Modelling hindered batch settling Part I: A model for linking zone settling velocity and stirred sludge volume index

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Abstract

A new type of model for describing the relationship between zone settling velocity (V_s) and stirred sludge volume index (SSVI) is developed. Compared to the earlier studies which link SSVI and V_s (to reduce multiple batch tests when collecting X-V_s data), the proposed model is more advantageous because it also describes regions of low solids concentrations. The model is derived from the model for batch settling curve and is expressed as $V_s = C\left(\frac{1000X}{(X^2+\beta)SSVI} - 1\right)$, where C and β are parameters describing zone settling velocity and X is the activated sludge concentration. The applicability of the relationship was tested by analysing data published earlier in the literature. Based on these data it was found that the model can be used in wide ranges of SSVI and sludge concentration. A modified Vesilind model is also introduced and written as $V_s = \frac{1000\nu}{SSVI}e^{-nX}$, where n and ν are parameters describing zone settling velocity.

Nomenclature

C, α and β	parameters describing solids settling
F	solids flux
f_{ns}	non-settleable fraction of X_{in}
$h(t, h_0)$	sludge blanket interface level at time t
h_0	initial sludge blanket interface level
n and ν	parameters describing zone settling velocity
r_h and r_p	parameters describing solids settling
SS_{reg}	sum of squares where the source is regression
SS_{res}	sum of squares where the source is residual
SSVI, $SSVI_{3.5}$	stirred sludge volume index
SVI	sludge volume index
t	time
V_0	maximum settling velocity
V_0'	maximum practical settling velocity
V_s	zone settling velocity
V_{sj}	settling velocity of the solids particles in layer j
X	solids concentration
X_{in}	solids concentration entering the settler
X_j	solids concentration in layer j
X_{max}	solids concentration after a long settling period
X_{min}	minimum attainable solids concentration in layer.

Introduction

Zone settling velocity is a well-known method used for describing sludge settleability but has not been used in daily operation of treatment plants as much as the sludge volume index (SVI), probably because measuring for zone settling velocity is more laborious than measuring for SVI. Zone settling has been widely utilized in research circles, especially in the settler models based on flux theory.

The solids flux theory is a generally accepted approach for describing the thickening function of activated sludge and applied in most of the clarifier models. The solids flux is defined as:

$$F = V_s X$$

where:
F is solids flux ($kg \cdot m^{-2} \cdot h^{-1}$)
V_s is zone settling velocity ($m \cdot h^{-1}$)
X is solids concentration ($kg \cdot m^{-3}$).

The solids flux theory requires a relationship to be established between different solids concentrations and zone settling velocities. The most well-known model describing the settling of activated sludge is presented by Vesilind (1968). The Vesilind model has the form:

$$V_s = V_0 e^{-nX}$$

where:
V_0 and n are parameters describing zone settling velocity having the units of ($m \cdot h^{-1}$) and ($m^3 \cdot kg^{-1}$), respectively
X is activated sludge concentration ($kg \cdot m^{-3}$).

Lately, the relationship proposed by Takács et al. (1991) has been considered more appropriate for purposes of settler modelling (see Jeppsson, 1996; Krebs, 1995 and Grijspeerdt et al., 1995) because of its applicability to regions of low concentration.

Takács's model can be presented as the sum of two exponential terms:

$$V_{sj} = V_0 e^{-r_h X_j^*} - V_0 e^{-r_p X_j^*}$$
$$0 \le V_{sj} \le V_0'$$

where:
V_{sj} is the settling velocity of the solids particles in layer j
V_0 is the maximum settling velocity
V_0' is the maximum practical settling velocity
r_h and r_p are parameters describing solids settling
$X_j^* = X_j - X_{min}$
X_j is the solids concentration in layer j
X_{min} is the minimum attainable solids concentration in layer
$X_{min} = f_{ns} X_{in}$
X_{in} is the solids concentration entering the settler
f_{ns} is the non-settleable fraction of X_{in}.

