping test was generated with a finite difference numerical model. Boundaries and a fracture were included in the model with hydraulic parameters. The drawdown with time was analysed with the method and the results compared favourably with the parameters assigned to the model. The method is in the form of a computer (Fortran) program called FTA (Fracture Test Analysis) and it can be used to analyse pumping tests in fractured aquifers. It also yields the distance to the boundary which influences the sustainability of the aquifer. By determining more reliable aquifer parameters from pumping tests, improved recommendations can be made concerning sustainable pumping rates for fractured aquifers.

Table of notations

S = Storage coefficient
S_f = Storage coefficient of the fracture
S_m = Storage coefficient of the matrix
ϕ = Piezometric pressure
sw = Drawdown in pumping borehole
∇ = Gradient operator
sw_f = Drawdown in fracture
μ = Viscosity of a fluid
sw_m = Drawdown in matrix
g = Acceleration of gravity
Tm' = Apparent matrix transmissivity
Tf' = Apparent fracture transmissivity
T = Transmissivity
T_f = Transmissivity of the fracture
T_m = Transmissivity of the matrix
xf = Fracture half-length
b = Fracture aperture
X_f = Fracture half-length
Q = Discharge of pumping borehole
Qf = Yield of the fracture
Qm = Yield of the matrix
vf = Fracture flow velocity

Introduction

The hydraulic behaviour of fractured aquifers in South Africa, like the Karoo and other aquifers, differs from the ideal well-known primary aquifers for which most of the analytical analysis techniques have been developed. One particular problem in the fractured aquifers in South Africa is the unsustainable long-term yield which is partly due to incorrect pumping test analysis. It was only during the 1970s that research was focused on the hydraulics of fractured reservoirs in the oil industry (Gringarten and Ramey, 1974).

The main geological formations in South Africa, and thus the aquifers, consist of very old, fractured rocks. Some of these aquifers, like the Karoo aquifers, have a very unique behaviour (Vivier, 1996; Van der Voort, 1996) and the standard techniques like Theis and Cooper-Jacob, fail to analyse these aquifers accurately as far as the determination of the storativity is concerned (Botha, 1993). A common feature is that these aquifers generally have low permeabilities and some large fractures. High- yielding boreholes and waterstrikes can only be obtained by drilling on fractures (like faults and dykes). The frequency and occurrence of these fractures are usually low and sometimes the fractures are isolated. During this investigation, the Cooper-Jacob formula was customised to evaluate single-fracture (fractured zone) aquifers.

The analytical method, FTA, (which is in the form of a Fortran program), is based on the principle that the fracture and aquifer matrix are considered as two separate systems which are interlinked to obtain the combined flow.

Overview of existing models

The conceptual model used, is in accordance with the model of Gringarten (1974) of a single vertical fracture. The model that Gringarten used does, however, not yield the storage coefficient directly; it can be obtained by curve fitting. In his model, Gringarten considers the flow from the matrix to the fracture to be linear during early pumping times and for later pumping times, it becomes radial.

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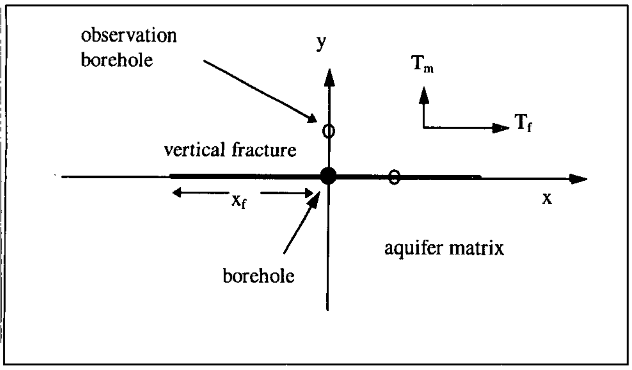

Figure 1
Schematic representation of a single plane vertical fracture in a homogeneous aquifer (after Gringarten, 1982)

