Laminar flow pipe hydraulics of pseudoplastic-thixotropic sewage sludges

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Abstract

The flow properties of heterogeneous sewage sludges are dependent on solids concentration and sludge type. General pipe flow design methods for sewage sludge applications are therefore unreliable. The non-Newtonian (pseudoplastic) and time-dependent (thixotropic) influence on the rheological characteristics of raw sludge was determined experimentally. These characteristics were used as basis to develop an empirical method to determine the head losses in pressure pipes conveying these sludges under laminar flow conditions. The method is illustrated by means of a design example.

Introduction

Sewage sludges are heterogeneous fluids which make the direct use of Newtonian fluid hydraulics in the design of pressure pipelines unreliable. Some factors that may affect the hydraulics are the sludge characteristics such as settling properties and concentration of solids in the sludge.

To overcome these uncertainties, design engineers are inclined to use a critical flow velocity (usually 1.5 to 2.0 m/s) above which flow is assumed to be turbulent. It is then assumed that no settling of solids will occur under turbulent flow conditions. For friction losses, the design is then based on Newtonian fluid hydraulics, with a so-called "sludge factor", which is usually based on the solids concentration of the sludge.

Although the assumption is always true that no settling of solids will occur under turbulent flow conditions (Dodge and Metzner, 1959), there is a minimum velocity (V_{min}) above which no settling of solids will occur, even under laminar flow conditions. This minimum velocity is determined by the settling properties of the sludge (Newitt et al., 1955).

For Newtonian fluids the Reynolds number (Re) is used to determine whether the flow is laminar or turbulent. This number is not only dependent on velocity (V) but also on dynamic viscosity (μ), pipe diameter (D) and fluid density (ρ) (Webber, 1971). In contrast, most sewage sludges with a solids concentration above 3% (mass per volume) conform to non-Newtonian fluid models, viz. pseudoplastic or Bingham plastic fluids (Frost, 1982; Rose-Innes and Nossel, 1983). This indicates that the viscosity is dependent on the shear rate (dv/dr) and thus an alternative method is required for determining the Reynolds number and friction headloss. To complicate matters further, Rose-Innes and Nossel (1983) indicated that these sludges are time-dependent, namely thixotropic (shear stress reduces with duration to shear).

The purpose of this study was to determine concentrated activated sludge flow characteristics required for the design of a pressure pipeline which conveys a sewage sludge under laminar flow conditions.

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Theoretical consideration

Four aspects of sludge hydraulics are of importance in the design of pressure pipelines, namely the minimum flow velocity, the type of fluid (Newtonian or non-Newtonian), the time behaviour of the sludge viscosity and the type of flow (laminar or turbulent).

Minimum flow velocity

Settling of solids inside a pipeline (and thus clogging) will be prevented if the flow velocity exceeds a minimum value (V_{min}) which is dependent on the relative densities of solids and liquids in the fluid (Newitt et al., 1955). Kapfer (1967) proposed the following relationship:

$$V_{min} = 1.9D^{0.2}[(\rho_p - \rho)/\rho]^{0.3} \qquad (1)$$

where:
V_{min} = minimum flow velocity to prevent settling of solids
ρ = fluid density
ρ_p = particle density (kg/m^3)

Newtonian and non-Newtonian flow characteristics: Herschel-Bulkley flow model

The Herschel-Bulkley model (also called the generalised Bingham model) is the most suitable model to describe the flow of non-Newtonian fluids (Frost, 1982):

$$\tau = \tau_y + K(dv/dr)^n \qquad (2)$$

where:
τ = shear stress (N/m^2)
τ_y = yield stress (N/m^2)
n = flow behaviour index (dimensionless)
K = fluid consistency coefficient ((N·s^n)/m^2)
dv/dr = shear rate (s^{-1})

This model is schematically shown in graphical form in Fig. 1.

