Groundwater management under uncertainty: A multi-objective approach

Ndambuki JM[1], Otieno FAO[2]*, Stroet CBM[1] and Veling EJM[1]

[1]Delft University of Technology, Faculty of Civil Engineering and Geosciences,Section for Hydrology and Ecology, Stevinweg 1, NL-2628 CN Delft, The Netherlands (PO Box 5048, NL-2600 GA Delft, The Netherlands)

[2]Department of Civil Engineering, University of Durban-Westville, Private Bag X54001, Durban 4000, South Africa

Abstract

A methodology is developed for determining a robust optimal strategy for a groundwater hydraulic control problem posed within the framework of stochastic multi-objective optimisation. The methodology explicitly considers uncertainty in hydraulic conductivity and allows decision- makers (DMs) to evaluate trade-offs among three conflicting objectives: aquifer yield, investment and operational cost, and recourse cost (penalties) incurred if the stipulated constraints are violated. The model includes a two-stage decision process in which the first-stage decision (commonly referred to as "here and now") on how many wells, their pumping rates and their locations is made and implemented immediately without the foreknowledge of the outcome of the uncertain parameters. At a later stage, when the uncertain parameters become known, a second-stage decision is taken using the updated data. Applicability of the methodology is demonstrated through a hypothetical (but realistic) example. A post-optimality Monte Carlo analysis is conducted to examine the performance of the model in terms of robustness (stability). Preliminary results show that robust optimisation can be useful in designing a strategy which performs reasonably well whatever the outcome of the uncertain parameters.

Introduction

In recent years, the use of management models has considerably grown in connection with the analysis of subsurface water resources as suggested by the large number of research papers in the literature. This scenario is understandable especially when one looks at the ever-increasing water consumption rates in agricultural, industrial and civil establishments. Closely related to this fact is the realisation that the available groundwater resources are limited to some extent and are to be carefully managed with the aid of appropriate scientific tools if we want to put them to the most beneficial use. To this end, numerical models have been used in combination with optimisation techniques to design optimal strategies. One of the most challenging problems associated with the simulation-optimisation approach to groundwater quantity management, especially when confronted with a problem encompassing multiple conflicting objectives, is how to incorporate the effects of flow modelling uncertainty into the optimal decision-making process. To date, most aquifer management models used to design optimal groundwater management in a multi-objective environment have been assumed to be deterministic. However, just like any other resource management, groundwater management is generally carried out in an environment of uncertainties. For example, natural geological formations that form aquifers are naturally heterogeneous. It is due to this heterogeneity, in combination with lack of data to fully characterise the aquifer, that solutions based on deterministic methods are put into question.

The objective of this paper is, therefore, to present a regional groundwater planning model which explicitly considers uncertainty in hydraulic conductivity in a multi-objective optimisation problem. The solution method is predicated on robust optimisation (which is essentially an extension of classical stochastic programming), which incorporates the DM's preferences (in terms of risk of penalties due to violation of constraints) and allows consideration of nearly feasible solutions in a multiple objective framework. This approach has been used by Wagner et al. (1992).

Aquifer management models that combine simulation with optimisation help in understanding how social and economic forces interact with the water resource allocation. Just as a simulation model is a tool to understand the physical/chemical behaviour of an aquifer system, a management model can be thought of as a tool, which provides insight into the economic and social consequences of institutional changes. The combination of these two methods has been achieved in at least two ways; through the response matrix approach, and through the embedding approach. In the response matrix approach, the influence of a unit change in an independent decision variable such as pumping or recharge at a pre-selected well location upon a variety of dependent variables like drawdown and velocity at specified observation points, is determined. Superposition is then performed to calculate their total response at specified points resulting from all decision variables. Its main drawback is the number of simulations required to generate the responses as well as recalculate the response matrix when the boundary conditions and well locations change. This approach has been used by Maddock and Lacher (1991); Heidari (1982); Wanakule et al. (1986); Willis and Finney (1985); and Van Tonder et al. (1998). Other applications of this method have been made by Gorelick and Remson (1982); Theodossiou and Tolikas (1995); Galeati and Gambolati (1988); Maddock (1972); Herrling and Heckele (1986); and Willis and Liu (1984). In the embedding approach, numerical approximations of the flow equations are included directly as constraints in the optimisation model. Discretisation is based on either the finite difference method or the finite element method. In this method, the unknown groundwater variables (heads and source/sink) become decision variables in the optimisation method. This method, not only solves the problem once (as opposed to the response matrix approach), but also provides lots of information regarding the behaviour of the aquifer.

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* To whom all correspondence should be addressed.