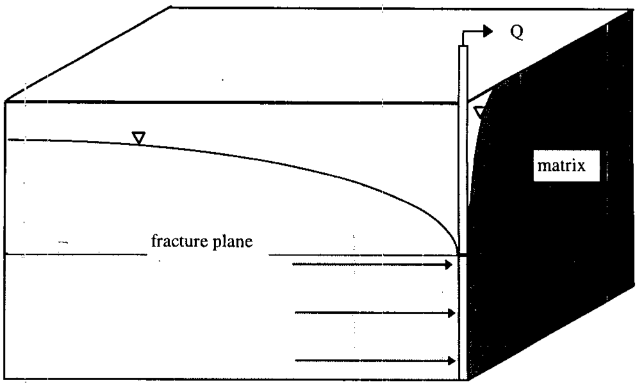

Figure 2
Block model of a fractured aquifer with the cone of depression in the fracture-plane and the cone of depression in the matrix, perpendicular to the fracture

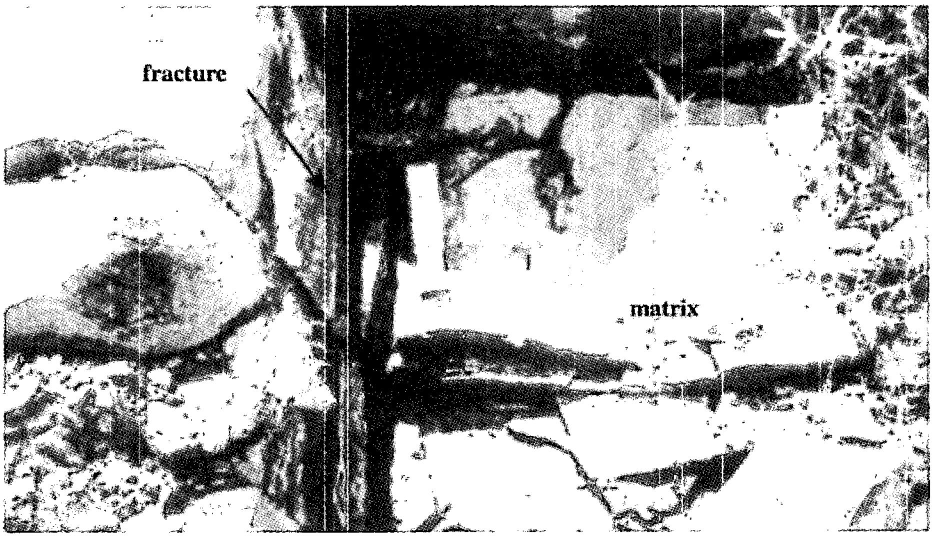

Figure 3
Photograph of a discrete fracture in a mine excavation with water flowing from it

According to Gringarten (1982) the drawdown in the pumping borehole that intersects a single vertical fracture is given by

$$sw = \frac{Q}{2\sqrt{\pi T S x_f^2}}\sqrt{t}$$

where:

sw is the drawdown in the pumped well
T the transmissivity
t the time
S the storativity
x_f^2 the fracture half-length (Fig. 1).

The conceptual model used during this investigation is similar to the single vertical fracture case, but the actual analysis method is different. Cooper-Jacob's formula was used firstly to solve the flow from the fracture to the borehole and secondly from the matrix to the fracture. This model must not be confused with other fractured-aquifer models like the double-porosity model (Kazemi et al., 1969) or the fracture skin model (Moench, 1984). According to Barenblatt et al. (1960), the fracture is considered to have a high transmissivity and low storativity and the aquifer matrix on the other hand has a high storativity and a low transmissivity. Although there already exist models for fractured aquifers, the method used here is technically different and it yields significant results.

Conceptual model

The conceptual aquifer consists of two separate systems in which groundwater flow takes place. The fracture and aquifer matrix have different hydraulic properties, as mentioned, the fracture has a high transmissivity and a low storativity and the matrix has a low transmissivity and a high storativity. When water is pumped from a borehole that intersects a fracture, the dominant flow would be from the fracture to the borehole during early pumping times, with some leakage from the matrix. In the matrix, the flow is perpendicular to the fracture and the shape of the cone of depression and extent of its influence, are thus determined by the direction and extent of the fracture.