Newtonian and non-Newtonian flow relationships for a fluid may be identified from the values of n, τ_y and K as shown in Fig. 1. Investigations by Frost (1982) and Rose-Innes and Nossel

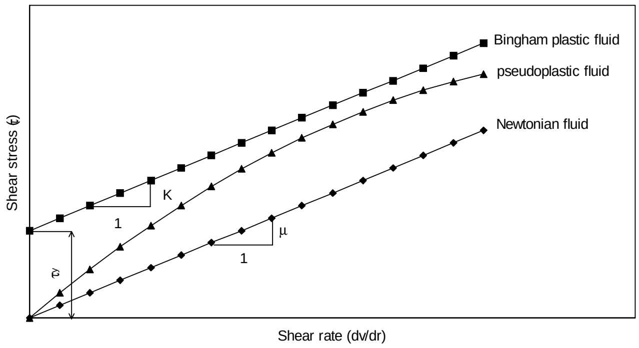

Figure 1
Shear stress / shear rate relationships of typical Newtonian and non-Newtonian fluids (Mulbarger et al., 1981)

(1983) indicated that most sludges conform to pseudoplastic behaviour ($\tau_y = 0$; $n < 1$), while a small number were tested as Bingham plastic ($n = 1$). The latter will not be discussed in this paper.

Laminar or turbulent flow characteristics: Generalised Reynolds number

A generalised Reynolds number (Re) has been developed to determine whether the flow of pseudoplastic fluids will be laminar or turbulent (Frost, 1982):

$$Re = \frac{\rho.V.D}{K[(3n+1)/(4n)]^n.(8V/D)^{n-1}} \qquad (3)$$

The critical Reynolds number (Re_c) for pseudoplastic fluids at which laminar flow conditions terminate, is dependent on n (Govier and Aziz, 1972) and may be calculated by:

$$Re_c = \frac{6464n}{(1+3n)^2[1/(2+n)]^{(2+n)/(1+n)}} \qquad (4)$$

Laminar flow terminates when Re exceeds Re_c.

Headloss due to friction

Many authors, including Reynolds, Von Karman, Colebrook and White have proposed empirical formulae for the calculation of headloss due to friction along a pipe (Webber, 1971) of which the following equation is generally used:

$$H_f = \frac{4fL}{D} \cdot \frac{V^2}{2g} \qquad (5)$$

where:
- H_f = head loss due to friction (m)
- L = length of pipe (m)
- V = mean velocity (m/s)
- g = gravitational constant (m/s^2)
- D = internal diameter of pipe (m)
- f = Fanning friction factor (dimensionless)

Govier and Aziz (1972) indicated that the relationship between the Fanning friction factor (f) and Reynolds number (Re) for the laminar flow of pseudoplastic fluids is equivalent to that of Newtonian fluids, namely:

$$1/Re = f/16 \qquad (6)$$

For thixotropic fluids they consider the following factors of importance:
- The headloss gradient (dH_f/dL) in the pipe is not a constant value, but decreases with time.
- The thixotropic effect decreases with time. The fluid approaches time-independent behaviour after being subjected to shear stress for a time period (t_c), which must be determined experimentally.

These two factors can be presented graphically as shown in Fig. 2.

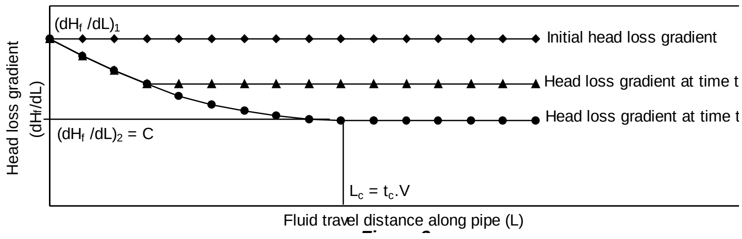

Figure 2
Head loss gradient decay along a pipeline for a typical thixotropic sludge

From Fig. 2 it can be seen that the initial headloss gradient is $(dH_f/DL)_1$. As the fluid is subjected to shear stress, the head loss gradient decreases until time-independent behaviour is approached at time t_c. Downstream of this point the headloss gradient is constant $(dH_f/dL)_2$. However, this is not applicable to "fresh" fluid entering the pipe which is still subjected to thixotropic behaviour. The distance L_c corresponds to the time t_c, where:

$$L_c = t_c.V \qquad (7)$$

Summary

Following on the theoretical considerations of the flow behaviour of pseudoplastic-thixotropic sludges, the following questions have to be addressed in order to design a pressure pipeline:

- What is V_{min}? (The minimum flow rate where settling will not occur).
- Is the sludge flow under consideration pseudoplastic and thixotropic?

