Recharge-incorporated approximate solution to tracer experiments under pumping conditions

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Abstract

Tracer experiments under radially convergent flow (or pumping) conditions are conducted to obtain effective porosity and dispersivity required for detailed evaluation of groundwater resources and pollution problems at selected sites. However, interpretation of the tracer experiment is not readily available. Existing Laplace transform solutions often need to be numerically evaluated. This paper derives an approximate analytical solution which is able to account for a scenario of a vertical recharge. As special cases, the two commonly used approximate analytical solutions are obtained from this recharge-incorporated approximate solution. The approximate analytical solutions are applied to a tracer experiment conducted in a Karoo aquifer on the campus site at the University of the Orange Free State, Bloemfontein. All the computations can be performed on a spreadsheet. This approach proves to be powerful and user-friendly. The problems associated with the interpretation are also discussed.

Introduction

The tracer experiment under a radially convergent flow field is appealing because ideally it should be possible to recover all of the tracer in the course of the experiment and consequently minimise contamination to the aquifer involved. However, no complete analytical solution is available for interpretation of such tracer experiments. An effort has been made to obtain Laplace transform solutions (Moench,1989, Maloszewski and Zuber, 1990 and Chen et al., 1996). Their solutions often need to be numerically evaluated. Therefore the existing Laplace transform solutions are not readily available to field hydrogeologists.

Based on the solution for a Dirac-pulse with constant dispersivity in one-dimensional uniform flow, this paper presents a general form of approximate analytical solutions to account for scenarios of both confined and unconfined aquifers. The special solutions of the general form, compatible with existing approximate solutions derived by Lenda and Zuber (1970) and Sauty (1978), are numerically evaluated. The approximate solutions are applied to a tracer experiment conducted in a Karoo aquifer on the campus site at the University of the Orange Free State, Bloemfontein.

Theory

A tracer experiment under radially convergent flow (or pumping) conditions is sketched in Fig. 1. The experiment has two components, i.e. a tracer dilution in the injection hole and the tracer being pumped out from the pumping hole. A tracer breakthrough curve (BTC) detected from the pumped water is normally used to estimate effective porosity and longitudinal dispersivity.

Derivation of approximate solutions for the tracer experiment is based on the solution for a Dirac-pulse with constant dispersivity in one-dimensional uniform flow. As shown in Fig. 1, the pulse is introduced along the line L_{ab} at time $t = 0$ (s).

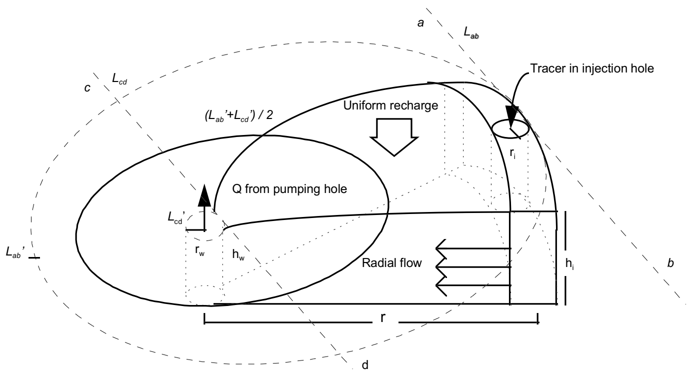

Figure 1
Sketch of tracer experimental configuration (not to scale)

The concentration monitored along the line L_{cd} at any time t (s) is given by

$$C(r,t) = \frac{M}{2An\sqrt{\pi a_L ut}} e^{-\frac{(r-ut)^2}{4D_L t}} \qquad (1)$$

where M (g) is the injected mass of the tracer; A (m^2) is the averaged cross-sectional area that the tracer travels through ($A = L_{ab}h_i = L_{cd}h_i$); n is the effective porosity; a_L (m) is the longitudinal dispersivity; u (m/s) is the averaged pore-water velocity of flow over a distance r (m); D_L (m^2/s) is the longitudinal dispersion coefficient ($D_L = a_L u$ if diffusion is negligible).