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The traditional V_s-X data collection procedure is not feasible at most treatment plants because it needs multiple batch tests and thus is time-consuming. One approach to avoid this problem is to link an easily determined index of settleability to zone settling velocity data. Correlating the settling test data with sludge volume index allows settling flux theory to be easily applied to plant design and operation (Daigger and Roper, 1985). In practice, the studies have been concentrated on linking the Vesilind model's parameters estimated from experimental zone settling velocity data to sludge volume indexes, i.e. sludge volume index (SVI), stirred sludge volume index (SSVI) and diluted sludge volume index (DSVI) (e.g. Pitman, 1984; Daigger and Roper, 1985; Catunda and Van Haandel, 1992; Ozinsky and Ekama, 1995b).

The present study proposes a totally new type of relationship between zone settling velocity (V_s), stirred sludge volume index (SSVI) and activated sludge concentration (X). The aim was to develop a model which links SSVI and V_s. The relationship is derived from the sludge blanket interface settling model (Renko, 1996) with simple assumptions, thus having a sound base. The purpose of the model is the same as the earlier attempts to link SSVI and V_s, i.e. to reduce the need for multiple batch tests. Since the proposed model also describes regions of low solids concentrations, unlike the previous models, it can be utilised in advanced settler models.

Model derivation

Settling of the sludge blanket interface as a function of time can be described as (Renko, 1996):

$$h(t, h_0) = \frac{C(X^2 + \beta)h_0}{\alpha X} + \left(h_0 - \frac{C(X^2 + \beta)h_0}{\alpha X}\right)e^{-t\alpha X/((X^2+\beta)h_0)} \quad (1)$$

where:

$h(t,h_0)$ is the sludge blanket interface level at time t (m)
h_0 is the initial sludge blanket interface level (m)
t is time (h)
X is the activated sludge concentration (kgm^{-3})
α, C and β are the parameters describing activated sludge settling having the units of ($m \cdot h^{-1}$), ($kg \cdot m^{-2} \cdot h^{-1}$), and ($kg^2 \cdot m^{-6}$), respectively.

Velocity can be thought of as a rate of change and it can be calculated as a derivative of the $h(t, h_0)$. Thus, sludge settling velocity as a function of $h(t, h_0)$ can be determined as a derivative of Eq. (1):

$$\frac{dh(t, h_0)}{dt} = -\frac{\alpha X h(t, h_0)}{(X^2 + \beta)h_0} + C \quad (2)$$

In a well-settling activated sludge, the maximum settling velocity is reached soon after the start of the settling test. By taking this into account, it can be assumed that $t \approx 0$ and $h(t,h_0) \approx h_0$. With these assumptions, the equation for initial settling velocity can be formulated as:

$$\frac{dh(t, h_0)}{dt} = -\frac{\alpha X}{(X^2 + \beta)} + C$$

where:

$$\frac{dh(t, h_0)}{dt} = -V_s$$

Besides, activated sludge concentration after a long settling period can be approximated as:

$$X_{max} = \frac{\alpha}{C}$$

and as a consequence the stirred sludge volume index as:

$$SSVI = \frac{C}{\alpha}\ 1000 \quad (3)$$

By solving α from Eq. (3) and by substituting it in Eq. (2) we get the following relationship linking V_s, SSVI and X:

$$Vs = C\left(\frac{1000X}{(X^2 + \beta)SSVI} - 1\right) \quad (4)$$

It should be noted that because of the initial assumption ($t \approx 0$ and $h(t, h_0) \approx h_0$) and the extension of Eq. (1) to cover a large range of X and SSVI the parameter estimates determined from a single batch settling curve would not describe the relationship between V_s, SSVI and X very accurately and vice versa.

Model applicability

The model accuracy was demonstrated by analysing the results reported by Pitman (1984) which presented typical values of settling property constants, viz. the Vesilind's parameters, for various SVI and $SSVI_{3.5}$ measured during more than a six-year period at four biological waste-water treatment plants in Johannesburg (Table 1). There is a considerable difference in settleability between the sludge at the plants and settling properties at most of the plants showed a fairly wide variation (Pitman, 1984). For more information on the data and the data collection see Pitman (1984).