Answers to these questions for a particular sludge can be experimentally obtained.

Experimental

Sludge samples

Samples from settled activated sludge were obtained from Zeekoegat Wastewater Treatment Works which treats mainly domestic sewage. Sludge samples were tested within 1h after sampling in order to ensure minimal structural changes.

Sludge concentration and density

The solids concentration and particle density were determined by

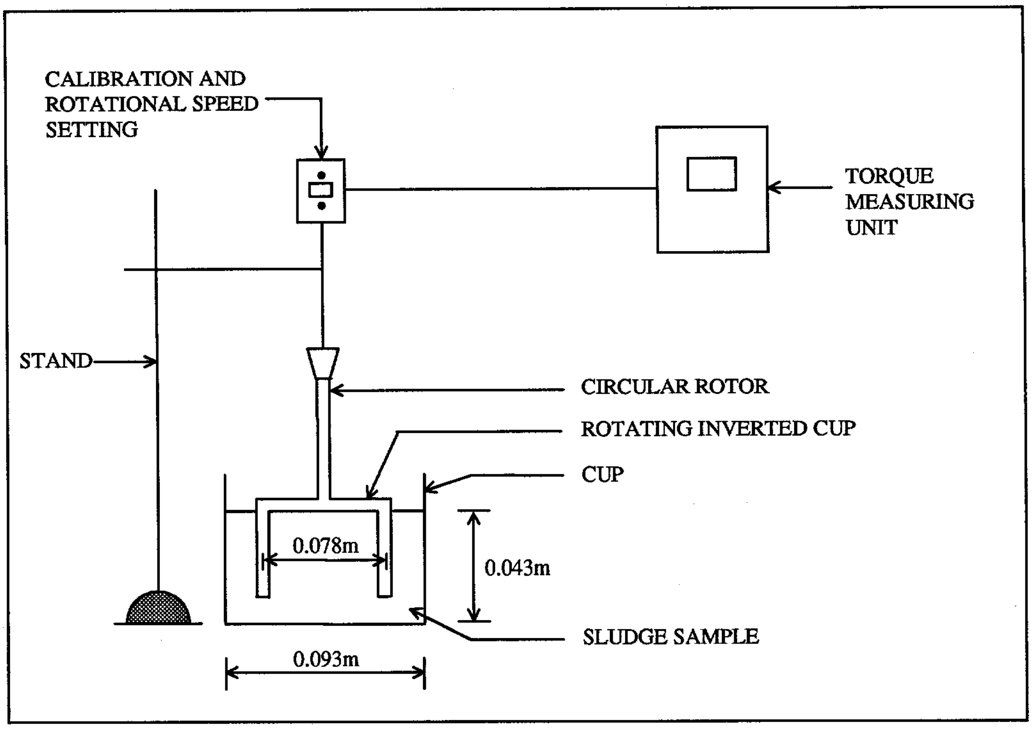

Figure 3
Schematic diagram of RZR 2102 stirrer with torque meter

a combination of methods 209 A and 213 E of *Standard Methods* (1981).

Rheological parameters

An experimental set-up as schematically shown in Fig. 3 was used to determine the rheological parameters. The apparatus consisted of a variable-speed stirrer with torque meter (Heidolf No. RZR 2102, Starenstrape 23, Kelhein 8420, Germany) provided with a rotating inverted cup (rotor) fitted inside a static cup with dimensions as shown.

Method

The static cup with rotor in place was filled to a predetermined level with the sludge to be tested. A rotor speed R was fixed and the torque (Md) was measured at t = 0, 50, 100, 200, 400 and 800 s intervals respectively. The whole procedure was repeated for R = 40, 65, 130 and 195 r/min respectively, each time using a new sample.

