Biological sludge stabilisation
Part 1: Kinetics of aerobic sludge digestion

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Abstract

The Marais and Ekama (1976) activated sludge model describes *inter alia* the aerobic digestion process as a first order process with respect to the active (live) sludge concentration. Since the active sludge concentration cannot be measured directly, the decay constant of the first order process relationships can only be calculated after deriving expressions linking changes of measurable parameters to the change of active sludge. Relationships are derived between the change of the active sludge concentration and of four parameters that can be determined by simple methods: Oxygen uptake rate (OUR), volatile suspended solids (VSS) concentration, nitrate concentration and alkalinity.

The aerobic batch digester is particularly useful to evaluate the validity of the model and the expression linking change of the four parameters to active sludge decay. From observations of aerobic batch digesters it is concluded that the changes of the four parameters all lead to the same value of the decay constant. In practice aerobic digesters are often operated in series and cyclic (daily) loads of excess sludge are applied. It is shown that the behaviour of such a series reactor system can also be described by the same model, using the same relationships between the parameters and the active sludge concentration.

Introduction

When sludge from an active sludge system is kept in an aerobic environment without feed of organic substrate, a gradual decrease of the sludge concentration is observed. This is attributed to aerobic digestion, a process in which part of the cell protoplasm is oxidised to produce the energy the micro-organisms require. The oxygen uptake for protoplasm oxidation is called endogenous respiration to distinguish it from exogenous respiration, that takes place when extracelluar organic material is metabolised.

In the first attempts to model aerobic digestion (Lawrence and McCarty, 1970), the process was considered a first order process with respect to the volatile sludge concentration. Not surprisingly these models failed to describe the process properly because a fraction of the sludge is not amenable to the digestion process: it is inactive material and does not exhibit biochemical activity. The magnitude of the inactive fraction in a sludge sample depends on the operational conditions of the system in which it was generated. In later models for aerobic digestion (Randall, 1975; Marais and Ekama, 1976, Benefield and Randall, 1978), a distinction is made between active sludge and inactive sludge. Aerobic digestion was assumed to affect only the active sludge fraction and a first order decay process was assumed.

Marais and Ekama (1976) showed experimental evidence that their model could be applied not only to describe the behaviour of aerobic sludge operating as batch reactors, but also to the biological reactor of the activated sludge system itself, in other words, it was shown that endogenous respiration always occurs when active sludge is in an aerobic environment, independent of whether exogenous respiration takes place or not.

A difficulty to verify the validity of the Marais and Ekama model is that there is no test to determine directly the active sludge concentration in a sample. However, Marais and Ekama (1976) showed that two variables can be linked to two parameters that can be measured directly: the oxygen uptake rate (OUR) and the volatile suspended solids (VSS) concentration. In this paper it is shown that the variation of another two parameters in aerobic digesters can be used to evaluate the Marais and Ekama model: the nitrate concentration and the alkalinity. Nitrate is generated and alkalinity is consumed, if the mineralised nitrogen from the decayed active sludge is nitrified, as will normally be the case when the temperature is not very low in the aerobic digester. Thus there are four variables available to verify the validity of the Marais and Ekama (1976) model. In this paper an experimental investigation is described, in which batch reactors were used to digest sludge and the decay constants were calculated with the aid of expressions for the change of the four measured variables: OUR, VSS concentration, nitrate and alkalinity. The experimental results show that the decay constants calculated from the behaviour of different variables all lead to the same value.

The batch digester is very convenient to verify the validity of the digestion model, but in practice aerobic digestion normally is not carried out in batch digesters but in one or more completely mixed digesters. The aerobic digesters typically operate in series and receive intermittent feeding, when excess active sludge is discharged from the active sludge system. The sludge composition as well as the nature of identifiable organisms, varies strongly in a series of aerobic digesters, but is spite of this it is shown that the decay constant in such a digester system remains constant and has the same value as in a batch digester.

Aerobic digestion theory can also be used to develop a convenient parameter to express the degree of stabilisation of aerobically digested sludge. It is shown that both the specific oxygen uptake rate (SOUR) and the BOD/VSS ratio of a sludge sample can be used for this purpose.

Kinetics of aerobic digestion

In order to give a consistent description of aerobic digestion of waste activated sludge, it is necessary to distinguish an active sludge fraction (X_a), composed of live organisms and amenable

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to aerobic digestion, and an inactive fraction (X_{na}), that is not. Models that have successfully predicted aerobic digestion (Marais and Ekama, 1976; Benefield and Randall, 1978) are based on two main assumptions: Decay of the active sludge is a first order process; and part of the decayed active sludge is mineralised whereas the remainder stays as an unbiodegradable solid: the endogenous residue. A constant fraction of the decayed active sludge becomes endogenous residue. The decay of active sludge can be expressed as:

$$r_d = -(dX_a/dt)_d = b_h X_a \quad (1)$$

The generation of endogenous residue can be written as:

$$(dX_e/dt)_d = -f(dX_a/dt)_d = fb_h X_a \quad (2)$$

where:

X_a = active sludge concentration
X_e = endogenous residue generated
b_h = decay constant
f = fraction of decayed active sludge transformed into endogenous residue
t = digestion time.

