Modelling hindered batch settling Part II: A model for computing solids profile of calcium carbonate slurry

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Abstract

This study suggests a hypothesis for determining and a model for computing suspended solids behaviour during batch settling aimed at better understanding of the settling process and more accurate and convenient determination of solids profiles. The approach is built on the graphical approach of Work and Kohler taking into account the effect of scale (i.e. initial height of suspension) on the settling of the surface of suspension. It is shown that the proposed approach includes the same information as the traditional Kynch theory of sedimentation. The proposed model, derived from the hypothesis and a model for a batch settling curve, introduces a mathematical tool for determining the solids layers and it also eliminates the disadvantage of the Kynch theory which only offers a graphical procedure for the determination. The applicability of the model was tested with calcium carbonate ($CaCO_3$) suspension. The experimental study proves that the model can be successfully utilised in computing the behaviour of calcium carbonate during a batch settling test.

Nomenclature

α and C	parameters describing solids settling
$CaCO_3$	calcium carbonate
$\dfrac{dh(t, h_0)}{dt}$, v, V	settling velocity of the solids surface
∂h_0	difference between the starting heights of two curves
$\partial h(t, h_0)$	difference in height between the two curves at time t
h_0, H_0	initial height of settling curve
$h(t)$, $h(t_1)$, $h(t_2)$, $h(t, h_0)$, $H(t)$, $H(t_1)$, $H(T_1)$, $H(T_2)$	height of settling curve "
$h_{tg}(t)$	point where tangent crosses the vertical axis
$Oh(t)$, $Oh(t_1)$, $Oh(t_2)$, $OH(t_1)$, $OH(T_1)$, $OH(T_2)$	line showing the constant concentration "
t, T_1, T_2, t_1 and t_2	time
X_0	initial solids concentration
$X_{Oh(t)}$, $X(t,h)$, $X(t,H)$	solids concentration

Introduction

Particles settle independently when solids concentration of suspension is low and the particles are at a considerable distance from each other. This type of particle settling is referred to as clarification and can be accurately explained by Stoke's law. As a particle settles independently from others, only the characteristics of the particle and the properties of the fluid affect the settling velocity of the particle.

Settling is different when the concentration of solids is high and the distance between particles is so small that the particles come into contact with one another. In such a case, Stoke's law

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fails to describe the settling of particles. If the concentration is high enough there is significant interaction between the particles and in a batch settling test they start to settle together having a clear interface between the suspension and the supernatant liquid. This type of settling is typically observed with suspensions like activated sludge, $CaCO_3$ and metal hydroxide slurries.

A theory for explaining settling of suspensions with high concentrations was suggested by Kynch (1952). The theory predicts behaviour of solids during a batch settling test and is applicable to certain materials like caolinite clay but not for all types of suspension like activated sludge (Cole, 1968). However, the theory is still, probably, the best interpretation suggested for zone settling phenomena despite its shortcomings.

On the other hand, the disadvantage of the Kynch theory is that its graphical approach for determining solids behaviour is time-consuming. This paper suggests a model to facilitate the prediction, which is postulated from the Work and Kohler (1940) approach, taking into account the effect of initial height of suspension on the settling curve. The applicability of the model was tested with a $CaCO_3$ suspension.

Determination of the solids behaviour

The Kynch (1952) theory of sedimentation

The Kynch (1952) theory of sedimentation is derived from a continuity equation ignoring the details of forces on the particles. The basic assumption of the theory is that at any point in the suspension the settling velocity of a particle depends only on the local concentration of the suspension. Other essential assumptions of the theory are that the wall effect is ignored and the particles are of the same size and shape.