Calculations

Shear stress and shear rate

The shear stress (τ_w) and shear rate $(dv/dr)_w$ at the rotor wall are given in the following equations (Rose-Innes and Nossel, 1983; Metzner, 1956) respectively:

$$\tau_w = Md/(2\pi h.rr^2) \qquad (8)$$

$$(dv/dr)_w = k_3[1 + k_1(1/n_r - 1) + k_2(1/n_r - 1)^2].R/60 \qquad (9)$$

where:

τ_w = shear stress at rotor wall (N/m²)
$(dv/dr)_w$ = shear rate at rotor wall (s^{-1})
k_1 = $[(u^2 - 1)/(2u^2)][1 + 2ln(u)/3]$
k_2 = $[(u^2 - 1)/(6u^2)].ln(u)$
k_3 = $4\pi/(1 - 1/u^2)$
u = rc/rr = 1.192
rc = cup radius = 0.0465 m
R = rotor speed (r/min)
n_r = slope of log-log plot of torque vs. rotational speed

Test for pseudoplastic behaviour

Since τ_w and $(dv/dr)_w$ are directly proportional to Md and R respectively, n is equal to n_r. A log-log plot of Md vs. R yielding a straight line will therefore confirm pseudoplasticity. Linearity of log Md vs. log R was checked by regression.

Fluid consistency coefficient (K) and flow behaviour index (n)

The linearised form of the Herschel-Bulkley model is used to calculate K and n from the respected shear stress and shear rate values.

Linearised Herschel-Bulkley:

$$\log(\tau_w - \tau_y) = \log(K) + n.\log(dv/dr)_w \qquad (10)$$

where:

τ_w = shear stress at wall (N/m²)
$(dv/dr)_w$ = shear rate at wall (s^{-1})

If τ_y is equal to 0 (for pseudoplastic fluids), this equation simplifies to:

$$\log(t_w) = \log(K) + n.\log(dv/dr)_w \qquad (11)$$

Log (K) and n are the intercept and slope of the graph respectively. Log (K) and thus K and n values were determined by means of regression.

Results

Physical properties of activated sludge

The physical properties of activated sludge are shown in Table 1.

TABLE 1
PHYSICAL PROPERTIES OF ACTIVATED SLUDGE (MEASURED AT 20°C)

Property	Value
Concentration of sludge	5% (mass/volume)
Sludge particle density (ρ_p)	1 300 kg/m^3
Liquid density (ρ)	1 015 kg/m^3

Rotor and cup properties

From the measurements given in Fig. 3 and Eq. (9), the rotor and cup properties were calculated and are shown in Table 2.

TABLE 2
EXPERIMENTAL ROTOR AND CUP PROPERTIES

Parameter	Value
Rotor height (h)	0.043 m
Rotor radius (rr)	0.039 m
Cup radius (rc)	0.0465 m
u	1.192
k1	0.1657
k2	0.00868
k3	42.37

Torque measurements

The measured changes of torque with time and rotational speed are shown in Table 3.

TABLE 3
EFFECT OF TIME AND ROTATION SPEED ON TORQUE

R (r/min)	Md (N.m) for various t (s)					
	0	50	100	200	400	800
40	0.016	0.016	0.015	0.013	0.011	0.009
65	0.020	0.018	0.016	0.014	0.012	0.010
130	0.028	0.025	0.023	0.019	0.015	0.013
195	0.033	0.029	0.026	0.021	0.017	0.015

Check for thixotropy

As Md (and therefore τ_w) decreases with time at constant R (and therefore $(dv/dr)_w$) the sludge is thixotropic.

Check for pseudoplasicity

Table 4 shows the slopes of the log-log plots of Md vs. R (n_r in Eq. (9)), as well as the regression coefficients for each of the time intervals tested.

TABLE 4
LOG Md VS. LOG R SLOPES (=n_r) AND REGRESSION COEFFIENTS FOR EACH TIME INTERVAL

t (s)	n_r=n	Regression coefficient
0	0.462	0.9996
50	0.390	0.9936
100	0.374	0.9810
200	0.324	0.9860
400	0.282	0.9943
800	0.331	0.9949

Shear stress (τ_w) and shear rate $(dv.dr)_w$

Sample calculations of τ_w and $(dv/dr)_w$ for t = 0 and 100 s, using the corresponding Md values from Table 3 and Eqs. (8) and (9) respectively, are shown in Table 5 and the plots of τ_w vs. $(dv/dr)_w$ in Fig. 4.