The validity of the aerobic digestion model cannot be verified directly because there is no direct measure for the activated sludge concentration. However, relationships can be derived for parameters that are influenced by the active sludge concentration.

By observing the changes of these parameters in a batch of sludge under digestion, the validity of the model can be verified and the value of the decay constant, b_h, and the endogenous mass fraction f can be calculated. Four parameters can be used to characterise aerobic sludge digestion:

- the VSS concentration
- the OUR
- the nitrate concentration
- the alkalinity change.

The relationships between these parameters and the decay of active sludge will now be derived for two types of aerobic digesters: the batch reactor and the complete-mix digester with a daily feed batch. The first type of reactor is particularly adequate to check the validity of the model, while the second type approaches more the usual operational conditions of an aerobic digester in practice.

The batch digester

For a batch of waste activated sludge, the decay of the active fraction can be expressed by direct integration of Eq. (1):

$$X_a = X_{ai}\exp(-b_h t) \quad (3)$$

where:

X_a = active sludge concentration
X_{ai} = initial active sludge concentration

The relationships between VSS decrease, nitrate increase, alkalinity change and OUR and the decay of active sludge will now be derived for a batch reactor. The variation of volatile sludge is the result of active sludge decay and endogenous residue generation in the sludge batch:

$$X_v = X_{vi} - (X_{ai} - X_a) + f(X_{ai} - X_a) \quad (4a)$$

After a long digestion period, decay is complete ($X_a = 0$) so that:

$$X_{v\infty} = X_{vi} - (1-f)X_{ai} \quad (4b)$$

Hence:

$$X_v - X_{v\infty} = X_{ai}(1-f)\exp(-b_h t)$$

or:

$$\log(X_v - X_{v\infty}) = \log[X_{ai}(1-f)] - 2.3b_h t \quad (5)$$

where:

X_v = the volatile sludge concentration (active + inactive sludge)
X_{vi} = initial volatile sludge concentration
$X_{v\infty}$ = volatile sludge concentration after a long digestion period (complete decay of X_a)
t = digestion time

Equation (5) shows how the decay constant may be determined experimentally:

- Submit a batch of sludge to aerobic digestion at a constant temperature.
- Take samples at regular intervals and determine the volatile sludge concentration (X_v) until a constant value is reached ($X_{v\infty}$). This takes 2 to 3 weeks.
- Plot the (X_v-$X_{v\infty}$) values as a function of the digestion time on semi-log paper.
- The slope of the best straight line through the points is 2.3*b_h.

The variation of the nitrate concentration in a sludge batch under digestion can also be linked to the decay of active sludge as follows: a fraction of f_n = 0.1 mgN·mg^{-1}VSS of volatile sludge is nitrogen (Marais and Ekama, 1976) and when the sludge is mineralised, the nitrogen is released to the liquid phase and usually will be nitrified in the aerobic environment of the digester. In that case there is a direct relationship between the decrease of the VSS concentration and the increase of the nitrate concentration of the sludge batch:

$$N_n - N_{ni} = f_n(X_{vi} - X_v)$$

Hence:

$$\log(N_{n\infty} - N_n) = \log[f_n(1-f)X_{ai}] - 2.3b_h t \quad (6)$$

Similarly the alkalinity change can also be related to the active sludge decay: when nitrogen is released and the organic nitrogen is converted into nitrate the alkalinity change can be calculated from the reaction equation:

$$RNH_2 + 2O_2 \rightarrow ROH + NO_3^- + H^+ \quad (7)$$

From Eq. (7), for each mol of nitrate (14 gN) produced, there is an acidity production of 1 equivalent or 50 g $CaCO_3$. Hence there is an alkalinity consumption of 50/14 = 3.57 ppm $CaCO_3$ per mgN nitrified. Therefore:

$$\log(Alk - Alk_\infty) = \log[3.57f_n(1-f)X_{ai}] - 2.3b_h t \quad (8)$$

Equations (6) and (8) can be used to determine the decay constant from the nitrate concentration change and the alkalinity change respectively of a sludge batch under aerobic digestion. The

procedure is *mutatis mutandis* the same as indicated above for VSS.