In order to estimate the parameters C and α the V_s -X data were computed based on parameter values (V_0 and n) in Table 1. The zone settling velocities were computed with the Vesilind model in each range from the concentration range 1 to 14 kgm^{-3} by 1 $kg \cdot m^{-3}$ interval. The average value of each $SSVI_{3.5}$ group was applied to determine a value for X_{max} attainable. All concentrations (and the corresponding V_s value) exceeding X_{max} value were removed. For example, with the $SSVI_{3.5}$ value of 90 $g \cdot m\ell^{-1}$ the

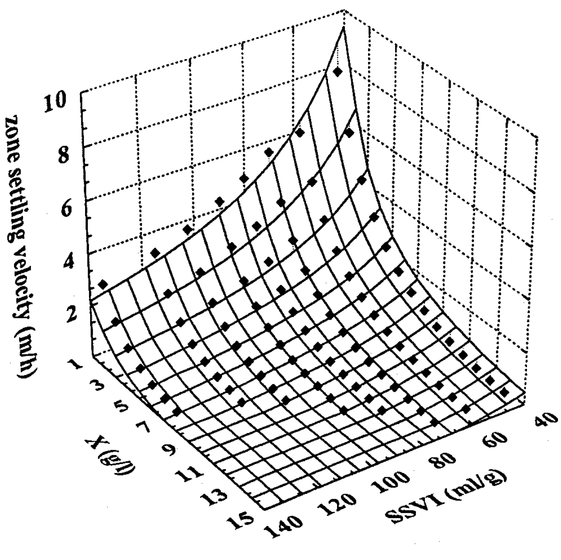

Figure 1
Zone settling velocities as a function of activated sludge concentration and $SSVI_{3.5}$ (♦ the Vesilind model , -- the proposed model)

SSVI$_{3.5}$	V_0	n	C	α	SS_{reg}	SS_{res}
35-50	10.5	0.30	0.64	0.77	133.23	0.85
50-65	8.06	0.31	0.71	0.86	75.46	0.16
65-75	7.82	0.34	0.81	0.81	62.73	0.06
75-85	7.03	0.37	0.79	0.75	45.06	0.02
85-95	6.40	0.40	0.77	0.69	33.40	0.01
95-110	5.63	0.44	0.72	0.62	22.45	0.006
110-120	5.09	0.48	0.69	0.57	16.06	0.004
120-150	4.47	0.52	0.71	0.56	10.91	0.01

TABLE 1
TYPICAL VALUES OF V_0 AND N FOR VARIOUS RANGES OF SSVI$_{3.5}$ BY PITMAN (1984) AND RESULTS OF PARAMETER ESTIMATION OF THE PROPOSED MODEL

concentrations 12, 13 and 14 kg·m^{-3} were removed from the data set. The manipulation was made because of the presumption of the model that when the X_{max} is reached no settling occurs, V_s is zero and thus higher concentrations cannot be obtained.

Parameters C and α were estimated for the whole reduced data set and also separately for each SSVI$_{3.5}$ group with the multivariate secant method (DUD method). The estimation of the whole data set resulted in a value of 0.69 m·h^{-1} for C and 0.71 kg^2·m^{-6} for α. The sum of squares for the regression (SS_{reg}) was 395.4016 and for the residual (SS_{res}) 5.0276 m^2·h^{-2}. The result is shown in Fig. 1. The result of the estimation for the eight SSVI$_{3.5}$ groups is shown in Table 1 and demonstrated in Fig. 2.

Discussion and conclusions

A model for combining zone settling velocity and stirred sludge volume indexes is derived. The model is a direct consequence of the batch settling curve model proposed by Renko (1996). The model applicability was tested with the settleability results published by Pitman (1984). The model gave a good fit with large SSVI$_{3.5}$ and concentration ranges offering a promising relationship linking zone settling velocity and stirred sludge volume index.

Ozinsky and Ekama (1995a) stated that one of the shortcomings of SVI is that it has no rational relationship to the zone settling velocity. The reason for this is most probably that these two measures are detected in different procedures; zone settling velocity is determined with stirred and SVI non-stirred methods (*Standard Methods*, 1985). For that reason in this study only the stirred sludge volume index is examined. It is obvious that the relationship between zone settling velocity and SVI is more complicated than the relationship between V_s and SSVI proposed here.