TABLE 5
τ_w AND $(dv/dr)_w$ AT t = 0 AND t = 100 s

R (r/min)	t = 0 s		t = 100 s	
	t_w (Pa)	$(dv/dr)_w$(s^{-1})	t_w (Pa)	$(dv/dr)_w$(s^{-1})
40	38.935	34.035	36.502	30.049
65	48.669	55.307	38.935	45.934
130	68.137	110.614	55.969	91.869
195	80.304	165.922	63.270	137.803

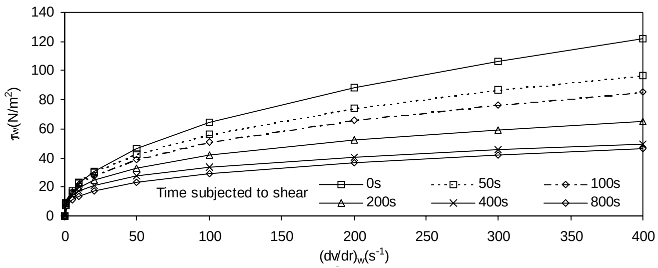

Figure 4
Activated sludge: True shear rate at wall (dv/dr)w vs. shear stress (tw)

Determination of consistency coefficient and flow behaviour index

Table 6 shows K and n for various time periods.

Discussion and conclusions

From the measured physical properties data of the sludge shown in Table 1, the minimum flow velocity above which no settling in a particular diameter pipe will occur, can be calculated with Eq. 1.

With the experimental set-up as shown in Fig. 3, it was possible to determine the rheological characteristics of the sludge. The fact that log-log plots of torque vs. rotational speed subjected to different time intervals yield straight lines (see regression coeffi-

TABLE 6
CONSISTENCY COEFFICIENTS AND BEHAVIOUR INDEXES FOR ACTIVATED SLUDGE

t (seconds)	K ($(N.s^n)/m^2$)	n (dimensionless)
0	7.648	0.462
50	9.335	0.390
100	9.065	0.374
200	9.356	0.324
400	9.168	0.282
800	6.392	0.331

cients > 0.8 in Table 4) shows that the activated sludge was pseudoplastic. This pseudoplasicity is also demonstrated in Fig. 4 where it is shown that all curves pass through the origin. Since the shear stress decreases with time at a specific shear rate (see Fig. 4), this activated sludge is also thixotropic. Time-independent behaviour was approached after 800 s, irrespective of the shear rate (no further decrease in shear stress was observed after 800 s of subjection). It is thus apparent that the activated sludge has a pseudoplastic-thixotropic flow behaviour which confirms similar finds by Mulbarger et al. (1981).

By knowing the physical and rheological properties of the sludge, the rheological constants K and n can be determined as shown in Table 6.

These values can be used to determine the initial headloss and reduction in headloss of pressure pipes which convey this sludge under laminar flow conditions. The various calculations required are included by means of a design example in the Appendix.

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APPENDIX

Design example of headloss profiles for the laminar flow of pseudoplastic-thixotropic sewage sludges

A Experimental

1 Rotor and cup properties

Rotor height h = 43 mm
Rotor radius rr = 39 mm
Cup radius rc = 46.5 mm

2 Determine experimentally as described in text

(a) Determine physical properties

Concentration of sludge X = 5% (mass/volume)
Sludge particle density ρ_p = 1 300 kg/m^3

(b) Determine torque (Md) vs. time (t) values at various rotor speeds (R)

TABLE A1
TYPICAL VALUES OBTAINED WITH EXPERIMENTAL SET-UP

R (r/min)	Md (N*m) for various t (s)					
	0	50	100	200	400	800
40	0.016	0.016	0.015	0.013	0.011	0.009
65	0.020	0.018	0.016	0.014	0.012	0.010
130	0.028	0.025	0.023	0.019	0.015	0.013
195	0.033	0.029	0.026	0.021	0.017	0.015

3 Calculations

(a) Calculate rotor and cup constants

u = rc/rr u = 1.192
k_1 = $(u^2-1)/(2*u^2)*(1+2*\ln(u)/3)$ k_1 = 0.166
k_2 = $(u^2-1)/(6*u^2)*\ln(u)$ k_2 = 0.009
k_3 = $(4*p)/(1-1/u^2)$ k_3 = 42.373

(b) Calculate liquid density

Bulk density $[\rho_p*X+\rho_w*(100-X)/100]$ ρ = 1 015 kg/m^3

(c) Calculate rheological parameters (τ_y, K and n)

For each time interval, calculate the following:
- slope of log-log plot of Md vs. R (n_r).
- correlation coefficient of the log-log plot of Md vs. R
- intercept of Md when R equals 0 for each time interval (τ_y)

The liquid is regarded pseudoplastic when the correlation coefficient is above 0.9, n_r is between 0 and 1 and the intercept is below 0.1.

