Management of a reservoir for drought

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Abstract

The Min-Der Reservoir in Taiwan is undersized for meeting the full demand from it, so it is necessary to manage the water in the reservoir to meet demands in the best possible way. The monthly inflow series was extended using a rainfall-runoff deterministic model. Probabilities of end storages, given different initial and operating conditions, were used to optimise the release for various conditions. The operating rule was derived using linear programming optimisation methods with an objective function derived from questionnaires which aimed to minimise the cost to the economy.

The operating rule thus obtained indicated that the draft should be reduced when the reservoir level is low at the beginning of dry seasons. Water requirements should be curtailed for at least one or two seasons each year, with the irrigation sector being restricted more than industrial and domestic users. Simulations using both the newly developed operating rule and the old operating rule as suggested by the dam designers, revealed that the new operating rule results in much lower costs to the economy and fewer zero-storages. Alternative criteria for decision-making exist, i.e. frequency of running dry, maximum volume of water used, or minimum economic effect of restrictions.

Introduction

The Republic of China (Taiwan) receives the majority of its precipitation from typhoons from the Pacific Ocean requiring seasonal storage. The demand for water in Taiwan is increasing at 14%/a. The scope for constructing new reservoirs is, however, limited because the rivers are steep and in addition there is an environmental awareness deterring the construction of new dams. In the interim therefore, existing reservoirs will have to be operated on a variable draft basis.

The Min-Der Reservoir on the Houlong River south of Taipei supplies the Miaoli basin with water. The bulk of the water is used for irrigation, but the town of Miaoli and a growing industry, including a fertiliser factory and petro-chemical industry, also receive water from the reservoir. The reservoir catchment area is 61 km^2, the mean annual rainfall is 1 992 mm and the mean annual runoff is 90.6 m^3. The effective reservoir capacity is 14 x 10^6 m^3 and the average annual supply is 28 x 10^6 m^3 which is limited by the reservoir storage and could be considerably larger if enough water were available over the dry season (October to February).

Rainfall and river flow data

Only 22 years of streamflow records exist, so the data were extended using a deterministic rainfall-runoff model (RAFLER) (Stephenson and Paling, 1992). Rainfall data were available for 94 years in 10 d intervals for each of four rain stations surrounding the Min-Der Reservoir catchment. The rainfall data were processed into usable form as follows:

- the 10 d values were summed for each month to produce a monthly figure for each rain station;
- the data from the four rain stations were averaged.

The program was calibrated by comparing generated data with actual data. Once the most accurate solution was achieved by adjusting parameters, the rainfall data for 1901 to 1994 were processed. The output was read into a text file which was then used in various sections of a computer program for the optimisation of the reservoir operation. The program was incorporated in an expert system including the following steps using a menu-driven VisualBasic system:

- Selection of data
- Extension of time series for river inflow using RAFLER
- Analysis of each period to determine frequency of operation at different levels, for different drafts and initial storages
- Set up probability matrix
- Use optimum operating rule in a simulation
- Plot resulting drafts and storage levels over time series

Optimisation model

Any draft that does not meet the demand has a cost associated with it. If an industry/farmer does not receive the amount of water needed for the product/crop, their production/yield is going to decrease, thereby inducing financial losses. The model to be optimised consists of an objective function to minimise the probable cost to the economy of damages due to water restrictions (hereafter termed the cost):

$$\text{minimise } Z = \sum_{d} (C_d \sum_{s,i} h(s,i,d) \qquad (1)$$

where:

s is the season
d is the draft as a percentage of total demand
i is the initial storage as a percentage of total capacity
C_d is the cost associated with draft d;
h(s, i, d) is the probability of having draft d given an initial storage i in season s. h(s, i, d) is what is being solved for and will ultimately define the operating rule.

The constraints are as follows:

- The storage at the end of one season must equal the storage at the beginning of the following season (continuity):

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TABLE 1 INITIAL STORAGES, DRAFTS AND END STORAGE CATEGORIES ILLUSTRATING THE SET-UP FOR OPERATING RULES 1 AND 2		
	Operating Rule 1	**Operating Rule 2**
Initial storages	10, 30, 50, 70, 90% of capacity	0, 20, 40, 60, 80, 100% of capacity
Drafts	20, 40, 60, 80, 100% of demand	0, 20, 40, 60, 80, 100% of demand
End-storage categories	$0 \leq es_j < 20$ $20 \leq es_j < 40$ $40 \leq es_j < 60$ $60 \leq es_j < 80$ $80 \leq es_j <= 100$	0 $0 < es_j \leq 20$ $20 < es_j \leq 40$ $40 < es_j \leq 60$ $60 < es_j \leq 80$ $80 < es_j \leq 100$

$$\sum_{d,i} [P(s,i,d,f) \times h(s,i,d)] = \sum_{d,i'} h(s+1, i',d) \quad (2)$$

where:

P(s,i,d,f) is the probability that the final storage of the season is f, having started with an initial storage i and having a draft d in season s

i' is the initial storage for season s + 1 and has the same value as the end storage f for season s, i.e. i' = f in Eq. (2).