The results of this study are compressed in Fig. 1: the model describes V_s in wide ranges of SSVI$_{3.5}$ and X with two parameters. Pitman (1984) computed parameter estimates for each of the SSVI$_{3.5}$ group and the proposed model utilises only one estimate for C and one for α. This result supports the presumption that the batch settling curve model from which the model for linking V_s and SSVI is derived is applicable in describing solids settling. If the SSVI$_{3.5}$ range is changed into SVI values it covers approximately the range between <50 and 375 mℓ$^{-1}$·g. Hence, the model gave a good fitting from very well settling sludge to bulking sludge. Also the sums of squares verify the applicability of the model. Collection of V_s - X data is considered to be the bottleneck in utilising the solids flux theory in practice. Based on the results of this study it can be stated that SSVI$_{3.5}$ is a measure to avoid the collection of the V_s - X data and it is an accurate method to assess settleability using a simple procedure.

It is obvious that a single universal parameter vector linking all the V_s - SSVI data cannot be detected. However, since zone settling velocity and SSVI describe the same sludge settling phenomena it is probable that typical parameter estimates can be found for certain process types at least within a limited time period. If so, the determination of settleability would be facilitated considerably. This is specially interesting in process control and settler operation because the tedious and slow multiple batch technique for measuring zone settling velocity as a continuous measurement routine could be avoided and there would be, perhaps, only need to conduct the zone settling velocity tests on a weekly basis for re-estimating the parameters.

When utilising the proposed model in practice V_s-SSVI-X data have to be collected first and the parameters C and α are estimated from these data. Then zone settling velocity as a function of sludge concentration will be determined by the model and by measuring SSVI only. The frequency to update the parameter estimates depends on the variations in the process and on the purpose for which the model is used. Whenever the parameters are being updated V_s-SSVI-X data have to be collected again.

The Vesilind model is generally accepted in describing a V_s - X relationship because it is very accurate and the structure of the model is simple. The parameter estimates for C and α were computed separately for each SSVI$_{3.5}$ group to demonstrate the applicability of the model. It can be seen that V_s - X data can be described accurately with a totally different type of model (Table 1 and Fig. 2). It can be stated that the proposed model is accurate because it gives practically identical results with the given ranges of activated sludge concentration compared to the Vesilind model which is considered to be superior to all others. Although the models give identical results they have at least four major differences:

a) The Vesilind model describes zone settling in the traditional way only as a function of activated sludge concentration whereas additional information (SSVI) is included in the proposed model. Thus, the structure of the models and the data needed are different.
b) As a consequence of a), the proposed model describes settling also as a function of SSVI whereas the Vesilind model does