The fracture system can be a discrete fracture or a narrow fractured zone. However, observations made in mine excavations and core samples from boreholes, show that some of the water-yielding fractures are discrete (Fig. 3). All of the pumping tests in fractured aquifers in South Africa may not behave like single, bounded fractures (or fractured zones), but in some cases, they do

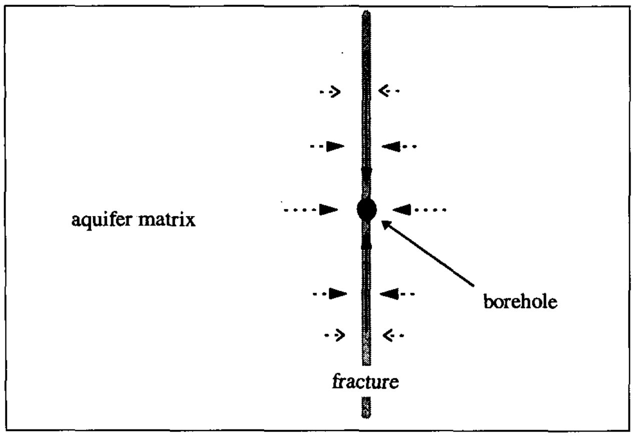

Figure 4
Borehole located on the axis of a vertical fracture with the flow perpendicular to the fracture

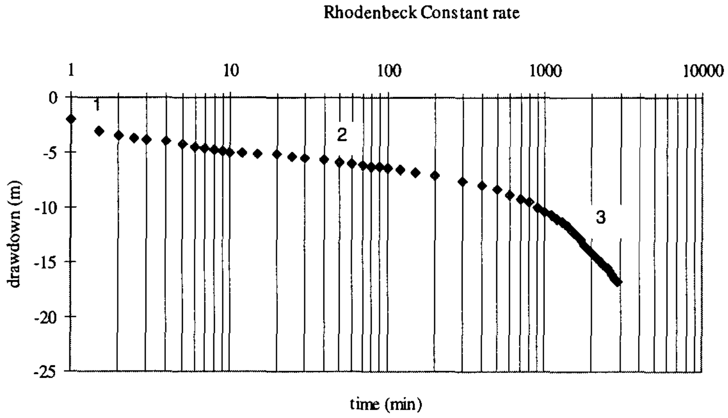

Figure 5
Constant discharge test at Rhodenbeck, Bloemfontein (Karoo aquifer Q=1334 $m^3{\cdot}h^{-1}$ or 15.44 ℓ·s^{-1})

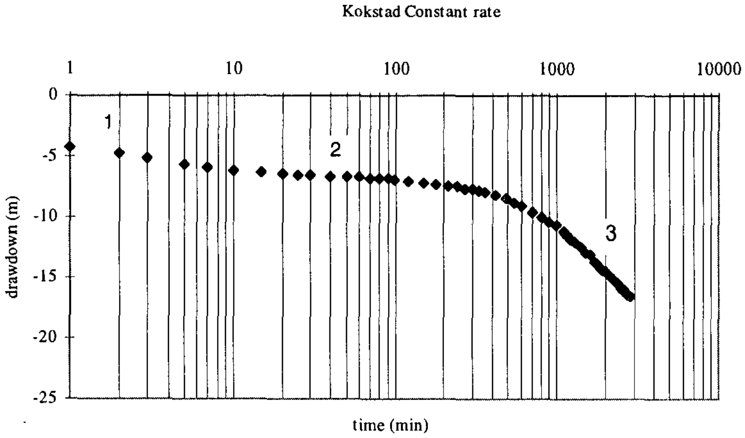

Figure 6
Constant discharge test at Kokstad (Karoo aquifer Q=1123.2 $m^3{\cdot}h^{-1}$ or 13 ℓ·s^{-1})

exist and with the proposed method, it is possible to analyse the tests.

The fracture acts as a sink that pumps from the matrix. The flow from the matrix to the fracture will depend on the transmissivity of the matrix and the piezometric gradient, according to Darcy's law:

$$q = -K\nabla\phi \qquad (1)$$

where:
q is the Darcy flux
$\nabla\phi$ is the piezometric gradient.

The highest flow rate from the matrix will be in the vicinity of the fracture (Fig. 4).