Furthermore, the liquid is regarded thixotropic if the value of Md (as indicated in Table 1) decreases with time for a specific R.

TABLE A2 VALUES OF n_r, CORRELATION COEFFICIENT AND INTERCEPT						
Time(t)	**0**	**50**	**100**	**200**	**400**	**800**
n_r=n	0.462	0.390	0.374	0.324	0.282	0.331
Correlation coefficient	0.999	0.987	0.962	0.972	0.989	0.990
Intercept (τ_y) (Pa)	0.003	0.004	0.004	0.004	0.004	0.003
Is liquid pseudoplastic?	Yes	Yes	Yes	Yes	Yes	Yes
Is liquid thixotropic?	Yes for all R values					

(d) For each time interval, calculate the shear stress (t_y) and shear rate $(dv/dr)_w$ (see text for formulae):

TABLE A3 τ_w AND $(dv/dr)_w$ AT DIFFERENT T						
	t = 0 s		**t = 50 s**		**t = 100 s**	
R(r/min)	τ_w**(Pa)**	$(dv/dr)_w$	τ_w**(Pa)**	$(dv/dr)_w$	τ_w**(Pa)**	$(dv/dr)_w$
40	38.935	34.035	38.935	36.154	36.502	36.755
65	48.669	55.307	43.802	58.751	38.935	59.727
130	68.137	110.614	60.836	117.502	55.969	119.454
195	80.304	165.922	70.570	176.252	63.270	179.180
	t = 200 s		**t = 400 s**		**t = 800 s**	
R(r/min)	τ_w**(Pa)**	$(dv/dr)_w$	τ_w**(Pa)**	$(dv/dr)_w$	τ_w**(Pa)**	$(dv/dr)_w$
40	31.635	39.072	26.768	41.789	21.901	38.731
65	34.068	63.491	29.201	67.907	24.335	62.938
130	46.236	126.983	36.502	135.815	31.635	125.875
195	51.102	190.474	41.369	203.722	36.502	188.813

(e)For each time interval, calculate the following:

- liquid consistency coefficient (K) (log(K) is the intercept of log(τ_w) when log$(dv/dr)_w$ equals 0).
- flow behaviour index (n) (n is the slope of the log-log plot of (τ_w) vs. $(dv/dr)_w$).

TABLE A4 VALUES OF K AND n FOR EACH TIME INTERVAL						
Time(s)	**0**	**50**	**100**	**200**	**400**	**800**
K(($N \cdot s^n/m^2$)	7.648	9.335	9.065	9.356	9.168	6.392
n(dimensionless)	0.462	0.390	0.374	0.324	0.282	0.331

B Design

1 Design requirement

Required pumping rate Q = 0.08 m^3/s
Pumping distance L = 2 000 m
Secondary losses Ls = 6.5*V^2/(2*g)

2 Determine initial conditions

Select initial flow velocity Vin = 1.5 m/s
Calculate initial pipe diameter from required pumping rate and selected initial flow velocity: Din = 0.2606 m
Selected standard size pipe: D = 0.25 m
Minimum velocity (from text): Vmin = 0.9837 m/s
Is chosen Vin >Vmin? Yes (Note: If No, choose Vin = Vmin)

3 Calculate pipeline losses for V=Vmin

(a) Calculate L_f, Re_c, R, f, dH_f/dL, C and ln(dH_f/dL)-C for each time interval.