The important contribution of the theory is that it shows that the solids concentration layers travel upwards at a uniform velocity and that such velocities are constant for each concentration. A typical settling curve with lines of constant concentration is shown in Fig. 1. Lines parallel to the line $Oh(t_1)$ show constant concentrations, representing the initial concentration of the suspension. As the solids settle to the bottom there is an extremely rapid increase of concentration from the initial to the maximum
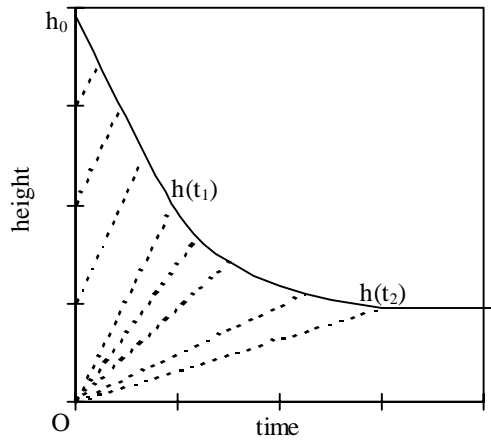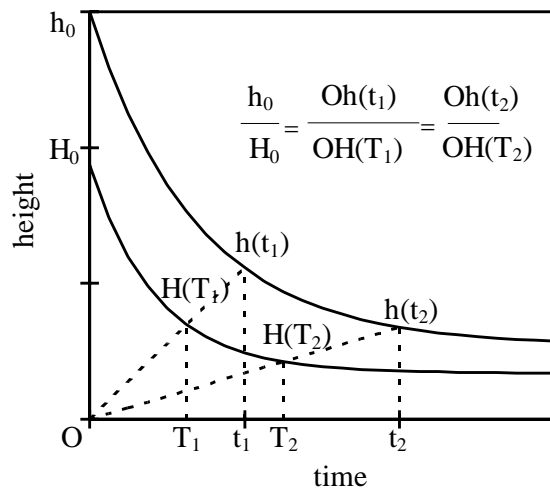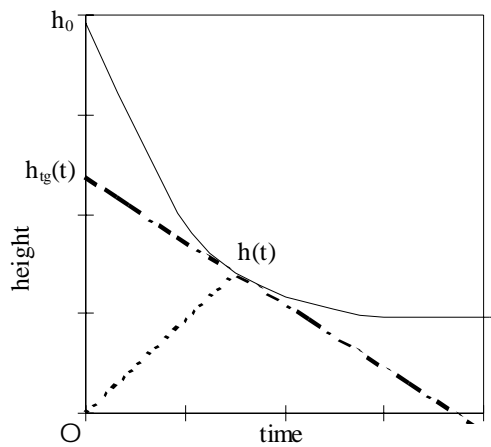
Figure 1
Settling of suspension surface with the lines of constant concentration (Kynch, 1952)

Figure 2
Graphical interpretation of the Kynch theory of sedimentation

possible concentration which is depicted with the line $Oh(t_2)$. The theory suggests that the concentration remains at its maximum under this line meaning that the possible consolidation of the material is ignored. The spray of lines between the initial and the maximum concentration lines represents intermediate concentrations.

The theory submits a simple graphical approach to determine the concentration layers. A tangent drawn to the settling curve from point $h(t)$ cuts the vertical axis at point $h_{tg}(t)$ (Fig. 2). The correlation between the initial conditions and concentration corresponding to line $Oh(t)$ is expressed as:

$$X_{Oh(t)} = \frac{X_0 h_0}{h_{tg}(t)} \qquad (1)$$

where:

X_0 is the initial solids concentration,
$h_{tg}(t)$ is the point where the tangent crosses the vertical axis
h_0 is the initial height of the settling curve.

With this relationship it is possible to determine the concentration at any point on the settling curve.

Figure 3
The effect of the initial height on settling curve (Work and Kohler, 1940)

The new proposed approach

According to Work and Kohler (1940), if a settling curve of suspension with a fixed concentration is plotted as a function of time for two different initial heights, the curves shown in Fig. 3 are obtained where the ratio $h_0:H_0=Oh(t_1):OH(T_1)=Oh(t_2):OH(T_2)$ is a constant everywhere. Thus, if the settling curve is obtained for any initial height, curves can be drawn for any other height.

Here the Work and Kohler (1940) approach is used as a basis for a hypothesis which assumes that curves drawn according to their approach can also be used for determining the solids profile during a batch test. The hypothesis suggests that in a settling column with a constant area of cross-section and vertically uniform initial concentration X_0, the amount of suspended solids remains constant between the drawn curves. The following description will demonstrate the proposed approach. In Fig. 4 the above curve starting from h_0 depicts a measured curve and the lower curve starting from H_0 is drawn according to the Work and Kohler (1940) approach. Now, according to the proposed approach the average concentration between the curves at the moment t can be calculated by the relationship:

$$(h_0 - H_0)X_0 = (h(t) - H(t))X(t, h)$$

and similarly, the average concentration between the lower curve and the horizontal axis with equation:

$$H_0 X_0 = H(t)X(t, H)$$

where:

h(t) and H(t) are the heights of the curves at time t, and
X(t,h) and X(t,H) are the average concentrations between h(t) and H(t), and H(t) and the vertical axis at time t.