Pumping tests in fractured aquifers

The techniques for the execution and evaluation of pumping tests in fractured aquifers, were originally developed for ideal (primary) aquifers. In fractured aquifers, however, the yield to a borehole is supplied by one or more fracture(s) that dominate the groundwater flow to a borehole. Most of the pumping tests are conducted for short durations of less than 48 h, often because of the expenses associated with longer tests. The behaviour of the water levels with time in boreholes, drilled in Karoo aquifers during short-duration pumping tests, differed significantly from long-duration tests (Vivier, 1996). After longer pumping times, the true behaviour of the aquifer matrix and the transition between the fracture- and matrix-dominant flow periods can be distinguished. In some of the fractured aquifers, borehole yields are controlled by meso-fractures and the yield might drop suddenly once the fracture is dewatered. The sudden drop in yield occurs when the cone of depression reaches a boundary and the yield is dependent on the flow from the matrix.

Reaction of water levels with time during constant-rate pumping tests

Although the term water level is used in the following description of the behaviour of the aquifers during constant discharge pumping tests, most of the water levels are actually piezometric levels, representing the pressure in the fracture. The water level in a borehole is more representative of the piezometric pressure in the fracture than that of the aquifer matrix (Van der Voort, 1996).

Several examples of pumping tests in Karoo and other fractured aquifers in South Africa are given in Figs. 5 to 8. The four tests display the same characteristic time-drawdown curves. On all the curves, two distinct slopes are present that represent different flow regimes. This behaviour is not restricted to Karoo aquifers only. Some pumping tests in other fractured aquifers also exhibit the same behaviour. All of these boreholes are located on large-scale fractures, like faults or dykes. The tests are constant discharge tests that lasted for 48 h and longer.

The semi-log drawdown curves of these pump-

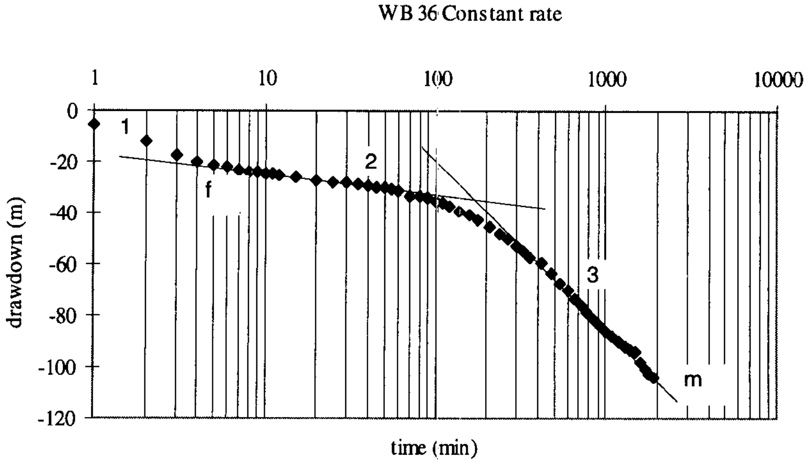

Figure 7
Constant discharge test at Grootegeluk, Ellisras
(Karoo aquifer Q=1593.2 m³·h⁻¹ or 18.44 ℓ·s⁻¹).
f = fracture fit and m = matrix fit.

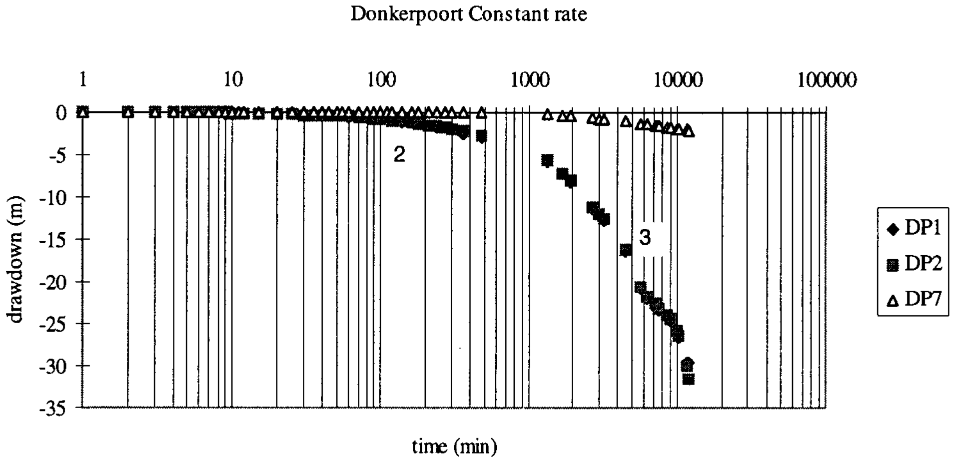

Figure 8
Donkerpoort Constant discharge test (at Thabazimbi)
Banded Iron Formation/Dolomitic aquifer
(Q=1555.2 m³·h⁻¹ or 18 ℓ·s⁻¹)