Equation (2) is based on the storage at the end of one season (f_s) equalling the storage at the beginning of the following season (i'_{s+1}). Thus the probability of ending a season with a certain storage must equal the probability of beginning the following season with the same storage. Serial correlation is therefore not accounted for, necessitating simulations at a later stage. The right-hand side is the probability of starting a season with a storage of i'. The left-hand side is the probability of ending the previous season with a storage f. The probability that the end storage is f is based on two things happening simultaneously - namely the probability of starting with a certain storage (P) multiplied by the probability of having a certain draft (h).

The probability of any mutually exclusive event happening is the sum of the individual probabilities i.e. $\Sigma(P \times h)$.

It will be noted that the formulation is based on the queuing theory of Langbein (1958). It does not require the lumping of initial storage and inflow used by Thomas and Watermeyer (in Maass et al., 1962). The formulation was initially used by Stephenson (1968).

- Ensure that there is some initial storage:

$$\sum_{s,i} h(s,i,d) = 1 \text{ for all } s \quad (3)$$

- The probabilities must be limited to a fraction:

$$0 \leq h(s,i,d) \leq 1 \text{ for all } s, i, d \quad (4)$$

Probabilities of end storages

The optimisation model requires the probabilities of ending a time interval with various end storages having started the season with a certain initial storage and having had a set draft throughout the season (P(s,i,d,f)).

The probability P(s,i,d,f) is calculated for every possible combination of initial storage representative, draft increment, season and end-storage category. This is obtained by simulating reservoir operation for a season using the following equation:

$$es_j = es_{j-1} + flow(j) - draft - evap(jm) \times area \quad (5)$$

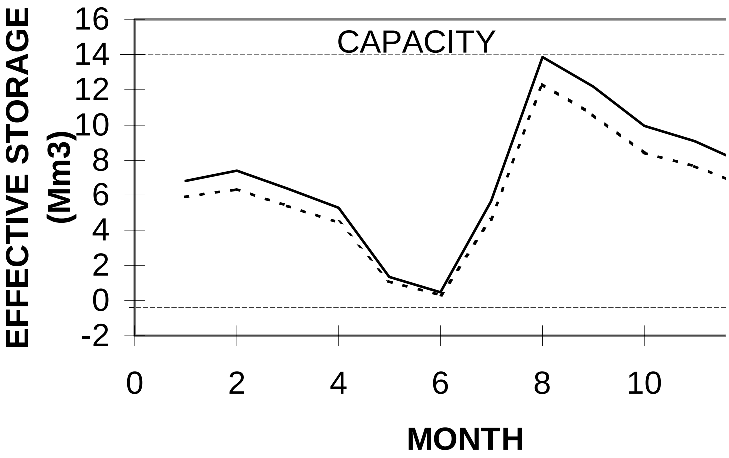
where:

j is the month number (1 - 12, 13 - 24 ...)
jm is the numerical value for each month in the year (jm = 1 = January etc.)
es_j is the end storage for month j (Mm^3)
es_{j-1} is the initial storage for month j
es_{j-1} = i for the first month of the season
flow(j) is the river flow into reservoir in month j (Mm^3/month)
draft is the allowable supply from the reservoir for the season (Mm^3/month)
evap(jm) is the evaporation for month jm (m)
area is the surface area of the reservoir (Mm^2).

The section of the computer program following the rainfall-runoff calculations calculates the above probabilities for two alternatives of an operating rule. They both work on the same principle, but with slightly different values. The values for the initial storages, drafts and end-storage categories used can be compared in Table 1. Apart from the different values (categories) the way in which the calculations proceed is the same.

Running the optimisation

The cost to the economy of any draft below the demand was estimated by issuing all consumers (industry, agriculture and domestic) with a questionnaire. The costs per month to each individual sector of different levels of rationing were established.