By drawing several curves solids concentration can be determined at any time and height.

The Kynch (1952) theory and the proposed approach for determining solids concentration are graphical methods and for that reason time-consuming and neither very practical nor accurate. To facilitate the determination a model is suggested.
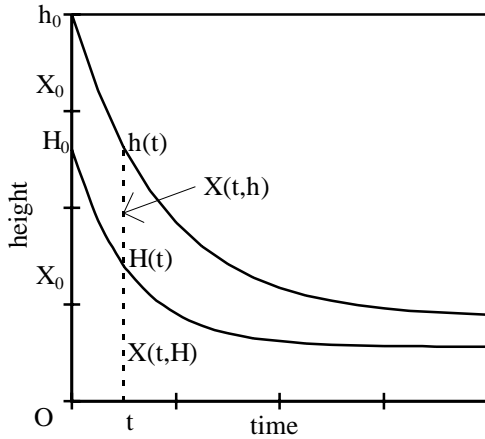
Figure 4
Graphical interpretation of the proposed approach

Figure 5
Measured and computed (by Eq. (2)) settling curves of CaCO$_3$

The model

If at time t the difference between the starting heights of two curves (drawn according to the Work and Kohler (1940) approach) h_0-H_0 = ∂h_0 is small, the expression for the proposed approach reduces to:

$$X(t, h) = \frac{X_0}{\dfrac{\partial h(t, h_0)}{\partial h_0}}$$

where:

X(t, h) is solids concentration at time t and height h

h(t, h$_0$) is the difference between the drawn curves at time t.

At certain times the concentration profile depends only on the initial solids concentration (X_0) which is a constant and the partial derivative of a function h(t, h$_0$), i.e. $\partial h(t, h_0)/\partial h_0$. Thus, modelling of the concentration profile turns out to be modelling of the settling curve where the effect of the initial height on the curve is taken into account according to the Work and Kohler (1940) approach.

Here, the model for the batch settling curve proposed by Renko (1996) is used for further modelling while any model describing accurately the settling curve and agreeing with the Work and Kohler's (1940) findings can be utilised.

The model for the batch settling curve can be written as:

$$h(t, h_0) = \frac{CX_0h_0}{\alpha} + (h_0 - \frac{CX_0h_0}{\alpha})e^{-t\alpha/(X_0h_0)} \qquad (2)$$

where:

t is time (h)

h (t, h$_0$) is the height of the settling curve at time t (m)
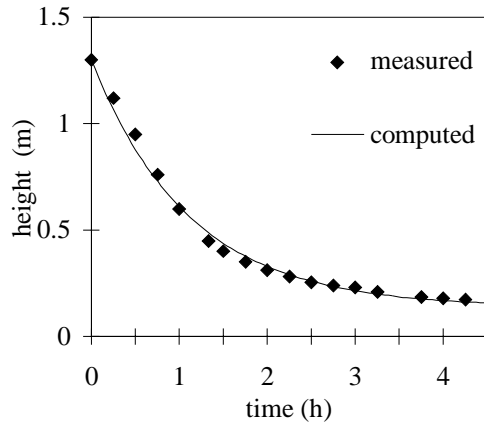
h$_0$ is the initial height of the settling curve (m)

α and C are the parameters describing solids settling (kg m^{-2} h^{-1} and m h^{-1})

X$_0$ is the initial solids concentration (kg m^{-3}).

It is shown in **Appendix I** that the model agrees with the Work and Kohler (1940) approach.