For adaptation of Eq. (1) in the pumping condition of Fig. 1, approximate solutions may be derived from modification of the cross-sectional area A in Eq. (1). Notice that the line L_{ab} becomes a circumference $L_{ab}' = 2\pi r$ while L_{cd} reduces to $L_{cd}' = 2\pi r_w$. Therefore the cross-sectional area A may be averaged as $(2\pi rh + 2\pi r_w h)/2 \approx \pi rh$ where $h = (h_i + h_w)/2$. Eq. (1) becomes:

$$C(r,t) = \frac{M}{2\pi rhn\sqrt{\pi a_L ut}} e^{-\frac{(r-ut)^2}{4D_L t}} \qquad (2a)$$

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Received 22 August 1997; accepted in revised form 13 October 1997

where:

$u = Q/(\pi rhn)$

In Eq. (2a), the velocity u is equal to the radial distance r divided by the time t the tracer takes to arrive at the pumping hole. Since t is a changing factor, it has to be integrated over r. This method of velocity averaging will apply to all cases that follow.

Based on a cylindrical flow concept, Eq. (2a) may be slightly modified and written in terms of the pumping rate Q. Notice that the πrh in Eq. (2a) is approximated by Qt/nr. Substitution of Qt/nr in Eq. (2a) yields:

$$C(r,t) = \frac{Mr}{2Q\sqrt{\pi a_L ut^3}} e^{-\frac{(r-ut)^2}{4D_L t}} \quad (2b)$$

where:

$u = Q/(\pi rhn)$

For the unconfined flow without consideration of influence of regional flow as seen in Fig. 1, a water balance may be established in terms of a simple differential equation: $Qdt = \pi r^2 Pdt + d(\pi r^2)hn$ where P is an effective recharge (m/s). Integration of this equation gives $\pi r^2 P = Q/(1-exp(-Pt/hn))$.

Taking into account induced recharge or leakage through a confining layer, the A in Eq. (1) may be replaced by $\pi rh(1-exp(-R_e Qt/\pi r^2 hn))/R_e$ where $R_e = \pi r^2 P/Q$ *with* $0<R_e<1$, a ratio of the amount of vertical recharge on the area πr^2 to pumping rate. Due to recharge, the concentration would be diluted. The mass balance would be $QC(r,t,R_e) = \pi r^2 PC_{recharge} + (Q - \pi r^2 P)C(r,t)$, where $C(r,t,R_e)$ is the concentration from the pumping hole, $C_{recharge}$ the concentration of recharge water and $C(r,t)$ the concentration due to the injected tracer. Since $C_{recharge}$ is taken as zero, we have $C(r,t,R_e) = (1 - R_e)C(r,t)$. This implies that the dilution may be calibrated by a factor $(1-R_e)$. Therefore Eq. (1) changes to:

$$C(r,t,R_e) = \frac{(1-R_e)R_e}{1-e^{\frac{-R_e Qt}{\pi r^2 hn}}} \frac{M}{2\pi rhn\sqrt{\pi a_L ut}} e^{-\frac{(r-ut)^2}{4D_L t}} \quad (2c)$$

where:

$u = -R_e Q/((\pi rhn)ln(1-R_e))$

Eq. (2c) may be regarded as a general expression of the approximate solutions, for it may account not only for the scenario of unconfined aquifers but also for confined ones. For unconfined aquifers, R_e ranges between > 0 to < 1, indicating the extent of aquifer confinement or leakage. When $R_e \rightarrow 1$, the concentration $C(r,t,R_e) \rightarrow 0$, implying that the location of tracer injection hole coincides with the outer edge of the pumping influence, i.e. no flow boundary where the tracer is not supposed to move towards the pumping hole. For confined aquifers, Eq. (2c) will approach Eq. (2b) when $R_e \rightarrow 0$, meaning that the cylindrical aquifer is tightly confined without any vertical recharge being added on to it. If the t in the exponential term of Eq. (2c) is further replaced by t_{ad} $(= \pi r^2 hn/Q)$, the mean transport time by pure advection, Eq. (2c) is reduced to Eq. (2a). When Eq. (2a) is used for experiments conducted under a confined flow condition, h in Eq. (2a) is the thickness of the confined aquifer instead of $(h_i+h_w)/2$ as averaged thickness from Fig. 1. But both equations are special cases of Eq. (2c) under the circumstances.