Alternatively, the OUR of the sludge batch can be used to determine the decay constant. In that case it is recognised that oxygen is used for the oxidation of decayed organic material and for nitrification. The oxidation rate of organic material can be expressed as:

$$OUR_c = f_{cv}(1-f)(dX_a/dt)_d = f_{cv}(1-f)b_hX_a = f_{cv}(1-f)b_hX_{ai}\exp(-b_ht) \quad (9)$$

Knowing that there is a release of f_n nitrogen per unit mass of mineralised sludge and that there is a stoichiometric demand of 4.57 $mgO \cdot mg^{-1}N$ in the nitrification process, one has:

$$OUR_n = 4.57f_n(dX_a/dt)_d = 4.57f_n(1-f)b_hX_{ai}\exp(-b_ht) \quad (10)$$

Hence the total OUR can be expressed as:

$$\begin{aligned} \log OUR_t &= \log(OUR_c+OUR_n) \\ &= \log[(f_{cv}+4.57f_n)(1-f)b_hX_{ai}] - 2.3\ \log b_ht \end{aligned} \quad (11)$$

Equations (5), (6), (8) and (11) are independent and each one can be used to determine the decay constant in a batch of waste activated sludge under aerobic conditions. In the next section is will be shown that, within experimental error, the four parameters all yield the same decay constant.

Completely mixed aerobic digester with a cyclic feed pattern

In practice aerobic digesters usually are not operated as batch reactors. Normally there is a daily discharge of waste activated sludge to the aerobic digester system composed of one or more reactors in series. The waste activated sludge flows through the reactor series and eventually is discharged as stabilised sludge for liquid solid separation and final destination of the solid fraction. If it is assumed that daily batches of waste activated sludge are instantaneously fed to a completely mixed aerobic digester, the decay of the active fraction can be calculated as follows:

- Upon discharge of the digested sludge and introduction of a feed batch, the concentration of active sludge is the weighted average of the active sludge concentrations of the feed batch and of the sludge remaining in the aerobic digester.
- After the batch of waste activated sludge has been received, the reactor operates as a batch reactor until one day later more waste sludge is discharged.

Hence just after introducing the feed batch one has:

$$V_rX_{a0} = (V_r-V_b)X_{a1} + V_bX_{ai} \quad (12)$$

Between successive feeds, the active sludge concentration decays exponentially and after one day:

$$X_{a1} = X_{a0}\exp(-b_h*1) \quad (13)$$

where:

X_{a0} = active sludge concentration just after the discharge of waste activated sludge

X_{a1} = activated sludge just before the discharge of the next batch, one day later

X_{ai} = activated sludge concentration in the daily batches.

By rearranging the active sludge concentration in the effluent of the digester is given as:

$$X_{a0} = (V_b/V_r)/[1-(1-(V_b/V_r))\exp(-b_h)]*X_{ai} \quad (14)$$

The average retention time in the reactor with a volume of V_r fed with daily batches of sludge V_b is given by $R_d = V_r/V_b$. The active sludge concentration in the effluent from the digester can now be expressed as:

$$\begin{aligned} X_{a1} &= (V_b/V_r)/[1-(1-(V_b/V_r))\exp(-b_h)]*X_{ai}*\exp(-b_h) \\ &= X_{ai}/\{R_d[\exp(b_h)-1]+1\} \end{aligned} \quad (15)$$

The decrease of the activated sludge concentration can now be expressed as:

$$X_{ad} = X_{ai}-X_{a1} = X_{ai}[\exp(b_h)-1]/[\exp(b_h)-1+1/R_d] \quad (16)$$

In the case of a series of aerobic digesters, the digested sludge of the first reactor (with an active sludge concentration of X_{a1}) is used as feed for the second reactor and so forth.

Having established the expression for the concentration of digested active sludge in the digester, the corresponding expression for VSS removal, nitrate concentration, alkalinity change and oxygen uptake are now readily deduced:

$$\begin{aligned} X_{vd} &= (1-f)X_{ad} \\ &= (1-f)X_{ai}[\exp(b_h)-1]/[\exp(b_h)-1+1/R_d] \end{aligned} \quad (17)$$

$$\begin{aligned} N_{nd} &= f_nX_{vd} \\ &= f_n(1-f)X_{ai}[\exp(b_h)-1]/[\exp(b_h)-1+1/R_d] \end{aligned} \quad (18)$$

$$\begin{aligned} Alc_d &= -3.57N_{nd} \\ &= -3.57*f_n(1-f)X_{ai}[\exp(b_h)-1]/[\exp(b_h)-1+1/R_d] \end{aligned} \quad (19)$$

$$\begin{aligned} OUR_1 &= (f_{cv}+4,f7f_n)*(1-f)b_hX_{a1} \\ &= (f_{cv}+4,f7f_n)*(1-f)b_hX_{ai}/\{R_d[\exp(b_h)-1]+1\} \end{aligned} \quad (20)$$

The validity of the model characterised by Eqs. (16) to (20) was tested by determining measurable parameters in both batch digesters and in completely mixed digesters fed with daily sludge batches and checking if the measured values correspond to the theoretical values predicted by theory.