☎(031) 204-4571; fax (031) 204-4291; e-mail Otieno@pixie.udw.ac.za

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Nevertheless, this method has a disadvantage in that it results to a large program (Gorelick, 1983). This approach has been used by Gharbi and Peralta (1994); Yazicigil and Rasheeduddin (1987); Magnouni and Treichel (1994); Aguado and Remson (1980); Alley et al. (1976); and Prathapar (1989).

In the planning of most water resource systems, there are many possible objectives to consider, and the future performance of each alternative is uncertain as a result of uncertainty in input parameters. Solution methods, which adopt a single objective approach, presuppose that the DM's preferences are satisfactorily addressed through the considered single objective. However, it can be argued that in decision contexts, numerous, usually conflicting objectives are considered, hence in such situations, the optimisation problem should be posed within a multi-objective framework. Within this framework, the analyst seeks to identify the Pareto set (set of non-inferior solutions). This set is then used to aid the DM state his preferences. Among the works addressing groundwater management in a multi-objective framework is that of Kaunas and Haimes (1985); Willis and Liu (1984); Shafike et al. (1992); Gharbi and Peralta (1994); and Magnouni and Treichel (1994).

Like any other resource management, groundwater management is generally carried out in an environment of uncertainties. Thus, the question of reliability of the model output is of paramount importance. Heterogeneity in natural aquifer formations is widely recognised as one of the major factors contributing to uncertainty in predicting groundwater flow behavior and management strategies.

Several methods have been used in an attempt to deal with uncertainty in parameters. They include post-optimality sensitivity analysis (e.g. Aguado et al., 1977; Gorelick, 1982; Bredehoeft and Young, 1983), chance constrained programming (see Tung, 1986; Mayer, 1992) and stochastic optimisation with recourse method as presented by Wagner et al. (1992). Recent works on stochastic modelling include that of Van Leeuwen et al. (1998), Loll and Moldrup (1998), and Huang and Mayer (1997).

In this paper, we present a methodology for solving multi-objective problems when the aquifer parameters are considered uncertain. Applicability of the methodology is demonstrated through a hypothetical example.

Problem formulation

We consider the spatially variable hydraulic conductivity, k, to be the only uncertain parameter. All other parameters are assumed to be known. Uncertainty is included in the optimisation formulation (Eqs. (1-6)) by sampling the stochastic field of continuous hydraulic conductivity values. The sampling is done by obtaining a set of realisations of this distribution, with every realisation having a distinct value for the hydraulic conductivity of each cell. If we denote the realisations (maps) of the hydraulic conductivity, k, as ω, $\omega = 1,...,\Omega$, then each realisation, ω, will result to different responses, $a_{i,j,\omega}$ ($a_{i,j,\omega}$ are components of the response matrix, A_ω) and hence different optimal solutions. Since this optimisation problem (Eqs. (1) - (6)) is stochastic because the constraints are stochastic, we cannot guarantee that a selected management plan will actually not violate the stipulated constraints. If $\Sigma^{Nw}_{j=1} a_{i,j,\omega} p_j$, $i = 1,\ldots, Nc$ (Nc is the number of control points and p_j are the decision variables) are the computed drawdowns due to a selected realisation, ω, and b_i are the specified constraining values, then for a control point i, if $(\Sigma^{Nw}_{j=1} a_{i,j,\omega} p_j) - b_i \geq 0$ it means that the constraint has been violated (we denote the violation by $v_{i,\omega} = (\Sigma^{Nw}_{j=1} a_{i,j,\omega} p_j) - b_i$) and hence a recourse action has to be taken to decrease the damage caused by this violation. Associated with this recourse is a cost to carry out the recourse action.

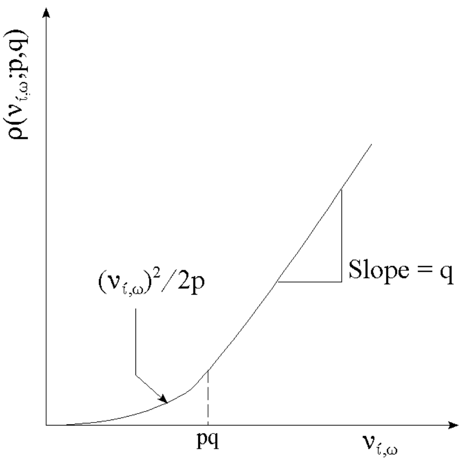

Figure 1
Linear-quadratic penalty function

Ideally, because of the uncertainty in hydraulic conductivity, it is difficult to design a solution stable enough to safeguard against violation of constraints, whatever the outcome of the uncertain parameters. However, not to over-emphasise on feasibility, a DM can choose to allow optimal solutions having slight violations by penalising them softly (or not penalising them at all) and discourage solutions having extreme violations by imposing heavy penalties. A linear-quadratic penalty function as shown in Fig. 1 is suitable for this purpose (Rockafellar and Wets, 1986). The shape of this curve allows the DM great flexibility in that he can control the shape of the entire function through choice of parameters p and q. This means that based on the analyst's results for various values of parameters p and q, the DM can choose a solution considering both the optimality and penalties, thus determining the overall shape of the curve.