Eq. (2a) through Eq. (2c) may be conditioned to a point (t_{max}, C_{max}) where a tracer BTC has a maxima. The t_{max} is obtained by solving $\partial C/\partial t = 0$. The corresponding equations in terms of t_{max} and C_{max} are presented as follows:

$$C(r,t) = C_{\max}\left(\frac{t_{\max}}{t}\right)^{\frac{1}{2}} e^{\frac{(r-ut_{\max})^2}{4D_L t_{\max}} - \frac{(r-ut)^2}{4D_L t}} \quad (2a')$$

where:

$t_{max} = ((r/u)^2+(D_L/u^2)^2)^{1/2} - D_L/u^2$

$$C(r,t) = C_{\max}\left(\frac{t_{\max}}{t}\right)^{\frac{3}{2}} e^{\frac{(r-ut_{\max})^2}{4D_L t_{\max}} - \frac{(r-ut)^2}{4D_L t}} \quad (2b')$$

where:

$t_{max} = ((r/u)^2+(3D_L/u^2)^2)^{1/2} - 3D_L/u^2$

$$C(r,t,R_e) = C_{\max}\frac{1-e^{\frac{-R_e Qt_{\max}}{\pi r^2 hn}}}{1-e^{\frac{-R_e Qt}{\pi r^2 hn}}}\left(\frac{t_{\max}}{t}\right)^{\frac{1}{2}} e^{\frac{(r-ut_{\max})^2}{4D_L t_{\max}} - \frac{(r-ut)^2}{4D_L t}} \quad (2c')$$

where:

$t_{max} \approx ((r/u)^2+((D_L/u^2)(1+(2R_e exp(-R_e))/(1-exp(-R_e))))^2)^{1/2} - (D_L/u^2)(1+(2R_e exp(-R_e))/(1-exp(-R_e)))$

Obviously in the above three cases, $t_{max} \rightarrow t_{ad} = r/u$, the mean transport time if $D_L \rightarrow 0$. The difference between t_{ad} and t_{max} increases with increasing a_L.

Again, Eq. (2c') will approach Eq. (2b') when $R_e \rightarrow 0$. Despite different appearance of the Eq. (2a') and Eq. (2b'), they are identical to the two solutions presented by Lenda and Zuber (1970) and Sauty (1978). Sauty's solutions have been popular for tracer experiments under radially convergent flow conditions (Sauty et al., 1992). Wang and Crampon (1995) calibrated Sauty's solutions with correction factors after comparison with numerical solutions. They also noted that Eq. (2a') should be used for overall tracer BTC fitting while Eq. (2b') should be used for the ascending part of the tracer BTC.

The relationship between Eq. (2b) and Eq. (2c) is illustrated in Fig. 2. In general, introduction of the recharge would result in delay of the BTC arrival time and a decrease in the peak concentration.

Numerical evaluation

The numerical evaluation of the approximate solution was based on the finite difference method (FDM) and finite element method (FEM). The finite difference model adopted an implicit iteration with the upstream weighting.

Unlike the approximation, the numerical simulation did not use a constant velocity but it used a constant flux. The radial coordinate intervals were chosen such that one ring segment would just empty completely into the next ring segment at one time step. That means the ring segments were increasing in thickness towards the pumping hole. The velocity in a ring segment was the average between the upper and lower boundary of the ring.