The partial derivative of Eq. (2) gives:

$$\frac{\partial h(t, h_0)}{\partial h_0} = \frac{CX_0}{\alpha} + (1 - \frac{CX_0}{\alpha})(1 + \frac{t\alpha}{Xh_0})e^{-t\alpha/(X_0h_0)}$$

and consequently, the model for the concentration profile can be written as:

$$X(t, h) = \frac{X_0}{\dfrac{CX_0}{\alpha} + (1 - \dfrac{CX_0}{\alpha})(1 + \dfrac{t\alpha}{X_0h_0})e^{-t\alpha/(X_0h_0)}} \qquad (3)$$

or equivalently as:

$$X(t, h) = \frac{X_0h_0}{h(t, h_0) - (C - \dfrac{\alpha}{X_0h_0}h(t, h_0))t} \qquad (4)$$

If h$_0$ is kept constant during computation (or in simple words if the solids concentration in the water-to-solids interface is computed) it can be seen that a model for the Kynch (1952) graphical approach is written (see **Appendix II**).

Experimental study

Data collection

The settling experiments to test the applicability of the model were conducted in a graduated cylinder with a total height of 1.5 m. The diameter of the cylinder was 0.2 m and it was equipped with seven sampling ports with the diameter of 12.5 mm at heights 0.075, 0.15, 0.25, 0.50, 0.75, 1.00, and 1.25 m (from the bottom) for measuring the solids profile.

The suspension examined was a CaCO$_3$ suspension which was prepared by mixing the known quantity of CaCO$_3$ with tap water. The test column was filled with the suspension up to 1.3 m and agitated with compressed air for about 1 min to provide uniform concentration throughout the depth. The solids profiles were determined in successive tests at the moments 0.0, 0.35, 0.78, 1.17, 1.87, 2.25 and 4.0 h after the agitation. The samples were collected starting from the top port downwards. The sample volumes were 50 to 100 mℓ. The solids concentrations of the samples were determined according to *Standard Methods* (1985).

In addition to the profile determination settling of the interface was measured as a function of time for estimating settling parameters. The parameters were estimated with SAS institute software for data analysis by using Eq. (1) as a non-linear
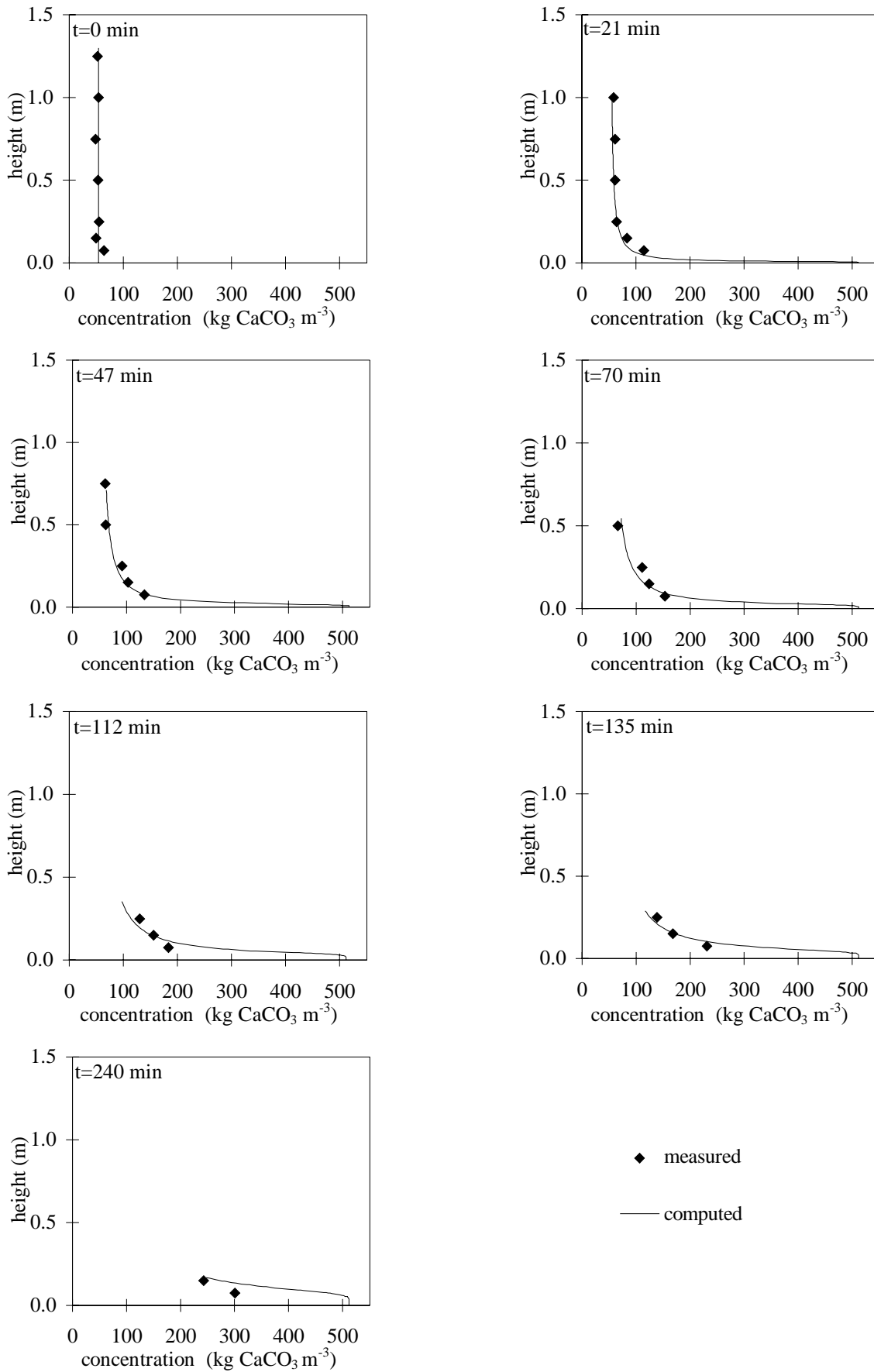
Figure 6
Measured and computed (by Eq. (2) and (3)) solids profiles as a function of height at different moments