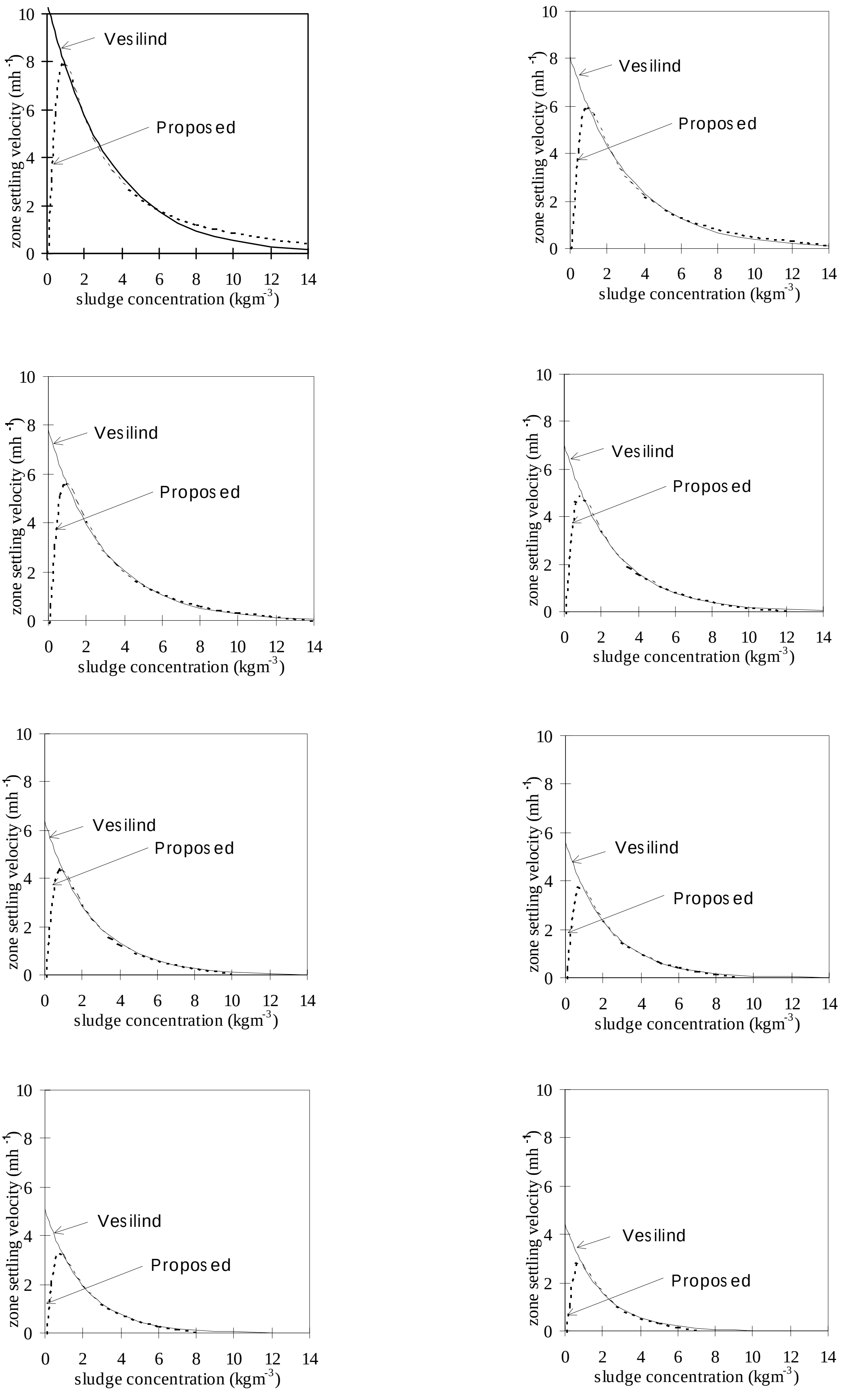

Figure 2

V_s as a function of X computed with the Vesilind and the proposed model

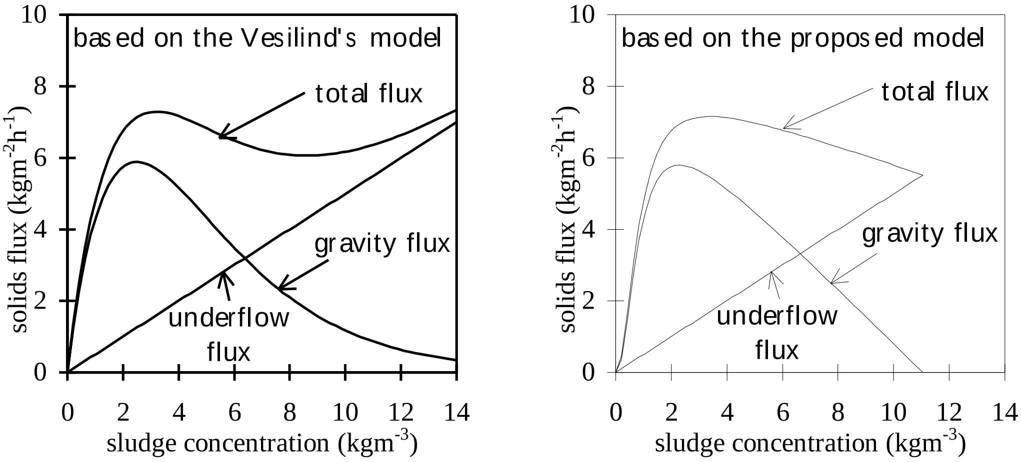

Figure 3
Graphical interpretation of the solids flux theory

not take into account SSVI values. Thus, the purpose for which the models are formulated is different.