Experimental investigation and results

Batch digesters

An activated sludge system at bench scale was operated under constant flow and load conditions, using raw sewage from the main outfall of Campina Grande, Brazil. When steady state was reached the sludge was used for batch digestion experiments. The batch digesters were 2 ℓ beakers, stirred by a jar tester and aerated with the aid of aquarium air pumps. Experiments were carried out at different (controlled) temperatures (21 to 30°C), but always the sludge was digested at the same temperature as it had been generated. The sludge age in the generation unit was varied between 3 and 10 d. In each experiment six batches were placed in the beakers and digested under identical conditions. A total of 13 experiments with 6*13 sludge batches was carried out to determine aerobic digestion kinetics, measuring OUR, the volatile sludge and nitrate concentrations and alkalinity as functions of time. In each experiment, samples for VSS, nitrate and alkalinity tests were taken at 12 h intervals for about a week. OUR tests in the digesters were carried out more frequently during the same period. As an example Table 1 shows the experimental data

TABLE 1
EXPERIMENTAL RESULT OF A BATCH DIGESTER EXPERIMENT NO 1. (AVERAGE VALUES OF 6 IDENTICAL BATCHES GENERATED AT 21°C AND A SLUDGE AGE OF 5 D)

time	OUR	time	X_v	N_n	Alk
0	43.6	0	4560	37	725
0.18	20.4	0.5	4100	69	585
0.82	33.5	1	3880	92	520
1.10	28.7	1.5	-	109	488
1.35	27.8	2	3840	110	435
1.87	25.1	2.5	3710	126	365
2.32	20.2	3	3490	138	355
2.87	18.9	3.5	3380	142	335
3.09	16.0	4	3320	158	295
3.33	16.5	4.5	3080	162	245
3.88	13.8	5	3080	178	222
4.17	12.4	5.5	-	185	175
4.41	13.0	6	2980	192	155
4.87	11.4				
5.26	10.1				
6.00	10.5				

of Experiment 1 carried out at 21°C with sludge generated at a sludge age of 5 d. The values in Table 1 are the arithmetic average of the results in the six batches of Experiment 1. The data of Table 1 were used to construct the diagrams of Fig. 1, which in turn were used to calculate the decay constant of the sludge. The following steps were taken:

- The OUR data were plotted as a function of the digestion time on semi-log paper and the inclination of the best straight line though the experimental points was used to determine the decay constant, while the intersection with the ordinate axis was used to calculate the initial active sludge concentration. Extrapolating for t = 0 in Fig. 1a: OUR = 40 mg·ℓ$^{-1}$·h^{-1} and with the aid of Eq. (11): X_{ai} = 2 355 mg/ℓ.
- Estimates of the final values of VSS, nitrate and alkalinity from the initial values and the calculated value of X_{ai} were made. For t = 0 the sludge concentration is X_v = 4 435. After complete decay the decrease of VSS will be: $X_{ai}(1\text{-}f)$ = 1 885 mg/ℓ so that the final concentration is estimated at: $X_{v\infty}$ = 4 435 - 1 885 = 2 550 mg/ℓ. Similarly: $N_{n\infty}$ = 240 mgN/ℓ and Alk_{∞} = 10 ppm.
- Plot the experimental data of (X_v-$X_{v\infty}$) as a function of digestion time on semi-log paper and determine the decay constant from the inclination. If the experimental points in