regression model. The method used in the optimisation of the parameters was a multivariate secant method.

Results

The initial solids concentration of the $CaCO_3$ suspension was 53.8 kg $CaCO_3 \cdot m^{-3}$. The estimation resulted in the value of 62.95 kg$\cdot m^{-2} \cdot h^{-1}$ for α and 0.12 m$\cdot h^{-1}$ for C, respectively. The sum of square for the regression was 5.7469 and for the residual 0.0140 m^2, respectively. The measured settling curve and computed (Eq. (2)) curve as a function of time are shown in Fig. 5, and the measured and computed solids profiles as a function of height at different moments in Fig. 6. The solids concentration is computed by Eq. (3) and the corresponding height by Eq. (2) (Fig. 6).

Discussion and conclusions

This study suggests a hypothesis for determining solids behaviour during batch settling. A model based on the hypothesis is also proposed. The preliminary experimental study shows that the model can be successfully utilised for computing the behaviour of $CaCO_3$.

Behaviour of solids during batch settling is traditionally described by the Kynch theory of sedimentation which was published already in 1952. Even if the theory has been criticised it is still the most elementary explanation for the phenomenon of zone settling. The main criticism is the fact that it is not for all types of suspended materials. However, this is a direct consequence of the initial assumption of the theory that particles have the same size and shape. Kynch himself understood the limitation and stated that part of further developments of the theory would seem to be attempts to remove that assumption.

Another apparent disadvantage of the theory is that it only offers a graphical procedure for determining the solids layers. This study suggests a model for eliminating this shortcoming. The Kynch (1952) theory is based on solids flux and the continuity equation having a strong mathematical background while the proposed model is derived from the Work and Kohler (1940) graphical approach for taking into account the effect of scale. Since the proposed procedure is derived from simple assumptions it is easy to comprehend, and also to doubt. However, despite the different origins the proposed method also includes the same information as the Kynch (1952) theory (see **Appendix II**). This is particularly interesting because Work and Kohler (1940) had published their study more than a decade before Kynch proposed his theory but as they did not apply the results on the Behaviour of solids, the correlation was not discerned. In fact, the Kynch (1952) theory of sedimentation can be seen as logical consequence of the Work and Kohler (1940) approach written in a form of theory.