The same parameters had to be used in simulations for comparison. All the numerical simulations were implemented on an Excel spreadsheet. The spreadsheet allows for adjusting simulation parameters while a corresponding colour graph is simultaneously generated for curve-matching purposes. The numerical results were compared with Eqs. (2a'), (2b') and (2c') as seen in Fig. 3. In general, the two different approaches give comparable results. The curve (3) in Fig. 3 does not match the other because its flow field is assumed to be an unconfined condition with $R_e = 0.25$. For a uniform aquifer, the conditioned

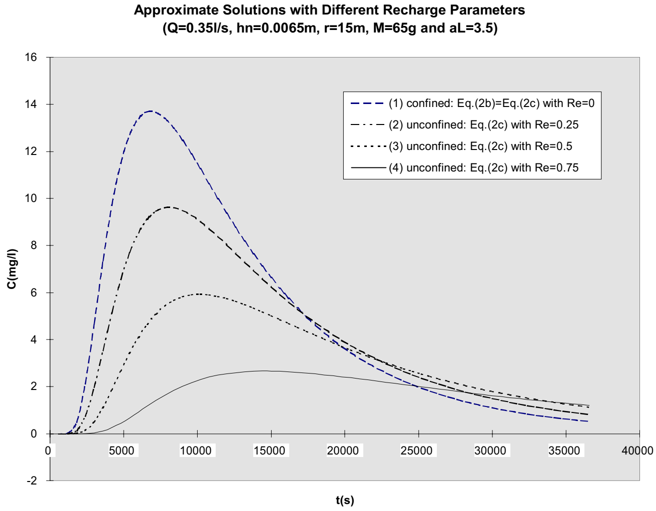

Figure 2
Effect of recharge on BTC

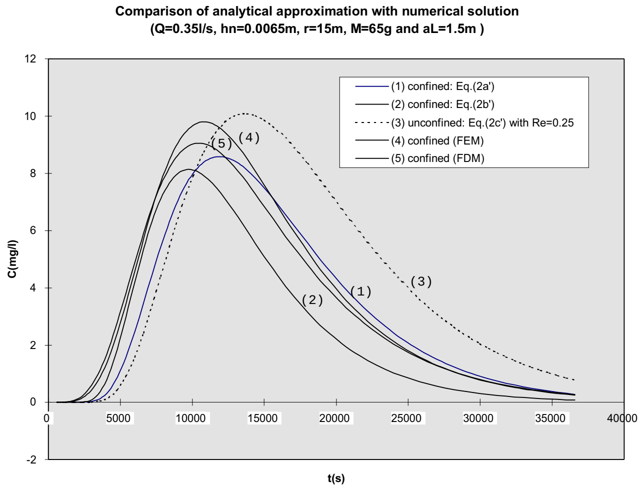

Figure 3
Numerical evaluation of approximate solutions

forms, i.e. Eqs. (2a'), (2b') and (2c'), would be more appropriate for estimating effective porosity and dispersivity.

It must be pointed out that the theoretical boundary condition proposed by Wang and Crampon (1995), which improved Sauty's (1978), is not appropriate in this case. The outer boundary condition used in this simulation is directly based on the actual dilution experiment in the injection hole.

Application

The approximate solutions are applied to the tracer experiment conducted in a Karoo aquifer on the campus site at the University of the Orange Free State, Bloemfontein. The experimental set-up is basically same as that shown in Fig. 1.

The campus site aquifer is underlain by a Karoo sequence of interbedded mudstone, siltstone and sandstone. A horizontal fractured zone at a depth of 22 m below ground surface was selected for the tracer experiment. A pumping hole termed UO5 and a tracer injection hole termed UO20, which both intersect the horizontal fracture zone, are 15 m away from each other. Since a really steady state of radial flow in a fractured aquifer was difficult to attain, a pseudo steady state had to be used for the experiment. UO5 was pumped at a rate of 0.35 l/s for about 25 h until it was safely assumed that a steady, horizontal, radially convergent flow field had been established. Then 65 g bromide used as tracer was injected in UO20 in such a manner that the flow field was not disturbed. The tracer pulse was allowed to be pumped out from UO5 where a complete tracer BTC was detected over 20 h.