The parameters p and q are related to the penalties $\rho(v_{i,\omega}; p,q)$ using the relationships:

$$\rho\left(v_{i,\omega};\, p,q\right) = 0,\, v_{i,\omega} \leq 0$$

$$\rho\left(v_{i,\omega};\, p,q\right) = v_{i,\omega}^{\,2} \big/ 2p,\, 0 \leq v_{i,\omega} \leq pq$$

$$\rho\left(v_{i,\omega};\, p,q\right) = qv_{i,\omega} - \frac{1}{2} pq^2,\, v_{i,\omega} \geq pq$$

If $\rho_{i,\omega} = \rho\,(v_{i,\omega}; p,q)$ expresses the penalty associated with the violation $v_{i,\omega}$ at the ith control point, in order to minimise the cost of violation of the stochastic constraints, a recourse can be carried out by means of a supplementary objective function. This supplementary objective function (noted by Z_3) which considers the economic consequences of the violations of the constraints can be written as $[(1/\Omega)\ \Sigma^{\Omega}_{\omega=1} \Sigma^{Nc}_{i=1} \rho_{i,\omega}]$. This approach requires calculation of the violations for all possible maps of the uncertain parameters, resulting to a deterministic program which is larger than the initial stochastic program. Wagner et al. (1992) has used this approach to solve a single objective optimisation problem. The formulation of the multi-objective problem can then be written as:

$$\underset{p}{Min} \left[Z_1 = \sum_{j=1}^{N_w} \lambda_j r_j p_j \right] \qquad (1)$$

$$\underset{p}{Max}\left[Z_2 = \sum_{j=1}^{N_w} p_j \right] \quad (2)$$

$$\underset{p}{Min}\left[Z_3 = \frac{1}{\Omega}\sum_{\omega=1}^{\Omega}\sum_{i=1}^{N_c} \rho\left(v_{i,\omega}; p, q\right)\right] \quad (3)$$

subject to:

$$\sum_{j=1}^{N_w} a_{i,j,\omega}\, p_j - b_i = v_{i,\omega}, i = 1,..., N_c \quad (4)$$

$$\sum_{j=1}^{N_w} p_j \geq W_D \quad (5)$$

$$p_j \geq 0, \; j = 1,..., N_w, \omega = 1,..., \Omega \quad (6)$$

where:

N_W is the number of pumping wells
N_c is the number of control points
λ_j is daily cost of pumping and transportation in monetary units per unit volume per unit lift for lacation j
p_j is pumping rate in cell j, j=1,…,N_w
r_j is the pumping lift given by (H-h_j)
H is height of ground surface (measured from bottom of aquifer)
h_j is the head in pumping well j
$a_{i,j}$ is response at cell i due to pumping in cell j
b_i is the constraining value at control point i
W_D is total water demand.

Objective Z_1 addresses the issue of cost of pumping and delivery of water, objective Z_2 ensures maximum extraction of water while objective Z_3 ensures minimum penalties due to violation of constraints. The constraints place limitations on maximum drawdowns, minimum water demand and minimum pumping rate (i.e. Eqs. (4) to(6)). Different objectives and constraints can as well be specified depending on the problem at hand. However, for this paper, we consider the above objectives and constraints. Note that the constraints are stochastic because of their dependence on realisation ω. The problem, therefore, involves identifying a solution which minimises the cost of pumping and transportation, maximises the pumping rates, and at the same time minimises the recourse costs (penalties) subject to the specified constraints (Eqs. (1) to (6)).

Methodology

Solution methods for multi-objective programs aims at arriving at a representation of the non-inferior set of solutions for consideration by a DM. This set of non-inferior solutions is usually large, and it is not immediately clear to the DM on how to choose one of them for implementation. Consequently, decision aid techniques, also known as decision support systems (DSSs), are required to assist the DM to identify one of the solutions as the most preferred compromise for implementation. The DSS used in this work is predicated on utility theory, which involves a procedure which aims at finding a value function as compatible as possible with the subjective ranking defined by the DM on a set of reference alternatives. The DSS requires a DM to subjectively rank a set of reference solutions presented to him by the analyst. The ranking is done considering both the information provided by the analyst and any other information (criteria) the DM might consider important even though such criteria may not be explicitly stated. Using this subjective ranking, a value function consistent with the ranking is determined. This value function is applied to all non-inferior solutions to obtain a global ranking. The alternative solution which ranks first is taken as the most satisfactory for implementation. For more details see Kostkowski and Slowinski (1996).