By studying the Work and Kohler (1940) approach more closely the similarities become more evident. In Fig. 3 two points $(h(t_1),t_1)$ and $(h(t_2),t_2)$ belong to a settling curve starting from h_0, and the points $(H(T_1),T_1)$ and $(H(T_2),T_2)$ belong to a settling curve drawn from H_0, respectively. The settling velocity between the points can be calculated as:

$$v = (h(t_1)-h(t_2))/(t_1-t_2) \text{ and}$$
$$V = (H(T_1)-H(T_2))/(T_1-T_2).$$

Because:

$$h(t_1)/Oh(t_1) = H(T_1)/OH(T_1),$$
$$h(t_2)/Oh(t_2) = H(T_2)/OH(T_2),$$
$$t_1/Oh(t_1) = T_1/OH(T_1), \text{ and}$$
$$t_2/Oh_{t2} = T_2/OH(T_2)$$

then equation for V can be rewritten as:

$$V = \frac{h(t_1)\dfrac{OH(T_1)}{Oh(t_1)} - h(t_2)\dfrac{OH(T_2)}{Oh(t_2)}}{(t_1)\dfrac{OH(T_1)}{Oh(t_1)} - (t_2)\dfrac{OH(T_2)}{Oh(t_2)}}$$

Since $Oh(t_1):OH(T_1) = Oh(t_2):OH(T_2)$ is constant, equations for velocities are equal, i.e. $v = V$. Now it can be deduced that a line drawn according to the Work and Kohler (1940) approach shows equal settling velocity of two settling curves starting from different initial heights. Thus, the solids interface settling velocity is the same at the points $h(t_1)$ and $H(T_1)$, and at the points $h(t_2)$ and $H(T_2)$, respectively. By making the basic assumption that the settling velocity of a particle depends only on the local concentration of the suspension it is found that the lines used for drawing the settling curves starting from different initial heights also show the constant concentrations.

The solids profile is computed from Eqs. (2) and (3), where Eq. (3) shows the solids concentration and Eq. (2) the corresponding height. The solids concentration is not computed straight as a function of height (i.e. $h(t, h_0)$), but with the help of h_0. This is not always practical, but otherwise very complicated equations would have to be used. In addition, the advantage of this procedure is that the examined points are always under the surface and hence, the illogical situations where a point is out of range can be avoided. However, efforts to develop a modified model where the solids profile is written as a function of height could be made.

The settling curve can be accurately computed with Eq. (2) (Fig. 5) and the parameters can be satisfactorily used for computing the solids concentration behaviour during batch settling with Eq. (2) and (3) (Fig. 6). The difference between the measured and the computed settling curves as well as the difference between the measured and the computed solids profiles are small. The sum of squares verifies the accuracy of the settling model. In addition, the model is structurally identifiable (**Appendix III**) revealing the validity of the structure of the model. *A posteriori* identifiability of the model also supports the choice of the used model structure (Renko and Sirviö, 1997).

The solids distribution was not totally uniform in the test column at the moment t = 0, i.e. at the beginning of the test (Fig. 6). Since the samples were collected starting from the top port downwards, all the samples were not taken exactly at the same moment 0.0 h. This is probably the reason for the small variation in the results. Despite the applied sampling procedure attempts to minimise the effect of the preceding sampling on the following one, it is obvious that the preceding sampling affected at least the samples taken from the bottom port when the height of the interface of the suspensions was low. The accuracy of the measurements can be checked from the fact that the amount of solids is constant during batch settling and this does not seem to be exactly the case when the solids surface is low. For these reasons better sampling techniques should be developed.

The model describes accurately the behaviour of a $CaCO_3$ suspension. This fact offers an interesting boundary condition for

modelling the settling of separate particles when the concentration is high since the model describes the average solids concentrations and settling velocities at certain heights and times.

This study focused on the distribution of solids during batch settling. It is obvious that the proposed model, like the Kynch (1952) theory, is only applicable to certain types of suspensions like a $CaCO_3$ suspension. In addition, successful use of the model presumes that the settling curve is accurately described. For these reasons the approach and the model should be considered not as a final result but as a step toward a more general model and a basis for further modelling of solids behaviour.