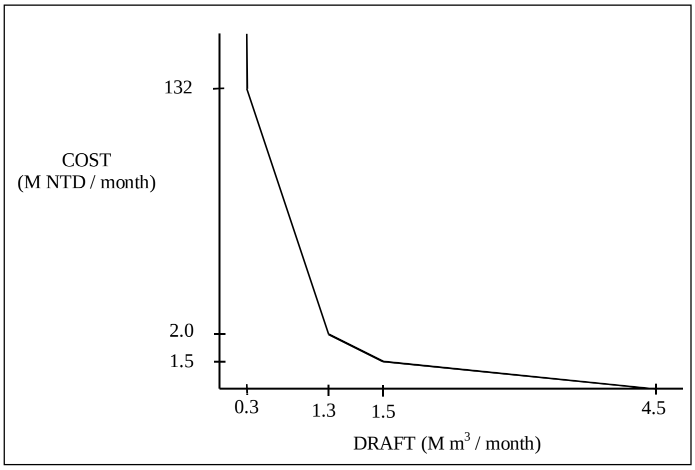


Figure 1 (top)
Old operating rule

Figure 2 (right)
Average monthly economic loss function (not to scale)

That is, the cost to the economy due to receiving insufficient water in times of drought was estimated.

The resulting cost function can be represented by the following set of equations (Fig. 2):

$1.5 < x < 4.5$	$y = -0.5\,x + 2.25$
$1.3 < x < 1.5$	$y = -2.5\,x + 5.25$
$0.3 < x < 1.3$	$y = -130\,x + 171$

where:
x = water use,
y = cost per month.

Draft is given as a volume in 10^6 m^3/month. Cost is given in million New Taiwan Dollars (NTD)/month. A punitive cost of 1 000 000 x 10^6 NTD/month for a draft of zero was used in the model for Operating Rule 2, i.e. it prevented the reservoir from running dry to result in zero draft.

The objective function to minimise probable economic loss due to rationing water over all seasons - $Z = \sum_d (C_d \sum_{s,i} h(s,i,d))$ - for Operating Rule 1 becomes:

$$\min Z = 0.45 \sum_{s,w} h(s,w,80) + 0.9 \sum_{s,x} h(s,x,60) + 1.35 \sum_{s,y} h(s,y,40) + 54 \sum_{s,z} h(s,z,20) \qquad (6)$$

where:
w (w = 90, 70), x (x = 90, 70, 50), y (y = 90, 70, 50, 30), and z (z = 90, 70, 50, 30, 10) are the sets of feasible initial storages that can support the given draft.

$w \neq x \neq y \neq z$ because certain drafts are not feasible given the initial storage. A low initial storage could not possibly sustain a large draft for the season e.g. an initial storage of 10% could not sustain a draft of 100%. A simulation was run for each of 94 years to determine the end storage of every combination of initial storage and draft for every season. Those combinations that often resulted in the reservoir running dry were omitted.

The objective function for Operating Rule 2 becomes:

$$\min Z = 0.45 \sum_{s,w} h(s,w,80) + 0.9 \sum_{s,x} h(s,x,60) + 1.35 \sum_{s,y} h(s,y,40) + 54 \sum_{s,z} h(s,z,20) + 1\,000\,000 \sum_{s,v} h(s,v,0) \qquad (7)$$

where:
w (w = 100, 80), x (x = 100, 80, 60), y (y = 100, 80, 60, 40), z (z = 100, 80, 60, 40, 20), and v (v = 100, 80, 60, 40, 20, 0) are the sets of feasible initial storages that can support the given draft.

Constraint 1 is applied for all but one category of end-storage f, namely f = 80 - 100 because for each combination of initial storage and draft, the sum of the probabilities of end storage equals unity.

The variables to be optimised in the model are the probabilities of having a draft d given an initial storage i in each of the seasons s (h(s,i,d)). The models developed above were solved using the QuattroPro optimiser.

The resulting Operating Rule 1 is represented graphically in Fig. 3. The probabilities h(s, i, d) for each initial storage are the values just above or below the curve. Fig. 3 gives the operating draft for any storage for each season.