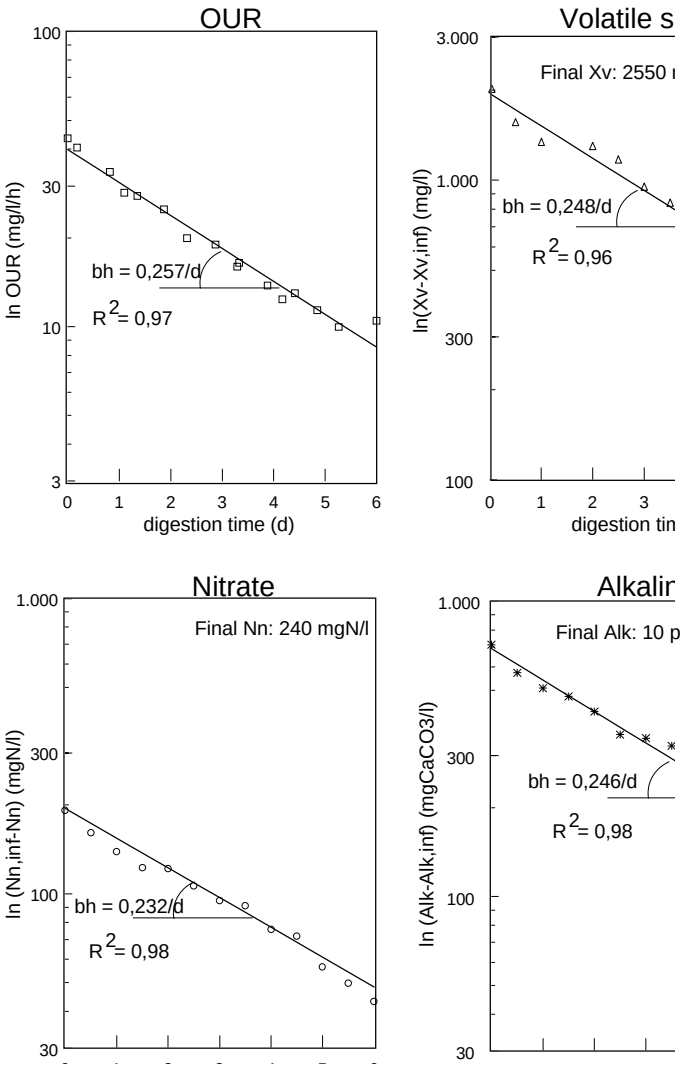

Figure 1
Experimental result of aerobic digestion in batch reactors: Variation of OUR, VSS concentration, nitrate concentration and alkalinity as a function of digestion time. The assumed final values are also indicated.

TABLE 2
RESULTS OF BATCH EXPERIMENTS OF AEROBIC SLUDGE DIGESTION (NITRIFICATION AT 28°C WAS INHIBITED BY ALLYL THIOUREUM ADDITIONS)

Experiment No. (temperature)	b_h values calculated from:				Average of the 4 parameters
	OUR	Xv	Nn	Alk	
1 (21°C)	0.257	0.248	0.232	0.245	0.246
2 (21°C)	0.266	0.276	0.245	0.248	0.259
3 (21°C)	0.240	0.247	0.260	0.252	0.250
4 (21°C)	0.260	0.254	0.248	0.239	0.250
5 (21°C)	0.253	0.257	0.254	0.241	0.251
6 (21°C)	0.254	0.257	0.239	0.258	0.252
7 (21°C)	0.236	0.248	0.265	0.256	0.251
8 (21°C)	0.262	0.252	0.246	0.248	0.252
Average 21°C	**0.253**	**0.255**	**0.249**	**0.248**	**0.251**
9 (28°C)	0.331	0.327			0.329
10 (28°C)	0.356	0.309			0.332
11 (28°C)	0.308	0.327			0.318
12 (28°C)	0.296	0.329			0.312
Average 28°C	**0.323**	**0.323**			**0.323**
13 (30°C)	0.357	0.369	0.363	0.335	0.356

the diagram show a systematic tendency of curvature (convex or concave), the estimate of the final concentration is inadequate and must be reviewed.

- Similarly treat the data of the nitrate concentration and alkalinity as a function of time and determine the decay constant on the basis of these parameters.

Figure 1 shows diagrams to calculate values of the decay constant from batch experiments carried out at 21°C, as well as the corresponding value of the correlation coefficients for each of the four parameters: OUR, X_v, NO_3^-, and alkalinity. The decay constants calculated from different parameters all have very similar values and that the correlation coefficients are all approaching the value of 1.00. These are clear indications that the kinetic model is adequate to describe sludge batch digestion behaviour. For the experiment the arithmetic average of the decay constant is: $b_h = (0.257 + 0.248 + 0.232 + 0.245)/4 = 0.246\ d^{-1}$. Table 2 shows the experimental values of b_h in all 13 experiments. On the basis of these results and assuming a Van't Hoff-Arrhenius relationship for the temperature, the following expression was found for the decay constant in all experiments, taking into consideration all four parameters:

$$b_h = 0.24(1.04)^{(t-20)} \qquad (21)$$

This expression is very close to the expression presented by Marais and Ekama (1976) for temperatures between 12 and 20°C: $b_h = 0.24(1.029)^{(t-20)}$.

Completely mixed aerobic digesters with cyclic feed

To investigate the behaviour of completely mixed aerobic digesters under a cyclic feed pattern, an experimental investigation was carried out at pilot scale. Active sludge was generated in an aerated lagoon of 1 000 ℓ and four aerobic digesters with useful volumes of 45 ℓ were operated in series at a constant temperature (25 ± 2°C) to digest the generated sludge. Every day 500 ℓ of the mixed liquor was withdrawn from the lagoon and settled in a batch settler of 500 ℓ. The volume of the lagoon was then completed with 500 ℓ of raw sewage. After discharging 470 ℓ of supernatant from the settler, a daily volume of 30 ℓ of sludge was available for digestion.

The series of aerobic digesters was fed with the sludge in a particular way that is resumed as follows (See also Fig. 2):

- A daily volume of 8 ℓ was withdrawn from the fourth and last aerobic digester (R_4) and this mixed liquor was used to determine the OUR and the VSS concentration. The volume withdrawn from R_4 was substituted with sludge from the third aerobic digester (R_3). The average retention time in R_4 was 45/8 = 5.6 d.
- Along with the 8 ℓ/d withdrawn form digester R_3 an additional volume of 7 ℓ/d was withdrawn and the 8+7 = 15 ℓ/d were substituted with mixed liquor from reactor R_2. Hence R_3 operated at a retention time of 45/15 = 3.0 d.