ing tests display three main stages or slopes. The first stage is a rapid drawdown, after which it enters the second stage. During this stage, the water level seems to stabilise. At longer pumping times, the third stage becomes evident when the slope steepens again. The first stage can be ascribed to storage effects in the borehole and it only lasts for a few minutes, depending on the extraction rate (this stage is not present at the Donkerpoort test (Fig. 8).

The stabilising effect of the second stage is due to the contribution of the fracture storativity. Its contribution lasts only for a limited time because of the limited extent of the fracture. Although the internal porosity of the fracture is high, its volume is negligible when compared to the volume of water contained in the aquifer matrix. The steepening of the slope is caused by a boundary. The contribution of the fracture lasts only until the cone of depression reaches a no-flow boundary. These boundaries can be due to geological barriers or the termination of the fracture. The last stage prevails at longer pumping times and it is representative of flow from the matrix to the fracture over the surface area of the fracture. If the Donkerpoort test was to be stopped after three days, this behaviour would not be visible on the time-drawdown curve.

The difference between the classical Cooper-Jacob and the proposed method is in the calculation of the parameters and the yield that is used to calculate the matrix storativity. With this method, the parameters are calculated separately by using firstly Stage 2 to obtain the parameters of the fracture and then Stage 3 to obtain the matrix parameters.

Modified Cooper-Jacob's method for flow in fractured aquifers

Cooper and Jacob (1946) derived their formula from the Theis equation. The preconditions, are however, that the aquifer must be horizontal, homogeneous and of infinite extent. The well-known formula of Cooper-Jacob:

$$sw = \frac{2.30Q}{4\pi T}\log\frac{2.25T\,t}{r^2S} \qquad (2)$$

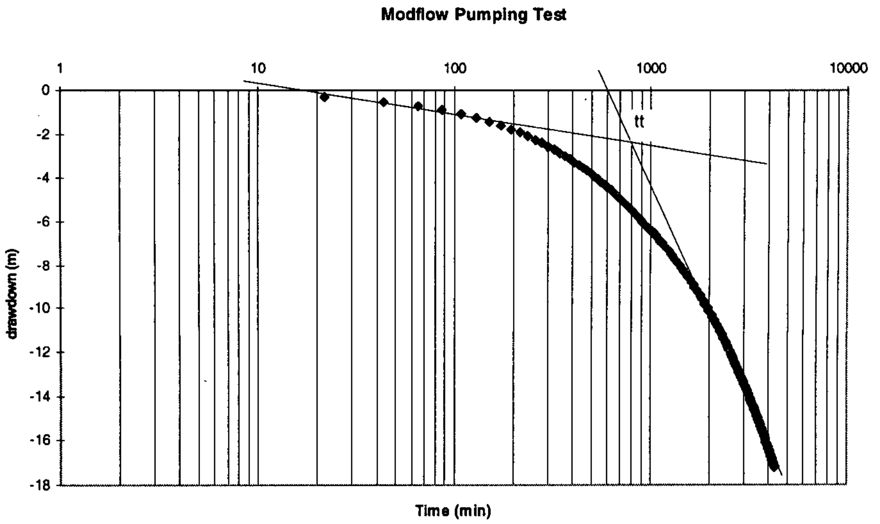

Figure 9
Time-drawdown graph generated by the Modflow pumping test

where:
sw = drawdown
Q_w = discharge rate of the borehole
r = distance from pumping borehole
S = storage coefficient
T = transmissivity
t = time in days

was also developed for an ideal (homogeneous, infinite extent) aquifer (Kruseman and De Ridder, 1992). The reason why unrealistic results are obtained for parameters such as storativity, is that the formula does not account for the variation in hydraulic parameters such as between the fracture and the aquifer matrix. As shown in Table 2, very high values for the matrix storativity are obtained with the classical method. This is because when the Cooper-Jacob line is fitted, it is being done with a discharge rate of the pumping borehole. In the next section, it will be shown that the true storativity for the aquifer matrix can be determined with some alterations to the method of Cooper-Jacob.