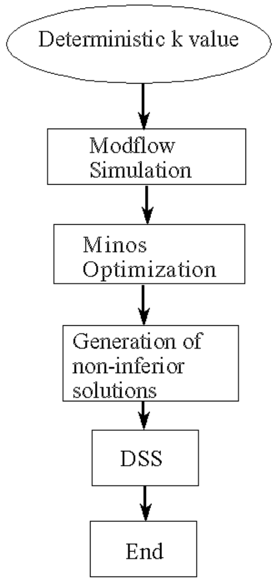

Figure 2
Flow diagram for a deterministic case

To solve the stochastic problem, the groundwater flow simulation software, *MODFLOW* (McDonald and Harbaugh, 1984), and a single objective optimisation software, *MINOS* (Murtagh and Saunders, 1995), are used. The simulation software is used to generate the response matrix (constraints for the optimisation problem) while *MINOS* together with user written programs are used to solve the multi-objective optimisation problem. To assist the DM identify one of the non-inferior solutions as the most preferred, *UTA+* software (Kostkowski and Slowinski, 1996), a decision support system, is used. The methodology used to solve the optimisation problem defined by Eqs. (1) to (6) is developed systematically starting with a deterministic scheme through multiple scenario scheme as follows:

Deterministic scheme

This scheme refers to a case when the parameters are assumed to be known without error, hence the problem is deterministic. When no data are available, this problem can be examined by solving it with all hydraulic conductivities set to their expected value resulting to a homogeneous problem (since the hydraulic conductivity distributions are all assumed to have the same mean). With availability of data, the problem can be formulated in two ways: the simplest way is to consider it as homogeneous by taking the mean of all available measurements, and considering it as heterogeneous by assuming that each or a group of the available sparse data represents a certain area of the model domain.

Scenario with second-stage decision scheme

Scenario with second-stage decision arises from a situation where a first-stage ("here and now") decision is made and implemented based on the available information about the uncertain parameters. At a later stage when more information on the uncertain parameters becomes available, a second-stage decision is made using the updated information so as to minimise violation of constraints. This scheme results to as many sets of optimal solutions as the number of realisations (maps) considered. As such, the choice of which

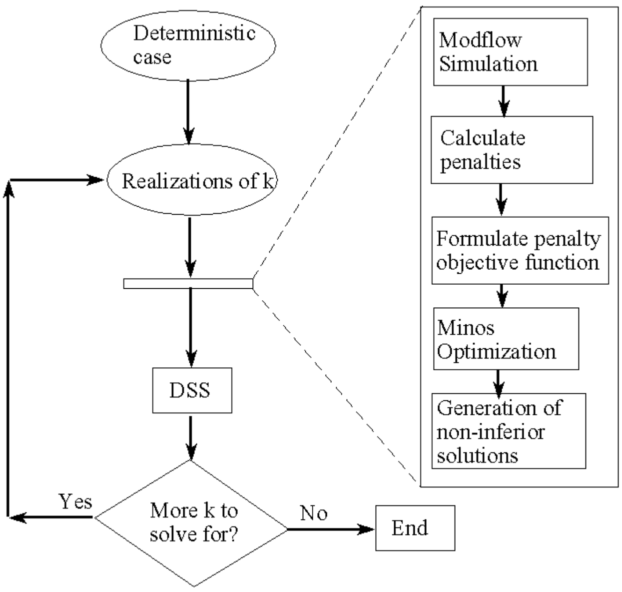

Figure 3
Flow chart for scenario analysis with second-stage decision

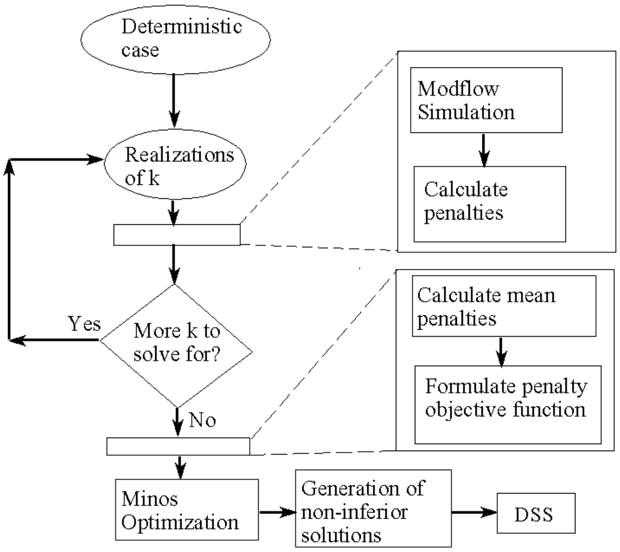

Figure 4
Flow chart for multiple scenario analysis

plan to implement remains a formidable task to the DM, hence the main disadvantage of scenario analysis in general. Nevertheless, solutions from this scheme are certainly better than the deterministic scheme because they are evaluated taking uncertainty of the parameters into consideration. They, therefore, provide information on how the optimal solution changes as the uncertain parameters assume different values. A flow diagram for the numerical scheme is shown as Fig. 3.