TABLE A5 CALCULATION OF L_f, Re_c, R, f, dH_f/dL, C AND ln(dH_f/dL)-C (SEE TEXT FOR EQUATIONS)						
Time (s)	L_f	Re_c	**Re**	**f**	(dH_f/dL)	ln((dH_f/dL)-C)*
0	0.000	2371	186	0.086	0.068	-3.466
50	49.184	2395	192	0.083	0.066	-3.547
100	98.368	2391	209	0.077	0.060	-3.746
200	196.737	2367	240	0.067	0.053	-4.141
400	393.473	2323	282	0.057	0.045	-4.839
800	786.947	2371	343	0.047	0.037	Infinitive

* Constant C equals dH_f/dL when time-independent behaviour is reached. In this instance at t = 800 s.

(b) Calculate constants A and B by means of linear regression of the following equation (A and B are the intercept and slope respectively of the equation):

$$\ln[(dH_f/dL) - C] = \ln(A) - B.L_f \quad \text{(A1)}$$

A = 0.032846509
B = 0.003605818

(c) Calculate total maximum friction head loss (Hf_{max}) which occurs at t=0s viz. when pump is switched on. For this purpose Eq. (5) in the main text is used:

$$H_{fmax} = (4f_{t=0}LV^2)/(2gD) \quad \text{(A2)}$$

H_{fmax} = 136.041 m

(d) Calculate total minimum friction losses (H_{fmin}) by integrating Eq. (A1). For pipe lengths (L) shorter than L_c (L_c=time when time-independent behaviour is approached):

$$H_{fmin} = (A/B).[1-\exp(-B.L)]+C.L \quad \text{(A3)}$$

For pipe lengths (L) longer than L_c:

$$H_{fmin} = (A/B).[1-\exp(-B.L_c)]+C.L \qquad (A4)$$

H_{fmin} = 82.153 m

(e) Calculate secondary losses:

H_{sec} = 0.326 m

(f) Calculate total losses(friction + secondary losses):

H_{totmax} = 136.367 m
H_{totmin} = 82.479 m

4 Calculate pipeline losses for increased V

Repeat steps 3(a) to 3(f), each time with an increased flow velocity(V), until Re>Rec at any time in Table A5. At this point laminar flow is terminated.

TABLE A6
Q AND V VS. H_{totmax} AND H_{totmin} IN THE ACCEPTABLE LAMINAR FLOW RANGE

Q (m^3/s)	V (m/s)	H_{totmax}(m)	H_{totmin}(m)
0.0483	0.9837	136.0	82.2
0.0491	1	137.4	83.1
0.0614	1.25	152.5	91.8
0.0736	1.5	166.1	100.0
0.0859	1.75	178.5	107.9
0.0982	2	190.1	115.6
0.1104	2.25	201.0	123.2
0.1227	2.5	211.4	130.6
0.1350	2.75	221.2	137.7
0.1473	3	230.7	144.7
0.1595	3.25	239.8	151.6

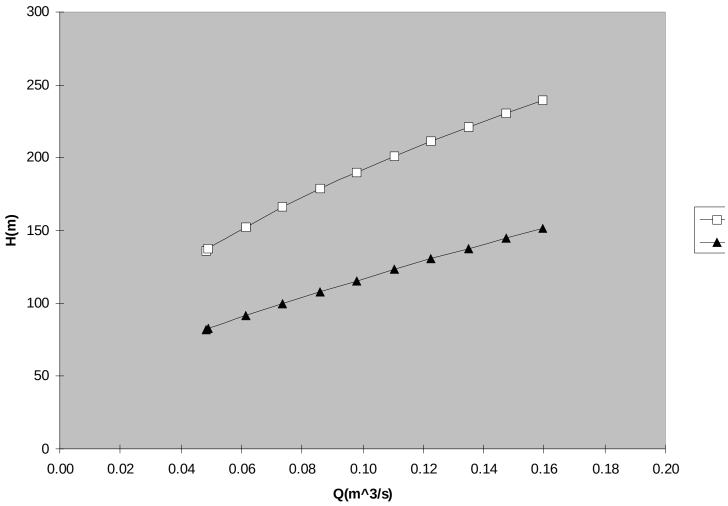

Figure A1
H_{totmax}, H_{totmin} vs Q in the acceptable laminar flow range for activated sludge (5% solids) in a 2 000 m long by 250 mm diameter pipeline