Identification of aquifer parameters

The method makes use of the same formula but the fracture and the matrix are considered separately. The computer program FTA (Fracture Test Analysis), makes use of the principle that the flow is primarily from the fracture towards the borehole and then from the matrix perpendicular towards the fracture. The fracture acts as a sink for the matrix.

This means that the water at any point in the aquifer matrix does not necessarily flow towards the borehole, but linearly towards the fracture (Gringarten, 1974). Only once the water enters the fracture, does it flow to the borehole. The fracture has a large surface area so that it is able to tap water from the matrix.

The program mainly has two parts. The first part is used to identify the parameters of the fracture system and the second part determines the matrix parameters. The program uses the data of a pumping test for the analysis. It will then determine the parameters of the two flow periods. The apparent fracture transmissivity ($T_f' = T_f + T_m$) is determined from the second slope (Fig. 7) by:

$$T_f' = \frac{2.30\,Q_w}{4\pi\,\Delta s_{wf}} \tag{3}$$

(Kruseman and De Ridder, 1992). The transmissivity of each observation is determined by calculating the gradient per log cycle and an average transmissivity is calculated for the final value. This is important because the values for early and late time data can differ even on the same slope. The transmissivity calculated here, is actually the transmissivity of the fracture plus that of the matrix (from leakage). Once the fracture transmissivity is known, the fracture storativity is determined from

$$\log S_f = \log\frac{2.25\,T_f}{r^2} - \frac{s_{wf}\,4\pi\,T_f}{2.30\,Q_w} \tag{4}$$

The storativity is calculated for each observation of the fracture flow period and the average value is again determined. The time between the fracture and matrix flow periods can be obtained by fitting two lines on the respective slopes (see Fig. 9). The transitional time (t_t) is determined from the intersection and it represents the transition between the fracture- and matrix-dominated flow periods since it happens when the boundary is reached. It can be used to determine the distance to the boundary. The following formula is used to determine the equivalent fracture aperture:

$$b = \sqrt[3]{-\frac{12\mu\,T_f}{\rho\,g}} \tag{5}$$

(Tsang, 1992 ; Novakowski, 1996). From the fracture aperture, the average fracture flow velocity can be calculated .

$$\overline{v_f} = \frac{Q_f}{b} \tag{6}$$

(Berkowitz and Bear, 1992). Once the fracture flow velocity is known, the distance to the boundary (x_f) can be obtained by:

$$x_f = v_f \times t_t \tag{7}$$

TABLE 1
COMPARISON OF RESULTS OBTAINED WITH FTA AND THE MODEL
(THE DATA OF THE PUMPING BOREHOLE WERE USED)

Method	Q ($L s^{-1}$)	T_f (m^2d^{-1})	S_f	T_n' (m^2d^{-1})	T_m (m^2d^{-1})	S_m
Model	33	50	$1.0x10^{-5}$		0.001	$2.0x10^{-3}$
FTA	33	201	$6.5x10^{-5}$	43	0.02	$1.0x10^{-3}$
AQUITEST	33	244	1.06	22.6		$2.4x10^{11}$

TABLE 2
ANALYSIS OF PUMPING TESTS WITH FTA AND AQUITEST

Test	Q ($L s^{-1}$)	T_f ($m^2 d^{-1}$)	S_f	$T_{m'}$ ($m^2 d^{-1}$)	T_m ($m^2 d^{-1}$)	S_m	X_f (m)
FTA							
Rhodenbeck	15.40	91	$3.13x10^{-8}$	39	$3.7x10^{-2}$	$6.22x10^{-4}$	539
Kokstad	13.00	71	$8.3x10^{-6}$	23	$3.1x10^{-2}$	$3.59x10^{-4}$	380
WB36	18.40	19	$4.2x10^{-7}$	8.02	$1.61x10^{-2}$	$7.8x10^{-4}$	243
DP 9	18.00	406	$1.8x10^{-6}$	19	$1.60x10^{-2}$	$1.2x10^{-3}$	537
Aquitest							
Rhodenbeck	15.40	270	0.14	15.55		>1	
Kokstad	13.00	271	$3.3x10^{-6}$	15.7		>1	
WB36	18.40	33.7	$5.0x10^{-4}$	4.1		0.5	
DP 9	18.00	278	>1	11.37		>1	