Multiple scenario scheme

In this scheme, a "here and now" plan is implemented based on the available information (deterministic scheme). For all realisations of the uncertain parameter, violations and hence penalties are calculated. The matrix of penalties is then used to formulate a

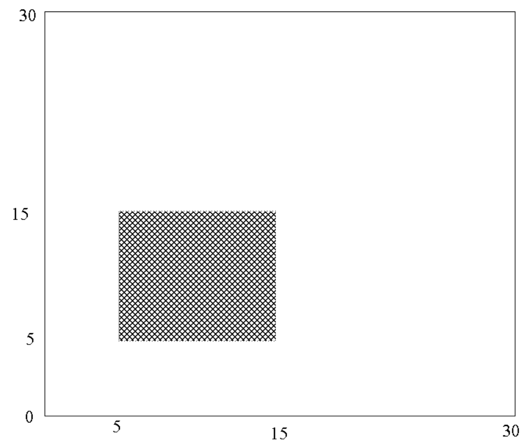

Figure 5
Ecological protection zone

penalty objective function, i.e. Z_3 (see Eq. 3) which is then included in the set of objective functions and re-optimisation of the problem done. A flow diagram for the numerical scheme is shown as Fig. 4.

Application of methodology to an example

The proposed methodology is applied to a hypothetical example modified from that of Magnouni and Treichel (1994). The example has the following dimensions and properties:

An island of 30 km x 30 km.
Groundwater resources are accumulated in a single confined aquifer of thickness 35 m that will be used for water supply to a distribution centre situated at the middle of the island.
The aquifer is heterogeneous with the spatial distribution of hydraulic conductivity fields of mean value of 2.25 x 10^{-3} m/s, a standard deviation (log) of 0.5 and a correlation length of 3 000 m in x-direction and 7 500 m in y-direction.

Objectives

Three objectives are considered and include:
- Maximisation of total water pumping rates
- Minimisation of operating costs (pumping and transportation costs),
- Minimisation of recourse costs due to violation of constraints resulting from uncertainties in the aquifer parameters.

Constraints

The DM expresses constraints on minimum water levels in an ecological protection zone (Fig. 5), flow direction along the aquifer/ocean border to avoid salt-water intrusion and maximum pumping yields at each cell as follows:

On the border of the island, the hydraulic head is equal to sea level. While in the other nodes it is bounded by bottom of the aquifer.
In the specified ecological protection zone, the minimum water level equals 5 m a. s. l. To avoid salt-water intrusion, head in cells next to the ocean are not allowed to fall below 0.2 m a. s. l.
There is minimum water demand of 3 m^3/s at the distribution centre which should be satisfied. Pumping rates from

P_1	P_2	P_3	P_4	P_5	P_6	P_7
P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}
P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}
P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{27}	P_{28}
P_{29}	P_{30}	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}
P_{36}	P_{37}	P_{38}	P_{39}	P_{40}	P_{41}	P_{42}
P_{43}	P_{44}	P_{45}	P_{46}	P_{47}	P_{48}	P_{49}

Figure 6
Location of potential pumping wells

potential wells (shown in Fig. 6) are limited to a maximum yield of 1.5 m^3/s.

Unit costs of exploitation, defined at each potential well are calculated as a combination of the water pumping costs and water transport costs which are dependent on the distance from the well to the distribution center. The costs are assumed to be higher towards the boundaries. The aggregated unit cost coefficients at each cell take on values from 0.1 at the centre and increase at a rate of 0.1 for every 200 m distance.

The problem is to find the location of wells and the corresponding pumping rates, satisfying all or nearly all the constraints on the hydraulic heads and pumping rates, and the minimum demand at the distribution center. Moreover, the solution should be the best compromise among the multiple conflicting criteria in an environment of uncertain spatial hydraulic conductivity values.

Discussion of results

For this example, 20 unconditional realisations (maps) of hydraulic conductivity were generated using a mean value of 2.25×10^{-3} m/s, a standard deviation (log) of 0.5 and a correlation length of 30 000 m in x-direction and 7 500 m in y-direction. Each of the realisations is assumed equally likely to exist in reality. The penalty function parameters used are p = 0, and q = 0.2 (see Fig. 1), i.e. the penalty is a linear function of violation.

Deterministic scheme

For this scheme, 20 non-inferior solutions were generated (see Table 1). For purposes of discussion on the application of UTA+ (a DSS), a subset of seven representative solutions was arbitrarily chosen and given subjective ranks. Note that ordinarily, the set is presented to a DM who is then asked to articulate his preferences in the form of ranking them. Table 2 shows these reference solutions with their subjective ranks.