Acknowledgements

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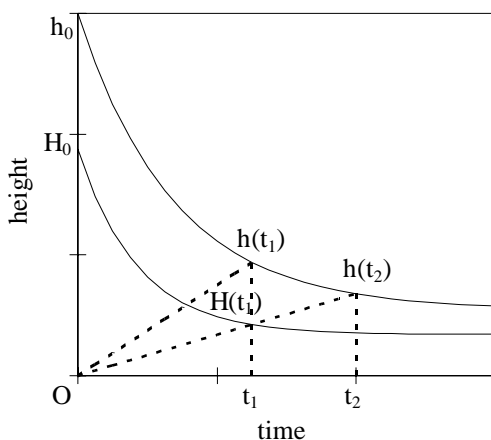
Appendix I

According to the Work and Kohler (1940) approach $h_0/H_0 = Oh(t_2)/OH(t_1)$. Because $t_1/OH(t_1) = t_2/Oh(t_2)$ and $H(t_1)/OH(t_1) = h(t_2)/Oh(t_2)$ then $h_0/H_0 = h(t_2)/H(t_1) = Oh(t_2)/OH(t_1) = t_2/t_1$. By replacing $h(t_2)$ and $H(t_1)$ with Eq. (2) the ratio h_0/H_0 can be written as:

$$\frac{h_0'}{H_0} = \frac{\dfrac{CX_0h_0}{\alpha} + (h_0 - \dfrac{CX_0h_0}{\alpha})e^{-t_2\alpha/(X_0h_0)}}{\dfrac{CX_0H_0}{\alpha} + (H_0 - \dfrac{CX_0H_0}{\alpha})e^{-t_1\alpha/(X_0H_0)}}$$

and by replacing t_2 with h_0t_1/H_0 in the exponent, correspondingly:

$$\frac{h_0}{H_0} = \frac{\dfrac{CX_0h_0}{\alpha} + (h_0 - \dfrac{CX_0h_0}{\alpha})e^{-t_1\alpha/(X_0H_0)}}{\dfrac{CX_0H_0}{\alpha} + (H_0 - \dfrac{CX_0H_0}{\alpha})e^{-t_1\alpha/(X_0H_0)}}$$

which is equivalent to:

$$\frac{CX_0H_0h_0}{\alpha} + (H_0h_0 - \frac{CX_0H_0h_0}{\alpha})e^{-t_1\alpha(X_0H_0)} = \frac{CX_0H_0h_0}{\alpha} + (H_0h_0 - \frac{CX_0H_0h_0}{\alpha})e^{-t_1\alpha(X_0H_0)}$$

Appendix II

The point $h_{tg}(t)$ where the tangent drawn to the settling curve cuts the vertical axis (see Fig. 2) is mathematically defined as [*]:

$$h_{tg}(t) = h(t, h_0) - \frac{dh(t, h_0)}{dt}t$$

where:

$\dfrac{dh(t, h_0)}{dt}$ is the settling velocity of the surface.

Since the first derivative of Eq. (2) can be defined as:

$$\frac{dh(t, h_0)}{dt} = C - \frac{\alpha}{X_0h_0}h(t, h_0)$$

the model for the concentration in the water-solids interface can be written:

$$X(t, h) = \frac{X_0h_0}{h(t, h_0) - (C - \dfrac{a}{X_0h_0}h(t, h_0))t}$$

[*] NOTE: h_0 is considered as constant in **Appendix II**.

Appendix III

One method for examining the structural identifiability of models that are non-linear in parameters is the power series approach (Pohjanpalo 1978). The approach analyses the Taylor series expansion of the model response around time zero checking if the successive derivatives have a unique solution for the parameters to be identified.

The two first derivatives of the model for batch settling curve are:

$$\frac{dh(0, h_0)}{dt} = -\frac{\alpha}{X} + C \quad \text{and} \quad \frac{d^2h(0, h_0)}{dt^2} = -\frac{\alpha^2}{X^2h_0} + C\frac{\alpha}{Xh_0}$$

Since both of the parameters can be solved from the above equations, α as:

$$\alpha = \frac{d^2h(0, h_0)Xh_0}{dt^2} \quad \text{and C as:}$$

$$C = \frac{dh(0, h_0)}{dt} - \frac{\dfrac{d^2h(0, h_0)}{dt^2}}{\dfrac{dh(0, h_0)}{dt}}h_0$$

the model is structurally identifiable under the experimentation. In fact the model is even globally identifiable except for the case when $\alpha/X = C$ which indicates the solids blanket interface is not settling at all and $\dfrac{dh(0, h_0)}{dt} = 0$ and $\dfrac{d^2h(0, h_0)}{dt^2} = 0$.