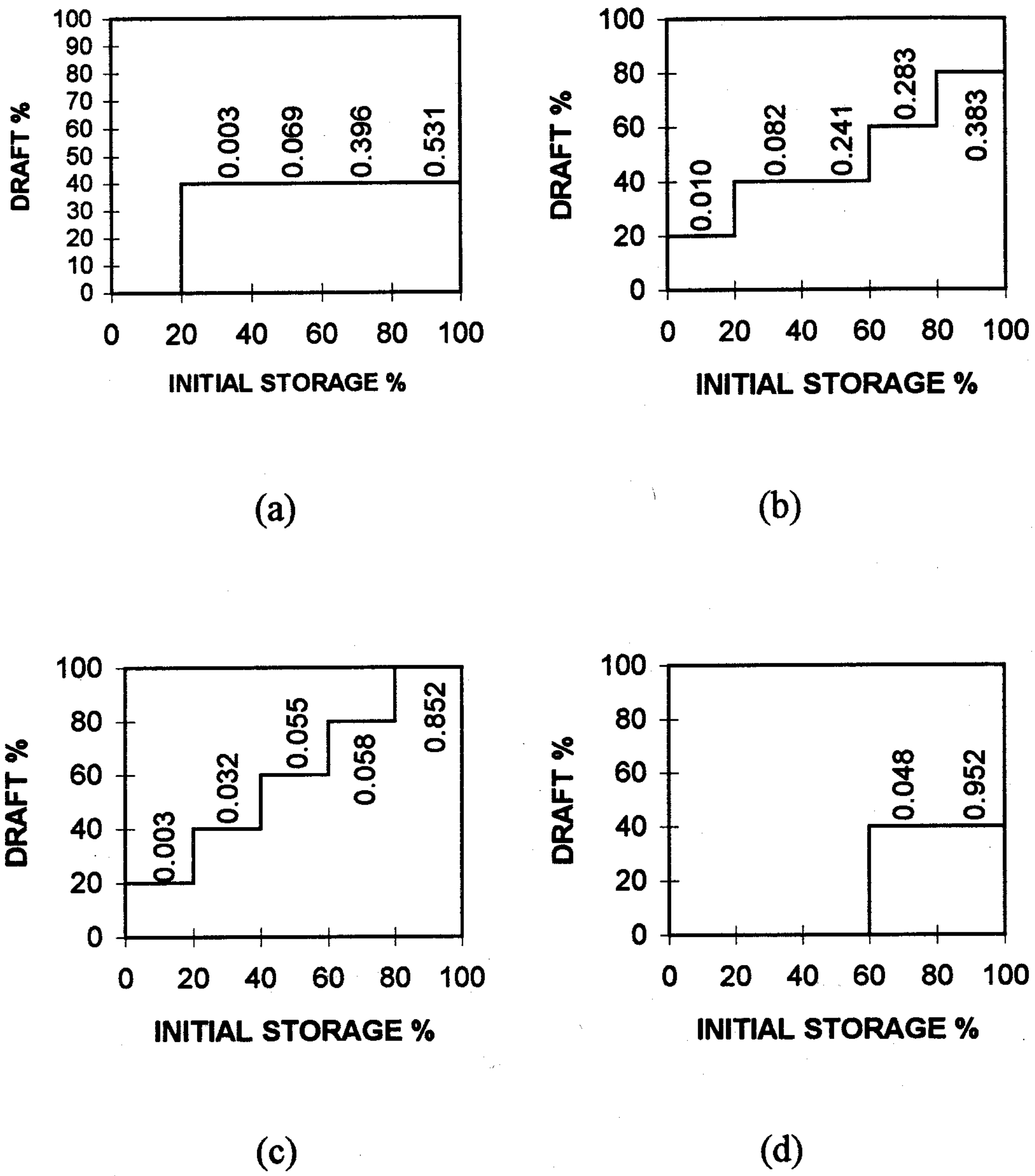


Figure 3 (top)
Graphical representation of Operating Rule 1 as defined by QuattroPro and LINDO: (a) season 1; (b) season 2; (c) season 3; (d) season 4

TABLE 2
COMPARISON OF AVERAGE ANNUAL COST, AVERAGE ANNUAL WATER USED AND CUMULATIVE STORAGES LESS THAN OR EQUAL TO ZERO FOR THE THREE OPERATING RULES

Operating rule	Average cost (10^6 x NTD / a)	Average water used (10^6 m^3 / a)	Storages = 0
1	31 930	31.9	3
2	42 591	32.3	0
Old	244 686	42.6	30

Simulation

Simulation of the Min-Der Reservoir operation was used as a tool to investigate and compare the impacts of each operating rule.

Equation (5) was used with a monthly time increment to simulate the operation of the reservoir:

Should the end storage for any month be zero, the draft for the following month is set to zero. If the draft has been set to zero and the storage in the reservoir increases sufficiently during the season, a small draft is allowed for the rest of the season. The cost incurred by each draft and the total amount of water used are summed for the series and the number of times the reservoir runs dry are counted.

Results

The costs, amounts of water used and number of storages less than or equal to zero for the simulation of all three operating rules can be found in Table 2.