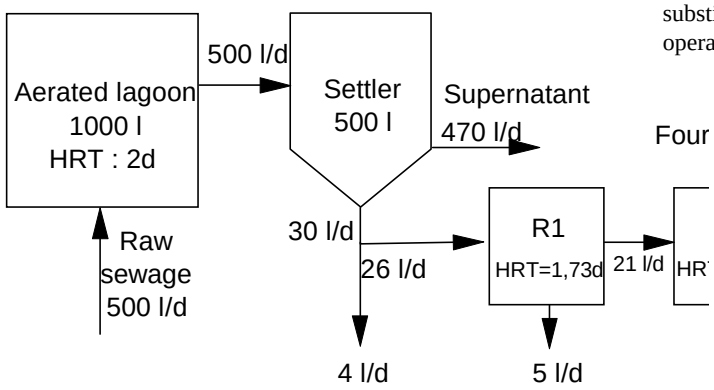

Figure 2
Schematic representation of the experimental set-up of the system composed of digesters with cyclic feed

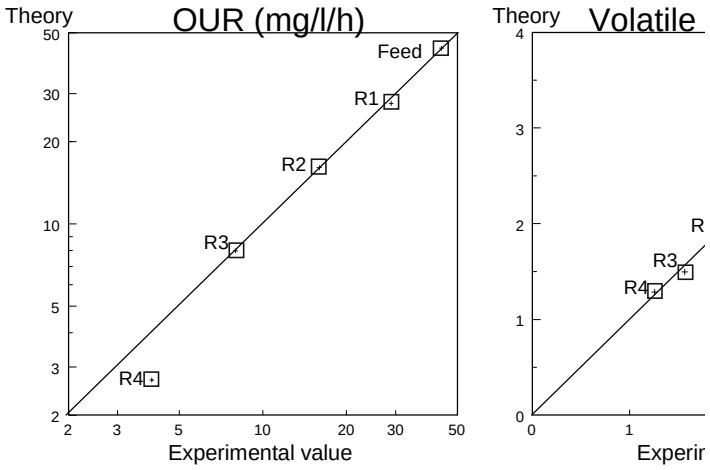

Figure 3
Observed and calculated variation of the OUR and the volatile sludge concentration in a series of four completely mixed aerobic digesters operated with a cyclic feed pattern of 1 sludge batch per day

- From reactor R_2 an extra volume of 6 ℓ/d was withdrawn in addition to the 15 ℓ/d for R_3, so that the retention time in R_2 was 45/21 = 2.14 d. The 15 + 6 = 21 ℓ/d were substituted by mixed liquor from R_1.
- From reactor R_1 an extra volume of 5 ℓ/d was withdrawn in addition to the 21 ℓ/d for R_2, so that the retention time in R_2 was 45/26 = 1.73 d. The 21 + 5 = 26 ℓ/d were substituted by mixed liquor from the batch settler, i.e. sludge from the aerobic lagoon.

TABLE 3
EXPERIMENTAL DATA OF OUR AND VOLATILE SOLIDS CONCENTRATIONS AND CORRESPONDING THEORETICAL VALUES CALCULATED WITH THE AID OF THE MODEL IN A SERIES OF FOUR COMPLETELY MIXED DIGESTERS WITH A CYCLIC (DAILY) LOAD

Sludge origin	Experimental data		Theoretical values		Active sludge fraction (X_{a1}/X_v)
	OUR ($mg \cdot \ell^{-1} \cdot h^{-1}$)	X_v ($g \cdot \ell^{-1}$)	OUR ($mg \cdot \ell^{-1} \cdot h^{-1}$)	X_v ($g \cdot^{-1}$)	
From settler	44	3.01	44	3.01	0.76
From R_1	29	2.52	27.7	2.33	0.65
From R_2	16	1.89	16.1	1.84	0.44
From R_3	8	1.57	8.0	1.50	0.26
From R_4	4	1.26	2.7	1.29	0.16