Using the information on subjective ranking on the reference set, and two linear segments for cost-marginal value function and one linear piece for the volume-marginal value function, a global value function (using a DSS) compatible with the subjective ranking with Kendall's coefficient value of 0.94 was found.

TABLE 1
GENERATED NON-INFERIOR SOLUTIONS

Solution No.	Cost	Volume
DSOL1	1.384	3.000
DSOL2	1.617	3.281
DSOL3	1.852	3.563
DSOL4	2.096	3.844
DSOL5	2.345	4.126
DSOL6	2.593	4.407
DSOL7	2.848	4.688
DSOL8	3.107	4.970
DSOL9	3.365	5.251
DSOL10	3.628	5.533
DSOL11	3.902	5.814
DSOL12	4.181	6.096
DSOL13	4.467	6.377
DSOL14	4.768	6.658
DSOL15	5.080	6.940
DSOL16	5.400	7.221
DSOL17	5.724	7.503
DSOL18	6.053	7.784
DSOL19	6.386	8.065
DSOL20	6.726	8.347

TABLE 2
REFERENCE SOLUTION SET

Solution	Rank
DSOL1	*5*
DSOL4	3
DSOL8	1
DSOL10	2
DSOL13	4
DSOL15	6
DSOL20	*7*

Kendall's coefficient is a measure of consistency between the subjective ranking and ranking due to the estimated value function. Note that, it is only when the Kendall's coefficient is equal to 1, that the estimated value function exactly fits the subjective ranking. Otherwise for a Kendall coefficient different from 1, it means that the ranking due to the estimated global value function does not rank the alternatives in exactly the same way as the DM. Therefore, some alternatives may end up being ranked differently (interchanged). The high value of Kendall's coefficient obtained indicates that the assessed value function is acceptable. This is because the ranking of the alternatives due to this value function is fairly consistent with the given one. This value function was applied to the whole set of pre-generated non-inferior solutions, resulting to a global ranking of the efficient solutions as shown in Table 3.

The DSS therefore identified solution *DSOL11*, as the most preferred one for implementation (Table 3). This solution resulted to an optimal objective function value of 3.902 MU (MU = monetary units), and pumping rates of $p_4 = 1.500$, $p_5 = 0.932$, $p_{13} = 0.596$, $p_{14} = 0.074$, $p_{21} = 0.466$, $p_{27} = 1.50$, $p_{28} = 0.240$ and $p_{35} = 0.506$ m^3/s. This optimal pumping scheme is shown as Fig. 7. As is evident from this figure, the pumping wells are generally located at the north-east side of the model domain, away from the ecological protection zone.

Scenario with second-stage decision

The "here and now" plan to be implemented is similar to that of the deterministic case (solution *DSOL11*). Then, depending on the outcome of the uncertain parameter, a second-stage decision is taken to minimise the penalties. Results for scenario analysis with second-stage decision corresponding to the 20 conductivity maps result in optimal objective function values ranging from a minimum value of 4.079 MU to a maximum value of 4.497 MU (see Table 4). Compared with the objective function value of the

TABLE 3
FINAL RANKING

Solution	Utility value	Rank
DSPL11	*0.885*	*1*
DSOL10	0.872	2
DSOL12	0.862	3
DSOL9	0.859	4
DSOL8	0.846	5
DSOL7	0.833	6
DSOL6	0.821	7
DSOL5	0.808	8
DSOL4	0.795	9
DSOL13	0.794	10
DSOL3	0.782	11
DSOL2	0.769	12
DSOL1	0.757	13
DSOL14	0.721	14
DSOL15	0.646	15
DSOL16	0.568	16
DSOL17	0.489	17
DSOL 18	0.408	18
DSOL19	0.327	19
DSOL20	*0.243*	*20*