If it is assumed that the borehole is situated in the centre of the fracture, the distance to the boundary is also the fracture half length. The next step is to determine the matrix parameters. The apparent matrix transmissivity which is the matrix transmissivity over the length of the fracture, is determined by calculating the gradient per log cycle on the last slope for every observation by:

$$T_m' = \frac{2.30\, Q_w}{4\pi\, \Delta s_{w_m}} \tag{8}$$

This value represents the matrix transmissivity (leakage) over the length of the fracture. An average value is determined from all the observations. Because the fracture half-length is known, the true matrix transmissivity can be determined by:

$$T_m = \frac{T_m'}{2x_f} \tag{9}$$

The true fracture transmissivity can also be obtained from:

$$T_f = T_f' - T_m' \tag{10}$$

In order to determine the matrix storativity, the yield of the matrix (Q_m) must be determined. The yield of the matrix will be according to Darcy's law:

$$Q_m = T_m \nabla\phi \tag{11}$$

where $\nabla\phi$ is the gradient in piezometric pressure (Bear, 1979) which can be determined for every time-drawdown measurement as,

$$Q_m = T_m \left(\frac{sw_{m(i)}}{r} \right) \tag{12}$$

where $sw_{m(i)}$ is the drawdown at each observation during the matrix flow period. From Eq. (12.) it can be seen that the yield of the matrix will increase as the drawdown or gradient increases. The next step is to determine the matrix storativity (S_m) with:

$$\log S_m = \log \frac{2.25\, T_m}{r^2} - \frac{sw_m 4\pi\, T_m}{2.30\, Q_m} \tag{13}$$

The matrix storativity is again determined for each observation on the last slope and the average value is obtained. This is also essential where this method differs from the classical method. Storativity values obtained with this method are much more in line with values determined from numerical models.

Results

The following constant-rate pumping test (Fig. 9) was generated with the Modflow numerical model (Zhang et al., 1996). A block model aquifer was constructed in Modflow. The aquifer with dimensions (2 000 m x 2 000 m) was isolated with no-flow boundaries and the borehole was situated in the middle on a high transmissivity zone which represents a fractured zone.

A constant discharge pumping test was simulated by pumping the aquifer for 4 320 min. From the time-drawdown curve, the two prominent flow regimes can be seen (Fig. 9). The test was analysed with FTA and the results were compared with the values assigned to the model. The test was also analysed with Aquitest (Waterloo Hydrogeologic Inc., 1996) which uses the standard Cooper-Jacob method.

The distance to the boundary (x_f), calculated with FTA, is 853 m whereas the known distance in the model is 1 000 m. This is important because the approximate dimensions of the reservoir can be determined from the pumping tests. From the results listed in Table 1, the fracture transmissivities determined by FTA and Aquitest are too high. This is due to storage effects (or storativity) of the fracture. However, by using the fracture transmissivity, its storativity can be determined. The apparent matrix transmissivity, given in Table 1, actually represents the fracture transmissivity assigned to the model The results show that with this method, reliable values for the fracture and matrix transmissivity and storativity can be obtained.

The storativity values obtained from FTA are more accurate than existing analytical techniques. The pumping tests in Figs. 5 to 8 were analysed with FTA and the following results were obtained. A minimum distance (r) of 1 m for pumping boreholes was used.

Discussion and conclusion

The analytical methods used to analyse pumping tests in fractured aquifers, were developed for ideal (homogeneous and infinite extent) aquifers. The method of Cooper-Jacob is one of these techniques. The techniques still succeed in determining the transmissivity of fractured aquifers, but the storativity values obtained, are unreliable. Pumping test data are the most basic information obtained during any geohydrological investigation. It is therefore important to obtain reliable parameters. Some time-drawdown curves display two distinct slopes on a semi-log plot. The two slopes are representative flow periods dominated during early pumping times by the fracture and at later pumping times by the aquifer matrix.