The bromide concentration arriving in the pumping hole was determined in the laboratory by analysing water samples collected from the hole while dilution of electrical conductivity (EC) induced by bromide in the injection hole was detected with an EC probe.

During the experiment, drawdowns in boreholes in the vicinity were monitored. The pseudo steady state prevailed long enough for completion of the experiment. The 2 d experiment resulted in the dilution and breakthrough curves obtained from the injection hole and pumping hole, respectively.

Since the horizontal fracture zone was separated by straddle packers with a 1 m interval, data obtained from the bromide experiment between pumping hole UO5 and injection hole UO20 would reveal some characteristics of the fracture zone. Based on the dilution curve, hydraulic conductivity (K), aquifer effective thickness (h) as well as effective porosity (n) can be inferred. The results can be used as initial input values for estimation of effective porosity and dispersivity using the approximate solutions.

Prior to tracer BTC simulation using the approximation solution, dispersivity may be roughly estimated as an initial input in the simulation. Disregarding the diffusion process, i.e. $D_L = a_L u$, rearrange Eq. (2a) or Eq. (2b) by assuming $t_{max} = t_{ad}$, so dispersivity a_L can be approximated as follows:

$$a_L \approx (r/4\pi)\,(C_{av}/C_{max})^2 \qquad (3a)$$

where:

$C_{av} = M/\pi r^2 hn$

It is clear that the dispersivity a_L indicates a degree of tracer dissipating away from mass center, C_{av}/C_{max} and is also related to the scale of experiment r. In Eq. (3a), Peclet number P_e may be defined as:

$$P_e = r/a_L \approx 4\pi(C_{max}/C_{av})^2 \qquad (3b)$$

A simple simulation model using approximate solutions was built on a spreadsheet. Advantage of this approach was that simulation results were simultaneously depicted in colour graphs. It is a user-friendly approach.

Available hydrogeological information indicates that the horizontal fracture zone is dominantly of confined condition. So both Eq. (2a') and Eq. (2b') were used for initial estimation of effective porosity and dispersivity.

The results simulated using Eqs. (2a') and (2b') do not seem to reflect characteristics of the fracture flow although Eq. (2b') appears to give a relatively better fit in terms of mass recovered. Then the unconfined model, i.e. Eq. (2c') was used for a best fit. Simulated results were depicted in Fig. 4.

Interpretation of the tracer BTCs is complicated by the principle of equivalence. For a tracer BTC with a single peak, effective porosity and thickness hn often cancel each other and appear as a pair in the model. They cannot be readily distinguished unless either of two is specified. If the aquifer thickness of 0.175 m, as estimated from the dilution experiment, increases