The experimental values of OUR and VSS concentrations in the mixed liquors withdrawn from the different aerobic digesters are in Table 2 (Columns 2 and 3). On the other hand these values can also be calculated from theory, if it is assumed that the decay constant can be expressed by the value determined above for batch digesters: $b_h = 0.24*1.04^{(25-20)} = 0.292\ d^{-1}$. In the case of OUR in the digesters, if the experimental value in the feed (44 $mg \cdot \ell^{-1} \cdot h^{-1}$) is taken as the basis for calculation, the OUR in the effluent from the first digester is calculated as 27.7 $mg \cdot \ell^{-1} \cdot h^{-1}$ with the aid of Eq. (20) for the retention time of 1.73 d in R_1. On the other hand the active sludge concentration in the feed can also be calculated from the OUR (Eq. 10): for t = 0: $X_{ai} = OUR_t/[(f_{cv}+4.57f_n)(1-f)b_h] = 2\ 310\ mg \cdot \ell^{-1}$. If this value is accepted, the VSS concentration in the digesters can be calculated with the aid of Eq. (17). The experimental and theoretical values of OUR and X_v are presented in Table 3 and in Fig. 3. It can be seen that the only significant deviation between experimental results and theoretical values is observed for OUR in R_4. However, the OUR in this reactor is so small (4 $mg \cdot \ell^{-1} \cdot h^{-1}$) that this discrepancy may be attributed to experimental error. It is concluded that the model for aerobic digestion predicts very accurately the behaviour of completely mixed digesters with a daily cyclic feed pattern over a very wide range of sludge compositions. The variability of the sludge can be assessed by calculating for each sludge the active fraction as the ratio of the active sludge concentration from the experimental OUR (Eq. 10) and the experimental VSS concentration. The active sludge fractions are calculated in Table 2 (last column). The range of the active fractions (0.16 to 0.76) shows the large differences in sludge compositions.

During the experimental investigation, the differences in sludge composition in the aerobic digesters were not only apparent from the SOUR, but also from occasional microscopic observations of the sludges. While in the sludge feed few micro-organisms other than bacteria were observed, in the series of aerobic digesters a much more varied population emerged, in conformity with the increasing degree of sludge stabilisation. In spite of the very different composition of micro-organisms in the series of reactors, the decay constant remained essentially constant and equal to the value observed in aerobic batch digesters.

Discussion

The results of the experimental investigations show that the aerobic digestion model of Marais and Ekama (1976) describes the aerobic digestion process of active sludge very accurately even under very different operational conditions. This accuracy is almost surprising, when it is considered that the composition of micro-organism populations in activated sludge is extremely dependent on the operational conditions.

In order to assess the degree of stabilisation of activated sludge in an aerobic digestion system it is necessary to have a simple and accurate test that can be used to determine the active sludge fraction of the sludge. From the Marais and Ekama (1976) model it would appear that such a parameter is the specific oxygen uptake rate (SOUR) defined as:

$$SOUR = OUR/X_v = (f_{cv}+4.57f_n)(1-f)b_hX_a/X_v = (f_{cv}+4.57f_n)(1-f)b_hf_{av} \quad (24)$$

or:

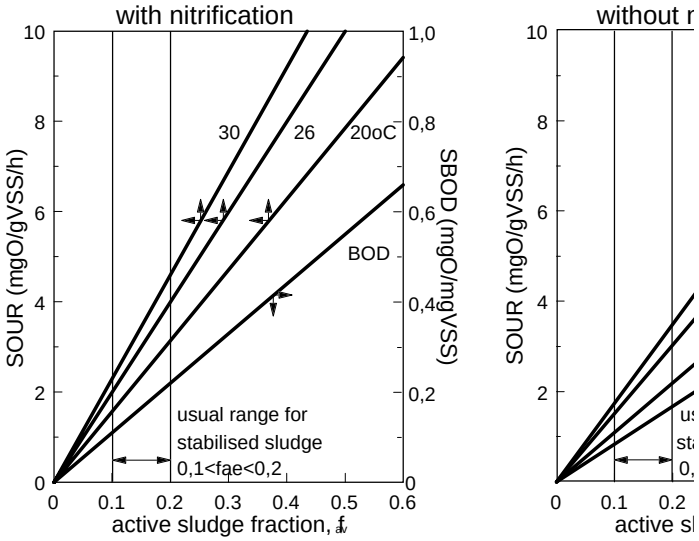

Figure 4
Relationship between active sludge fraction in aerobically digested sludge and the specific oxygen uptake rate, SOUR, as well as the BOD/VSS ratio for sludges with (a) and without (b) nitrification developing

$$f_{av} = SOUR/[(f_{cv}+4.57f_n)(1-f)b_h] \quad (25)$$

In case nitrification does not occur, there is no oxygen uptake relative to this process and the expression is changed correspondingly:

$$f_{av} = SOUR/[f_{cv}(1-f)b_h] \quad (26)$$

where:

SOUR = specific oxygen uptake rate ($mgO \cdot mg^{-1}X_v \cdot d^{-1}$)
f_{av} = active sludge fraction ($mgX_a \cdot mg^{-1}X_v$)