A pumping test was simulated by constructing a model of a bounded block model aquifer with a vertical fractured zone. The data were analysed with FTA and the results show that by analysing the fracture-dominated flow period, a very high fracture transmissivity is obtained. This is due to the storage effect of the fracture before the cone of depression reaches the boundary, but this transmissivity can be used to determine the fracture storativity. After the storage effects of the fracture are depleted, the true fracture transmissivity becomes apparent (during the last slope). The transmissivity computed from the matrix-dominated flow period corresponds with the fracture transmissivity assigned to the model and it can be used to determine the matrix storativity. From the fracture flow equations, the average fracture flow velocity and consequently the distance to the boundary can be determined from the transitional time (Fig. 9). The distance that is calculated to the boundary corresponds with the boundary distance included in the model.

Advantages of the method are:

- The aquifer parameters of the fracture and matrix systems can be obtained separately.
- Reliable storativity values can be determined, especially for the aquifer matrix.
- Parameters can be obtained for observation boreholes which are situated in the matrix.
- The approximate distance to the aquifer boundary can be determined.
- The true transmissivity of the matrix can be obtained.
- The program is very simple to use.

One of the remaining problems is sustainable aquifer management in South Africa. The major uncertainty is the existence of boundaries that restrict groundwater flow. One particular advantage of this method is that it yields the distance to the boundary and the actual yield of the matrix. Further research is in the process to develop a method that will use the output of FTA to simulate the effect of no-flow boundaries and to determine sustainable pumping rates for bounded, fractured aquifers.

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References

AQUITEST (1996) *Aquifer Test* Version 2.0 by Waterloo Hydrogeologic.

BARENBLATT GE, ZHELTOV IP and KOCHINA IN (1960) Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks. *J. of Appl. Math. and Mech.* 1286-1303.

BEAR J (1979) *Hydraulics of Groundwater.* Mc Graw-Hill International. New York.

BERKOWITZ B and BEAR J (1992) Hydrological Modelling of Flow and Pollution in dry Regions. Transport Processes in Porous and Fractured Media. Lecture notes for course on transport processes.

BOTHA JF (1993) Analysis of aquifer tests in hard rock aquifers. In: *Proc. of the Groundwater Symp.* University of the Witwatersrand.

COOPER HH and JACOB CE (1946) A generalised graphical method for evaluating formation constants and summarising well field history. *Am. Geophys. Union. Trans.* **27** 526-534.

GRINGARTEN AC (1982) Flow-test evaluation of fractured reservoirs. *Geol. Soc. Am. Special Paper.* 237-263.

GRINGARTEN AC (1974) *Unsteady-State Pressure Distributions Created by a Well With a Single Infinite-Conductivity Vertical Fracture.* American Institute of Mining, Metallurgical, and Petroleum Engineers. San Antonio, Texas. 347-360.

GRINGARTEN AC and RAMEY HJ Jr. (1974) Unsteady state pressure distributions created by a well with a single horizontal fracture, partial penetration or restricted entry. *Soc. Pet. Eng. J.* 413-426.

KAZEMI H, SETH MS and THOMAS GW (1969) The interpretation of interference tests in naturally fractured reservoirs with a uniform fracture distribution. *Soc. Pet. Eng. J.* 463-472.

KRUSEMAN GP and DE RIDDER NA (1991) *Analysis and Evaluation of Pumping Test Data.* (2nd edn.) International Institute for Land Reclamation and Improvement, Wageningen, The Netherlands.

MOENCH AF (1984) Double-porosity models for a fissured groundwater reservoir. *Water Resour. Res.* **20** 1031-1045.

NOVAKOWSKI KS (1996) Hydraulic Testing in Fractured Rocks. Course presented at the Institute for Groundwater Studies, Bloemfontein.

TSANG YW (1992) Usage of "Equivalent Apertures" for rock fractures as derived from hydraulic and tracer tests. *Water Resour. Res.* **28** (5) 1451-1455.

VAN DER VOORT I (1996) An Investigation Into the Geometry of Karoo Aquifers and the Effect this has on their Ability to Yield Water. Unpublished M.Sc. Thesis. Institute for Groundwater Studies, University of the Orange Free State, Bloemfontein.

VIVIER JJP (1996) The Influence of Geology on the Geohydrology of Karoo Aquifers. Unpublished M.Sc. Thesis. Institute for Groundwater Studies, University of the Orange Free State, Bloemfontein.

ZHANG Y, ZHENG C, NEVILLE CJ and ANDREWS CB (1996) Modime, An integrated Modelling Environment for MODFLOW, PATH3D and MT3D. Version 1.1.