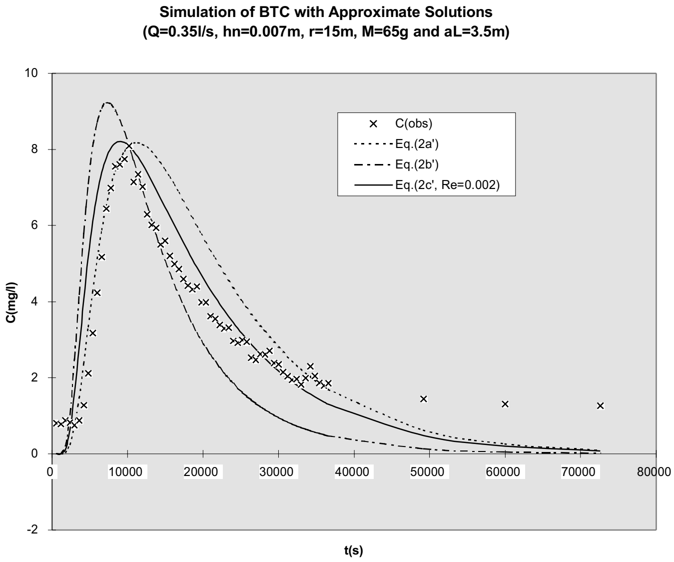

Figure 4
Tracer BTC simulations

to 1 m, the thickness sealed off during the experiment, the effective porosity would increase to 0.007. It makes sense to define product *hn* as effective thickness which controls pore-water velocity. The bigger the effective thickness is, the smaller the pore-water velocity would be. For a given effective thickness, many combinations of *n* and *h* are possible. Thanks to the tracer dilution analysis, the thickness of the horizontal fracture zone may be estimated. Therefore a unique *n* value could be obtained.

Discussion

The discrepancy between Eq. (2a') and Eq. (2b') as seen in Fig. 4 is due to different approximations of the cross-sectional area *A*. In fact, Eq. (2a) may be used to simulate unconfined conditons because the result obtained by Eq. (2a) can be obtained by using Eq. (2c') with $R_e = 0.005$. In recharge simulations using Eq. (2c'), we used $R_e = 0.002$. The simulation result may be used to calculate the radius of influence of the cone of depression *R*. Recall $R_e = \pi r^2 P/Q$, so $P = R_e Q/\pi r^2$. Since the steady state was maintained through recharge during the experiment, the pumping rate *Q* must be equal to the recharge rate induced over the whole cone of depression $\pi R^2 P$. Hence the radius of influence may be calculated by the following formula: $R = (r^2/R_e)^{1/2}$. Substitution of all known parameters yielded $R = 335$ m.

Another form similar to Eq. (2c) may also be derived by replacing *A* in Eq. (1) with $R_e Qt/(hn(1-exp(-R_e Qt/\pi r^2 hn)))$. The result may be discussed the same way as presented above. A similar result should be obtained.

The tracer experiment conducted at the campus site aquifer was interpreted using the approximate solutions. In spite of improvement of the simulation by introduction of the recharge model, the one layer simulation still did not adequately describe the tracer BTC of the fracture zone. Further work should be focused on the effect of imperfect mixing in the injection hole, multi-layer transport and matrix diffusion in an attempt to account for fat-tailing recorded in the experiment.

The interpretation presented in this paper would be refined, should gradients between two boreholes be established properly. One obvious shortcoming is that regional flow is not taken into consideration at all. Another important factor which is not accounted for is the mixing process in both pumping and injection holes. The approximation solutions do not take into account borehole radius of either injection hole or pumping hole. This implies that no consideration is given to the influence of borehole mixing on the tracer BTC. This would affect accuracy of parameters inferred. For a detailed discussion on this aspect, the interested reader is referred to Moench (1989). The Dirac-type approximation also implies that no consideration is given to the tracer-labeled water volume injected. The solution may not be valid when the injected volume is large. Nevertheless, the simple model presented here would provide preliminary results and may also serve as an educational tool for understanding the effect of recharge on the tracer experiment in a user-friendly manner.

Conclusion

The general form of approximate analytical solutions for interpretation of the tracer experiment under pumping conditions is presented. The general solution may account for the scenario of both confined and unconfined aquifers. The existing approximate solutions are special cases when no recharge is considered. The introduction of recharge would generally delay and dilute the peak concentration of a tracer BTC. Based on numerical evaluation, the conditioned forms of the approximate solutions are acceptable for use in the preliminary estimation of effective porosity and dispersivity in uniform aquifers.

For the case study, the simulation of the one-layer model indicates that dewatering in the fracture zone would induce the vertical recharge but the one-layer model does not adequately represent the characteristics of the fracture zone because the fat-tailing as seen in Fig. 4 is still not accounted for. To explain this discrepancy, further work is still needed, including assessment of the effect of the imperfect mixing, the multi-layer aquifer and the matrix diffusion.

Acknowledgement

The Water Research Commission, the sponsor of this work, is acknowledged for its permission to publish this paper. The authors would like to thank JM Nel of DWAF for his careful review of a draft.

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