The Old Operating Rule is indicated in Fig. 1. Operating Rule 1 clearly produces the least cost solution, almost one eighth of the cost of the Old Operating Rule. The reservoir, however, runs dry

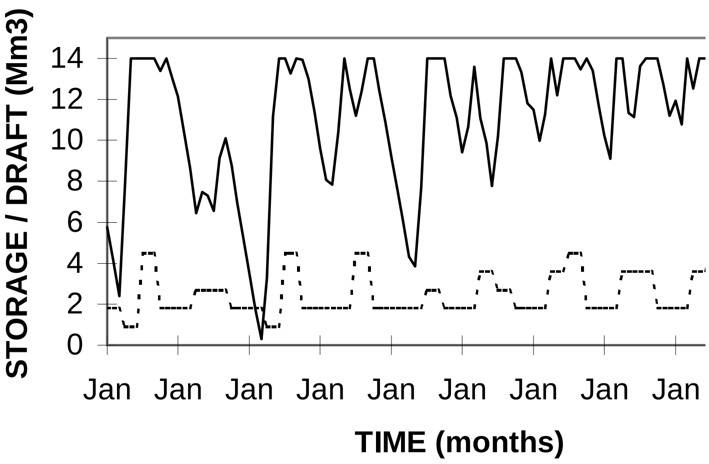


Figure 4

Graph Illustrating Operating Rule 1 storage and draft for the hydrological years 1951 - 1960

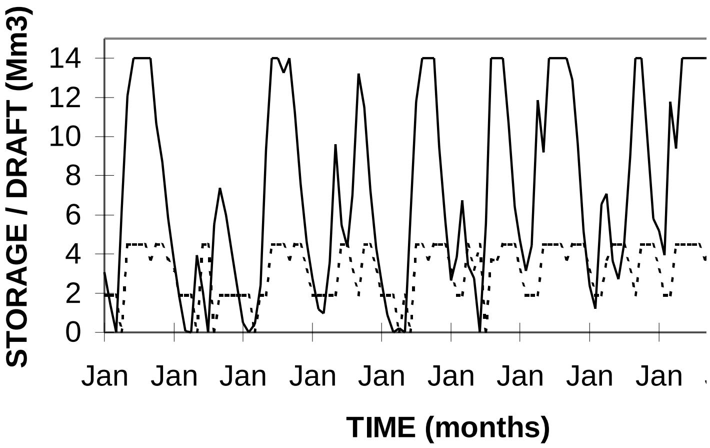


Figure 5

Graph Illustrating Old Operating Rule storage and draft for the hydrological years 1951 - 1960

for one period of three months under Operating Rule 1 (January to March of 1931), but not at all under Operating Rule 2. This is despite the fact that Operating Rule 2 has more zero drafts than Operating Rule 1 (hence the greater cost). The Old Operating Rule runs dry 21 times for a total of 23 months. As the cost of the operating rule increases so does the amount of water used.

Figures 4 to 5 illustrate the storages and drafts in the reservoir resulting from Operating Rule 1 and the Old Operating Rule respectively. As can be seen, the Old Operating Rule resulted in the reservoir running dry several times in the 1951 to 1960 time interval. There are also corresponding zero drafts in this time interval. Operating Rule 1 did not have any zero drafts or storages.

Discussion

The amount of water used in terms of an operating rule increases with an associated increase in cost. The likely reason for this is that if more water is utilised in any season, there is likely to be less water left in the reservoir for the following season. Therefore a harsher restriction will have to be implemented for the following season, resulting in a higher cost to the economy. This is because the cost function is non-linear. Operating Rule 1 results in a lower cost than the Old Operating Rule, but less water is used. This is contrary to the anticipated result that the new operating rule would increase the yield. As the objective to minimise cost is achieved, however, this is not important.

It must be noted that **the simulation is a hypothetical enactment of what may occur**. This is important since it is likely that the simulation involving the Old Operating Rule does not produce a very accurate scenario. It is possible that if the drought was extremely severe, the reservoir operator may have restricted more vigorously than prescribed by the operating rule and at more frequent times. This may possibly have led to fewer incidents of the reservoir running dry and a resulting decrease in cost to the economy. However, this reduction in cost would not be enough to bring into doubt the superiority of Operating Rules 1 and 2. The same argument may even be applied to Operating Rules 1 and 2 - that the operating rules may not be strictly adhered to - which would influence the cost to a degree.

The water demands and costs are relevant for the 1990s. As the population increases and industry grows, they will place a greater demand on the water resources which are barely sufficient at present. At the same time a decrease in farming as well as improved crops and irrigation methods might result in a decrease in water demand from agriculture. It is difficult to gauge and predict these changes as historical records are required. The historical records of water supplied to the different sectors however, do not necessarily reflect changes in demand, but rather fluctuations in availability of water.

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