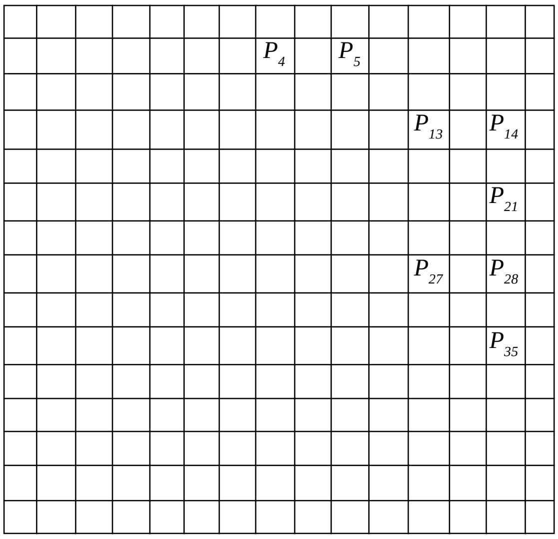

Figure 7
*Optimal pumping scheme (*DSOL11*)*

deterministic scheme (3.902 MU), all the solutions for the scenario with the second-stage decision scheme have higher initial investment cost than the deterministic case. Thus, it can be argued that the price to pay for a more robust solution, which will be able to withstand slight variations of the system parameters is the difference in investment cost between the deterministic solution and the scenario with second-stage decision solutions, which range between a minimum value of 0.177 (i.e. 4.079-3.902) and 0.595 MU (i.e. 4.497-3.902). The optimal pumping schemes corresponding to the 20 conductivity maps are different from one another as well as from the implemented deterministic solution (Table 4).

As mentioned before during our discussion on methodology, when dealing with scenario analysis (i.e. scenario with second-

TABLE 4
OPTIMAL SOLUTIONS FOR SCENARIO WITH SECOND-STAGE DECISION

Maps	Objective value	Wells in use
Map1	*4.114*	$P_4, p_5, p_{20}, p_{21}, p_{27}, p_{28}$
Map2	*4.115*	$P_4, p_6, p_{14}, p_{21}, p_{27}, p_{28}$
Map3	*4.292*	$P_4, p_5, p_6, p_7, p_{20}, p_{28}, p_{35}$
Map4	*4.468*	$P_4, p_5, p_6, p_7, p_{20}, p_{28}, p_{42}, p_{49}$
Map5	*4.158*	$P_5, p_6, p_{14}, p_{21}, p_{27}, p_{28}$
Map6	*4.082*	$P_4, p_5, p_{14}, p_{21}, p_{27}, p_{28}$
Map7	*4.121*	$P_4, p_5, p_{20}, p_{21}, p_{27}, p_{28}$
Map8	*4.353*	$p_6, p_7, p_{14}, p_{20}, p_{21}, p_{27}, p_{28}, p_{35}, p_{42}$
Map9	*4.384*	$P_4, p_5, p_6, p_7, p_{20}, p_{28}, p_{42}, p_{49}$
Map10	*4.079*	$P_4, p_5, p_{14}, p_{21}, p_{27}, p_{28}$
Map11	*4.268*	$p_4, p_5, p_6, p_{20}, p_{21}, p_{28}$
Map12	*4.278*	$P_6, p_7, p_{14}, p_{20}, p_{21}, p_{27}, p_{28}, p_{35}, p_{42}, p_{49}$
Map13	*4.132*	$p_5, p_6, p_{14}, p_{21}, p_{27}, p_{28}, p_{35}$
Map14	*4.305*	$p_4, p_5, p_6, p_7, p_{20}, p_{28}$
Map15	*4.137*	$p_4, p_5, p_{20}, p_{21}, p_{27}, p_{28}$
Map16	*4.211*	$p_4, p_5, p_6, p_{20}, p_{21}, p_{28}$
Map17	*4.187*	$P_4, p_5, p_6, p_{20}, p_{21}, p_{28}$
Map18	*4.424*	$P_4, p_5, p_6, p_7, p_{20}, p_{21}, p_{42}$
Map19	*4.497*	$P_3, p_6, p_7, p_{20}, p_{21}, p_{27}, p_{28}, p_{35}, p_{42}, p_{48}$
Map20	*4.206*	$P_5, p_6, p_{20}, p_{21}, p_{27}, p_{28}$

stage decision), the question of which optimal plan to choose among the many scenarios for implementation is not very obvious. This is because, as Table 4 demonstrates, there are changes in the optimal pumping strategy from realisation (map) to realisation which involves changes in well locations and pumping rates. This means that the optimal scheme corresponding to, for example, Map19 (worst-case scenario), will not necessarily satisfy the constraints for all realisations, because individual realisations control individual flow patterns and consequently dictate optimal well placement and pumping patterns. This limitation was also pointed out by Wagner and Gorelick (1989).

Multiple scenario results

The optimal solution for multiple scenario scheme is shown in Fig. 8 with the objective function taking on a value of 4.749 MU. Comparison of the optimal objective function values of the scenario with second-stage decision scheme (see Table 4, column 2) and that of multiple scenario scheme of 4.749 MU suggests that the latter scheme is more expensive than the former. Compared with the optimal pumping strategy of the deterministic scheme, the optimal pumping strategy of the multiple scenario scheme is more spread out (see Fig. 7 and Fig. 8)indicating that solutions obtained by assuming that the parameters are perfectly known are quite different from those obtained when the problem is posed within a stochastic framework.