In Fig. 4 the relationships between f_{av} and SOUR are presented for different temperatures, with (4a) or without nitrification (4b). The usual range of the active fraction in aerobically digested sludge (f_{av} = 10 to 20%) is also indicated. The determination of SOUR is attractive because the necessary tests (OUR and X_v) are straightforward. However, if equipment is not available for OUR tests, the BOD of the sludge can be used as an alternative for the assessment of the stability. The relationship between the active fraction and the BOD of a sludge sample can be derived by considering that the BOD is equal to the oxygen consumption during the 5 d incubation period:

$$BOD = (f_{cv}+4.57f_n)(1-f)(X_{ai}-X_{a5}) = (f_{cv}+4.57\ f_n)(1-f)X_{ai}(1-\exp(-b_{h20}*5)) \quad (27)$$

As $\exp(-b_{h20}*5) = \exp(-0.24*5) = 0.30$, it follows that:

$$BOD = 0.70(f_{cv}+4.57f_n)(1-f)X_{ai} \quad (28)$$

Hence the ratio BOD/X_v or specific BOD can be expressed as:

$$SBOD = BOD/X_v = 0.70(f_{cv}+4.57f_n)(1-f)f_{av} = 1.10f_{av} \quad (29a)$$

If nitrification does not develop in the BOD bottle, the expression becomes:

$$SBOD = BOD/X_v = 0.70f_{cv}(1-f)f_{av} = 0.84f_{av} \quad (29b)$$

The graphical representations of Eqs. (29 a) and (b) are also in Fig. 4a and b respectively.

The present paper has only dealt with aerobic digesters, in which the Marais and Ekama (1976) model was shown to be very accurate. The model becomes less adequate when the environment is not aerobic. Warner et al. (1986) showed that in digesters operated under anoxic and aerobic conditions, the consumption of nitrate, expressed as equivalent oxygen consumption, during the anoxic periods was lower than the oxygen consumption in an aerobic digester under comparable conditions. By contrast when the anoxic environment was changed to aerobic the OUR was superior to that in a comparable aerobic digester. Similarly when a sludge batch from an aerobic digester is placed in an anaerobic environment (without dissolved oxygen or nitrate), endogenous respiration is impossible because there is no oxidant. However, as soon as oxygen is introduced the OUR is higher than before the batch was placed in the anaerobic environment. The phenomena above led Dold et al. (1980) to develop a new model called the death-regeneration model in which no endogenous respiration as such is considered. Instead it is assumed that when bacteria cease to exist as living organisms, a large part of their mass becomes available as organic substrate to the bacteria still alive and these will regenerate cellular mass with this substrate. The death-regeneration approach can be used in aerobic, anoxic and anaerobic environments and as such it is superior to the endogenous respiration approach of the Marais and Ekama model. However, conceptually the latter approach is simpler and the calculations are also less complicated. This is the reason the approach was preferred to describe aerobic digester behaviour.

Conclusions

- Aerobic digestion of active sludge can be described as a first order decay process with respect to the active (live organism) fraction of the sludge. In the decay process of the active sludge a fraction of the decayed material is oxidised to mineral compounds, whereas the remainder is transformed into an inactive organic fraction: the endogenous residue.
- The validity of the aerobic digestion model can not be shown directly because the active fraction cannot be measured. However relationships between four measurable parameters and the active sludge concentration in aerobic digesters can be derived: These parameters are:

- the VSS concentration;
- the nitrate concentration;
- the alkalinity; and
- the OUR.

- By observing the variations of the four parameters in batch reactors values of the decay constant of active sludge were calculated. It was shown that each of the four parameters will give the same value for the decay constant.
- From experiments in batch digesters, the temperature dependency was established for the range of 20 to 30°C:

  $b_h = 0.24*(1.04)^{(t-20)}$
- Aerobic decay of active sludge in a series of aerobic digesters with a cyclic load pattern was very well described with the same decay constant, even though the sludge composition and the nature of the organisms varied strongly in the series digester system.
- A convenient way to express the degree of stability of aerobically digested sludge is the SOUR, which value is directly proportional to the active sludge fraction. If equipment for SOUR determination is not available, the specific BOD value (SBOD) is an alternative measure.

References

BENEFIELD LD and RANDALL CW (1979) Design relationships for aerobic digestion. *J. Water Pollut. Contr. Fed.* **50** 518-523.

DOLD PL, EKAMA GA and MARAIS GvR (1980) A general model for the activated sludge process. *Prog. Water Technol.* **12** 47-77.

LAWRENCE AW and McCARTY PL (1970) Unified basic for biological treatment design and operation. *J. Sanit. Eng. Div., ASCE*. **96** SA3 757-778.

MARAIS GvR and EKAMA GA (1976) The activated sludge process Part I: Steady state behaviour. *Water SA* **2** (4) 163-200.

RANDALL CW (1975) Temperature effects on aerobic digestion. *J. Environ. Eng. Div. ASCE* **101** 95-811.

WARNER APC, EKAMA GA and MARAIS GvR (1986) The activated sludge process Part 4 - Application of the general kinetic model to anoxic-aerobic digestion of waste activated sludge. *Water Res.* **20** (8) 943-958.