c) The models have a different interpretation of high concentrations. The proposed model suggests that there is a maximum concentration attainable which can be determined from SSVI. In other words, the term $\left(\frac{1000X}{(X^2+\beta)SSVI}-1\right)$ cannot be smaller than zero. In the Vesilind model such a limit cannot be determined. This has an important consequence on the interpretation of solids flux theory depicted in Fig. 3. It can be seen that a limiting flux exists with the models but with the proposed model it is obtained with the maximum concentration. In practice, this does not have a big effect on clarifier models because the settling velocities of high concentrations computed by the Vesilind model are so slow that much higher concentrations than X_{max} cannot be reached with the typical operation range (retention time) of settlers.

d) The models have different interpretations of low concentrations. If the models are extrapolated into low concentration (Fig. 2) it can be seen that the Vesilind model has its maximum at concentration zero whereas the proposed model has it at around 1 $kg \cdot m^{3}$. This feature is more theoretical because zone settling does not occur with low concentrations and the model applicability cannot be assessed based on data.

The proposed model has a similar type of interpretation of low concentrations as the model introduced by Takács et al. (1991). This model has lately been of great interest in applications for settler models (Jeppsson, 1996; Krebs, 1995; Grijspeerdt et al., 1995) especially because of its description of low concentrations. The advantage of the proposed model compared to that of Takács et al. (1991) is the low number of parameters. The proposed model has two parameters and the model of Takács et al. (1991) has five parameters. The low number of parameters reduces the computational and identifiability problems in parameter estimation. It should be noted that neither of these models are comparable because the model by Takács et al. (1991) describes V_s only as a function of X and additional information is needed (i.e. SSVI) in the proposed model. However, since the determination of SSVI is much less tedious than determination of V_s the extra work can considered negligible.

By applying the findings of this study, i.e. an inversely proportional relationship between V_s and SSVI, to Vesilind model a modified Vesilind model, can be written as:

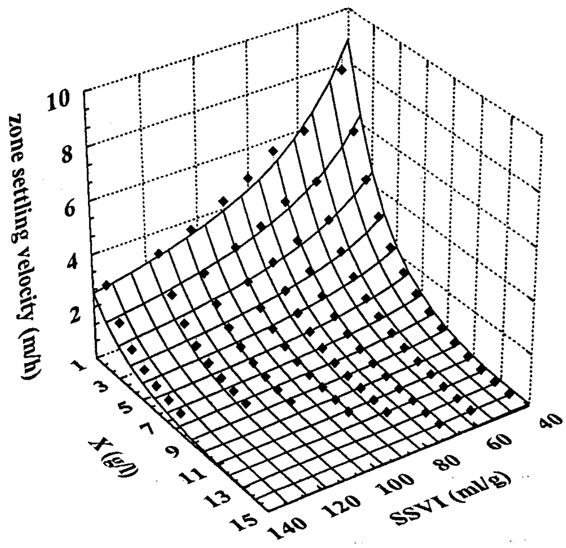

Figure 4
Zone settling velocity as a function of sludge concentration and $SSVI_{3.5}$ (♦ the Vesilind model, -- the modified Vesilind model)

$$Vs = \frac{1000\nu}{SSVI} e^{-nX} \tag{5}$$

where:

ν is the parameter describing settleability ($kg \cdot m^{-2} \cdot h^{-1}$).

This interpretation suggests that n is a constant and V_0 is a function of $SSVI_{3.5}$. Figure 4 shows zone settling velocity as a function of X and $SSVI_{3.5}$ computed with the modified (ν = 0.50 $m^4 \cdot kg^{-1} \cdot h^{-1}$ and n= 0.34 $m^3 \cdot kg^{-1}$) and the original (V_0 and n from Table 1) Vesilind models. The effect of $SSVI_{3.5}$ can be similarly taken into account in the model proposed by Takács et al. (1991).

The proposed model combines the accuracy of the Vesilind model, interpretation of low concentration of the Takács model and the effect of SSVI into one simple model. The results of the preliminary analysis appear to be promising. However, the data set analysed here is limited based on the Vesilind parameters and according to Pitman (1984) the original data showed a fairly wide variation. If the results by Pitman (1984) present a typical relationship between zone settling velocity and $SSVI_{3.5}$, the proposed model should also be applicable more generally. In the future, the model should be investigated in more detail with experimental data including different types of activated sludges in order to determine the applicability of the model and the variation of the parameters. Also the applicability of the modified Vesilind model should be checked with larger data sets.

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