Robustness of the multiple scenario solution

Ideally, the solution of the multiple scenario scheme would require consideration of a very large set of realisations (maps), if one is to gain a thorough knowledge of its stability (robustness) against the effects of uncertainty due to spatial variability of the hydraulic conductivity. To achieve such a solution, enormous computational

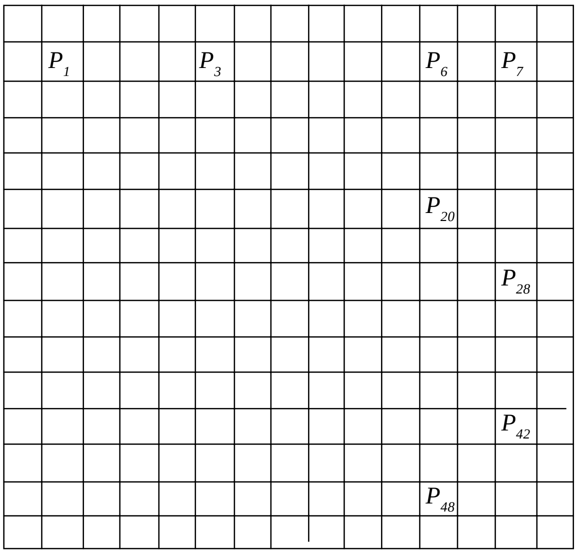

Figure 8
Optimal pumping scheme for multiple scenario scheme

effort is required which might be unrealistic, hence limitation of the number of realisations to be considered is unavoidable. This limitation, therefore, leads to an optimal (most satisfactory) solution which cannot guarantee feasibility for all possible realisations of the hydraulic conductivity.

To investigate whether the multiple scenario solution was any different from the deterministic (the "here and now") plan in terms of robustness, a post-optimality Monte Carlo analysis was performed on the multiple scenario scheme using the 20 pre-generated realisations. The corresponding penalties associated with these conductivity values before and after taking a second-stage decision are shown in Table 5. If RC_A and RC_B are the sum of penalties after and before second-stage decision respectively (Table *5*), then the percentage reduction in penalty is calculated as:

$$\left(\frac{RC_B - RC_A}{RC_B}\right)\times 100$$

and amounts to about 46.4% corresponding to a reduction of penalties amounting to about 12.5 MU. The percentage increase in investment cost is calculated as:

$$\left(\frac{OBJ_M - OBJ_H}{OBJ_H}\right)\times 100$$

where OBJ_M is the objective function value of the multiple scenario solution and OBJ_H is the objective function value of the implemented deterministic solution (*DSOL11*, see Table 1). This percentage increase in investment cost for the more robust scheme amounts to about 21.7% (corresponding to an increase in investment cost of about 0.9 MU). Comparison of the percentage reduction of penalties due to second-stage decision taking, with percentage increase in investment cost suggests that the returns from a second-stage decision taking outweigh the opportunity cost of implementing the more stable solution.

Since robust optimisation aims at establishing a solution which can perform reasonably well under all scenarios (i.e. a solution which is nearly optimal under most conditions and at least reasonably safe under the worst conditions), it allows a DM to search for solutions considering both solution robustness (solution should remain close to optimal for any realisation of uncertain parameter), and model robustness (solution should remain almost feasible for any realisation of the uncertain parameter). As it is apparent from Table *5*, even with this robust solution, not all the constraints can be met for all scenarios. However, the violations are substantially reduced, and since the aim of robust optimisation is to design a solution which performs rather well under all scenarios, the optimal multiple scenario solution will not be declared useless under the circumstances of the remaining violations.

TABLE 5
PENALTIES FOR MULTIPLE SCENARIO SCHEME BEFORE AND AFTER SECOND-STAGE DECISION TAKING

Maps	Before second-stage decision	After second-stage decision
Map1	0.185	0.541
Map2	0.142	0.000
Map3	1.418	2.065
Map4	2.770	0.603
Map5	0.476	0.119
Map6	0.017	0.000
Map7	0.453	0.218
Map8	1.774	0.639
Map9	2.593	2.404
Map10	0.000	0.038
Map11	1.505	0.214
Map12	2.307	0.711
Map13	0.370	0.044
Map14	1.403	1.648
Map15	0.200	0.528
Map16	0.854	0.599
Map17	0.573	1.519
Map18	2.328	1.644
Map19	6.547	0.592
Map20	1.057	0.327
	ΣRC_B = 26.972	ΣRC_A = 14.448

Conclusion

The results obtained from this example clearly demonstrate the importance of robust optimisation in water management decision-making when confronted with a multi-objective problem in an environment of uncertainty. Though robust optimisation, which is predicated on a two-stage decision model, assumes the knowledge of the uncertain parameters before the second-stage decision is taken, it can be argued that perfect information is never acquired in reality, and therefore robust optimisation aims at assisting a DM to find solutions which hedge against the uncertainties which remain.

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